Suppression of superconductivity at a nematic critical point in underdoped cuprates

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A nematic quantum critical point is anticipated to exist in the superconducting dome of some high-temperature superconductors. The nematic order competes with the superconducting order and hence reduces the superconducting condensate at $T = 0$. Moreover, the critical fluctuations of nematic order can excite more nodal quasiparticles out of the condensate. We address these two effects within an effective field theory and show that superfluid density $\rho^s(T)$ and superconducting temperature T_c are both suppressed strongly by the critical fluctuations. The strong suppression of superconductivity provides a possible way to determine the nematic quantum critical point.

I. INTRODUCTION

The strong electron correlation in high-temperature superconductors (HTSC) is able to drive an electronic nematic phase,[1–4] which preserves translational symmetry but breaks rotational symmetry. In the past decade, there have been a number of experimental signatures pointing to the presence of nematic ordering transition in some HTSCs.[3–8] On the basis of these experiments, a zero-temperature nematic quantum critical point (QCP) is supposed to exist at certain doping concentration x_c in the superconducting (SC) dome.[3,4,9–15] Generally, the nematic order has two impacts on the SC state. First, it competes with the SC order.[2–4] Second, the nematic order parameter couples to the gapless nodal quasiparticles (QPs), which are believed to be the most important fermionic excitations in unconventional superconductors with $d_{x^2-y^2}$ energy gap. The latter coupling is singular at the nematic QCP x_c, and has stimulated considerable theoretical efforts.[9–15] A recent renormalization group analysis[11] showed that it leads to a novel fixed point at which the ratio between gap velocity v_Δ and Fermi velocity v_F of nodal QPs flows to zero, $v_\Delta/v_F \to 0$.

Although a zero-temperature nematic QCP is expected to exist somewhere in the SC dome,[3,9–15] shown schematically in Fig. 1, its precise position, and even its very existence, has not been unambiguously confirmed by experiments so far. It is therefore always interesting to seek evidence which can help convincingly confirm or disconfirm the existence of such a point. In this paper, we study the superfluid density $\rho^s(T)$ and the SC temperature T_c at the supposed nematic QCP x_c. If $\rho^s(T)$ and T_c exhibit sharply distinct behaviors at x_c, then the nematic QCP may be detected by measuring these quantities.

HTSCs are known to be doped Mott insulators, so their superfluid density is much smaller than that of conventional metal superconductors. At $T = 0$, the superfluid density in the underdoping region depends[16,17] linearly on doping x as $\rho^s(0) = x/a^2$, where a is the lattice spacing. At finite T, a certain amount of nodal QPs are thermally excited out of the SC condensate. Lee and Wen argued that these normal nodal QPs can efficiently deplete the superfluid density.[18] Formally, the superfluid density contains two terms, $\rho^s(T) = \rho^s(0) - \rho^n(T)$, where $\rho^n(T)$ is the normal QPs density. Setting $\rho^s(T_c) = 0$ allows for an estimate of the critical temperature T_c. Employing a phenomenological approach, Lee and

Wen[18] obtained $T_c \propto \rho^s(0) \propto \frac{v_\Delta}{v_F} x$, reproducing the Uemura plot.[19]

Once a nematic ordering transition occurs at x_c, the superfluid density and T_c will be substantially changed. As $v_\Delta/v_F \to 0$ due to the critical nematic fluctuations, it seems that $T_c \to 0$, i.e., superconductivity would be completely suppressed at x_c. This argument is certainly oversimplified since the above expression of T_c is obtained in the noninteracting limit. However, this qualitative analysis does indicate the importance of the critical nematic fluctuations, and indeed motivates us to perform a quantitative computation of the renormalized $\rho^s(T)$ and T_c after taking into account the nematic fluctuations.

The nematic order affects $\rho^s(T)$ in two ways. On the one hand, since the nematic order competes with the SC order, it reduces $\rho^s(0)$. This reduction can be examined by studying the competitive interaction between nematic and SC order parameters. On the other, the critical nematic fluctuations can excite more nodal QPs out of the condensate, compared with the case without nematic order. As a consequence, $\rho^n(T)$ is enhanced and the superfluid density is further suppressed. We shall access this effect by generalizing the phenomenological approach proposed in Ref. 18. The velocity anisotropy plays an essential role in these considerations. After explicit calculations, we find that superfluid density $\rho^s(T)$ and T_c are both significantly reduced due to critical nematic fluctuations, indicating a strong suppression of superconductivity at nematic QCP x_c (see Fig. 1).

