Strong enhancement of Rashba spin-orbit coupling with increasing anisotropy in the Fock-Darwin states of a quantum dot

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We have investigated the electronic properties of elliptical quantum dots in a perpendicular external magnetic field, and in the presence of the Rashba spin-orbit interaction. Our work indicates that the Fock-Darwin spectra display a strong signature of Rashba spin-orbit coupling even in a low magnetic field, as the anisotropy of the quantum dot is increased. An explanation of this pronounced effect with respect to the anisotropy is presented. The strong spin-orbit-coupling effect manifests itself prominently in the corresponding dipole-allowed optical transitions and hence is susceptible to direct experimental observation.

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In recent years our interest in understanding the unique effects of the spin-orbit interaction (SOI) in semiconductor nanostructures[1] has peaked, largely due to the prospect of possible realization of coherent spin manipulation in spintronic devices,[2] where the SOI is destined to play a crucial role.[3] As the SOI couples the orbital motion of the charge carriers with their spin state, an all-electrical control of spin states in nanoscale semiconductor devices could thus be a reality. In this context the Rashba SOI (Ref. 4) has received particular attention, largely because in a two-dimensional electron gas the strength of the Rashba SOI has already been shown to be tuned by the application of an electric field.[5] Interestingly, the Rashba effect has also been a major player in topological insulators and in the search for Majorana fermions.[6] While the earlier studies of Rashba SOI were primarily in a two-dimensional electron gas, our collective attention has now shifted to the role of SOI in a single InAS quantum dot.[7] The quantum dot (QD),[8] a system of few electrons confined in a nanometer region, has the main advantage that the shape and size of the confinement can be externally controlled, which provides a unique opportunity to study the atomiclike properties of these systems.[8,9] In fact, SO coupling in quantum dots generates anisotropic spin splitting[10] that is related to the SO-coupling strength.

Extensive theoretical studies of the influence of Rashba SOI in circularly symmetric parabolic confinement have already been reported earlier,[11] where the SO coupling was found to manifest itself mainly in multiple level crossings and level repulsions. They were attributed to an interplay between the Zeeman and the SO interactions present in the system Hamiltonian. Those effects, in particular the level repulsions, were weak, however, and as a result, would require extraordinary efforts to detect the strength of SO coupling[12] in those systems. Introducing anisotropy in a QD, we show that a major enhancement of the Rashba SO-coupling effects can be generated in a quantum dot. As shown below, this can be observed directly in the Fock-Darwin states of a QD, and therefore should be experimentally observable.[8,9] We find that the Rashba SO-coupling effects are manifestly strong in an elliptical QD,[13] which should provide a direct route to unambigiously determine (and control) the SO-coupling strength. The anisotropy of a quantum dot can in turn be tuned by an in-plane magnetic field.[14]

The Fock-Darwin energy levels in elliptical QDs subjected to a magnetic field were first reported almost two decades ago,[13] when it was found that the major effect of anisotropy was to lift the degeneracies of the single-particle spectrum.[15] The starting point of our present study is the stationary Hamiltonian

$$\mathcal{H}_S = \frac{1}{2m^*}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}_S\right)^2 + V_{\rm conf}(x,y) + \mathcal{H}_{SO} + \mathcal{H}_z = \mathcal{H}_0 + \mathcal{H}_{SO} + \mathcal{H}_z,$$

where the confinement potential is chosen to be of the form $V_{\rm conf} = \frac{1}{2}m^*(\omega_x^2 x^2 + \omega_y^2 y^2)$, $\mathcal{H}_{SO} = \frac{\alpha}{\hbar}[\boldsymbol{\sigma} \times (\mathbf{p} - \frac{e}{c}\mathbf{A}_S)]_z$ is the Rashba SOI, and $\mathcal{H}_z$ is the Zeeman contribution. Here m^* is the effective mass of the electron, $\boldsymbol{\sigma}$ are the Pauli matrices, and we choose the symmetric gauge vector potential $\mathbf{A}_S = \frac{1}{2}(-y,x,0)$. We introduce the rotated coordinates and momenta[13]

