Universal low-temperature tricritical point in metallic ferromagnets and ferrimagnets

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An earlier theory of the quantum phase transition in metallic ferromagnets is revisited and generalized in three ways. It is shown that the mechanism that leads to a fluctuation-induced first-order transition in metallic ferromagnets with a low Curie temperature is valid, (1) irrespective of whether the magnetic moments are supplied by the conduction electrons or by electrons in another band, (2) for ferromagnets in the XY and Ising universality classes as well as for Heisenberg ferromagnets, and (3) for any systems with a nonzero homogeneous magnetization, such as ferrimagnets or canted ferromagnets. This vastly expands the class of materials for which a first-order transition at low temperatures is expected, and it explains why strongly anisotropic ferromagnets, such as UGe_2, display a first-order transition as well as Heisenberg magnets.

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I. INTRODUCTION

Quantum phase transitions are a subject of great interest.[1,2] In contrast to classical or thermal phase transitions, which occur at a nonzero temperature $T_c > 0$ and are driven by thermal fluctuations, quantum phase transitions occur at zero temperature, $T = 0$, as a function of some nonthermal control parameter and are driven by quantum fluctuations. In this paper we will focus on quantum phase transitions in metallic systems. For reasons discussed below, these transitions are especially interesting.

A prototypical quantum phase transition is the one from a paramagnetic metal to a ferromagnetic metal. Indeed, the earliest theory of a quantum phase transition was the Stoner theory of ferromagnetism.[3] Stoner assumed that the conduction electrons are responsible for the ferromagnetism and developed a mean-field theory that describes both the classical and the quantum ferromagnetic transition. In an important paper, Hertz later derived a Landau-Ginzburg-Wilson (LGW) functional for this transition by considering a simple model of itinerant electrons that interact only via a contact potential in the particle-hole spin-triplet channel.[1] Hertz analyzed this (dynamical) LGW functional by means of renormalization-group (RG) methods. He concluded that the critical behavior in the physical dimensions $d = 2$ and $d = 3$ is mean-field-like. That is, as far as the static critical exponents of the transition at $T = 0$ are concerned, he concluded that Stoner theory is exact in $d = 2$ and $d = 3$.

In the mid 1990s, it was realized that the above conclusion is not correct. The problem is that in metals at $T = 0$ there are gapless particle-hole excitations that couple to the magnetic order-parameter fluctuations and influence the quantum critical behavior for all dimensions $d \leqslant 3$. In Hertz's theory, this coupling is taken into account only in an approximation that does not suffice for yielding the leading critical behavior. Technically, Hertz theory treats the fermionic soft modes in a tree approximation, whereas describing their influence on the critical behavior requires taking into account fermionic loops. Physically, a correct description of any phase transition must treat the order parameter fluctuations and all soft modes that couple to them on equal footing.

A theory that takes into account these effects was developed by the present authors and T. Vojta. In Ref. 4, it was shown that the quantum phase transition from a metallic paramagnet to an itinerant ferromagnet in the absence of quenched disorder in $d = 2$ and $d = 3$ is generically discontinuous, or of first order, in contrast to the second-order transition with mean-field critical behavior predicted by Hertz theory.[5] The mechanism behind this phenomenon is analogous to what is known as a fluctuation-induced first-order transition in superconductors and liquid crystals.[6] There, soft fluctuations of the electromagnetic vector potential (in superconductors) or the nematic order parameter (in liquid crystals) couple to the order parameter and effectively change the sign of the cubic term in the equation of state, leading to a first-order transition. In the quantum magnetic case, the role of the additional soft modes is played by the fermionic particle-hole excitations mentioned above that are massless at $T = 0$. Since these modes acquire a mass at $T > 0$, the tendency toward a first-order transition diminishes with increasing temperature. This leads to a tricritical point at a temperature $T_{tc} > 0$ that separates a line of continuous transitions at $T > T_{tc}$ from a line of first-order transitions at $T < T_{tc}$. In a later paper with Rollbühler, the effects of a magnetic field H were investigated.[7] It was found that in the space spanned by T, H, and the control parameter, tricritical wings, or surfaces of first-order transitions, emanate from the tricritical point and terminate in a pair of quantum critical points in the $T = 0$ plane. The wing boundaries at $T > 0$ are given by lines of critical points that are reminiscent of a conventional liquid-gas critical point and connect the tricritical point with the quantum critical points at $T = 0$. The resulting generic phase diagram is shown in Fig. 1. This general picture is in good agreement with experimental results for low-Curie-temperature metallic ferromagnets, including $ZrZn_2$,[8] UGe_2,[9] URhGe,[10], MnSi,[11,12] and CoS_2.[13]

In this paper we generalize our previous theory in three important ways. First, we show that our previous results, which had been derived under the same assumption made by Stoner and by Hertz, namely, that the magnetism is caused only by itinerant electrons, remain valid in metallic systems where the magnetism is caused by electrons in a different band than the conduction electrons.

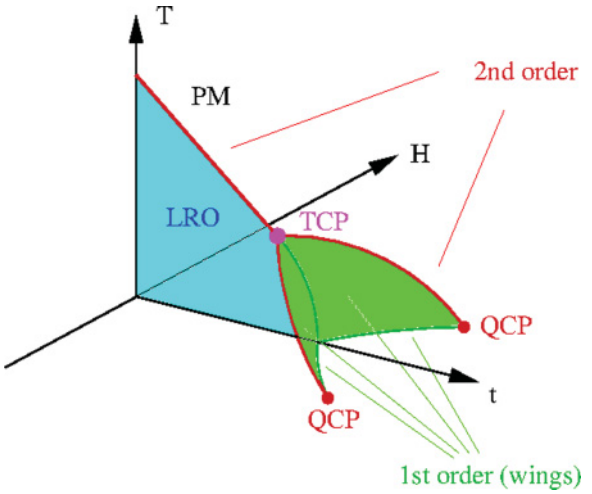

FIG. 1. (Color online) Generic phase diagram of a metallic magnet in the space spanned by temperature (T), magnetic field (H), and the control parameter (t). Shown are the long-range ordered magnetic (LRO) and paramagnetic (PM) phases, lines of second-order transitions, surfaces of first-order transitions ("tricritical wings"), the tricritical point (TCP), and the two quantum critical points (QCP). The long-range order can be of ferromagnetic or ferrimagnetic type, and the electrons causing the long-range order can be in the same band as the conduction electrons, or in a different band. See the text for further explanation.

Second, we show that the results are *not* restricted to Heisenberg ferromagnets, contrary to what was implied in Refs. 4 and 14. Rather, they apply equally well to metallic XY or Ising magnets, since the magnetic moments couple to conduction electrons whose spins have three degrees of freedom. This is an important point, since some of the relevant materials are strongly anisotropic magnets, including UGe_2 (easy axis) and URhGe (easy plane).

Third, we show that the phase diagram shown in Fig. 1 also applies to generic metallic ferrimagnets. Ferrimagnets are materials that spontaneously develop both a homogeneous and a staggered magnetization at the same critical value of either the temperature (for a classical transition) or a nonthermal control parameter (for a quantum transition). Physically, this can happen when magnetic moments of unequal magnitude on a bipartite lattice align in opposite directions.[15] More generally, it applies to any system with a nonvanishing homogeneous magnetization, for instance, canted ferromagnets.

