Understanding the electric-field enhancement of the superconducting transition temperature for complex oxide interfaces

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We examine the enhancement of the interfacial superconductivity between LaAlO_3 and SrTiO_3 by an effective electric field. Through the breaking of inversion symmetry at the interface, we show that a term in the free energy, coupling the superfluid density and an electric field, can augment the superconductivity transition temperature. Microscopically, we show that an electric field can also produce changes in the carrier density by relating the measured capacitance to the density of states. Through the standard BCS (Bardeen, Cooper, and Schrieffer) weak-coupling interaction in bulk SrTiO₃, we estimate the transition temperature.

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I. INTRODUCTION

Superconductivity is typically thought to be produced by the pairing of electrons below a critical temperature T_c .^{[1](#page-5-0)} The overall pairing interaction is dependent on a subtle balance of the repulsive Coulomb interaction and any attractive interaction supplied by the exchange of collective excitations in the system. $1,2$ That balance depends, in a complex way, on the number of carriers within the material. $2-4$ This effect becomes all the more important in the case of spatial inhomogeneity, e.g., at an interface where the spatial distribution of the carrier density will be important.^{[5,6](#page-5-0)} Therefore, it is of great interest to have a tuning parameter like an external electric field producing a change to the carrier distribution and effectively maximizing the transition temperature T_c .

In recent years, oxide heterostructures have attracted much attention due to a rich spectrum of unexpected behavior discovered at interfaces.^{7–9} Specifically, the emergence of a two-dimensional (2D) electron gas that becomes superconducting at around 200 mK at the epitaxially grown interface of the insulator $SrTiO₃$ (STO) and a few atomic layers of LaAlO₃ (LAO) (illustrated in Fig. [1\)](#page-1-0) has sparked an intense search for the mechanism of superconductivity as well as for improved materials. $7-12$ Studies investigating the electron mobility have demonstrated that intrinsic doping may be key to understanding the underlying mechanisms. 13 While the superconductivity occurs at much lower temperatures than in high- T_c superconductors, this finding shows the importance of investigating interfacial physics in heterostructures. Even though there are limitations due to lattice matching and strain within the materials, $14,15$ the coupling between these materials may provide information that is critical to the understanding of these heterostructures.

Bulk STO has been shown to become superconducting through the doping of oxygen vacancies and by niobium (Nb) at about the same temperature as observed in the LAO-STO system.^{[16,17](#page-5-0)} This observation highlights the effects of carrier density on the superconducting state.[18,19](#page-5-0) Research into the enhancement of the superconducting transition temperature through an electric-field effect has been observed and discussed for many systems. Further studies of the LAO-STO system have also shown that T_c can be increased by an applied electric field.^{20,21} Furthermore, STO has been found to exhibit interfacial superconductivity when brought in contact with an oxide gel in the presence of an external electric field. 22 These studies show that T_c can be induced and/or enhanced through the increase of carriers pulled from a source (i.e., LAO, oxide gel, or doping)[.20–22](#page-6-0) The similarity between the bulk and interfacial transition temperatures signals comparable pairing mechanisms.

The nature of the superconducting state at the interface is a subject of ongoing discussion. One possibility is that the superconducting state is conventionally electron-phonon driven, but there is also a growing list of possible uncon-ventional states including magnetism-driven ones.^{[23,24](#page-6-0)} Indeed, recent data from scanned probes point to a complicated spatial pattern of coexisting superconductivity and magnetism at the interfaces.^{[25](#page-6-0)} Early reports on ferromagnetic order coexisting with superconductivity²⁶ have been independently confirmed. 27 For an assessment of the present situation, see Ref. [28.](#page-6-0)

For the following, it will be important to know the thickness of the mobile-electron layer at the interface. Early estimates of the 2D density of mobile carriers have yielded values far below the theoretical prediction of 0.5 electrons per unit cell.^{[29](#page-6-0)} Apparently, most of the added charge is not mobile and is trapped within a few unit cells of the surface as observed by high-energy-photoemission experiments^{[30](#page-6-0)} while the mobile electrons may cover a region of thickness on the order of 10 nm.