$$x = q_1\cos\chi - \chi_2 p_2 \sin\chi, \quad y = q_2\cos\chi - \chi_2 p_1 \sin\chi,$$
$$p_x = p_1\cos\chi + \chi_1 q_2 \sin\chi, \quad p_y = p_2\cos\chi + \chi_1 q_1 \sin\chi,$$

where

$$\chi_1 = -\left[\frac{1}{2}\left(\Omega_1^2 + \Omega_2^2\right)\right]^{1/2}, \quad \chi_2 = \chi_1^{-1},$$
$$\tan 2\chi = m^*\omega_c\left[2\left(\Omega_1^2 + \Omega_2^2\right)\right]^{1/2}/\left(\Omega_1^2 - \Omega_2^2\right),$$
$$\Omega_{1,2}^2 = m^{*2}\left(\omega_{x,y}^2 + \frac{1}{4}\omega_c^2\right), \quad \omega_c = eB/m^*c.$$

In terms of the rotated operators introduced above, the Hamiltonian $\mathcal{H}_0$ is diagonal,[13]

$$\mathcal{H}_0 = \frac{1}{2m^*}\sum_{\nu=1,2}\left[\beta_\nu^2 p_\nu^2 + \gamma_\nu^2 q_\nu^2\right],$$

where

$$\beta_1^2 = \frac{\Omega_1^2 + 3\Omega_2^2 + \Omega_3^2}{2\left(\Omega_1^2 + \Omega_2^2\right)}, \quad \gamma_1^2 = \frac{1}{4}\left(3\Omega_1^2 + \Omega_2^2 + \Omega_3^2\right),$$
$$\beta_2^2 = \frac{3\Omega_1^2 + \Omega_2^2 - \Omega_3^2}{2\left(\Omega_1^2 + \Omega_2^2\right)}, \quad \gamma_2^2 = \frac{1}{4}\left(\Omega_1^2 + 3\Omega_2^2 - \Omega_3^2\right),$$
$$\Omega_3^2 = \left[\left(\Omega_1^2 - \Omega_2^2\right)^2 + 2m^{*2}\omega_c^2\left(\Omega_1^2 + \Omega_2^2\right)\right]^{1/2}.$$

Since the operator $\mathcal{H}_0$ is equivalent to the Hamiltonian of two independent harmonic oscillators, the states of the electron can

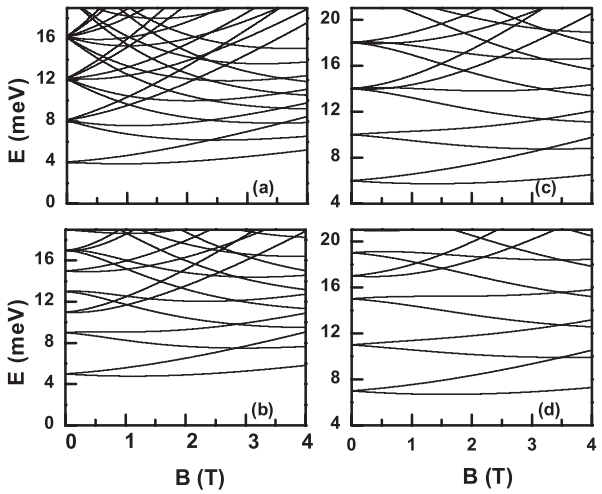

FIG. 1. Magnetic field dependence of the low-lying Fock-Darwin energy levels of an elliptical dot without the Rashba SO interaction ($\alpha = 0$). The results are for (a) $\omega_x = 4$ meV and $\omega_y = 4.1$ meV, (b) $\omega_x = 4$ meV and $\omega_y = 6$ meV, (c) $\omega_x = 4$ meV and $\omega_y = 8$ meV, and (d) $\omega_x = 4$ meV and $\omega_y = 10$ meV. The symbols are explained in the text.

