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Universal line shape of the Kondo zero-bias anomaly in a quantum dot

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Encouraged by recent real-time renormalization group results, we carried out a detailed analysis of the nonequilibrium Kondo conductance observed in an InAs nanowire-based quantum dot and found them to be in excellent agreement. We show that in a wide range of biases the Kondo conductance zero-bias anomaly is scaled by the Kondo temperature to a universal line shape predicted by the numerical study. The line shape can be approximated by a phenomenological expression of a single argument $eV_{sd}/k_B T_K$. The knowledge of an analytical expression for the line shape provides an alternative way to estimate the Kondo temperature in a real experiment, with no need for time-consuming temperature dependence measurements of the linear conductance.

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In quantum dots an unpaired spin of the localized electron may act as a single magnetic impurity, which interacts with electrons in metallic leads, forming a Kondo-correlated state.[1] Quantum dots have been recognized as a versatile system for probing the many-body nature of the Kondo effect.[2,3] The main advantage of a quantum dot hosting the Kondo state is the ability to tune its main characteristic parameter, the Kondo temperature (T_K). Thereby, it is possible to study the reaction of the many-body system to various external perturbations, such as temperature T, bias V, and magnetic field.

One of the remarkable properties of the Kondo effect is that the response of the Kondo-enhanced conductance to a perturbation is governed by a set of universal laws independent of the physical system in which the Kondo state is realized. It has been experimentally verified that in the low-energy limit ($\{T, e|V|/k_B\} \ll T_K$, where k_B is the Boltzmann constant and e is the elementary charge) the Kondo conductance as a function of T and V is described by a universal quadratic law[4,5] expected from the Fermi-liquid theory.[6,7] At higher temperatures ($T \sim T_K$) the Kondo conductance has been shown to scale to a universal dependence of a single parameter T/T_K found from a numerical renormalization group (NRG) study.[6] These NRG calculations were later approximated by the phenomenological expression[3] of a single parameter T/T_K, which became the main method for the estimation of the Kondo temperature in real systems. Recently, we also attempted to check the universal behavior of the magnetic field dependence in both the low- and intermediate-field range.[5] Despite all the theoretical advances, the problem of the nonequilibrium Kondo model at intermediate bias ($|V| \sim k_B T_K/e$) remained unsolved. As a result, the line shape of one of the main hallmarks of the Kondo effect in quantum dots, known as the zero-bias conductance anomaly (ZBA),[8] had not been fully described by the theory. Only very recently, the 2-loop real-time renomalization group (RTRG) calculations developed for the Kondo model[9] provided a numerical description of the nonequilibrium Kondo conductance in a wide range of biases. The result of these calculations agreed very well with the experimental data and linked previously developed analytical theories made in the low-[6] and high-energy[10] limits. The development of the above-mentioned RTRG calculations gave us an additional impulse to study the Kondo ZBA in more detail and test the universality of its line shape.

In this Rapid Communication we present a detailed study of the ZBA associated with the Kondo effect observed in an InAs nanowire-based quantum dot. This different analysis of our previously published results[5] shows that in a wide range of applied biases the Kondo ZBA can be scaled into a universal line shape predicted by the RTRG calculations.[9] We found our experiment to be well described by the proposed phenomenological expression, which accurately approximates the RTRG results in the experimentally relevant range of bias. We also show that this expression can be employed as an alternative method for a quick and accurate estimation of the Kondo temperature.

Experiment. The quantum dot used in the experiment was formed in a 50-nm-diam high-quality InAs nanowire grown by the vapor-liquid-solid method on a (011) InAs substrate using molecular-beam epitaxy. The as-grown nanowires were randomly deposited onto a p^+-Si/SiO$_2$ substrate and individually connected by ohmic (Ni/Au) source and drain electrodes using e-beam lithography. To avoid undesirable effects of the underlying substrate the nanowires were suspended in vacuum over grooves predefined in the substrate, and fixed from the sides by the contacts. The lateral size of the dot was defined by the contact separation (~450 nm) due to the electrons in the nanowire being localized between two Schottky barriers formed at the nanowire-contact interface. The experiment was performed in a ^{3}He-^{4}He dilution refrigerator with a base temperature $T_{base} \approx 10$ mK. The transport measurements were made by a standard lock-in technique. Depending on the temperature the ac excitation bias was kept smaller or equal to $k_B T$. The dc bias V_{sd} was applied to the source with respect to the drain, "virtually" grounded through the transimpedance preamplifier. The dot occupancy was tuned by the backgate voltage V_g applied to the p^+-Si substrate. More details on growth, fabrication, and the experimental setup can be found in Ref. 5.