The unifying principle behind these generalizations is the realization that coupling a homogeneous magnetization to conduction electrons will produce the same results irrespective of the microscopic origin of the magetization.[16] As a result, the phase diagram depicted schematically in Fig. 1 is valid for generic metallic ferromagnets in addition to itinerant ones, for ferromagnets of XY or Ising type in addition to Heisenberg magnets, and for ferrimagnets as well as for ferromagnets.[17] In all cases we also consider the effects of nonmagnetic quenched disorder. In Ref. 4 it was shown that this type of disorder leads to an interesting phase diagram with a number of multicritical points and that sufficiently strong quenched disorder causes the first-order paramagnetic-to-ferromagnetic transition in metals to become second order. We will see that the same result holds for metallic ferrimagnets. Experimentally, the effects of disorder on either one of these transitions have not yet been studied systematically.

II. THEORY

We now derive the results listed in Sec. I. To this end, we are interested in a theory that describes the magnetization or order-parameter (OP) field $\boldsymbol{M}$, the fermionic degrees of freedom described by Grassmann-valued fields $\bar{\psi}$ and ψ, and the coupling between them. Accordingly, the action will have three parts:

$$\mathcal{A}[\boldsymbol{M};\bar{\psi},\psi] = \mathcal{A}_{\rm OP}[\boldsymbol{M}] + \tilde{\mathcal{A}}_{\rm F}[\bar{\psi},\psi] + \tilde{\mathcal{A}}_{\rm c}[\boldsymbol{M};\bar{\psi},\psi], \quad (2.1a)$$

and the partition function is given by

$$Z = \int D[\boldsymbol{M}]\, D[\bar{\psi},\psi]\, e^{-\mathcal{A}[\boldsymbol{M};\bar{\psi},\psi]}. \quad (2.1b)$$

We are, however, not interested in a complete description of the fermionic degrees of freedom; rather, we want to restrict ourselves to the fermionic soft modes and integrate out the massive modes in the simplest approximation that respects the symmetries of the problem to arrive at an effective Landau-Ginzburg-Wilson (LGW) theory in terms of soft modes only. If we denote the soft fermionic degrees of freedom collectively by q, and the massive ones by P, we formally have

$$Z = \int D[\boldsymbol{M},q]\, e^{-\mathcal{A}_{\rm LGW}[\boldsymbol{M},q]}, \quad (2.2a)$$

where

$$\begin{aligned}\mathcal{A}_{\rm LGW}[\boldsymbol{M},q] &= \mathcal{A}_{\rm OP}[\boldsymbol{M}] - \ln \int D[P]\, e^{-\tilde{\mathcal{A}}_{\rm F}[q,P]} \\ &\quad \times e^{-\tilde{\mathcal{A}}_{\rm c}[\boldsymbol{M};q,P]} \\ &\equiv \mathcal{A}_{\rm OP}[\boldsymbol{M}] + \mathcal{A}_{\rm F}[q] + \mathcal{A}_{\rm c}[\boldsymbol{M},q]. \end{aligned} \quad (2.2b)$$

As we will see later, the q are matrices formed by bilinear products of the fermion fields, $q_{nm}(\boldsymbol{x},\boldsymbol{y}) = \bar{\psi}_n(\boldsymbol{x})\,\psi_m(\boldsymbol{y})$ with $(n+1/2)(m+1/2) < 0$, and the P are given by the same products with $(n+1/2)(m+1/2) > 0$. Here, $\psi_n(\boldsymbol{x}) \equiv \psi(\boldsymbol{x},\omega_n)$ is the temporal Fourier transform of the Grassmann field $\psi(x)$, where $x \equiv (\boldsymbol{x},\tau)$ comprises the real-space position $\boldsymbol{x}$ and the imaginary-time variable τ in a Matsubara formalism, and $\omega_n = 2\pi T(n+1/2)$ is a fermionic Matsubara frequency. $\bar{\psi}_n(\boldsymbol{x})$ is defined analogously.

This separation of soft and massive fermionic modes q and P, respectively, integrating out P in a suitable approximation, and determining the consequences of the coupling between q and $\boldsymbol{M}$, is the central objective of this paper. For the separation we will make use of the general theory developed in Refs. 18 and 19.

A. Order parameter and coupling to fermions

We are interested in magnetic order, and, hence, the appropriate order-parameter field is the magnetization $\boldsymbol{M}(x)$. We write the magnetization as a part $\boldsymbol{m}(x)$ whose average is the homogeneous magnetization and a part $\boldsymbol{n}(x)$ whose average is

a staggered magnetization,

$$\boldsymbol{M}(x) = \boldsymbol{m}(x) + \boldsymbol{n}(x) \sum_{j=1}^{N} \cos(\boldsymbol{k}_j \boldsymbol{x}). \tag{2.3}$$

Here, the $\boldsymbol{k}_j$ are N wave vectors that characterize the staggered magnetic order, and both $\boldsymbol{m}(x)$ and $\boldsymbol{n}(x)$ are slowly varying in space and time. In particular, their Fourier expansions contain only wave numbers that are small compared to the norms of the $\boldsymbol{k}_j$.

In a paramagnetic state, the expectation values of $\boldsymbol{m}$ and $\boldsymbol{n}$ are both zero. At a transition to a ferromagnetic state, the expectation value of $\boldsymbol{m}$ becomes nonzero while that of $\boldsymbol{n}$ remains zero; at a transition to an antiferromagnetic state the converse is true. A ferrimagnetic transition is characterized by both $\boldsymbol{m}$ and $\boldsymbol{n}$ acquiring a nonzero expectation value at the same point in parameter space. In this sense there is only one order parameter field for a ferrimagnetic transition; this fact will be important later. For the purposes of the present paper, a crucial question is the coupling of the order-parameter fluctuations to the soft fermionic degrees of freedom. Since the soft parts of the latter are soft at zero wave number, the leading coupling is to $\boldsymbol{m}$. The fermions also couple to $\boldsymbol{n}$, but this leads to subleading effects since the staggered magnetization is soft at a nonzero wave number. We will neglect this coupling in what follows. We also mention that by the same reasoning our conclusions still apply if the nonhomogeneous part of the magnetization is not of a staggered type. For instance, they apply to canted ferromagnets.

Physically, the near-homogeneous magnetization fluctuations act as a magnetic field proportional to $\boldsymbol{m}$ that couples to the electronic spin density

$$\boldsymbol{n}_s(x) = \sum_{a,b} \bar{\psi}_a(x)\, \boldsymbol{\sigma}_{ab}\, \psi_b(x). \tag{2.4a}$$

Here, $\boldsymbol{\sigma} = (\sigma^x,\sigma^y,\sigma^z) \equiv (\sigma^1,\sigma^2,\sigma^3)$ denotes the Pauli matrices, and $a,b = (\uparrow, \downarrow) \equiv (+1, -1)$ are spin indices. The coupling takes the form of a Zeeman term

$$\tilde{\mathcal{A}}_c[M;\bar{\psi},\psi] = c \int dx\ \boldsymbol{m}(x)\boldsymbol{n}_s(x), \tag{2.4b}$$

with c a coupling constant. As we will see, the spin density contains both massive and massless modes, so only part of Eq. (2.4b) contributes to $\mathcal{A}_c[\boldsymbol{M},q]$ in Eq. (2.2b). We will discuss this separation next.

B. Fermionic soft modes

In this subsection we separate the massless fermionic modes from the massive ones by means of the technical apparatus developed in Ref. 19. Here we will quote only as much of this formalism as is necessary for the further development; see Ref. 19 for additional details.