Recent experiments²⁵ suggest that the spatial separation of superconducting and ferromagnetic regions may allow spin-singlet pairing to coexist with ferromagnetic domains. The lateral spatial extension of regions with mobile carriers reaching over several tens or even hundreds of unit cells speaks against a prominent role of the on-site Coulomb interaction and therefore a pairing mechanism induced by magnetic fluctuations. These experimental observations may point to a possible conventional*s*-wave superconducting state. However, the precise nature of the superconducting state at the interface remains an open issue that can only be settled experimentally.

FIG. 1. (Color online) An illustration of a LaAlO3/SrTiO3 interface, detailing the interface region. The interfacial (lighter color) area is a small region from $-\delta z$ to δz . The bulk (darker color) region is that volume that is essentially unaffected by the interface interactions. For the purpose of this study, we examine the effect of an applied electric field produced throughout the sample. Therefore, all calculations are to mimic this general setup.

In this study, we investigate first the effect of an electric field applied externally on the 2D and 3D carrier density. We calculate the variation of the 2D carrier density, capacitance, and 3D carrier-density-depth profile with increasing electric potential for LAO-STO heterostructures. From changes of the overall carrier density at the interface, we deduce that the nonlinear capacitance of the layer at the interface implies an energy-dependent density of states (DOS). Second, using modifications of the charge-carrier density and DOS as tunable parameters, we calculate changes in the superconducting transition temperature T_c . We assume that T_c can be described though a simple Bardeen, Cooper, and Schrieffer $(BCS)^1$ $(BCS)^1$ weak-interaction formula as suggested by studies of doped bulk STO.^{[17](#page-5-0)} Using this model assumption, we show that basic electrostatics provides an explanation for the observed change in T_c for the band-insulator STO with an electric field, which is due to changes in the charge density. We also find that T_c does exhibit a maximum at a certain carrier density determined by the peak in the capacitance. A complete understanding of the mechanisms involved in the experimental observations of the electric-field enhancement of superconductivity could lead to a possible mechanism for a tunable functional response: dramatic changes in the superconducting properties with the application of external fields.

II. FIELD EFFECT ON THE GINZBURG-LANDAU FREE ENERGY

The breaking of inversion symmetry at the interface of heterostructures introduces the interface normal vector as a preferred direction of the system.^{[31](#page-6-0)} The normal component of the electric field F_z is therefore compatible with the symmetries of the free energy, allowing a coupling term of the electric field and superconducting density $F_z|\psi|^2$ with the superconducting order parameter ψ . This is a simple consequence of the screening effect on the charge density within the superconductor. 32 The charge distribution at the interface would be dependent on the distance from the interface, typically on the order of nanometers. On the other hand, the relevant superconducting-length scale is set by a fairly large coherence length of about 60 nm, 33 typically on the scale of tens to hundreds of nanometers.³ Therefore, we may use a Ginzburg-Landau (GL) functional in our approach when we address the coupling between superconducting and electronic degrees of freedom.^{[2,4](#page-5-0)}

Since the interface has the additional constraints of boundary conditions and strain (as shown in Fig. 1), the standard heterostructure free energy can be broken into two main components consisting of bulk *B* and interfacial *I* regions. The effect of the interface decays as one moves into the bulk. In the case of the LAO/STO interface, the total free energy *E*_{free} is then given by

$$
E_{\text{free,tot}} = \int_{-L}^{-\delta z} F(z)_{B} \text{mod} z + \int_{-\delta z}^{0} F(z)_{I} \text{mod} z + \int_{0}^{\delta z} F(z)_{I} \text{mod} z + \int_{\delta z}^L F(z)_{B} \text{mod} z, \quad (1)
$$

where the depth-dependent free energy for the bulk and interface are then given by

$$
E_{\text{free}}(z)_B = \alpha_B |\psi(z)_B|^2 + \frac{\beta_B}{2} |\psi(z)_B|^4 \tag{2}
$$

and

$$
E_{\text{free}}(z)_I = \alpha_I |\psi(z)_I|^2 + \frac{\beta_I}{2} |\psi(z)_I|^4 + g |\nabla \psi(z)_I|^2
$$

+ $\lambda F_z |\psi(z)_I|^2 + \gamma \eta^2 |\psi(z)_I|^2,$ (3)

where $\alpha_{B,I}$ and $\beta_{B,I}$ are the standard GL-energy parameters. In the standard GL formalism, the parameter $\alpha_B = a_B(T - T_c^0)$, with $a > 0$, determines the normal ($\alpha_B > 0$) and superconducting states ($\alpha_B < 0$) while β_B is always positive and helps stabilize the ground state.