Large bias scaling. To investigate the Kondo ZBA in more detail we used the experimental results previously examined in our study of the low-bias and magnetic field scaling of the spin-1/2 Kondo conductance [see Ref. 5, Fig. 1(c)]. Figure 1(a) presents the linear differential conductance G through the quantum dot as a function of V_g taken at different T. The region of V_g at around −2.83 V corresponds to the conductance enhanced at lower temperatures due to the presence of the many-body Kondo state. There are two Coulomb

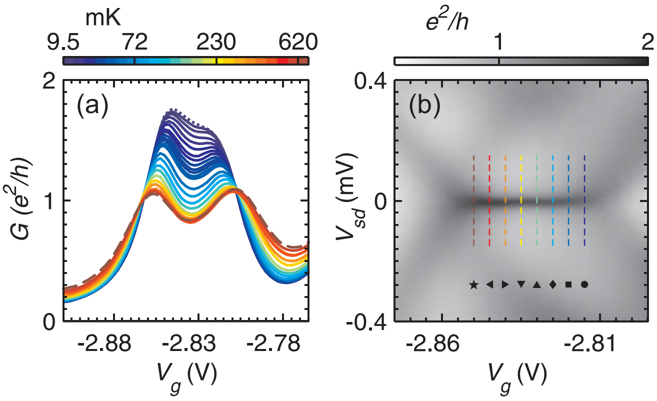

FIG. 1. (Color online) (a) The temperature dependence of the linear conductance observed in the InAs nanowire-based quantum dot. The dotted trace was taken at the lowest temperature (10 mK), and the dashed trace was taken at the highest temperature (620 mK). (b) The two-dimensional grayscale plot of the nonequilibrium conductance measured in a V_g-$V_{\rm sd}$ plane at $T = T_{\rm base}$ for the same range of V_g as in (a). The vertical dashed lines mark the conductance traces at fixed values of V_g used for the scaling analysis ($V_g = -2.85$ V, star; $V_g = -2.845$ V, left-pointing triangle; $V_g = -2.84$ V, right-pointing triangle; $V_g = -2.835$ V, downward-pointing triangle; $V_g = -2.83$ V, upward-pointing triangle; $V_g = -2.825$ V, diamond; $V_g = -2.82$ V, square; $V_g = -2.815$ V, circle).

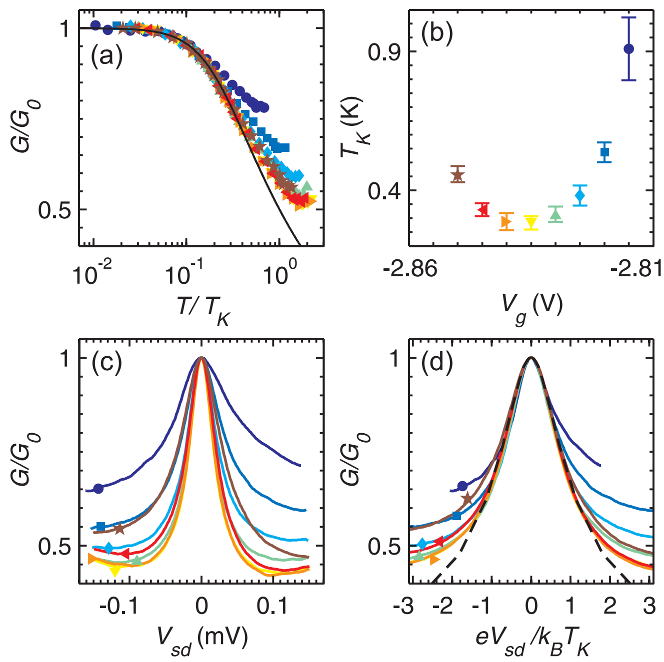

FIG. 2. (Color online) (a) Scaled temperature $G(T)$ dependence measured at different V_g (symbols) [see Fig. 1(b) for the symbols code], approximated with Eq. (1) (solid curve). (b) The dependence of the estimated value of $T_{\rm K}$ on V_g. The error bars here represent the 68% confidence interval. (c) The normalized conductance G/G_0 as a function of $V_{\rm sd}$ measured at different V_g and $T = T_{\rm base}$. (d) The same normalized conductance traces as in (c), but replotted as a function of $eV_{\rm sd}/k_B T_K$ (solid curves). The traces are shown to collapse onto the same universal dependence given by RTRG calculations (Ref. 9) (dashed curve).