The soft fermion excitations are all two-particle excitations; the related correlation functions are those of bilinear products of fermion fields. The latter commute with each other, and with individual fermion fields, and, hence, are isomorphic to classical fields. Denoting these classical fields by Q, we define a classical matrix field

$$Q_{nm}(\boldsymbol{x},\boldsymbol{y}) \cong \frac{i}{2} \begin{pmatrix} -\psi_{n\uparrow}(\boldsymbol{x})\bar{\psi}_{m\uparrow}(\boldsymbol{y}) & -\psi_{n\uparrow}(\boldsymbol{x})\bar{\psi}_{m\downarrow}(\boldsymbol{y}) & -\psi_{n\uparrow}(\boldsymbol{x})\psi_{m\downarrow}(\boldsymbol{y}) & \psi_{n\uparrow}(\boldsymbol{x})\psi_{m\uparrow}(\boldsymbol{y}) \\ -\psi_{n\downarrow}(\boldsymbol{x})\bar{\psi}_{m\uparrow}(\boldsymbol{y}) & -\psi_{n\downarrow}(\boldsymbol{x})\bar{\psi}_{m\downarrow}(\boldsymbol{y}) & -\psi_{n\downarrow}(\boldsymbol{x})\psi_{m\downarrow}(\boldsymbol{y}) & \psi_{n\downarrow}(\boldsymbol{x})\psi_{m\uparrow}(\boldsymbol{y}) \\ \bar{\psi}_{n\downarrow}(\boldsymbol{x})\bar{\psi}_{m\uparrow}(\boldsymbol{y}) & \bar{\psi}_{n\downarrow}(\boldsymbol{x})\bar{\psi}_{m\downarrow}(\boldsymbol{y}) & \bar{\psi}_{n\downarrow}(\boldsymbol{x})\psi_{m\downarrow}(\boldsymbol{y}) & -\bar{\psi}_{n\downarrow}(\boldsymbol{x})\psi_{m\uparrow}(\boldsymbol{y}) \\ -\bar{\psi}_{n\uparrow}(\boldsymbol{x})\bar{\psi}_{m\uparrow}(\boldsymbol{y}) & -\bar{\psi}_{n\uparrow}(\boldsymbol{x})\bar{\psi}_{m\downarrow}(\boldsymbol{y}) & -\bar{\psi}_{n\uparrow}(\boldsymbol{x})\psi_{m\downarrow}(\boldsymbol{y}) & \bar{\psi}_{n\uparrow}(\boldsymbol{x})\psi_{m\uparrow}(\boldsymbol{y}) \end{pmatrix}. \tag{2.5}$$

Here, "$\cong$" means "isomorphic to"; technically, the isomorphism is implemented by means of a Lagrange multiplier field; see below. We also define the Fourier transform of Q,

$$Q_{nm}(\boldsymbol{k},\boldsymbol{p}) = \frac{1}{V} \int d\boldsymbol{x}\, d\boldsymbol{y}\, e^{-i\boldsymbol{k}\boldsymbol{x}+i\boldsymbol{p}\boldsymbol{y}}\, Q_{nm}(\boldsymbol{x},\boldsymbol{y}). \tag{2.6a}$$

It is further useful to define

$$Q_{nm}(\boldsymbol{k};\boldsymbol{q}) = Q_{nm}(\boldsymbol{k}+\boldsymbol{q}/2,\boldsymbol{k}-\boldsymbol{q}/2) \tag{2.6b}$$

and

$$Q_{nm}(\boldsymbol{x}) = Q_{nm}(\boldsymbol{x},\boldsymbol{x}) = \frac{1}{V} \sum_{\boldsymbol{q}} e^{i\boldsymbol{q}\boldsymbol{x}} \sum_{\boldsymbol{k}} Q_{nm}(\boldsymbol{k};\boldsymbol{q}). \tag{2.6c}$$

The 4×4 matrix Q_{nm} can be expanded in a spin-quaternion basis

$$Q_{nm}(\boldsymbol{x},\boldsymbol{y}) = \sum_{r,i=0}^{3} (\tau_r \otimes s_i)\, {}^{i}_{r}Q_{nm}(\boldsymbol{x},\boldsymbol{y}), \tag{2.7}$$

where $\tau_0 = s_0 = \mathbb{1}_2$ is the unit 2×2 matrix, and $\tau_{1,2,3} = -s_{1,2,3} = -i\sigma^{1,2,3}$. An explicit inspection of the 16 matrix elements shows that $r = 0,3$ represents the particle-hole channel, i.e., products of the form $\bar{\psi}\psi$, whereas $r = 1,2$ represents the particle-particle channel, i.e., products of the form $\bar{\psi}\bar{\psi}$ or $\psi\psi$. For our purposes we will need only the particle-hole degrees of freedom.

It was shown in Ref. 19 (see also Ref. 20) that a crucial criterion for separating the fermionic degrees of freedom into soft and massive modes is given by the relative signs of the frequency arguments of the matrix elements Q_{nm}. Accordingly, we write

$$\begin{aligned} {}^{i}_{r}Q_{nm}(\boldsymbol{x}) = {}&{}^{i}_{r}q_{nm}(\boldsymbol{x})\,\Theta(-\omega_n\omega_m) \\ &+ {}^{i}_{r}P_{nm}(\boldsymbol{x})\,\Theta(\omega_n\omega_m)(i = 1,2,3). \end{aligned} \tag{2.8}$$

Here, Θ is the step function, and we use the fact that in the spin-triplet channel ($i = 1,2,3$) the expectation value of the Q matrix vanishes (this is since the fermionic degrees of freedom

described by Q do not by themselves have long-ranged magnetic order; see the discussion at the end of the current subsection), so that q and P represent fluctuations. In what follows we will absorb the step functions into the matrix fields q and P, i.e., writing q_{nm} implies $n \geqslant 0$ and $m < 0$ and P_{nm} implies either $n \geqslant 0$ and $m \geqslant 0$ or $n < 0$ and $m < 0$. The ${}^i_r q$ are the spin-quaternion elements of a matrix

$$q_{nm}(\boldsymbol{x}) = \sum_{r,i} (\tau_r \otimes s_i)\, {}^i_r q_{nm}(\boldsymbol{x}). \tag{2.9a}$$

It is also useful to define an adjoint matrix

$$q^+_{nm}(\boldsymbol{x}) = \sum_{i,r} (\tau^+_r \otimes s^+_i)\, {}^i_r q_{mn}(\boldsymbol{x}), \tag{2.9b}$$

where τ^+_r and s^+_i are the hermitian conjugates of τ_r and s_i, respectively. In addition, the theory contains a field $\not{q}_{nm}(\boldsymbol{x})$ that has the same properties as $q_{nm}(\boldsymbol{x})$ except for different propagators; see below. The origin of $\not{q}$ is the Lagrange multiplier field λ that constrains the bilinear products of fermion fields to the q. In various places in the theory $q - \lambda \equiv \not{q}$ appears, and the λ propagator equals minus the q propagator for noninteracting electrons, whereas cross-correlations between q and λ vanish. The net effect of λ is therefore to subtract the noninteracting part of the q propagator wherever the combination $q - \lambda$ occurs.