For the interface region, the gradient and strain *η* terms are produced by the interface and are typically positive as the gradient produces the boundary condition for the interface and can be assumed to be in the small-wave-vector limit. For the field interaction, we use $\mathbf{F} = (0,0,F_z)$. The coefficient λ will in general depend on the electric-field strength. Therefore, the local carrier density can increase or decrease depending on a positive or negative F_z , respectively. It should be noted that while the electric field is global, only the superconductivity at the interface will increase due to the accumulation of carriers from the carrier source. This may be different for a hightemperature superconductor, but given that STO is an insulator, the distribution of carriers is lessened.

Focusing on the effects of the additional quadratic terms, it can be easily shown that the standard GL definitions are changed. The inclusion of the electric-field interaction and strain terms induces a shift in T_c . Concentrating on the ψ^2 terms, we rewrite the free energy as

$$
E_{\text{free}}(z)_I^{\psi^2} = \alpha_I |\psi(z)_I|^2 + \lambda F_z |\psi(z)_I|^2 + \gamma \eta^2 |\psi(z)_I|^2
$$

= $\tilde{\alpha} |\psi(z)_I|^2$, (4)

where $\tilde{\alpha} = \alpha_I + \lambda F_z + \gamma \eta^2 = \tilde{a}(T - T_c^0)$ and describes the superconducting state when $\tilde{\alpha} < 0.2 - 4$ Therefore, the shifted critical temperature is given by

$$
T_c = T_c^0 - \left[\frac{\lambda_c F_z + (\gamma \eta^2)_c}{\tilde{a}}\right],\tag{5}
$$

where λ_c and $(\gamma \eta^2)_c$ are taken at $T = T_c^0$. From this we conclude that the electric field has to compete with the lattice strain. Even a large lattice strain may be overcompensated by a reasonably strong electric field, which therefore can still affect the superconducting order parameter. However, if the lattice matching is close, then the strain can be assumed to have a negligible effect on the superconducting state. Recent measurements indicate the various degrees of lattice mismatch may even produce an increase in the superconducting T_c .^{[34](#page-6-0)}

Thus, the addition of extra quadratic terms to the free energy modifies the critical temperature. While the strain terms usually produce a negative shift of T_c , a suitably directed electric field could give rise to a positive shift. This may be interpreted as being due to an increase in the carrier density of the superconductor. While this is shown here only in the phenomenological GL model, we will now turn to a modeling of the charge density induced by an applied electric field and show how that is correlated with T_c .

III. SEMIEMPRICAL MODELING OF THE ELECTROSTATIC-DIFFERENCE POTENTIAL

To examine the change in the electron density at an LAO/STO interface, we calculate the electrostatic-difference potential (EDP) ϕ^* using a semiempirical, extended Hückel model. The EDP is described as the difference between the electrostatic potential ϕ of the self-consistent valence-charge density and the electrostatic potential from atomic-valence densities[.35,36](#page-6-0) Therefore, the EDP is determined by solving the Poisson equation $\nabla^2 \phi^* = -\rho^* / \epsilon_r$, where is ρ^* is the electron-difference density.

Figure 2 shows the calculated EDP for various applied electric potentials. This calculation is produced by building a three-dimensional interface of LAO and STO that is $3 \times 3 \times 8$ unit cells for each side. The electric potential is produced by simulating a device in which positive and negative electrodes are place on the ends of the interface. For this simulation, the LAO electrode is negative, and the STO electrode is positive. As the electric potential is increased from 0 V to 10 V, the EDP is observed to diffuse from LAO into STO. This details the shifting of the electron-charge density through the interface. To gain a more detailed view of the change in the charge and carrier densities, we examine the general electrostatics of the interface and compare them to known experimental data.