blockade peaks emerging at higher temperatures, which mark the region of V_g with an odd dot occupancy. To identify the relevant Kondo ZBA in Fig. 1(b) we show the grayscale plot of the nonequilibrium differential conductance measured for the same range of V_g. The ZBA is seen here as a black horizontal line at around $V_{\rm sd} = 0$. To test the theoretically predicted scaling of the Kondo ZBA we chose several nonequilibrium conductance traces taken at fixed values of V_g, as shown in Fig. 1(b) by dashed lines. For each of the chosen values of V_g we estimated $T_{\rm K}$ by fitting the temperature dependence of the linear conductance $G(T)$ with the phenomenological expression[3]

$$G(T)/G_0 = \left[1 + (T/T'_{\rm K})^2\right]^{-s}, \qquad (1)$$

where $G_0 = G(T = 0, V_{\rm sd} = 0)$, $T'_{\rm K} = T_{\rm K}/(2^{1/s} - 1)^{1/2}$, and the parameter $s = 0.22$. Here the definition of $T_{\rm K}$ is such that $G(T = T_{\rm K}, V_{\rm sd} = 0) = 1/2G_0$. The experimental $G(T)$ dependencies fitted with Eq. (1) with $T_{\rm K}$ used as a fitting parameter are shown in Fig. 2(a). The symbols represent the experimental points taken at the values of V_g designated in Fig. 1(b), and the solid curve corresponds to Eq. (1). The deviation from the theoretical curve is related to additional mechanisms engaged in the transport at higher temperature. To avoid the effect of additional mechanisms the fitting procedure was made only for $T \leqslant 200$ mK. The fact that at $T < 200$ mK all the experimental data plotted as a function of $T/T_{\rm K}$ collapse on the same theoretical curve reflects the universality of the temperature dependence of the Kondo conductance.[3,6] The estimated value of $T_{\rm K}$ at chosen V_g is shown in Fig. 2(b) and follows the previously observed[5,11] paraboliclike dependence.[10]

As noticed in the early experiments[2,11,12] the width of the Kondo ZBA appears to be proportional to $T_{\rm K}$; the same qualitative behavior is observed in our experiment. Figure 2(c) shows the normalized nonequilibrium conductance G/G_0 at different V_g with a well-pronounced ZBA maximum at $V_{\rm sd} = 0$. The widest ZBA (dark blue curve marked with a circle) is associated with the highest $T_{\rm K} \approx 900$ mK, and the narrowest ZBA (orange and yellow curves marked with right- and downward-pointing triangles) corresponds to the lowest $T_{\rm K} \approx 300$ mK [see Fig. 2(b)]. In accordance with the RTRG calculations[9] the nonequilibrium conductance in the Kondo regime is scaled by $T_{\rm K}$ into a universal dependence (these calculations were done in the zero-temperature limit, as for our experiment, $T/T_{\rm K} \leqslant 0.03$, which was theoretically checked[9] to be close enough to zero). To verify this prediction we plotted the normalized conductance G/G_0 as a function of the scaled bias $eV_{\rm sd}/k_{\rm B}T_{\rm K}$ in Fig. 2(d). It is evident from Fig. 2(d) that all curves collapse onto the same universal dependence, which is given by the RTRG calculations and plotted as the dashed curve. The deviations from the prediction observed at higher biases occur due to the approaching resonant level of the dot. In this case the system switches from the Kondo to the mixed-valence regime where the scaling is no longer valid.[3,13]

Phenomenological formula. As shown above, the ZBA in the Kondo regime appears to have a universal dependence on bias, scaled by $T_{\rm K}$. This property of ZBA makes it potentially valuable for extracting $T_{\rm K}$ from the bias dependence of the conductance $G(V_{\rm sd})$, in the same way as for $G(T)$ by approximating it with Eq. (1). Here the use of numerical