The q correlation functions are the basic soft modes in the theory; see below. However, due to nonlinear couplings the P couple to the q and thus have a soft component. This effect can be expressed by expanding P in a power series in q. To quadratic order in q and to lowest order in the fermion interaction one finds

$$P_{12}(\boldsymbol{k}) \approx -2i \sum_3 \sum_{\boldsymbol{p}} \varphi^{(3)}_{132}(\boldsymbol{p}, \boldsymbol{k}-\boldsymbol{p})\, \varphi^{-1}_{13}(\boldsymbol{p})\, \varphi^{-1}_{32}(\boldsymbol{k}-\boldsymbol{p}) \times [\not{q}_{13}(\boldsymbol{p})\, \not{q}^+_{32}(\boldsymbol{k}-\boldsymbol{p}) + \not{q}^+_{13}(\boldsymbol{p})\, \not{q}_{32}(\boldsymbol{k}-\boldsymbol{p})]. \tag{2.10}$$

Here and in what follows we use a simplified notation for frequency indices, $1 \equiv n_1$, etc. We have dropped contributions to P of higher order in q, and a contribution that is linear in the interaction and linear in q (see Ref. 19); neither will be needed for our purposes. We also have omitted a term quadratic in q and quadratic in the interaction, which leads to less singular contributions to the free energy than the one we keep. Note the frequency restrictions inherent in Eq. (2.10): $\mathrm{sgn}\,(\omega_{n_1}) = \mathrm{sgn}\,(\omega_{n_2}) = -\mathrm{sgn}\,(\omega_{n_3})$. Here,

$$\varphi_{12}(\boldsymbol{k}) = \frac{1}{V} \sum_{\boldsymbol{p}} G_1(\boldsymbol{p})\, G_2(\boldsymbol{p}-\boldsymbol{k}), \tag{2.11}$$

with $\omega_{n_1}\, \omega_{n_2} < 0$ implied, and

$$\varphi^{(3)}_{132}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \frac{1}{V} \sum_{\boldsymbol{p}} G_1(\boldsymbol{p})\, G_3(\boldsymbol{p}-\boldsymbol{k}_1)\, G_2(\boldsymbol{p}-\boldsymbol{k}_1-\boldsymbol{k}_2), \tag{2.12}$$

where $G_1(\boldsymbol{p}) \equiv G(\boldsymbol{p}, i\omega_{n_1})$ is the single-particle Green function. φ_{12} has a scaling form

$$\varphi_{12}(\boldsymbol{k}) = N_{\mathrm{F}}\, \frac{2\pi G}{k}\, \varphi_d(G i \Omega_{1-2}/k) \equiv \varphi(\boldsymbol{k}, \Omega_{1-2}), \tag{2.13}$$

where G is a coupling constant whose bare value is the inverse Fermi velocity, $G = 1/v_{\mathrm{F}}$, N_{F} is the density of states per spin at the Fermi level, and $\Omega_{1-2} = \omega_{n_1} - \omega_{n_2}$. In $d = 2,3$, and for free electrons, we find explicitly

$$\varphi_{d=2}(z) = \mathrm{sgn}\,(\mathrm{Im}\, z)/\sqrt{1-z^2}, \tag{2.14a}$$

$$\varphi_{d=3}(z) = \frac{-i}{2}\, \ln\left(\frac{1-z}{-1-z}\right), \tag{2.14b}$$

which we recognize as the hydrodynamic part of the Lindhard function. Equations (2.13) and (2.14) reflect the soft particle-hole excitations with a linear momentum-frequency relation in a metallic electron system. In particular, $\varphi(\boldsymbol{k}, \Omega_n = 0) \propto 1/|\boldsymbol{k}|$ and $\varphi(\boldsymbol{k} = 0, \Omega_n) \propto 1/\Omega_n$.[21] For later reference, we also note the following identities that hold for a special form of $\varphi^{(3)}$:

$$\varphi^{(3)}_{121}(\boldsymbol{k}, -\boldsymbol{k}) = -\varphi^{(3)}_{212}(\boldsymbol{k}, -\boldsymbol{k}) = -\frac{\partial}{\partial i\omega_{n_1}}\, \varphi_{12}(\boldsymbol{k}) \equiv \varphi^{(3)}(\boldsymbol{k}, \Omega_{1-2}). \tag{2.15}$$

The fermionic action can be expressed in terms of q and P, and by using Eq. (2.10) and its generalizations to higher order, one obtains a fermionic soft-mode action entire in terms of q. For our purposes, we need only the Gaussian part of this action, which reads

$$\mathcal{A}_{\mathrm{F}}[q] = -8 \sum_{\boldsymbol{k}} \sum_{\substack{1,2\\3,4}} \sum_{r=0,3} \sum_{i=0}^{3} {}^i_r q_{12}(\boldsymbol{k})\, \Gamma^i_{12,34}(\boldsymbol{k})\, {}^i_r q_{34}(-\boldsymbol{k}). \tag{2.16a}$$

Here, $1 \equiv n_1$, etc., and the Gaussian vertex is given by

$$\Gamma^i_{12,34}(\boldsymbol{k}) = \varphi^{-1}_{12}(\boldsymbol{k}) + \delta_{1-2,3-4}\, 2T\gamma^i, \tag{2.16b}$$

with $\gamma^{i=0} = -\gamma_{\mathrm{s}}$ and $\gamma^{i=1,2,3} = \gamma_{\mathrm{t},i}$, where $\gamma_{\mathrm{s}} > 0$ and $\gamma_{\mathrm{t},i} > 0$ are the spin-singlet and spin-triplet interaction amplitudes. The fermionic Gaussian propagator is given by the inverse of the vertex. One finds

$$\left\langle {}^i_r q_{12}(\boldsymbol{k})\, {}^j_s q_{34}(-\boldsymbol{k}) \right\rangle = \frac{1}{16}\, \delta_{rs}\, \delta_{ij} \Big[\delta_{13}\, \delta_{24}\, \varphi_{12}(\boldsymbol{k}) - 2\gamma^i T\, \delta_{1-2,3-4} \times \frac{\varphi_{12}(\boldsymbol{k})\, \varphi_{34}(\boldsymbol{k})}{1 - 2\gamma^i \chi^{(0)}_{1-2}(\boldsymbol{k})} \Big], \tag{2.17a}$$

where

$$\chi^{(0)}_{1-2}(\boldsymbol{k}) \equiv \chi^{(0)}(\boldsymbol{k}, \Omega_{1-2}) = -T \sum_{34} \delta_{1-2,3-4}\, \varphi_{34}(\boldsymbol{k}). \tag{2.17b}$$

We see that the q propagator is given in terms of φ and, hence, is soft. The fields $\not{q}$ that enter P, Eq. (2.10), are characterized by Gaussian propagators

$$\left\langle {}^i_r \not{q}_{12}(\boldsymbol{k})\, {}^j_s q_{34}(-\boldsymbol{k}) \right\rangle = \left\langle {}^i_r q_{12}(\boldsymbol{k})\, {}^j_s \not{q}_{34}(-\boldsymbol{k}) \right\rangle = \left\langle {}^i_r q_{12}(\boldsymbol{k})\, {}^j_s q_{34}(-\boldsymbol{k}) \right\rangle \tag{2.17c}$$

and

$$\left\langle {}^i_r \not{q}_{12}(\boldsymbol{k})\, {}^j_s \not{q}_{34}(-\boldsymbol{k}) \right\rangle = \frac{-1}{8}\, \gamma^i T\, \delta_{1-2,3-4}\, \frac{\varphi_{12}(\boldsymbol{k})\, \varphi_{34}(\boldsymbol{k})}{1 - 2\gamma^i \chi^{(0)}_{1-2}(\boldsymbol{k})}. \tag{2.17d}$$

The last expression is just the interacting part of the q propagator, Eq. (2.17a), as was mentioned after Eq. (2.9b).