The rest of the paper is organized as follows. In Sec. II, we address the competitive interaction between SC and nematic order parameters and calculate zero-T superfluid density. In Sec. III, we calculate the density of normal QPs after taking into account fermion velocity renormalization due to critical nematic fluctuation. Based on these calculations, we predict a strong suppression of superconductivity at nematic QCP. In Sec. IV, we present a brief summary of our results, and also discuss the possible experimental determination of the nematic QCP.

II. COMPETING ORDERS AND ZERO-TEMPERATURE SUPERFLUID DENSITY

We first consider the renormalized zero-T superfluid density at nematic QCP. Based on phenomenological grounds,
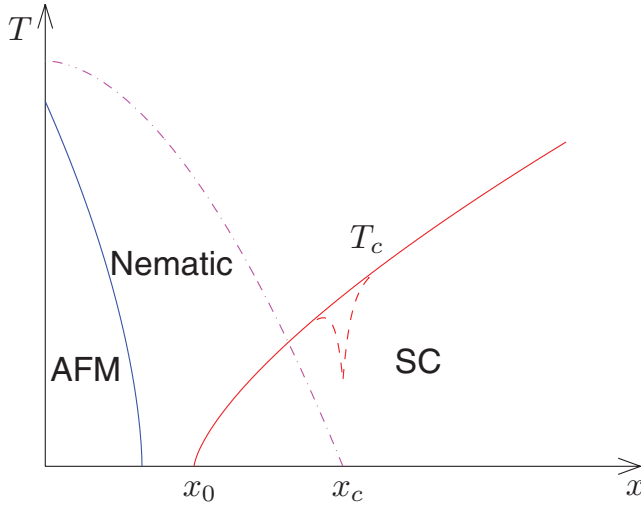
we write a free energy of two competing orders as

$$F = F_\psi + F_\phi + F_{\psi\phi}$$
$$= \frac{1}{2m}(\nabla\psi)^2 - \alpha\psi^2 + \frac{\beta}{2}\psi^4 + F_\phi + \gamma\psi^2\phi^2, \quad (1)$$

where ψ and ϕ are the SC and nematic order parameters, respectively. The strength of the competitive interaction between ψ and ϕ is represented by a positive constant γ. Such type of effective model has been adopted to describe competing orders in various superconductors.[20,21]

In the absence of nematic order, the mean value of ψ is $|\psi| = \sqrt{\alpha/\beta}$. To be consistent with experiments, the parameters α and β must be properly chosen such that $4|\psi|^2 = 4\alpha/\beta = \rho^s(0) = x/a^2$. In the presence of nematic order, $|\psi|$ will be renormalized by the $\gamma\psi^2\phi^2$ term. The quantum fluctuation of nematic order ϕ is very strong and actually singular at nematic QCP x_c, so ϕ should be regarded as a quantum mechanical field. However, we can consider ψ as a classical field and ignore its quantum fluctuations, provided x_c is not close to the SC QCP x_0.

The free energy of nematic order F_ϕ should be specified now. Analogous to the free energy of SC order ψ, F_ϕ contains a quadratic term ϕ^2 and a quartic term ϕ^4. However, the additional coupling between nematic order and nodal QPs introduces an extra term. The action describing this coupling is given by[9–11]

$$S_\Psi = \int \frac{d^3k}{(2\pi)^3}[\Psi_{1i}^\dagger(-i\omega + v_F k_1 \tau^z + v_\Delta k_2 \tau^x)\Psi_{1i}$$
$$+ \Psi_{2i}^\dagger(-i\omega + v_F k_2 \tau^z + v_\Delta k_1 \tau^x)\Psi_{2i}], \quad (2)$$

$$S_{\Psi\phi} = \int d^2x \, d\tau[\lambda_0\phi(\Psi_{1i}^\dagger\tau^x\Psi_{1i} + \Psi_{2i}^\dagger\tau^x\Psi_{2i})], \quad (3)$$