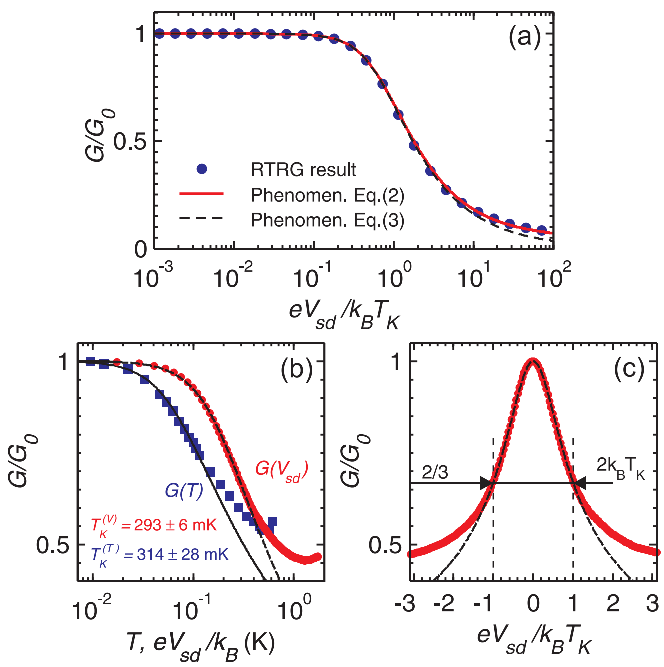

FIG. 3. (Color online) (a) The RTRG calculations of the nonequilibrium Kondo conductance as a function of bias taken from Ref. 9 (circles) and its approximation with Eq. (2) (solid curve) and Eq. (3) (dashed curve). (b) Extraction of T_K using two different measurements made at $V_g = -2.83$ V [see the upward-pointing triangle in Fig. 1(b)], $G(T)$ (squares) and $G(V_{sd})$ (circles) fitted with Eq. (1) (solid curve) and Eq. (3) (dashed curve), correspondingly. The two extracted values $T_K^{(T)}$ and $T_K^{(V)}$ are the same within the statistical error. (c) The experimental Kondo ZBA measured at $V_g = -2.83$ V (circles) and its theoretical approximation made with Eq. (3) (dashed curve). When the bias is such that $e|V_{sd}| = k_B T_K$, the Kondo conductance decreases to 2/3 of its zero-bias value G_0.

calculations is quite cumbersome, and the authors of Ref. 9 suggested a phenomenological formula for describing their RTRG results, which we utilized for our definition of T_K, [14]

$$G(T = 0,\nu)/G_0 = \left[1 + \frac{(2^{1/s_1} - 1)\nu^2}{\pi + b(|\nu|^{s_2} - 1)}\right]^{-s_1}, \qquad (2)$$

where $\nu = eV_{sd}/k_B T_K$. The best fit of Eq. (2) to the RTRG results gives $b = 0.05 \pm 0.01$, $s_1 = 0.32 \pm 0.01$, and $s_2 = 1.28 \pm 0.03$. Figure 3(a) illustrates the quality of this fit. As seen from the plot, Eq. (2) (solid curve) approximates the numerical results (circles) very well in a wide range of biases. However, in a real experimental situation the Kondo ZBA rarely extends beyond $\pm 2k_B T_K/e$ due to a dominating background, thus for the narrower range of bias ($e|V_{sd}|/k_B T_K < 10$) one can use a simplified expression

$$G(T = 0,\nu)/G_0 = [1 + \alpha\nu^2]^{-s_1}, \qquad (3)$$

where $\alpha = (2^{1/s_1} - 1)/\pi$. The simplified formula is identical to the one in Ref. 9. As can be seen from Fig. 3(a), the simplified expression, shown by the dashed curve, approximates the numerical result adequately up to $eV_{sd}/k_B T_K \sim 10$. Note that in the low-bias limit $e|V_{sd}|/k_B T_K \ll 1$, Eq. (3) reduces to the quadratic Fermi-liquid dependence $G(V_{sd})/G_0 \approx 1 - s_1\alpha (eV_{sd}/k_B T_K)^2 = 1 - c_V(eV_{sd}/k_B T_K)^2$, where the coefficient $c_V \approx 0.78$ is in a good agreement with both theory and experiment.[5]

