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One-channel conductor coupled to a quantum of resistance: Exact ac conductance and finite-frequency noise

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We consider a one-channel coherent conductor with a good transmission embedded into an Ohmic environment, the impedance of which is equal to the quantum of resistance $R_q = h/e^2$ below the RC frequency. This choice is motivated by the mapping of this problem to a Tomonaga-Luttinger liquid with one impurity, the interaction parameter of which corresponds to the specific value $K = 1/2$, allowing for a refermionization procedure. The "new" fermions have an energy-dependent transmission amplitude, which incorporates the strong correlation effects and yields the exact dc current and zero-frequency noise through expressions similar to those of the scattering approach. We recall and discuss these results for our present purpose. Then, we compute the finite-frequency differential conductance and the finite-frequency nonsymmetrized noise. Contrary to intuitive expectation, both can not be expressed within the scattering approach for the new fermions, even though they are still determined by the transmission amplitude. Even more, the finite-frequency conductance obeys an exact relation in terms of the dc current, which is similar to that derived perturbatively with respect to weak tunneling within the Tien-Gordon theory, and extended recently to arbitrary strongly interacting systems coupled eventually to an environment and/or with a fractional charge. We also show that the emission excess noise vanishes exactly above eV, even though the underlying Tomonaga-Luttinger liquid model corresponds to a many-body correlated system. Our results apply for all ranges of temperature, voltages, and frequencies below the RC frequency, and they allow us to explore fully the quantum regime.

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I. INTRODUCTION

Laws of electrical circuits are drastically modified when mesoscopic systems are incorporated. A coherent conductor embedded into a circuit sees the imposed voltage by the generator reduced by a fluctuating voltage associated with the impedance of the surrounding electromagnetic environment. This gives rise to a current reduction with a pronounced nonlinearity, called zero-bias anomaly (ZBA): transfer of electrons becomes inelastic as they exchange photons with the electromagnetic environment. This phenomena, called the dynamical Coulomb blockade (DCB), has attracted a tremendous interest both theoretically and experimentally.[1] Nevertheless, most of the works have been initially developed by focusing on the limit of a weakly transmitting conductor, i.e., the tunneling regime.[2] More recently, interest in the regime of few well-transmitting channels has emerged. On one hand, reducing the number of channels makes more apparent the effect of the circuit, as many channels could play as well the role of an out-of-equilibrium environment. On the other hand, highly transmitting conductors raise two interesting questions. First, whether the charge fluctuations wash out the DCB, and second, whether the reduction in the current is related to shot noise in the absence of the environment. Indeed, this intuitive and attractive relation had been proposed through a perturbative computation with respect to a very weak impedance, and has been checked experimentally.[3–5] Nevertheless, its validity domain is restricted to high enough energies, as logarithmic divergences arise: one needs to go beyond the weak feedback action. This was particularly the case for an Ohmic environment, i.e., having an impedance $Z(\omega) = R$ at frequencies $\omega < \omega_{RC} = 1/RC$ (where R is the resistance and C the capacitance): this situation has not only relevance to realistic experiments but also a fundamental interest, being related to the investigation of electronic interactions. The logarithmic divergences have been resummed using a renormalization group (RG) scheme by Kindermann and Nazarov,[6] dealing again with $R \ll R_q$, where $R_q = h/e^2$ is the quantum of resistance. It has been nevertheless possible, in a simultaneous and independent work, to deal with an arbitrary value of R: one of the authors, with Saleur,[7] has shown that the problem of a short-coherent conductor in series with a resistance R is equivalent to the impurity problem in a Tomonaga-Luttinger liquid[8,9] (TLL) with an interaction parameter

$$K = (1 + R/R_q)^{-1}. \tag{1}$$

When specified to the limit of small R, this mapping had led to recover the same results as those by Kindermann and Nazarov. These findings answer the two questions addressed above. First, the DCB still persists at good transmission, showing up below an energy scale eV_B (depending in a nonuniversal way on R and on transmission): it corresponds to the crossover voltage between the so-called weak backscattering (WBS) regime at high energy and strong backscattering (SBS) regime at low energy.[10] Second, the reduction in the current is related to the noise in the presence of the environment, and not to the noise of the isolated conductor as stated before [see Eq. (4)].