IV. CARRIER DENSITY AND CAPACITANCE

The basic connection of the charge density and an applied electric field is given by integrating Gauss's law for a dielectric medium, keeping the nonlinear dependence of the dielectric function $\epsilon(F) = \epsilon_0 \epsilon_r(F)$ on an electric field. This function may be determined through the measured capacitance at the LAO/STO interface. The change in the 2D carrier density n_{2D} in a semiconducting channel, induced by an electric field, is then given by

$$
n_{\rm 2D}(F) = \frac{2\epsilon_0}{e} \int_0^F \epsilon_r(F) dF,\tag{6}
$$

FIG. 2. (Color online) Model calculation for the shifting electrostatic-difference potential (EDP) due to an applied electric potential ranging from 0 V to 10 V using a semiempirical, extended Hückel model for the LAO/STO interface.^{[35,36](#page-6-0)} The model shows the diffusion of the potential (approximately 1 unit cell per 2 V) from the LAO region into the STO region. (The arrows are to help guide the eye.) In the simulation, the last two unit cells are used as the electrodes [LAO (cathode) and STO (anode)], producing an overall electric potential over the sample region.

dependent relative permittivity, and *e* is the basic unit charge. The field dependence of the relative permittivity for STO has been shown to be $\epsilon_r(F)^{STO} = 1/A(1 + \frac{B}{A}F)^{37}$ $\epsilon_r(F)^{STO} = 1/A(1 + \frac{B}{A}F)^{37}$ $\epsilon_r(F)^{STO} = 1/A(1 + \frac{B}{A}F)^{37}$ where *A* and *B* are temperature-dependent variables determined experimentally (4*.*⁰⁹⁷ [×] ¹⁰−⁵ and 4*.*⁹⁰⁷ [×] ¹⁰−¹⁰ ^m*/*V for STO at 4.2 K, respectively). This is a first-order fit to the permittivity and produces a carrier density which has a singularity and is discontinuous at $F = -A/B$. The singularity can be removed by expanding out the denominator and including a second-order term wherein the expanded parameter provides constraints that make the system continuous, assuming $C_2^2/4C_1C_3 < 1$. This produces a continuous function in carrier density over all *F*,

$$
\epsilon_r(F)^{\text{STO}} = \frac{1}{A \left[C_1 + C_2 \frac{B}{A} F + C_3 \left(\frac{B}{A} \right)^2 F^2 \right]},\tag{7}
$$

where *F* is the effective electric field within the sample, ϵ_0 is the permittivity of free space, $\epsilon_r(F)$ is the electric-fieldwhere C_i ($i = 1, 2,$ and 3) are determined from experimental fits. From this, we find that

FIG. 3. (Color online) Simulated capacitance and 2D carrier density n_{2D} as a function of the electric potential for the LAO/STO interface in Ref. [20.](#page-6-0) It should be noted that the change in curvature denotes an effective change in electron or hole doping.

$$
n_{2D}(F) = \frac{4\epsilon_0 \left\{ \tan^{-1} \left[\frac{(2C_3BF + C_2 A)}{A\sqrt{C_4}} \right] - \tan^{-1} \left(\frac{C_2}{\sqrt{C_4}} \right) \right\}}{eB\sqrt{C_4}},
$$
 (8)

where $C_4 = 4C_1C_3 - C_2^2$.

To examine the modulation of the carrier density along the LAO/STO interface, we compare it to data provided by Ref. [20.](#page-6-0) Here, a gate voltage is used to obtain an electric field where $F = F_0 + V/d$, *V* is the electric potential, and *d* is the sample thickness (0.5 mm). The 2D carrier density is also related to the capacitance $C(V)$ by

$$
C(V) = eS\left(\frac{\partial \delta n_{\text{2D}}}{\partial V}\right),\tag{9}
$$

where *S* is the area of the capacitor and $\delta n_{2D} = n_{2D}(V)$ – $n_{2D}(200V)$ (measuring *V* in volts). The cross-sectional area of the capacitor is determined by adjusting the simulated peak capacitance to match the observed capacitance. Using this method, the cross-sectional area is determined to be about 5 mm2. Therefore, the capacitance will be

$$
C(V) = \frac{2\epsilon_0 S}{Ad[C_1 + C_2 \frac{B}{A} (F_0 + \frac{V}{d}) + C_3 (\frac{B}{A})^2 (F_0 + \frac{V}{d})^2]}.
$$
\n(10)