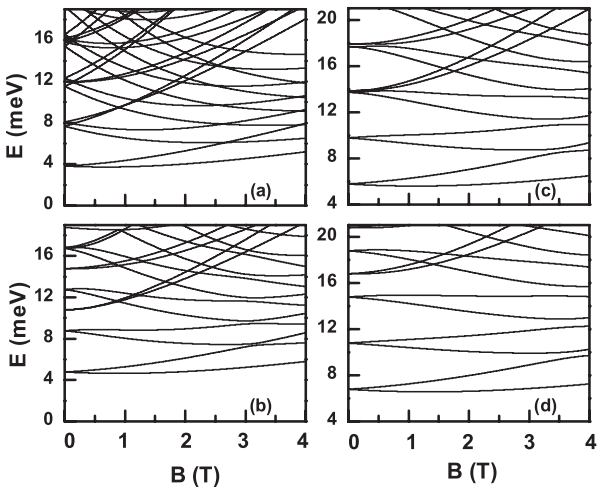

FIG. 2. As Fig. 1, but for $\alpha = 20$ meV nm. The symbols are explained in the text.

be described by the state vectors $|n_1,n_2;s_z\rangle$. Here the oscillator quantum numbers $n_i = 0,1,2,\ldots$ correspond to the orbital motion and $s_z = \pm\frac{1}{2}$ to the spin orientation of the electron. The Rashba SOI term is

$$\begin{aligned}\frac{\hbar}{\alpha}\mathcal{H}_{\rm SO} = {}& \sigma_x(\sin\chi\chi_1 - \cos\chi\omega_0)q_1 \\ & - \sigma_y(\sin\chi\chi_1 + \cos\chi\omega_0)q_2 \\ & - \sigma_y(\cos\chi - \sin\chi\omega_0\chi_2)p_1 \\ & + \sigma_x(\cos\chi + \sin\chi\omega_0\chi_2)p_2,\end{aligned}$$

where $\omega_0 = eB/2c$. The effect of the SO coupling is handled by resorting to the standard ladder operator formalism of harmonic oscillators and by diagonalizing $\mathcal{H}_{\rm SO}$ in the complete basis formed by the vectors $|n_1,n_2;s_z\rangle$.

The Fock-Darwin states in the absence of the Rashba SOI ($\alpha = 0$) are shown in Fig. 1, for $\omega_x = 4$ meV and $\omega_y = 4.1,6,8,10$ meV in Figs. 1(a)–1(d), respectively. We have considered the parameters of an InAs QD throughout,[11] because in such a narrow-gap semiconductor system, the dominant source of the SO interaction is the structural inversion asymmetry,[16] which leads to the Rashba SO interaction.[17] As expected, the breaking of circular symmetry in the dot results in the lifting of degeneracies at $B = 0$, which are otherwise present in a circular dot.[13,15] In Fig. 1(a), the QD is very close to being circularly symmetric, and as a consequence, the splittings of the zero-field levels are vanishingly small. As the anisotropy of the QD is increased [Figs. 1(b)–1(d)], splitting of the levels becomes more appreciable.

As the SO term is linear in the position and momentum operators it is also linear in the raising and lowering ladder operators. It is also off diagonal in the quantum number s_z. Hence the SOI can mix only states which differ in spin orientation, and differ by 1 either in the quantum number n_1 or in n_2 but not in both. For rotationally symmetric confinements, these selection rules translate to the conservation of the total angular momentum $j = m + s_z$ in the planar motion of the electron.