where $\tau^{x,y,z}$ are Pauli matrices and the flavor index i sums up 1 to N. Ψ_1 represents nodal QPs excited from $(\frac{\pi}{2},\frac{\pi}{2})$ and $(-\frac{\pi}{2},-\frac{\pi}{2})$ points, and Ψ_2 the other two. The effective action

of ϕ has the form[11]

$$S_\phi = \int \frac{d^3q}{(2\pi)^3}\left[\frac{1}{2}[q^2 + r + \Pi(q)]\phi^2 + \frac{u}{4}\phi^4\right], \quad (4)$$

which is obtained by integrating out N-flavor nodal QPs. Here, r is the tuning parameter for the nematic ordering transition, with $r = 0$ defining the nematic QCP x_c. The polarization function $\Pi(q)$ comes from nodal QPs, and, to the leading order of $1/N$ expansion, is defined as

$$\Pi(\epsilon,\mathbf{q}) = N \int \frac{d\omega \, d^2\mathbf{k}}{(2\pi)^3}\text{Tr}[\tau^x G_0(\omega,\mathbf{k})\tau^x G_0(\omega+\epsilon,\mathbf{k}+\mathbf{q})],$$
$$(5)$$

where

$$G_0(\omega,\mathbf{k}) = \frac{1}{-i\omega + v_F k_1 \tau^z + v_\Delta k_2 \tau^x} \quad (6)$$

is the free propagator for nodal QPs Ψ_1 (free propagator for nodal QPs Ψ_2 can be similarly written). The polarization function $\Pi(\epsilon,\mathbf{q})$ has already been calculated previously[11,15] and is known to have the form[22]

$$\Pi(\epsilon,\mathbf{q}) = \frac{N}{16v_F v_\Delta}\left[\frac{\epsilon^2 + v_F^2 q_1^2}{(\epsilon^2 + v_F^2 q_1^2 + v_\Delta^2 q_2^2)^{1/2}} + (q_1 \leftrightarrow q_2)\right]. \quad (7)$$

Note there is no direct interaction between ψ and nodal QPs Ψ. Indeed, Ψ are excited on top of a SC order ψ. Moreover, they have sharp peak and long lifetime in the SC dome in the absence of competing orders,[17] so their coupling to ψ must be quite weak.

Starting from Eqs. (1) and (4), we can compute the correction to zero-T superfluid density due to the competition between SC and nematic orders. To this end, we need to minimize the effective potential of SC order $V[\psi]$. The bare potential for ψ is simply $V_0[\psi] = -\alpha\psi^2 + \frac{\beta}{2}\psi^4$. It receives an additional term $V_1[\psi]$ due to the SC-nematic competition. This additional term will be calculated using the methods presented in Refs. 21 and 23. The corresponding partition function is

$$Z[\psi(\mathbf{r})] = \int \mathcal{D}\phi(\mathbf{r},\tau)\exp\left(-\frac{\mathcal{F}_\psi}{T} - S_\phi - S_{\psi\phi}\right), \quad (8)$$

where $\mathcal{F}_\psi = \int d^2\mathbf{r} \, F_\psi$. The saddle-point equation for ψ reads as

$$\frac{\delta \ln Z[\psi(\mathbf{r})]}{\delta\psi(\mathbf{r})} = 0, \quad (9)$$

which gives rise to

$$\left[-\alpha + \beta\psi^2(\mathbf{r}) - \frac{1}{m}\nabla^2 + \gamma f[\psi]\right]\psi(\mathbf{r}) = 0. \quad (10)$$

Here, $f[\psi]$ is the expectation value of ϕ^2. At the one-loop level, it has the form

$$f[\psi] \equiv \langle\phi^2\rangle = \int \frac{d^3q}{(2\pi)^3}\frac{1}{q^2 + \Pi(q) + \gamma\psi^2}, \quad (11)$$

where $\Pi(q)$ and $\gamma\psi^2$ represent the contributions due to nodal QPs and SC order, respectively. Although the q^2 term appearing in the denominator of $f[\psi]$ is much smaller than

FIG. 1. (Color online) Schematic phase diagram of HTSC. The suppression of T_c at nematic QCP x_c is represented by the dashed line.