The interaction amplitudes in the Gaussian fermionic vertex, Eq. (2.16b), warrant some comments. First, we note that the three spin-triplet amplitudes $\gamma_t^{1,2,3}$ are in general not identical in a cyrstalline solid, and they do not need to be for what follows. Second, we comment on the two cases that result from the magnetism being caused by the conduction electrons or by electrons in a band different from the conduction band, respectively. Let us first assume the latter case, which is the conceptually more straightforward one. Then, $\mathcal{A}_F[q]$, which describes the conduction electrons, is independent of the magnetism and contains interactions in both the spin-singlet and spin-triplet channels. The only restriction is that the latter are weak enough to not lead to magnetism by themselves. The conduction electrons are affected by the magnetization, which acts as an effective magnetic field, and this is described by the Zeeman coupling term, Eq. (2.4b). The other possibility, which is conceptually more complex, is that the magnetism is caused by the conduction electrons themselves. In this case, the magnetic order parameter and the soft modes q describe degrees of freedom for electrons in the same band. The magnetic order parameter then should be thought of as deriving from the spin-triplet interaction between the conduction electrons, e.g., via a Hubbard-Stratonovich decoupling of the latter. This leaves the bare action $\mathcal{A}_F$ with a spin-singlet interaction only. However, as long as the latter is present, a spin-triplet interaction will always be generated under renormalization. The action $\mathcal{A}_F$ will, therefore, again contain a spin-triplet interaction amplitude, albeit one that is much weaker than the one in the underlying action that describes the system before the separation of magnetic and fermionic degrees of freedom. This is the case that was discussed, for ferromagnetism, in Ref. 14, which used phenomenological and symmetry arguments to construct the fermionic part of the action. Finally, we mention that we assume the conduction electrons, in the absence of a nonzero magnetization [i.e., with the coupling constant c in Eq. (2.4b) put equal to zero], to indeed have three soft spin-triplet excitations at $T = 0$, which are given by Eqs. (2.17) with $i = 1,2,3$. This is not necessarily the case. For instance, an external magnetic field gives two of these three channels (the ones transverse to the field) a mass, and a small concentration of magnetic impurities will make all three channels massive without having significant other effects. However, in general, the energy scales associated with these effects will be small, and they will lead to a small reduction, but not a complete suppression, of the tricritical temperature in Fig. 1. We will discuss this point in more detail in Sec. III.

C. Coupling between the order parameter and the fermionic soft modes

We are now in a position to separate the Zeeman term, Eq. (2.4b), into parts where the order parameter couples to soft and massive fermionic modes, respectively. If we define a temporal Fourier transform of the magnetization field m by

$$\boldsymbol{m}_n(\boldsymbol{x}) = \sqrt{T}\int_0^{1/T} d\tau\; e^{i\Omega_n\tau}\,\boldsymbol{m}(\boldsymbol{x},\tau), \tag{2.18}$$

with $\Omega_n = 2\pi T n$ a bosonic Matsubara frequency, then we can write Eq. (2.4b) in the form

$$\tilde{\mathcal{A}}_c[\boldsymbol{M};Q] = 2c\sqrt{T}\int d\boldsymbol{x}\sum_n\sum_{i=1}^{3} m_n^i(\boldsymbol{x}) \times \sum_{r=0,3}(-1)^{r/2}\sum_m \mathrm{tr}\,[(\tau_r\otimes s_i)\,Q_{m,m+n}(\boldsymbol{x})]. \tag{2.19}$$

By expressing Q in terms of q and P by means of Eq. (2.8), and P in terms of $\not{q}$ by means of Eq. (2.10), we obtain the desired coupling $\mathcal{A}_c[\boldsymbol{M},q]$ between the order-parameter fluctuations and the fermionic soft modes q.

D. Generalized mean-field theory

An effective action, $\mathcal{A}_{\rm eff}[\boldsymbol{M}]$ in terms of the order parameter alone can be obtained by integrating out the fields q,

$$\mathcal{A}_{\rm eff}[\boldsymbol{M}] = \ln\int D[q]\; e^{\mathcal{A}_{\rm LGW}[\boldsymbol{M},q]}. \tag{2.20}$$

In general, the evaluation of this expression is very difficult. However, it can be evaluated exactly within a generalized mean-field approximation that was first employed in the context of liquid crystals and superconductors[6] and is defined as follows. First, we ignore temporal and spatial variations of the order parameter; i.e., we treat the fields $\boldsymbol{m}(x)$ and $\boldsymbol{n}(x)$ in Eq. (2.3) as numbers. If we assume ordering in the 3-direction, we have

$$M^i(x) \approx \delta_{i3}\left[m + n\sum_{j=1}^{N}\cos(\boldsymbol{k}_j\boldsymbol{x})\right], \tag{2.21a}$$

which implies

$$m_n^i(x) \approx \delta_{i3}\,\delta_{n0}\,m/\sqrt{T}. \tag{2.21b}$$

This mean-field approximation for the order parameter means that only the part of Q that is diagonal in frequency space, i.e., P_{mm}, contributes to Eq. (2.19). This in turn means that the contribution to P that is linear in q, which we had dropped from Eq. (2.10), does not contribute. Second, we restrict ourselves to quadratic order in q. That is, we treat the fermionic soft modes in a Gaussian approximation with a fixed magnetic order parameter. The validity of these approximations will be discussed in Sec. III B.

With these approximations, the action $\mathcal{A}_c$ that couples q and the order parameter is quadratic in q and can be written

$$\mathcal{A}_c[m,q] = 8\sum_{r,s=0,3}\sum_{i,j}{}^{i}_{r}q_{12}(\boldsymbol{k})\,{}^{ij}_{rs}\Gamma^{c}_{12,34}(\boldsymbol{k})\,{}^{j}_{s}q_{34}(-\boldsymbol{k}). \tag{2.22a}$$

Here,

$${}^{ij}_{rs}\Gamma^{c}_{12,34}(\boldsymbol{k}) = \delta_{13}\,\delta_{24}\,4\,c\,m\begin{pmatrix}0&1\\-1&0\end{pmatrix}_{rs}\begin{pmatrix}0&0&0&0\\0&0&1&0\\0&-1&0&0\\0&0&0&0\end{pmatrix}_{ij} \times\varphi^{(3)}_{121}(\boldsymbol{k},-\boldsymbol{k})\,\varphi_{12}^{-2}(\boldsymbol{k}), \tag{2.22b}$$

and we have used Eq. (2.15). The matrices give the values of ${}^{ij}_{rs}\Gamma^{c}$ for the 4 possible values of (r,s) and the 16 possible values of (i,j).