At the field $B = 0$ the ground states $|0,0;\pm\frac{1}{2}\rangle$ are twofold degenerate. Due to the selection rules, this degeneracy cannot be lifted either by the eccentricity of the dot or by the Rashba coupling. Many of the excited states, such as $|n_1,n_2;\pm\frac{1}{2}\rangle$, retain their degeneracy no matter how strong the SO coupling is or how eccentric the dot is (Kramers doublet), as seen in Figs. 1 and 2. At the same time, many other degeneracies are removed by squeezing or streching the dot. At nonzero magnetic fields some of the crossings of the energy spectra are turned to anticrossings by the Rashba term in the Hamiltonian. For example, the second and third excited states in Figs. 2(a)–2(d) are composed mainly of the states $|0,0;\frac{1}{2}\rangle$ (marked as • in Fig. 2) and $|1,0;-\frac{1}{2}\rangle$ (marked as ■ in Fig. 1), which are mixed by the $\mathcal{H}_{\rm SO}$ around $B = 3$ T, causing a level repulsion. Squeezing of the dot also enhances the SO coupling. The level repulsion gap due to the Rashba SOI is presented in Fig. 3 as a function ω_y while ω_x is kept fixed at 4 meV. Only the lowest two level repulsion gaps (around the magnetic field of $B = 3$–4 T are shown. A rapid increase of the gap in both cases is found as a function of the eccentricity of the dot. Such an effect has not been reported before to our knowledge.

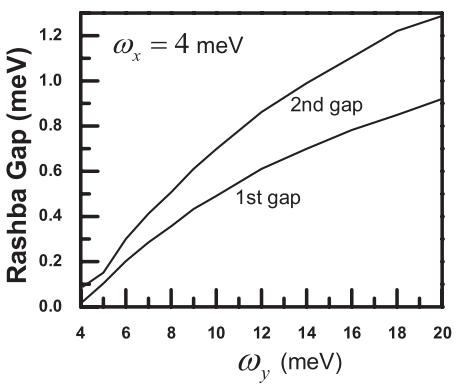

FIG. 3. The Rashba gap for the lowest level repulsion (first gap) and for the next higher energy (second gap) versus the anisotropy of the dot.

In fact the four lowest gaps shown in Fig. 2 at around 3–4 T are between the states with major components $|n,0;\frac{1}{2}\rangle$ and $|n+1,0;-\frac{1}{2}\rangle$, that is, between the states related to the oscillations parallel to the x axis. When the dot is squeezed in the y direction we would naively expect these states to experience only the trivial zero-point-energy shift of magnitude $\frac{1}{2}\hbar\Delta\omega_y$. In particular, we would expect the Rashba gap to approximately retain its width. There are, however, two indirect mechanisms that are influencing the size of the gap. The first one can be understood using simple perturbation-theoretical arguments: the squeezing moves the states $|n+1,1;\frac{1}{2}\rangle$ to such high energies that their tendency to repel the states $|n+1,0;-\frac{1}{2}\rangle$ downward and to close the gap is diminished. The second mechanism is attributed to the imposed magnetic field which rotates the oscillators in the phase space by mixing the high-momentum y-direction oscillation with the x-direction motion. Since the SOI is proportional to the momentum, this mixing will increase the Rashba effect.

The effects of anisotropy and spin-orbit interaction on the energy spectra above are also reflected in the optical absorption spectra. Let us turn our attention to the absorption spectra for transitions from the ground state to the excited states. For that purpose we subject the dot to the radiation field

$$\mathbf{A}_{\mathrm{R}} = A_0\hat{\boldsymbol{\epsilon}}(e^{i(\omega/c)\hat{\mathbf{n}}\cdot\mathbf{r}-i\omega t} + e^{-i(\omega/c)\hat{\mathbf{n}}\cdot\mathbf{r}+i\omega t}),$$

where $\hat{\boldsymbol{\epsilon}}$, ω, and $\hat{\mathbf{n}}$ are the polarization, frequency, and direction of propagation of the incident light, respectively. We let the radiation enter the dot along the direction perpendicular to the motion of the electron, that is, parallel to the z axis. Due to the transversality condition the polarization vector will then lie in the xy plane.