$\Pi(q)$ at low energy, it can not be simply neglected since $f[\psi]$ would be divergent without such a term. It is now straightforward to get

$$\frac{\delta V_1[\psi]}{\delta \psi} = 2\gamma \psi f[\psi], \tag{12}$$

which then leads to a renormalized potential

$$V[\psi] = -\alpha \psi^2 + \frac{\beta}{2}\psi^4 + V_1[\psi]. \tag{13}$$

To calculate the renormalized zero-T superfluid density $\rho_R^s(0)$, one needs to minimize the effective potential $V[\psi]$ by taking

$$\frac{\delta V[\psi]}{\delta \psi} = 0. \tag{14}$$

The renormalized $|\psi|^2$ and therefore $\rho_R^s(0)$ can be obtained from the solution of the following equation:

$$-\alpha \psi + \beta \psi^3 + \gamma \psi f[\psi] = 0. \tag{15}$$

In order to see the role of nodal QPs, we first ignore the fermion contribution in $f[\psi]$ by assuming $\Pi(q) = 0$. In this case, the integration over q in $f[\psi]$ can be exactly performed, yielding

$$f[\psi] = \frac{1}{4\pi}(\Lambda - \sqrt{\gamma}\psi). \tag{16}$$

The renormalized potential becomes

$$V[\psi] = V_0[\psi] + \frac{\gamma \Lambda}{2\pi}\psi^2 - \frac{\gamma \sqrt{\gamma}}{6\pi}\psi^3. \tag{17}$$

The cubic term induced by nematic order turns the SC transition to first order,[21,23] with the critical point being $\alpha_c = \frac{\gamma \Lambda}{4\pi} - \frac{\gamma^3}{64\pi^2 \beta}$. However, since the gapless nodal QPs are present even at the lowest energy, the polarization function $\Pi(q)$ should be included in the effective action S_ϕ. After including the polarization $\Pi(q)$ into $f[\psi]$, integration over q can not be carried out analytically, and a numerical method will be used. After numerically solving Eq. (15), we found a critical value γ_c for the competitive interaction between SC and nematic orders. The SC transition remains continuous if $\gamma < \gamma_c$, and is driven to first order when $\gamma > \gamma_c$.

We now discuss the effects of the competing nematic order on the zero-T superfluid density. When γ is zero or very small, the nematic order does not change $|\psi|^2$, which is expected and trivial. As γ increases continuously, $|\psi|^2$ and $\rho_R^s(0)$ both decrease rapidly. To estimate this effect more quantitatively, we assume an ultraviolet cutoff $\Lambda = 10$ eV, and choose $\alpha = 2.5 \times 10^{-3}$ eV. Moreover, we consider a representative bare velocity ratio $v_\Delta/v_F \approx 0.075$, which is an appropriate value for YBa$_2$Cu$_3$O$_{6+\delta}$.[24] We further choose $\rho^s(0)/m = 4\alpha/m\beta = 10^{-2}$ eV, which corresponds to $T_c \approx 20$ K. It is now useful to introduce a parameter given by $\gamma_0 = \beta/\xi_0^2 \alpha$ with $\xi_0^2 = 1/2m\alpha$, and define a dimensionless coupling constant γ/γ_0. Figure 2(a) shows that $\rho_R^s(0)$ is strongly suppressed by the nematic order and is completely destroyed when γ is large enough. It is also obvious that the bare velocity ratio plays a vital role: a larger anisotropy causes a smaller drop of $\rho^s(0)$. This property provides further evidence that nodal QPs can not be simply ignored.
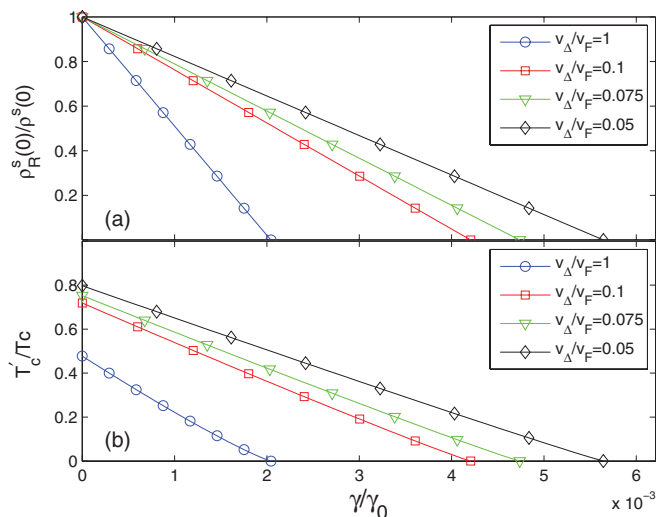