Recent pioneering experiments by Pierre's group,[11] where the strong feedback of an arbitrary impedance on a one-channel edge state has been investigated, has shown satisfactory agreement with the theoretical predictions of Refs. 6 and 7. Interestingly, even though one has to take strictly the limit of one channel in Ref. 7, the equivalence to a TLL thus established seems to extend to many channels when they are treated in a mean-field framework, provided one renormalizes the parameter K by including the resistance of the channels, and scales the voltage appropriately, as one can infer from a more recent study.[12]

By mapping a one-channel conductor in series with an Ohmic environment to a TLL, the parameter K of which can be controlled by tuning R [see Eq. (1)], one gets as well a promising alternative to test the theoretical predictions for the impurity problem in a TLL. Indeed, satisfactory experimental evidence has been lacking, even though partial success has been claimed to be obtained. In particular, as noticed in Ref. 7, the case of an environmental resistance equal to the quantum of resistance $R = R_q$ corresponds to a TLL with an interaction parameter $K = 1/2$. In that case, the impurity problem in a TLL can be solved exactly in a more transparent way compared to other values of K (see Ref. 13 for instance). Such an apparent simplicity is due to the introduction of new chiral fermions, which arise from a mathematical construction without any physical entity, and incorporate nontrivial strong correlation effects. The nonperturbative investigation has a fundamental interest: it explores crucial issues not reached through perturbative approaches. Nevertheless, the situation with $K = 1/2$ has never been achieved experimentally. Indeed, the ideal candidate to investigate the TLL's behavior is usually provided by edge states with a constriction in the fractional quantum Hall effect (FQHE). However, as K plays the role of the fractional filling factor ν, and as the description of the edges in terms of one-channel chiral TLL model is valid only for simple values, $\nu = K = 1/(2n + 1)$ with n an integer, it has not been possible to achieve a chiral TLL with $K = 1/2$. Other potential candidates are provided by quantum wires and carbon nanotubes. However, three main difficulties arise in those systems: (i) achieving only one backscattering center; (ii) tuning the interaction parameter K; and (iii) taking into account the connection to reservoirs and finite-size effects, where only a perturbative treatment of the impurity has been possible.[14–18] Thus, a coherent one-channel conductor connected to a quantum of resistance $R = R_q$, as achieved recently,[11] offers a unique opportunity to explore the properties of a TLL with an impurity at $K = 1/2$.

The specific value $K = 1/2$ is exciting as it allows us to obtain handy analytic expressions for dc current and the zero-frequency noise. Beyond the stationary regime, time-dependent transport probes even more the dynamics and tests in a precise way the underlying model.

This is the case of finite-frequency (FF) noise. In view of the mapping in Ref. 7, one can use previous results obtained in the FQHE. On the one hand, as far as the FF symmetrized noise is concerned, Chamon *et al.*[19] have computed it perturbatively with respect to the impurity strength, and nonperturbatively at the specific value $K = 1/2$. On the other hand, within the framework of the exact solution of Ref. 13, Lesage and Saleur[20] have obtained only its behavior for low frequencies or close to the "Josephson" singularity e^*V, where V denotes the voltage and $e^* = Ke$.[21] Notice that both approaches disagree, apart from the particular value $K = 1/2$.

Nevertheless, it has been possible to measure experimentally the FF nonsymmetrized noise,[22] which is more interesting to explore. It can be inferred from Ref. 23, dealing with edge states at simple filling factors both in the WBS and SBS regimes: the same results apply to a coherent conductor with a good bare transmission τ_0 connected to an arbitrary resistance at high or low enough energies, as well as for a low transmission.[24] Our aim is to go beyond this perturbative computation, offering a full description extending over all energy ranges below $\omega_c = \min\{\omega_F,\omega_{RC}\}$, where ω_F is the frequency cutoff associated to the fermionic degrees of freedom in the conductor. This is precisely possible at $R = R_q$. Thus, our study gives nonperturbative results for the FF nonsymmetrized noise, without assuming either weak resistance or high or low enough energies associated to the WBS or SBS regimes. In addition, this offers a benchmark for other values of the resistance.

One of the key steps within the refermionization procedure is that the chiral-independent fermions have now an energy-dependent transmission amplitude $t(\omega)$ [see Eq. (5)], which encodes the nontrivial many-body correlations. Indeed, the associated transmission coefficient $\mathcal{T}(\omega) = |t(\omega)|^2$ has nothing to do with the effective transmission obtained for $K \approx 1$ by the RG approach,[6,25] and can not be obtained adiabatically from that in the absence of interactions τ_0. It has been widely accepted that transport properties of the system can be obtained within the scattering approach using $t(\omega)$. This was shown to be valid both for the dc current, as well as for the full counting statistics (FCS).[26] Surprisingly, our study shows that the scattering approach fails when one deals with time-dependent transport: the FF noise can still be expressed in terms of $t(\omega)$, nevertheless it obeys a different relation. The same feature occurs for another quantity: the out-of-equilibrium FF dissipative conductance $\mathrm{Re}[G(V,\omega)]$, which depends on the applied dc voltage and the frequency of the superimposed modulation, in addition to its implicit dependence on temperature.