Comparing to the data in Ref. [20](#page-6-0) and adjusting only the *Ci* parameters, Fig. 3 shows the capacitance and 2D carrier density as a function of the gate voltage for STO. Through fitting of the δn_{2D} data from Ref. [20,](#page-6-0) the extended parameters (C_1 , C_2 , and C_3) are determined to be 4.25, -0.37 , and 0.29, respectively, with a value of $F_0 = 1.2 \times 10^5$ V/m. Here, we find that the change in the 2D carrier density shifts from electron to hole doping around 30 V. This produces the corresponding peak in the capacitance. Further analysis in Sec. V will correlate the capacitance to the change in T_c . However, in order to accurately model the transition temperature, an overall understanding of the relationship between the DOS and capacitance must be explored.

To compare the DOS to the measured capacitance, we consider

$$
\frac{\delta n_{\text{2D}}(V)}{\delta V} = N_{\text{2D}}[E_F(V), V] \frac{\delta E_F(V)}{\delta V} + \int_0^{E_F(V)} \frac{\delta}{\delta V} N_{\text{2D}}(E, V) dE. \tag{11}
$$

Here, it is shown that as the gate potential is increased, the carriers in the system are increased at the interface. This increases the overall density of carriers and shortens the interfacial depth. By using the above-determined Fermi energy $E_F(V)$, the density of states in 2D may be related to the derivative of $n_{2D}(V)$ with respect to V and hence to the capacitance as

$$
N_{2D}[E_F(V), V] = \left[\frac{\partial n_{2D}}{\partial V} - \int_0^{E_F(V)} \frac{\delta}{\delta V} N_{2D}(E, V) dE\right] \frac{\partial V}{\partial E_F(V)} = \left[\frac{1}{eS}C(V) - \int_0^{E_F(V)} \frac{\delta}{\delta V} N_{2D}(E, V) dE\right] \frac{\partial V}{\partial E_F(V)}.
$$
(12)

The integral over $N_{2D}(E, V)$ is dependent on the electric-field dependence of the effective masses and $E_{i,j}$. However, it is assumed that this contribution is small within the confines of the interface.

Since the DOS is dependent on $\partial V / \partial E_F(V)$, we need a method for which to approximate this value. This can be achieved through an investigation of the 3D carrier density. This will provide an understanding of the accumulation of charge at the LAO/STO interface as a gate voltage is applied as well as provide through a self-consistent methodology the change in E_F with respect to the electric potential. This will also yield an approximation for the carrier density as a function of depth.

The depth profile of the charge accumulation in STO is given by summing over the density profiles $|\xi_{i,j}(z)|^2$ of all occupied states:

$$
n_{3D}(F,z) = \sum_{j=l,h} \left[\frac{g_j m_j^*}{2\pi\hbar^2} \sum_i (\Delta E_{i,j}) \Theta(\Delta E_{i,j}) |\xi_{i,j}(z)|^2 \right],
$$
\n(13)

where $\Delta E_{i,j} = E_F - E_{i,j}$ and the sum over $j = l, h$ is over the light and heavy electronic bands in STO. The inner sum is over the occupied states labeled *i,j* for the energy of the eigenstates $E_{i,j}$ and E_F where the Θ function assures that all states $E_{i,j} < E_F$ are included. m_j^* is the effective mass $(m_l^* = 1.2m_0 \text{ and } m_h^* = 4.8m_0), m_0 \text{ is the electron mass, and}$ $g_i = 1,2$ is the band degeneracy.^{[21,38](#page-6-0)} The confining potential is well represented by a triangular shape, and the corresponding eigenstates are therefore normalized Airy functions:

$$
\xi_{i,j}(z) = n_c \text{Ai}\left[\alpha_j \left(z - \frac{E_{i,j}}{eF}\right)\right],\tag{14}
$$

where n_c is the normalization constant in units of inverse length, $E_{i,j} = \frac{eF}{\alpha_j} \left[\frac{3\pi}{2} (i - \frac{1}{4})\right]^{2/3}$, and $\alpha_j = (2m_j^* eF/\hbar^2)^{1/3}$.