FIG. 2. (Color online) (a) $\rho_R^s(0)/\rho^s(0)$ for four different values of bare ratio v_Δ/v_F; (b) T_c'/T_c. The open circles, etc., are guides to eyes.

III. VELOCITY RENORMALIZATION AND SUPPRESSION OF SUPERCONDUCTIVITY

We then turn to calculate the density of normal nodal QPs, $\rho^n(T)$. We first briefly outline the phenomenological approach of Lee and Wen[18] and then generalize it to the case with critical nematic fluctuation. In the scenario of Lee and Wen, a crucial problem is how to assess the roles played by nodal and antinodal QPs in the destruction of SC condensate. As revealed clearly by extensive experiments,[25] a pseudogap phase exists above T_c. The pseudogap turns out to have the same d-wave symmetry as the SC gap,[25] which implies the antinodal QPs remain gapped as temperature increases across T_c. It was argued[18] that the SC state is primarily destroyed by the thermal proliferation of the low-lying, gapless nodal QPs, and the antinodal QPs are only spectators. Lee and Wen further assume that the nodal QPs do not carry superflow,[18] so the fermion spectrum is shifted by the vector potential to $E(\mathbf{k}, \mathbf{A}) = E(\mathbf{k}) + \frac{e}{c}\mathbf{v}_k \cdot \mathbf{A}$, where the QPs energy is linearized as $E(\mathbf{k}) = \sqrt{v_F^2 k_1^2 + v_\Delta^2 k_2^2}$ near nodes $(\frac{\pi}{2}, \frac{\pi}{2})$ and $\mathbf{v}_k$ is the normal-state velocity. The electric current is $j_\mu = \frac{e}{cm}\rho_{\mu\nu}^s \mathbf{A}_\nu$, where the superfluid tensor $\rho_{\mu\nu}^s$ can be written as $\rho_{\mu\nu}^s(T) = \rho^s(0)\delta_{\mu\nu} - \rho_{\mu\nu}^n(T)$. The zero-$T$ superfluid density is $\rho^s(0) = x/a^2$, and the normal QPs density is derived from the free energy[18]

$$F(\mathbf{A}, T) = -T \sum_{\mathbf{k}, \sigma} \ln(1 + e^{-E(\mathbf{k}, \mathbf{A})/T}) \tag{18}$$

by the formula $\frac{1}{m}\rho_{\mu\nu}^n(T) = -2\sum_{\mathbf{k}}\frac{dE}{dA_\mu}\frac{dE}{dA_\nu}\frac{\partial n_F}{\partial E}$. In the noninteracting case, the fermion velocities are constants, so one gets

$$\frac{1}{m}\rho^s(T) = \frac{x}{ma^2} - \frac{2\ln 2}{\pi}\frac{v_F}{v_\Delta}T, \tag{19}$$

which exhibits a linear temperature dependence, in agreement with experiments.[26] At $T = 0$, $\rho^s(T)$ takes its maximum value $\rho^s(0)$. As T is increasing from zero, $\rho^s(T)$ decreases rapidly due to thermally excited nodal QPs and eventually

vanishes as $T \to T_c$. By taking $\rho^s(T_c) = 0$, it is easy to obtain

$$T_c \propto \rho^s(0) \propto \frac{v_\Delta}{v_F} \frac{x}{ma^2}. \qquad (20)$$

This linear doping dependence of T_c is well consistent with the Uemura plot. The same results were later reproduced by means of the Green's function method.[27] The Fermi-liquid (FL) corrections to these results were also investigated.[27,28] An important fact is that both $\rho^s(T)$ and T_c contain the velocity ratio v_Δ/v_F, so these two quantities are expected to be significantly affected by the critical nematic fluctuation, which can cause a nontrivial renormalization of the velocity ratio.