This last quantity has started to be studied only recently in few correlated systems, where its interest has been shown: quantum wires with an impurity[27] and Kondo problem.[28] This has become possible owing to a crucial result: $\mathrm{Re}[G(V,\omega)]$ is given by an out-of-equilibrium Kubo-type formula, i.e., by the asymmetry between the emission and absorption FF noise.[29–31] In this nonperturbative investigation, we show that $\mathrm{Re}[G(V,\omega)]$ does not fit with its expression obtained within the scattering approach. Even more, it obeys exactly a surprising relation [Eq. (14)], which determines it fully from the dc current, in a way similar to that obtained in tunnel junctions within the Tien-Gordon theory, but with a renormalized charge here.[32,33] Recently, such a relation has been generalized to full extent, including arbitrary interactions: it has been shown to be universal to lowest order with respect to a local or spatially extended tunneling at arbitrary dimension, as well as with respect to local or spatially extended backscattering if one-dimensional systems are considered.[34,35]

Notice that our results will depend on parameters such as the frequency cutoff ω_c, an effective transmission we call τ, and the voltage scale V_B, which is related in a nonuniversal way to ω_c and τ. Nevertheless, we will show that the scaling laws, known to be obeyed by the dc current, can be extended to the FF conductance and FF noise. More precisely, all these quantities depend only on V/V_B, k_BT/eV_B, and $\hbar\omega/eV_B$, where T denotes the temperature.

The paper is organized as follows: In Sec. II, we review the mapping to an impurity problem in the TLL model, which is used to take the Ohmic environment into account, and some of its general consequences on dc transport for arbitrary values of the environmental resistance.[7] Next, in Sec. III, we present our calculations performed in the case $R = R_q$, and give the formal expressions of the current, noise, and conductance in terms of transmission amplitude. In Sec. IV, we discuss in details the dc regime [dc current, differential conductance, and zero-frequency (ZF) noise] for which we recover known results.[36–39] In Sec. V, we explicit new results concerning the time-dependent transport, FF conductance, and FF nonsymmetrized noise, obtained in all energy ranges, and explore them in the quantum regime. We finally conclude in Sec. VI.

II. MODEL AND OUTLOOK

We consider a one-channel coherent conductor with bare transmission τ_0 in series with a dissipative environment, the effective capacitance C of which includes implicitly that of the conductor, thus whose impedance reads as $Z(\omega) = R/(1 + i\omega RC)$. However, we will restrict to energies below $\omega_{RC} = 1/RC$ in the following, thus one can approximate

$$Z(\omega) \approx R \quad \text{for} \quad \omega < \omega_{RC}. \tag{2}$$

We denote the voltage imposed by the generator by V, and the current through the circuit by I (see upper panel in Fig. 1). The voltage drop across the conductor is generally different from V. In the trivial limit of $\tau_0 = 1$, it is given simply by KV, where K is defined by Eq. (1), due to the resistance in series of the perfect conductor and the resistance R of the Ohmic environment. Whenever $\tau_0 < 1$, this is no more the case, apart from the perturbative regime with respect to $1 - \tau_0$, when τ_0 is close to one.

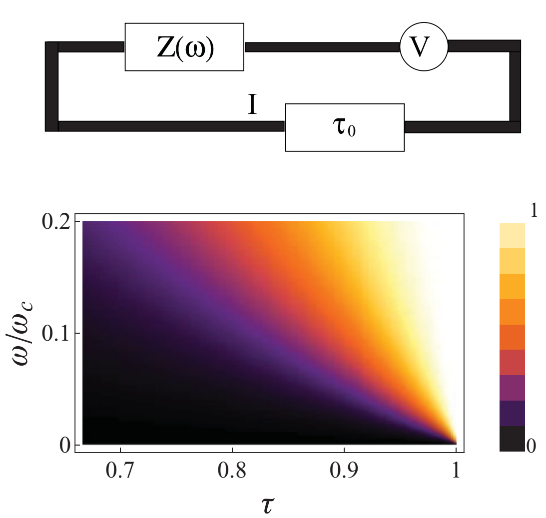

FIG. 1. (Color online) Upper panel: Schematic representation of a one-channel conductor with bare transmission τ_0 embedded in an electric circuit. Lower panel: Profile of the frequency-dependent transmission coefficient $\mathcal{T}$ of the equivalent system as a function of the frequency ω/ω_c and the effective transmission τ [see Eq. (6)].