Figure [4](#page-4-0) shows the n_{3D} electron density as a function of *z*. Here, the the Fermi energy $E_F(V)$ is determined

FIG. 4. (Color online) Simulated 3D carrier density n_{3D} as a function of depth for various electric potentials [0 V (solid black), 100 V (dashed red), and 200 V (dotted blue)] for the LAO/STO interface. This describes the increase in the overall carrier density at the interface as the electric potential is increased. As this occurs, the relative interfacial region reduces in depth. The inset shows the effective Fermi energy E_F as a function of the electric potential. A fitting of this data to a quadratic polynomial provides a general functional form.

self-consistently by calculating

$$
n_{2D}(V) = \int n_{3D}(z)dz = \int_0^{E_F(V)} N_{2D}(E, V)dE, \qquad (15)
$$

where $N_{2D}(E, V)$ is the density of states

$$
N_{2\text{D}}(E,V) = \sum_{j=l,h} \left[\frac{g_j m_j^*}{2\pi\hbar^2} \sum_i \Theta(\Delta E_{i,j}) \right],\qquad(16)
$$

and comparing it to the value from Eq. (8) . We can also determine the average n_{3D} by $\langle n_{3D} \rangle = \int_a n_{3D}^2 dz / \int n_{3D} dz$ and the average thickness $d_{\text{av}} = \int n_{3D}z dz / \int n_{3D}dz$.

The inset of Fig. 4 shows E_F as a function of *V*. Using a quadratic-polynomial fit of the data, we find that E_F (meV) = $4.30 + 0.0410V - 5.05 \times 10^{-5}V^2$. Therefore, we can use the derivative to find an approximation of $\partial V / \partial E_F(V)$ and determine the DOS.

V. EFFECT ON THE SUPERCONDUCTING TRANSITION TEMPERATURE *Tc*

To explore the consequences of the change in carrier density induced by an applied electric field on the superconducting transition temperature, we exploit the fact that the change in the DOS will modify the (dimensionless) pair coupling. We conjecture that the pairing is similar to that observed in bulk doped $STO¹⁶$ There, weak-coupling BCS theory provides a quantitative description of the observed transition temperature as a function of doping. Within weak-coupling theory, the critical temperature T_c below which a material becomes superconducting is given by

$$
T_c = T_D e^{-\frac{1}{\lambda_p \delta N(E_F)}},\tag{17}
$$

where T_D is the Debye temperature (513 K for LAO-STO multilayers), λ_p is the effective interaction, and $N(E_F)$ is the electronic density of states at the Fermi level in $d = 3$ dimensions. In the case of bulk doped STO, a detailed theoretical study of the transition temperature in the framework of Eliashberg's theory has been performed[.17](#page-5-0) The conclusion was that inter- and intravalley electron-phonon processes as well as Coulomb repulsion contribute to determining T_c . These calculations accounted quantitatively for the observed maximum of T_c as a function of carrier density. The initial increase of T_c at low density is connected with the increase in the density of states while the decrease beyond the maximum may be traced to the weakening of the attraction caused by enhanced screening. Comparing the values of density n_{3D} found above with the density at the maximum of T_c in Ref. [17](#page-5-0) $(n_{\text{3D}}^m = 10^{20} \text{ cm}^{-3})$, we conclude that our systems are on the low-density side wherein density-dependent-screening effects of the pair interaction are less relevant. We therefore assume that the pair interaction λ_p may be taken as a constant, which we deduce from the dimensionless coupling, $\lambda_p N(E_F) = 0.13$ and the measured DOS $N(E_F) = 2 \times 10^{47} \text{ J}^{-1} \text{m}^{-3}$ at density n_{3D}^m as $\lambda_p \approx 6.5 \times 10^{-49}$ Jm³. However, the pair interaction may also have a dependence on the applied potential. As shown above, the 2D electronic density of states can be directly measured through the capacitance and the dependence of the Fermi energy on applied voltage while the latter can be inferred from the observed density-voltage relation. This is due to the change in the capacitance produced by the 2D electron gas. Therefore, the changes in the 3D DOS can be approximately expressed as