The above approach of computing T_c is applicable when the nodal QPs are well defined. At the nematic QCP x_c, the critical fluctuation of nematic order couples strongly to gapless nodal QPs and may lead to breakdown of FL behavior. As a consequence, the nodal QPs might no longer be well-defined QPs. In a strict sense, the validity of the simple d-wave BCS theory[18] and its FL-interaction generalizations[27,28] are both doubtful. In order to proceed, here we assume that the basic approach of Ref. [18] is still valid after the free energy of QPs receives a singular correction due to nematic fluctuation.

The free fermion propagator shown in Eq. (6) is renormalized by nematic order to

$$G(\omega, \mathbf{k}) = \frac{1}{G_0^{-1}(\omega, \mathbf{k}) - \Sigma(\omega, \mathbf{k})}, \qquad (21)$$

where the self-energy is

$$\Sigma(\omega, \mathbf{k}) = \int \frac{d\epsilon \, d^2\mathbf{q}}{(2\pi)^3} G_0(\omega + \epsilon, \mathbf{k} + \mathbf{q}) \frac{1}{q^2 + \Pi(q)}. \qquad (22)$$

The pole in $G(\omega, \mathbf{k})$ determines the renormalized energy $\widetilde{E}(\omega, \mathbf{k}) = E(\omega, \mathbf{k}) + \delta E(\omega, \mathbf{k})$, where $E(\omega, \mathbf{k})$ is given by pole of free propagator $G_0(\omega, \mathbf{k})$ and $\delta E(\omega, \mathbf{k})$ represents the interaction correction to energy. To describe this renormalization procedure, it is convenient to expand the self-energy formally as

$$\Sigma(\omega, \mathbf{k}) = -i\omega \Sigma_0 + \Sigma_1 v_F k_1 \tau^z + \Sigma_2 v_\Delta k_2 \tau^x, \qquad (23)$$

which exhibits unusual logarithmic behaviors since $\Sigma_{1,2}(k)$ contains such a term as $\ln(\Lambda/k)$. We combine this self-energy with $G_0^{-1}(k)$, and find that the bare velocities acquire strong k dependence $v_{F,\Delta} \to v_{F,\Delta}(k)$. $v_{F,\Delta}(k)$ are very complicated functions of k, and thus can not be written analytically. After straightforward numerical computation, we show the k dependence of fermion velocities and their ratio in Fig. 3 for the representative bare value $v_\Delta/v_F = 0.075$. It is clear that both $v_F(k)$ and $v_\Delta(k)$ vanish as $k \to 0$. Nevertheless, $v_F(k)$ approaches zero much more slowly than $v_\Delta(k)$ does. Therefore, the velocity ratio vanishes at the lowest energy $v_\Delta/v_F \to 0$, which recovers the extreme velocity anisotropy.[11] Such velocity renormalization causes breakdown of FL behavior[12,29] and can faithfully characterize the non-FL nature of QPs at x_c.

Although the nodal QPs are strongly damped by the nematic fluctuations and thus no longer well defined at x_c, it is still reasonable to assume that they do not carry superflow,[18] which allows us to write $\widetilde{E}(\mathbf{k}, \mathbf{A}) = \widetilde{E}(\mathbf{k}) + \frac{e}{c} \mathbf{v}_k \cdot \mathbf{A}$, where the normal velocity $\mathbf{v}_k$ becomes strongly k dependent. Now
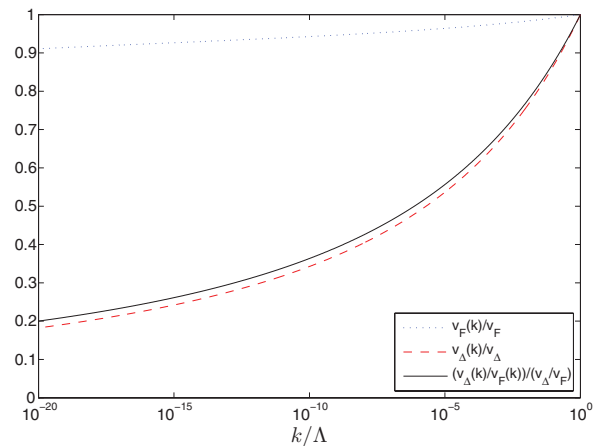