The integral over q in Eq. (2.20) can now easily be carried out. For the free-energy density $f = -T\mathcal{A}_{\rm eff}/V$, we obtain

$$f = f_0(m,n) + \Delta f(m). \tag{2.23a}$$

Here, $f_0 = -T\mathcal{A}_{\rm OP}/V$ is the mean-field free energy in the absence of a coupling to the fermionic soft modes. For $\Delta f(m)$, which is the contribution to the free energy due to this coupling, one finds

$$\Delta f(m) = \frac{2}{V}{\sum_{\boldsymbol{k}}}' T \sum_n \ln N(\boldsymbol{k},\Omega_n;m), \tag{2.23b}$$

where $\sum'_{\boldsymbol{k}}$ denotes a wave vector sum such that $|\boldsymbol{k}| < \Lambda$ with Λ an ultraviolet cutoff, and

$$N(\boldsymbol{k},\Omega_n;m) = -16\,c^2\,\gamma_{{\rm t},1}\gamma_{{\rm t},2}\,m^2\,\Omega_n^2\,[\varphi^{(3)}(\boldsymbol{k},\Omega_n)]^2\varphi^{-4}(\boldsymbol{k},\Omega_n) + \varphi^{-4}(\boldsymbol{k},\Omega_n)\prod_{i=1,2}[1-2\gamma_{{\rm t},i}\chi^{(0)}(\boldsymbol{k},\Omega_n)]. \tag{2.23c}$$

The equation of state is obtained by minimizing the free-energy density. In the absence of a coupling between the order parameter and the fermionic soft modes, this amounts to minimizing f_0, which yields the ordinary mean-field equation of state. For a ferromagnet, the latter has the usual Landau form. For a ferrimagnet, the equation of state depends on details of the magnetic order. It can be complicated and describe several different phases; see, e.g., Ref. 22. However, generically, the first phase encountered as one approaches from the paramagnetic state is entered via a second-order transition. After minimizing f_0 and expressing n in terms of m, one thus has again an ordinary mean-field equation of state given by

$$h = r\,m + u\,m^3 + O(m^5), \tag{2.24}$$

where h is an external magnetic field in the 3-direction, $u > 0$, and the transition occurs at $r = 0$.[23] In the Appendix we recall a very simple model that leads to this result. The second term on the right-hand side of Eq. (2.23a) gives an additional contribution to the equation of state, which then reads

$$h = r\,m + u\,m^3 - 64\,m\,c^2\gamma_{{\rm t},1}\gamma_{{\rm t},2} \times\frac{1}{V}{\sum_{\boldsymbol{k}}}' T\sum_{n=1}^{\infty}\frac{\Omega_n^2\,[\varphi^{(3)}(\boldsymbol{k},\Omega_n)]^2\,\varphi^{-4}(\boldsymbol{k},\Omega_n)}{N(\boldsymbol{k},\Omega_n;m)}. \tag{2.25}$$

This is the desired generalized mean-field equation of state, which takes into account the coupling of the order parameter to the fermionic soft modes.

E. Discussion of the generalized mean-field equation of state

With some effort, the integrals in Eqs. (2.23b) and (2.25) can be explicitly performed. However, the salient points can be seen by simple scaling considerations and dimensional analysis. Equations (2.11) and (2.13) imply that the frequency Ω_n scales as the wavenumber k, $\Omega_n \sim k$, and that $\varphi(\boldsymbol{k},\Omega_n) \sim 1/k \sim 1/\Omega_n$, which also can be seen explicitly from Eq. (2.23c). Equation (2.15) implies that $\varphi^{(3)}(\boldsymbol{k},\Omega_n) \sim 1/k^2 \sim 1/\Omega_n^2$. Equation (2.23c) then shows that there is a length scale L_m, or a corresponding frequency scale ω_m, that scales as $L_m \sim 1/\omega_m \sim 1/m$. If one attempts to expand $\Delta f(m)$, Eq. (2.23b), in powers of m at $T = 0$, then nonanalyticities will occur at next-to-leading order for all $d \leqslant 3$.

An alternative way to describe this mechanism is to say that of the three soft fermionic spin-triplet excitations, Eq. (2.17a) with $r = s = 0,3$ and $i = j = 1,2,3$, two (namely, the ones transverse to the order parameter direction) acquire a mass due to the coupling between the fermions and the order parameter m, as can be seen explicitly from Eq. (2.22b). This acquisition of a mass by a generic soft mode due the spontaneous breaking of a continuous symmetry is an example of the Anderson-Higgs mechanism,[24–26] even though the broken symmetry in this case is not a gauge symmetry; see the discussion in Sec. III A. It implies in turn that the free energy is a nonanalytic function of m.

At nonzero temperatures the singularities are cut off by T according to $m \sim T$. That is, a crossover occurs from m scaling to T scaling when the Zeeman splitting is comparable to the temperature or the thermal length scale $L_T \propto 1/T$ is comparable to the magnetic length scale L_m mentioned above. Taking into account the sign of N, Eq. (2.23c), one finds schematically, for $1 < d < 3$,

$$\Delta f(m) = -v\,m^2(m^2+T^2)^{(d-1)/2}, \tag{2.26a}$$

and for $d = 3$

$$\Delta f(m) = \frac{v}{8}\,m^4\ln(m^2+T^2), \tag{2.26b}$$

where $v > 0$ is a positive constant.

The most important aspects of this result, as far as the order of the transition is concerned, are the sign of v and the power of m at $T = 0$. For all $d \leqslant 3$, there is a negative term in the free energy that dominates the m^4 in the Landau free energy and, hence, necessarily leads to a first-order transition. Another way to see this is by expanding $\Delta f(m)$, Eq. (2.26a), in powers of m for $T > 0$. The leading term is proportional to $-m^4/T^{3-d}$. That is, there is a *negative* m^4 term whose prefactor diverges as $T \to 0$ for all $d \leqslant 3$, which implies that there will be a tricritical point at some temperature. The free energy for three different values of r is plotted schematically in Fig. 2. For this schematic free energy, the equation of state in the case $d = 3$, for which many experimental results exist, takes the form

$$h = r\,m + \frac{v}{2}\,m^3\ln(m^2+T^2) + m^3\left(u + \frac{v}{4}\,\frac{m^2}{m^2+T^2}\right), \quad (d=3). \tag{2.27}$$

Also of interest is the other physical dimensionality, $d = 2$, where the equation of state reads

$$h = r\,m - 2v\,m(m^2+T^2)^{1/2} + m^3\left(u - \frac{v}{(m^2+T^2)^{1/2}}\right), \quad (d=2). \tag{2.28}$$

Here, the analyticity is stronger than in the 3-d case, with a negative m^2 term in the equation of state at $T = 0$. This is particularly interesting in the case of Ising magnets, which display long-range order in $d = 2$ even at $T > 0$. The case of

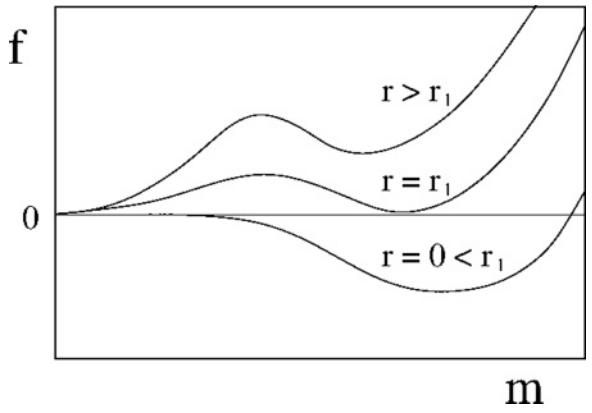

FIG. 2. Schematic sketch of the free energy for three values of the parameter r. The first-order transition occurs at $r = r_1 > 0$. It pre-empts the second-order transition of Landau theory, which would occur at $r = 0$.

Heisenberg and XY magnets, which do not show true long-range order in $d = 2$ except at $T = 0$, is more complicated.