$$
\delta N[E_F(V), V] = \frac{1}{Sed_{\text{av}}\gamma} \delta C(V) \frac{\partial V}{\partial E_F(V)},\tag{18}
$$

where d_{av} is the average thickness of the electron layer. As discussed above, the second term from Eq. (12) is small and can be ignored. Hence, a change in the capacitance with the applied electric field provides a direct effect on T_c within this system. For simplicity, we use the total capacitance to calculated T_c and assume that the contribution of the bulk STO is small. Figure [5](#page-5-0) shows the calculated T_c as a function of *V* for the LAO-STO heterostructure for which $\gamma \approx 0.0045$ has been adjusted to place the peak value of T_c at the experimentally determined value 0.37 K. This details the subtle significance of a gate voltage on the superconducting state. The inset of Fig. [5](#page-5-0) details and compares the experimentally observed change in T_c from Ref. [21](#page-6-0) (dashed blue) and Ref. [20](#page-6-0) (dotted red).

The observed transition into the superconducting state is most likely of the Kosterlitz-Thouless-Berezinski (KTB) type. In general, the transition temperature T_{KTB} is related to the bulk transition temperature T_c by 39

$$
\frac{T_{\text{KTB}}}{T_c} / f\left(\frac{T_{\text{KTB}}}{T_c}\right) = 2.18 \frac{R_Q}{R_{\text{sheet}}},\tag{19}
$$

where R_{sheet} is the sheet resistance and $R_Q = \hbar/e^2 \approx 4.12k\Omega$ is the resistance quantum. Here, the function *f* is defined in terms of the gap parameter $\Delta(T)$ as

$$
f\left(\frac{T}{T_c}\right) = \frac{\Delta(T)}{\Delta(0)} \tanh\left[\frac{\Delta(T)}{2T}\right].
$$
 (20)

FIG. 5. (Color online) Calculated superconducting transition temperature T_c as a function of the electric potential for the LAO/STO interface based from the n_{2D} fit of the data from Ref. [20.](#page-6-0) Here, it is shown that due to the variation of the density of states with voltage, tracing the observed capacitance [Eq. (18)], T_c shows a distinctive change. The inset shows the experimentally observed change in T_c from Ref. [21](#page-6-0) (dashed blue) and Ref. [20](#page-6-0) (dotted red). The difference in peak width and level can vary depending on the variation of the capacitance $C(V)$, the Fermi energy $E_F(V)$, and the pair interaction with *V* .

In the limit of weak disorder, $T_{\text{KTB}} \approx T_c$. At strong disorder when $R_{\text{sheet}} \gg 2R_Q$, one gets a suppression of the KTB transition temperature as $T_{\text{KTB}} = 2.18T_cR_o/R_{\text{sheet}}$.

VI. CONCLUSION

In conclusion, we examined the effect of an electric potential on the carrier densities and superconducting transition temperature for the conducting LAO/STO interfaces. Through the use of basic electrostatics, we find that the carrier density and superconducting temperature can be significantly modified by an external electric field. Recently, there have been experimental observations that have explored the relationship between an applied potential on STO heterostructures and the increased carrier density or capacitance. Here, we take this relationship further and assume it is the field effect that, through nonlinear capacitance, changes carrier density that in turn affects the superconducting coupling and ultimately T_c , and is intimately related to the T_c in Nb-doped SrTiO₃ and SrTiO_{3− δ}. We present a mechanism of this phenomenon assuming a standard BCS weakly interacting transition temperature at the interface. The modification of T_c seems to be mainly affected by the change in the carrier density. Recent two-band calculations of the critical density for this system have also shown a distinct connection to the change in T_c ^{[40](#page-6-0)} placing great emphasis on further experimental investigations to clarify the full nature of the superconductivity. In combination with experimental observations of this phenomena, this effect shows a possible route to tunable material properties and suggests an exciting perspective on the way to tunable superconductivity.

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