FIG. 3. (Color online) Flow of v_F, v_Δ, and v_Δ/v_F with momentum k for bare ratio $v_\Delta/v_F = 0.075$. As $k \to 0$, both velocities are reduced down to zero. However, v_Δ decreases much faster than v_F, so velocity ratio $v_\Delta/v_F \to 0$.

the new free energy is

$$\widetilde{F}(\mathbf{A}, T) = -T \sum_{\mathbf{k}, \sigma} \ln(1 + e^{-\widetilde{E}(\mathbf{k}, \mathbf{A})/T}), \qquad (24)$$

from which we obtain a renormalized superfluid density

$$\rho_R^s(T) = \rho_R^s(0) - \rho_R^n(T), \qquad (25)$$

$$\frac{\rho_R^n(T)}{m} = \frac{4}{T} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{v_F^2(k) e^{\sqrt{v_F^2(k)k_1^2 + v_\Delta^2(k)k_2^2}/T}}{(1 + e^{\sqrt{v_F^2(k)k_1^2 + v_\Delta^2(k)k_2^2}/T})^2}. \qquad (26)$$

At nematic QCP, T_c is changed from its original value to a renormalized value T_c'. When $T = T_c'$, the renormalized superfluid density vanishes, $\rho_R^s(T_c') = 0$, so that

$$\rho_R^s(0) = \rho_R^n(T_c'), \qquad (27)$$

which builds a relationship between T_c' and T_c. The ratios of $\rho_R^s(T)/\rho^s(T)$ and T_c'/T_c can be numerically calculated according to the above three equations. To include the influence of interaction corrections, the velocities $v_{F,\Delta}$ appearing in $\rho_R^s(0)$ are also replaced by $v_{F,\Delta}(k)$. The numerical results of T_c'/T_c are shown in Fig. 2(b), and those of $\rho_R^s(T)/\rho^s(T)$ given in Fig. 4. The suppression of superconductivity is prominent even when the competitive interaction between SC and nematic orders is quite weak, and is further enhanced by this interaction. As aforementioned, the SC phase transition at T_c' remains continuous as long as γ is small.

Clearly, the extent to which $\rho^s(T)$ and T_c are suppressed by the nematic order depends on a number of parameters, such as α, γ, and v_Δ/v_F. It is helpful to make a more quantitative analysis. The magnitudes of α as well as Λ have already been chosen. The bare velocity ratio is still fixed at $v_\Delta/v_F = 0.075$. Figure 2(b) shows that T_c is roughly suppressed by 25% when $\gamma = 0$, and by 50% when $\gamma/\gamma_0 \approx 1.5 \times 10^{-3}$. The drop of $\rho^s(T)$ can be analyzed in a similar way. An important implication of Fig. 4 is that $\rho^s(T)$ is more strongly reduced at higher temperatures. Experimentally, the suppression of T_c can be tested by measuring resistivity and Meissner effect, and the suppression of superfluid density may be probed by measuring
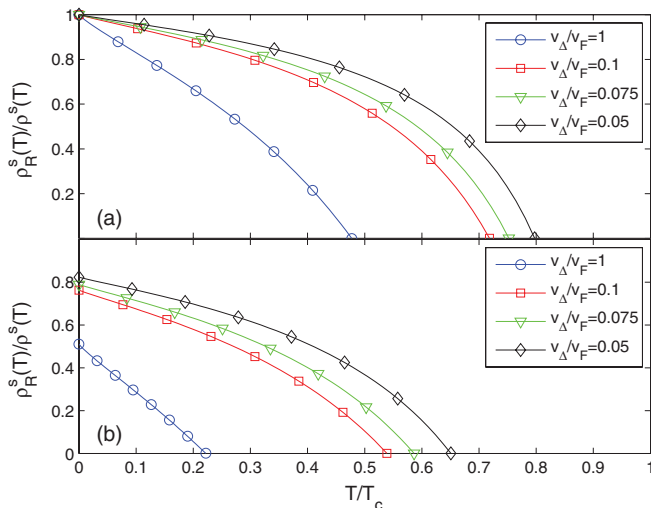
FIG. 4. (Color online) Superfluid density at finite temperature $\rho_R^s(T)$. (a) $\gamma = 0$; (b) $\gamma/\gamma_0 = 10^{-3}$.