Since the Kondo ZBA is scaled by T_K into a universal line shape, which is described by a phenomenological formula, it should be possible to perform the reverse operation and extract the value of T_K by fitting the experimental $G(V_{sd})$ dependence with Eq. (3). Figure 3(b) shows a semilogarithmic plot of the Kondo conductance measured as a function of T (squares) and the modified bias eV_{sd}/k_B (circles). Each set of data is approximated with its own phenomenological formula using T_K as a fitting parameter: $G(T)$ with Eq. (1) for $T \leqslant 200$ mK and $G(V_{sd})$ with Eq. (3) for $|V_{sd}| \leqslant 30\ \mu$V, correspondingly.[15] The two values of the Kondo temperature were found to be $T_K^{(T)} = 314 \pm 28$ mK for $G(T)$ and $T_K^{(V)} = 293 \pm 6$ mK for $G(V_{sd})$, and they are equal within the statistical error. Also, fitting of $G(V_{sd})$ provides a statistically more accurate value of T_K due to a larger number of experimental points available for analysis. In our opinion this method of estimating T_K may be more advantageous than the traditional one involving the measurement of the $G(T)$ dependence and approximating it with Eq. (1). First, it is much easier to reliably measure the $G(V_{sd})$ dependence at the lowest possible temperature, rather than performing time-consuming measurements of $G(V_g)$ in the linear regime at multiple temperatures. Normally, the low temperature ($T/T_K \ll 1$) required for Eq. (3) is easily satisfied, especially at submillikelvin temperatures, since for most systems $T_K \geqslant 200$ mK. Second, as pointed out, the value of T_K extracted from the experimental $G(V_{sd})$ dependence is potentially more accurate.

Width of ZBA. Finally, we would like to discuss an important practical implication of Eq. (3). So far, the width of the Kondo ZBA has been used as a rough experimental estimate for T_K,[2,11,12] and the full width at half maximum (FWHM) of the ZBA peak was assumed to be approximately equal to $2k_B T_K$. However, more careful measurements demonstrated an overestimation of T_K determined by this method, if compared to the values extracted from the $G(T)$ dependence.[11] To clarify the issue of the ZBA width we did a simple mathematical analysis of Eq. (3), which revealed that $G(V_{sd} = \pm k_B T_K/e) = 0.67G_0 \approx 2/3G_0$ and the ZBA peak FWHM $= 2k_B T_K^* \approx 2\sqrt{\pi}k_B T_K$.[14] By means of these simple relations a quick and accurate estimation of T_K from the raw data is straightforward. As an example, Fig. 3(c) shows the Kondo ZBA peak measured at $V_g = -2.83$ V and plotted as a function of the normalized bias $eV_{sd}/k_B T_K$, where T_K is found from fitting of the same data with Eq. (3) [see Fig. 3(b)]. The universal line shape is shown in Fig. 3(c) by the dashed curve, illustrating an excellent agreement with the experiment. It is seen that the width of the Kondo ZBA is $2k_B T_K$ at two thirds of its total magnitude, contrary to the earlier assumed half magnitude.[11,12] It also suggests an explanation for the discrepancy between the FWHM and the value of T_K reported in Ref. 11, alternative to the dephasing by bias.[8,16] Unfortunately, we were unable to measure the FWHM reliably because in our experiment the Kondo ZBA at $0.5G_0$ was already broadened by the background. The discussed scaling and universal line shape of the Kondo ZBA have also been confirmed for different Coulomb blockade peaks and experimental samples.[17]

In conclusion, using recent results of RTRG calculations of the nonequilibrium Kondo model, we experimentally verified the universal scaling of the Kondo conductance at intermediate ($e|V_{sd}| \sim k_B T_K$) bias. We established that the Kondo ZBA in a quantum dot can be scaled by T_K to a universal dependence predicted by the numerical calculations and approximated by the phenomenological formula. An excellent agreement with the experiment allowed us to use this formula to extract the value of T_K solely from the analysis of the Kondo ZBA line shape at the lowest temperature. This method appears to be quicker and more statistically accurate compared to the traditional one involving the measurement of the $G(T)$ dependence. Also, a closer look at the phenomenological formula revealed that when the applied bias is equivalent to $k_B T_K/e$ the Kondo conductance is at two thirds of its zero-bias value. At the same time the FWHM is about $2\sqrt{\pi} k_B T_K$, which is larger than it was previously thought to be. We demonstrated that those relations can provide an immediate and accurate estimate of the experimental T_K.

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[15] An approximation of $G(V_{sd})$ with the unsimplified Eq. (2) gives the same value of T_K as Eq. (3) within its statistical error.

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