We now assume that the intensity of the field is so weak that only the terms linear in $\mathbf{A}_{\mathrm{R}}$ have to be taken into account. Then the effect of the radiative magnetic field on the spin can be neglected as well. So we can replace in the stationary Hamiltonian $\mathcal{H}_{\mathrm{S}}$ the vector potential $\mathbf{A}_{\mathrm{S}}$ with the field $\mathbf{A} = \mathbf{A}_{\mathrm{S}} + \mathbf{A}_{\mathrm{R}}$. Discarding terms higher than linear order in $\mathbf{A}_{\mathrm{R}}$, we get the total Hamiltonian $\mathcal{H} = \mathcal{H}_{\mathrm{S}} + \mathcal{H}_{\mathrm{R}}$, with the radiative part

$$\mathcal{H}_{\mathrm{R}} = -\frac{e}{m_e c}\mathbf{A}_{\mathrm{R}}\cdot\left(\mathbf{p} - \frac{e}{c}\mathbf{A}_{\mathrm{S}}\right) - \frac{\alpha e}{\hbar c}[\boldsymbol{\sigma}\times\mathbf{A}_{\mathrm{R}}]_z.$$

Dipole approximation. We assume that the amplitude of radiation can be taken as constant within the dot, so that the field is written as $\mathbf{A}_{\mathrm{R}} \approx A_0\hat{\boldsymbol{\epsilon}}(e^{-i\omega t} + e^{i\omega t})$. Since the transition energies expressed in terms of radiation frequences are of the order of terahertz, the corresponding wavelengths are much larger than the typical size of a dot, thus justifying our approximation. The Fermi golden rule now leads to the dipole approximation form

$$\sigma_{\mathrm{abs}}(\omega) = 4\pi^2\alpha_{\mathrm{f}}\omega_{ni}|\langle n|\hat{\boldsymbol{\epsilon}}\cdot\mathbf{x}|i\rangle|^2\delta(\omega_{ni}-\omega)$$

of the absorption cross section for transitions from the inital state $|i\rangle$ to the final state $|n\rangle$. Here α_{f} is the fine structure constant and ω_{ni} is the frequency corresponding to the transition energy $\hbar\omega$.

The dipole selection rules for the oscillator states largely dictate the features in Fig. 4. Without the SOI, these rules—the spin state is preserved and either n_1 or n_2 is changed by unity—fully determine the two allowed transitions $|0,0;-\frac{1}{2}\rangle$ (marked as $\triangle$ in Fig. 1) $\rightarrow |1,0;-\frac{1}{2}\rangle$ and $|0,0;-\frac{1}{2}\rangle \rightarrow |0,1;-\frac{1}{2}\rangle$ (marked as $*$ in Fig. 1). In elliptical dots the absorption depends on the polarization since the oscillator strengths

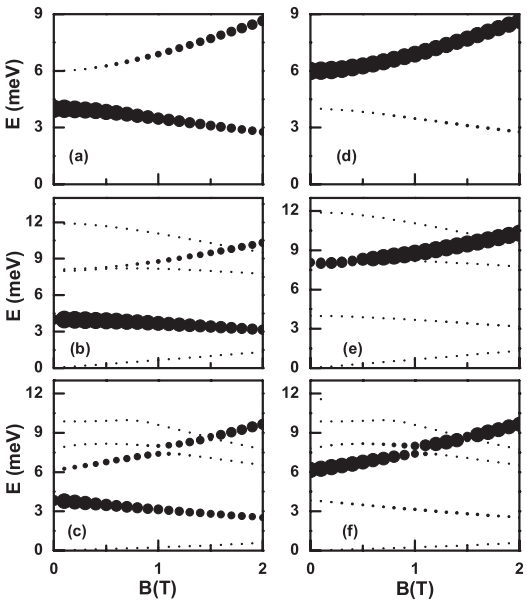

FIG. 4. Optical absorption (dipole-allowed) specta of elliptical QDs for various choice of parameters: (a) $\alpha = 0$, $\omega_x = 4$ meV, $\omega_y = 6$, (b) $\alpha = 20$ meV nm, $\omega_x = 4$ meV, $\omega_y = 8$ meV, and (c) $\alpha = 40$ meV nm, $\omega_x = 4$, $\omega_y = 6$. The polarization of the incident radiation is along the x axis. The parameters for (d)–(f) are the same, except that the incident radiation is polarized along the y axis. The areas of the filled circles are proportional to the calculated absorption cross section.