These are the same results that were obtained using a more phenomenological theory of the fermionic soft modes in Ref. 14. They were discussed extensively in that reference, as well as in Refs. 4 and 7. There is no need to repeat this discussion here, and the salient features are summarized by the schematic phase diagram shown in Fig. 1. The important conclusion of the current paper is that the validity of these results, in addition to itinerant Heisenberg ferromagnets, extends to metallic ferromagnets where the magnetism is not due to the conduction electrons, to metallic ferromagnets in the XY or Ising universality class, and also to metallic ferrimagnets. The only condition is that the conduction electrons are not subject to strong spin-symmetry breaking effects such as magnetic impurities. We note in passing that an interesting system is provided by the easy-plane ferromagnet URhGe, where an in-plane magnetic field transverse to the magnetization has been used to tune the transition, access the tricritical point, and map out the tricritical wings.[10] This situation requires a refinement of the theory presented above, which will be reported elsewhere.[27]

III. DISCUSSION AND CONCLUSION

We now discuss our results, before concluding with a summary.

A. The mechanism behind the first-order transition

The mechanism that leads to the first-order transition discussed in Sec. II E is precisely analogous to the fluctuation-induced first-order transition discussed in Ref. 6 for the BCS-superconductor transition and the nematic-to-smectic-A transition in liquid crystals. An important physical ingredient is an underlying "generic" soft mode, i.e., one that is not related to the phase transition in question, but couples to the order parameter. In the case of liquid crystals this soft mode is the nematic Goldstone mode, in the case of superconductors, the vector potential, in the present case, the spin-triplet particle-hole excitation. At the transition of interest, this soft mode acquires a mass that is given in terms of the nonzero expectation value of the order parameter. This general mass-generating mechanism was first pointed out by Anderson and is now known as the Anderson-Higgs mechanism.[24–26] This coupling of the order parameter to underlying soft modes leads to a nonanalytic term in the Landau free energy that is dominant over the usual quartic term and has a negative sign, leading to a first-order transition. It should be stressed that this is only one way to realize a fluctuation-induced first-order transition; another one, for instance, is realized by a ϕ^4 theory with a cubic anisotropy.[28] The current realization is analogous to the case of scalar electrodynamics studied by Coleman and Weinberg in a particle-physics context.[29] In either case, the mass generation eliminates the generic soft-mode fluctuations, which become energetically more costly with decreasing dimensionality. In the case of quantum magnets, the system takes advantage of this option to lower the free energy for $d \leqslant 3$. It is also worthwhile noting that the analogy between superconductors on one hand, and liquid crystals and quantum magnets on the other, breaks down in the ordered phase. In the former case, the Goldstone mode gets absorbed into the longitudinal component of the vector potential, which is massive, and there is no soft mode in the ordered phase. In the latter, there are Goldstone modes in the ordered phases, namely, a "smecton" with an anisotropic dispersion relation in the smectic-A phase (Ref. 30; see also Ref. 31) and magnons in the magnetic phase.

B. Universality of the first-order transition and the validity of the generalized mean-field theory

Experimentally, all examples of clean low-T_c ferromagnets (for disordered systems, see below; ferrimagnets so far have not been systematically studied from this point of view) show a first-order transition if the Curie temperature is suppressed far enough. There is not a single example of a quantum critical point in zero magnetic field. While this is consistent with the generalized mean-field theory presented in Sec. II, it is somewhat surprising when compared with the case of liquid crystals, where an analogous theory also predicts a first-order transition. In this case, in stark contrast to that of quantum magnets, the observed transition is usually of second order, and only recently have examples of a (weakly) first-order transition been found.[32] These observations beg the question whether in the case of quantum magnets the generalized mean-field approximation is more generally valid than in classical systems.

To discuss this point, we first observe that we have made three approximations to treat the action given by Eq. (2.1a). First, we have integrated out the fermionic massive modes in a saddle-point approximation that respects the Ward identity that governs the soft-mode structure of the system.[18,19] Second, we have kept the soft fermionic degrees of freedom only to Gaussian order in the soft modes q. Third, we have treated the order parameter in a mean-field approximation. These approximations are not independent of one another, and the first two simplifications do not constitute any additional approximation over and above the last one. This can be seen as follows.

The mean-field approximation for the order parameter means that the fermionic degrees of freedom describe an

interacting electron system that is spin-polarized by the coupling to the homogeneous magnetization, which acts as an effective external magnetic field. The state of the fermionic subsystem is, thus, described by a stable Fermi-liquid fixed point. Corrections to the fermionic soft-mode action due to massive degrees of freedom are irrelevant with respect to this fixed point by at least one-half power of frequency or wavenumber in all dimensions and, thus, cannot change the properties of system.[20] Similarly, only the terms quadratic in q contribute to the fixed-point action; all higher-order terms are irrelevant by power counting. Keeping terms of higher order in q will therefore renormalize the parameters of the theory, but it cannot change its structure. In particular, it cannot change the sign of the term in the equation of state, Eqs. (2.27), (2.28), which is due to the soft fermionic fluctuations and leads to the first-order transition.

This leaves the mean-field approximation for the order parameter to be discussed. If the first-order transition at $r = r_1$ occurs far from the second-order transition at $r = 0$ that is pre-empted by it (see Fig. 2), then order-parameter fluctuations are negligible and the results of the generalized mean-field theory are qualitatively correct. If, however, the first-order transition occurs close to the putative second-order one, i.e., if the minimum in the free energy in Fig. 2 is very shallow, then it is less clear whether order-parameter fluctuations can be neglected.[33] One key difference between classical liquid crystals and quantum magnets is that in the former case, the system is below the upper critical dimension $d_c^+ = 4$ for the (unrealized) phase transition that would occur in the absence of any coupling between the smectic order parameter and the nematic soft modes. In contrast, the quantum magnetic systems are above the corresponding upper critical dimension $d_c^+ = 1$ that follows from Hertz theory, and even with that coupling taken into account, ordinary mean-field theory becomes exact, as far as the description of the phase transition is concerned, for $d > 3$.[34] This strongly suggests that order-parameter fluctuations are of much less importance in the case of quantum magnets, and it provides a possible explanation of the fact that the observed transition is universally of first order.

Irrespective of these observations, the role of order-parameter fluctuations in quantum magnets is a topic that warrants additional work. For the case where the magnetism is not produced by the conduction electrons, this will require an action that properly describes localized magnetic moments and their fluctuations, e.g., the one given in Ref. 35. For itinerant magnets, i.e., if the magnetism is due to the conduction electrons themselves, the theory developed in Sec. II will apply, but the order-parameter fluctuations and the fermionic excitations both need to be kept, along the lines of the phenomenological theory of Ref. 14. The latter reference gave a scenario that can lead to a second-order transition in the magnetic case. It would also be interesting to experimentally study quantum ferromagnets or ferrimagnets in $d = 2$, where order-parameter fluctuations will be stronger than in $d = 3$.

C. The effects of quenched disorder

So far we have discussed the case of clean or pure magnets. Impurities, modeled by quenched disorder, have important effects that are both needed to understand experimental observations in certain systems and to predict effects that can serve to ascertain that the first-order transition in pure samples is indeed due to the posited mechanism.