London penetration length λ_L, preferably using microwave techniques[26,30] based on the relationship $\rho^s(T) \propto \lambda_L^{-2}(T)$.

The strong suppression effects rely on the divergence of correlation length ξ of the nematic field ϕ at the QCP x_c. As one moves away from the point x_c, the strong suppression of superfluid density and T_c will be rapidly diminished. In fact, once correlation length ξ becomes finite, the interaction between nematic fluctuation and nodal QPs is no longer singular. Consequently, the self-energy of nodal QPs, $\Sigma(k)$, does not display logarithmic behavior, which indicates that the fermion velocities only receive unimportant renormalizations. In this case, the density of normal QPs is not significantly modified by the competing nematic order since the velocity ratio does not depart far from its bare value. The influence of nematic order on $\rho^s(0)$ is also weakened. For $x > x_c$, the fluctuation of ϕ is rather weak and does not change $\rho^s(0)$ much if γ is not very large. For $x_0 < x < x_c$, the fluctuation of ϕ is gapped and also leads to much smaller change of $\rho^s(0)$ compared with what happens at nematic QCP x_c. Therefore, the T_c curve generally follows the Uemura plot in the whole underdoped region, but deviates from the linear behavior in the close vicinity of x_c. Since the suppression of superconductivity is most pronounced at x_c, it may help to find the position of the predicted nematic QCP if such a point really exists.

IV. SUMMARY AND DISCUSSION

In summary, we have considered the effects of nematic fluctuation on superfluid density and critical temperature T_c in the vicinity of nematic QCP in the contexts of some HTSCs. On one hand, the critical fluctuation of nematic order parameter reduces zero-T superfluid density as a result of ordering competition. On the other hand, it couples strongly to the gapless nodal QPs and excites more normal QPs out of the SC condensate by triggering an extreme fermion velocity anisotropy. Both of these two effects combine to significantly suppress superconductivity. Therefore, we have predicted a dip-shape reduction of T_c at nematic QCP x_c, which is schematically shown in Fig. 1.

The dip shape of T_c reminds us of 1/8 anomaly,[2–4,31–34] which refers to an anomalous suppression of superconductivity at doping $x = 0.125$. A sudden drop of $\rho^s(0)$ was also observed[35] in La$_{2-x}$Sr$_x$CuO$_4$ (LSCO) at $x = 0.125$. It appears that $\rho^s(T)$ and T_c near the nematic QCP bear some similarity to the basic features of these experiments. However, we refrain from identifying the 1/8 doping as the anticipated nematic QCP for several reasons: (i) 1/8 anomaly is usually attributed to the formation of static stripe order.[2,32] (ii) It is not clear why nematic QCP exists precisely at $x = 0.125$, and not elsewhere. (iii) Low-T dc thermal conductivity κ/T was predicted to be enhanced at nematic QCP,[13] whereas an early transport measurement found a drop of κ/T in LSCO at $x = 0.125$.[36]

Recently, the critical nematic fluctuations are shown to induce unusual behaviors in several quantities, such as QPs spectral function,[10] specific heat,[12] nuclear relaxation rate,[12] thermal conductivity,[13] and QP interference.[14] Normally, a single observation alone is not able to uniquely fix the nematic QCP. Should all or most of these predictions, including the suppression of superconductivity, be observed, the nematic QCP might be determined. However, the absence of a sharp nematic QCP does not necessarily mean the absence of nematic phase since the sharp QCP may be rounded by disorders and become a crossover.[10] In case this smearing occurs, many of the anomalous critical behaviors are weakened, or even destroyed, and the nematic phase most possibly shows its existence in the $\omega \ll T$ regime.

In the calculations presented in this paper, we have assumed that the SC order parameter ψ is a classical field and neglected its quantum fluctuation. This assumption is expected to be a good one if the nematic QCP x_c is deep inside the SC dome, namely, x_c is not close to the SC critical point x_0. When x_c is close to x_0, the quantum fluctuation of SC order ψ may become very important, and a fully quantum mechanical treatment will be necessary.

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