$$f_{ni} = \frac{2m^*\omega_{ni}}{\hbar}|\langle n|\hat{\boldsymbol{\epsilon}}\cdot\mathbf{x}|i\rangle|^2$$

probe the occupations of quantum states related to oscillations in the direction of the polarization $\hat{\boldsymbol{\epsilon}}$. In a circular dot all oscillation directions are equally probable at all energies, implying that the oscillator strengths are independent of the polarization and depend only slightly on the transition energy via ω_{ni}, and the final-state quantum numbers $n_{1,2}$. When the dot is squeezed in the y direction, say, the oscillator states related to the y-axis motion are pushed up in energy. The polarization being along the x axis, most of the oscillator strength comes from transitions to allowed states with the lowest energies. Similarly, when the incident radiation is polarized along the y axis most of the contribution is due to the transitions to the oscillator states pushed up in energy. In elliptical dots the oscillator states are not pure x and y oscillators but their superpositions. Therefore, in addition to the main absorption lines, other allowed final states also have nonvanishing oscillator strength. Furthermore, the phase-space rotation formulas above show that the external magnetic field tends to rotate the directions of the oscillator motion, causing a shift of the oscillator strength from one allowed transition to another. This is precisely what is seen in Figs. 4(a) and 4(d).

Even in the presence of the SOI the two allowed final oscillator states provide major contributions to the corresponding corrected states. Hence we still see two dominant absorption lines. However, now many other transitions are also allowed. The lowest absorption line corresponding to the transition between the Zeeman-split states with the main components $|0,0;-\frac{1}{2}\rangle$ and $|0,0;\frac{1}{2}\rangle$ provides a typical example. The transition involves a spin flip and is therefore strongly forbidden without the SOI. As the SOI mixes the state $|1,0;\frac{1}{2}\rangle$ (marked as □ in Fig. 2) with the former and $|0,1;-\frac{1}{2}\rangle$ with the latter, the transition is allowed. The appearance of other new lines can be explained by analogous arguments. There are also additional features involving discontinuities and anticrossings in Fig. 4, which are the consequences of the anticrossings present in the energy spectra.

The oscillator strengths satisfy the Thomas-Reiche-Kuhn sum rule,[18] $\sum_n f_{ni} = 1$. In terms of the cross section this translates to the condition

$$\int_{-\infty}^{\infty} \sigma_{\rm abs}(\omega)\, d\omega = \frac{2\pi^2\hbar\alpha_{\rm f}}{m^*}.$$

The absorptions in Fig. 4 practically saturate the sum rule, the saturation being, of course complete in the absence of the SOI in Figs. 4(a) and 4(d). The largest fraction (of the order of 1/10) of the cross section either falling outside of the displayed energy scale or having too low intensity to be discernible in our pictures is found at the strongest Rashba coupling in Figs. 4(c) and 4(f) for large magnetic fields.[19]

Our present work clearly indicates that the anisotropy of a QD alone causes lifting of degeneracies of the Fock-Darwin levels at $B = 0$, as reported earlier.[13] However, for large SO-coupling strengths α, the effects of the Rashba SOI, mainly the level repulsions at finite magnetic fields, are magnified rather significantly as one introduces anisotropy in the QD. This prominent effect of the Rashba SOI predicted here could be confirmed experimentally in optical spectroscopy[8,9,20] and in the Fock-Darwin spectra of few-electron QDs.[9,21] It would also provide a very useful step to control the SO coupling in nanostructures, en route to semiconductor spintronics.[2]

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