Quenched disorder changes the soft-mode spectrum of the fermions. It gives the ballistic soft modes that are represented by Eqs. (2.17) as mass and leads to new soft modes that are diffusive. In the context of the current theory, this change has two principal effects. First, it cuts off the nonanalyticity in the clean equation of state, Eqs. (2.27), (2.28). Second, it leads to a new nonanalytic term in the equation of state that has the opposite sign and whose prefactor vanishes in the clean limit.[34] The resulting schematic generalized Landau theory has been discussed in Ref. 4. A more detailed model discussion that allows for semi-quantitative predictions of the effects of disorder will be presented elsewhere;[27] here, we just present the most pertinent aspects of such a model calculation. A good representation of the mean-field equation of state for realistic values of the magnetization, the temperature, and the disorder is

$$h = r\,m + \frac{v^{1/4}}{4(k_F\ell)^{3/2}}\,\frac{m^3}{m^{3/2}+(bT)^{3/2}} + \frac{v}{2}\,m^3\,\ln[c\,m^2 + (1/k_F\ell + bT)^2] + u\,m^3, \qquad (3.1)$$

which generalizes Eq. (2.27) in the presence of quenched disorder.[36] Here, the magnetic field h and the temperature T are measured in units of the Fermi energy ϵ_F and the Fermi temperature T_F, respectively, and the magnetization m is measured in units of the conduction electron density (we put $\mu_B = 1$). The dimensionless coupling constant v is proportional to the fourth power of the effective spin-triplet interaction amplitude of the conduction electrons. It is a measure of how strongly correlated the conduction electrons are, and it is bounded above by a stability criterion that requires $v \lesssim 0.5$. k_F is the Fermi wave number of the conduction electrons, and ℓ is the elastic mean-free path. Within a Drude model, and for good metals, one has approximately $k_F\ell \approx 1{,}000/(\rho_0/\mu\Omega\mathrm{cm})$, with ρ_0 the residual electrical resistivity. c and b are dimensionless constants that are equal to $c = 1/45$ and $b = 3\pi$ in a model calculation.[27] The second factor in the second term on the right-hand side is a reasonable representation, for realistic parameter values, of a more complicated scaling function,

$$m^{3/2}\,g(k_F\ell\,m, bT/m) \approx \frac{m^3}{m^{3/2}+(bT)^{3/2}}, \qquad (3.2)$$

which depends on the disorder in addition to the temperature, and we have dropped the last term in Eq. (2.27) from Eq. (3.1) since one generically expects $v \ll u$.

At $T = 0$, and in a clean system, Eq. (3.1) yields a first-order transition at $r_1 = v\,m_1^2/4$, where the magnetization discontinuously jumps from $m = 0$ to $m = m_1 = e^{-(1+2u/v)/2}$. With $u \approx 0.14$ and $v \approx 0.02$, this yields $m_1 \approx 4 \times 10^{-3}$, which is reasonable for a weak ferromagnet. Similarly, there is a tricritical temperature given by $T_{tc}/T_F = (1/b)\exp(-u/v)$; with the same parameter values, this yields $T_{tc}/T_F \approx 10^{-4}$, or $T_{tc} \approx 10\,\mathrm{K}$ for $T_F = 100{,}000\,\mathrm{K}$, which is also reasonable. This tricritical point gets destroyed by quenched disorder on the order of $k_F\ell \approx bT_{tc}/T_F \approx 1{,}000$, or a residual resistivity on the order of $\rho_0 \approx 1\,\mu\Omega\mathrm{cm}$. At this point, the second term on the right-hand side of Eq. (3.1) is still very small, and the critical behavior at the resulting quantum critical point

is given by ordinary mean-field exponents except extremely close to the transition, where it crosses over to the critical behavior derived in Ref. 20. For instance, in this asymptotic region the critical exponents β and δ, defined by $m(h = 0) \propto |r|^{\beta}$ and $m(r = 0) \propto h^{1/\delta}$, respectively, are given by $\beta = 2$ and $\delta = 3/2$, as opposed to the mean-field values $\beta = 1/2$ and $\delta = 3$. Only for substantially larger values of the disorder, $\rho_0 \approx 100\,\mu\Omega$cm with the above parameters, does the asymptotic critical behavior extend over a sizable range of r values (up to $|r| \approx 0.01$). This observation explains why an experiment on Ni_xPd_{1-x}, which shows a ferromagnetic transition at a very small value of x ($x \approx 0.025$) corresponding to weak disorder, found mean-field exponents consistent with Hertz theory,[37] whereas Bauer *et al.*[38] found nonmean-field exponents, at least some of which were consistent with Ref. 20, in $URu_{2-x}Re_xSi_2$, where the ferromagnetic transition occurs at $x \approx 0.15$ with the residual resistivity on the order of $\rho_0 \approx 100\,\mu\Omega$cm.[39]

D. Conclusion

In conclusion, we have extended a previous theory of quantum ferromagnets in several important ways. We have shown that the mechanism that leads to the paramagnet-to-ferromagnet transition at low temperature in $d = 3$ and $d = 2$ to be generically of first order, which was first reported in Ref. 4, is valid in anisotropic ferromagnets, in ferrimagnets, and in metallic ferromagnets where the conduction electrons are not the source of the magnetization, in addition to the case of isotropic itinerant ferromagnets originally considered. Even more generally, it is valid for any metal with a nonvanishing homogeneous magnetization, e.g., canted ferromagnets. This explains why the low-temperature transition is observed to be of first order in highly anisotropic ferromagnets, and it much expands the class of materials for which this phenomenon is predicted. For clean magnets, an effective theory of soft fermionic modes recently developed in Ref. 19 has provided a technical basis that improves on the phenomenological theory of Ref. 14. In the presence of quenched disorder, the theory allows for a semiquantitative description of the suppression and ultimate destruction of the tricritical point. A sizable range of disorder exists where the observable critical behavior is predicted to be mean-field like, whereas for very large disorder the asymptotic critical region, which is characterized by nonmean-field Gaussian critical exponents, expands and eventually eliminates the mean-field region.

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APPENDIX: A SIMPLE MEAN-FIELD MODEL OF A FERRIMAGNET

Here we recall a very simple mean-field model of the transition from a paramagnet to long-range ferrimagnetic order.[15] Consider a one-dimensional chain of alternating magnetic moments μ_a, μ_b that are antiferromagnetically coupled. Weiss theory assumes that the a moments and b moments are subject to effective magnetic fields

$$B_a = -\lambda\, M_b \tag{A1a}$$

$$B_b = -\lambda\, M_a, \tag{A1b}$$

respectively, where $\lambda > 0$. The magnetizations $M_{a,b}$ are given by the Brillouin expressions

$$M_a = \nu\, \mu_a\, \tanh(\mu_a\, H/T + \mu_a\, B_a/T), \tag{A2a}$$

$$M_b = \nu\, \mu_b\, \tanh(\mu_b\, H/T + \mu_b\, B_b/T). \tag{A2b}$$

Here H is an external magnetic field, T is the temperature, and ν is the number of magnetic moments of each species. If one defines reduced magnetic fields $h_{a,b} = H/\nu\mu_{a,b}\lambda$, a reduced temperature $t = T/\nu\mu_a\mu_b\lambda$, and reduced moments $m_{a,b} = M_{a,b}/\nu\mu_{a,b}$, then one sees that the Weiss mean-field Eqs. (A1), (A2) have a solution $m_a = -m_b = \tilde{m}$, where $\tilde{m}$ is the solution of the usual mean-field equation of state

$$h = r\tilde{m} + \tilde{m}^3/3 + O(\tilde{m}^5), \tag{A3}$$

where $r = t - 1$. This simple model thus describes a transition at $t = 1$ to ferrimagnetic order where the homogeneous magnetization is given by $m = M_a + M_b = \nu(\mu_a - \mu_b)\tilde{m}$ and the staggered magnetization $n = M_a - M_b = \nu(\mu_a + \mu_b)\tilde{m}$ is proportional to m.

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