From now to the end of this section, we review the results of Ref. 7. The one-channel conductor in series with a resistance has been shown to be equivalent to an impurity problem in a TLL, the parameter K of which is given by Eq. (1). Accordingly, one can use the TLL model in the presence of backscattering with amplitude v_B, the Hamiltonian of which reads as

$$\mathcal{H} = \mathcal{H}_0 + \frac{\hbar\omega_F v_B}{4\pi\sqrt{\pi}} e^{i\phi(t) - ieKVt/\hbar} + \text{H.c.}, \tag{3}$$

where $\mathcal{H}_0$ is the TLL Hamiltonian (for simplicity, we do not include spin degrees of freedom). The bosonic field ϕ coincides with the charge that is transferred through the conductor $e\phi(t) = Q(t)$. This bosonized Hamiltonian is an effective one valid at energies below a typical energy $\hbar\omega_F$. In combination with the condition $\omega < \omega_{RC}$ for the mapping to hold, we denote the effective frequency cutoff by $\omega_c = \min\{\omega_F, \omega_{RC}\}$.[40] In Eq. (3), the parameter K in $\exp(-ieKVt/\hbar)$ results from the dc conductance of the ballistic conductor (as can be inferred for instance from Ref. 16). Moreover, the effective backscattering amplitude v_B is not related universally to the bare transmission τ_0 in the absence of the environment. Indeed, one can not, in general, express the parameters of a strongly correlated system in terms of those without interactions, lacking correspondence between them.[41] Nevertheless, we introduce an effective transmission $\tau = 1/(1 + v_B^2)$ keeping in mind that τ does not have to coincide with τ_0. Of course, when $\tau_0 = 1$, one has no backscattering, and $\tau = 1$ too. Also, we restrict v_B to be weak enough (thus τ close to one) in order to write the backscattering Hamiltonian in its bosonized form on the right-hand side of Eq. (3). It is reasonable, though not well established, that this would correspond to τ_0 close to one as well. It is, however, possible that the Hamiltonian in Eq. (3) could be extended beyond that restriction.[42]

The important effective parameter is indeed the scaling voltage V_B,[10] which appears in the Bethe-ansatz solution as nonuniversal.[13] It can be roughly related to the backscattering amplitude through $eV_B \simeq \hbar\omega_c v_B^{1/(1-K)}$. Indeed, V_B characterizes the crossover between the WBS and SBS regimes. At energies high enough compared to eV_B, one can use the perturbative RG analysis with respect to a weak impurity by Kane and Fisher.[43] In the opposite limit (i.e., at energies $\ll eV_B$), the wire is cut into two pieces, with weak tunneling between them. Thus, even though one starts from a weak impurity, lowering the energy drives the system from the WBS behavior, where the conductance is slightly reduced, into the SBS regime where the conductance is suppressed. This means that the DCB still takes place even when the effective transmission τ is close to one, but below an effective charging energy eV_B. In particular, at energies much smaller than eV_B, thus in the SBS regime, it has been shown that one recovers the $P(E)$ theory, which is rather obtained starting from a very weak transmission.[1] Even more, it is possible to obtain nonperturbative results not only with respect to R, but also to $1 - \tau$, and to describe the whole regime of energies below ω_c. Such an achievement was made possible by exploiting the Bethe-ansatz exact solutions of Fendley *et al.*[13] for the impurity problem in a TLL.

In particular, using these results for the current and noise, it was possible to derive a crucial and exact relation between the derivative of the differential conductance $G(V,\omega=0)=dI(V)/dV$ and the differential ZF noise $S(V,\omega=0)$ at zero temperature:

$$R_q|eV|\,dG(V,\omega=0)=2R\,dS(V,\omega=0). \tag{4}$$

Let us remark that in the limit of small $R\ll R_q$, Eq. (4) fits perfectly with the RG equation obtained by Kindermann and Nazarov. Within the Bethe-ansatz solution, the FCS can be computed exactly as well. Using its derivation at zero temperature,[44] the mapping permits to extend Eq. (4) to higher cumulants. It has also motivated partly the recent investigations of the FCS at finite temperature.[27,45–48]

More recently, in an interesting work, Golubev *et al.*[12] have studied a mesoscopic interacting multichannel conductor connected to an Ohmic environment, using a method based on Keldysh action. They have confirmed the results presented above (of Ref. 7) in the perturbative regime they restrict to (with respect to $1-\tau$), which corresponds to high energies in the WBS domain.[49] Even more, when treating many channels in a mean-field approach, the agreement holds too, provided the TLL's parameter in Eq. (1) incorporates the resistance of the conductor. This shows that the mapping can be extended to many channels, as well as its consequences discussed above (at least at high enough energy), and motivates further the computation done in this work.

In the following, we consider the situation where the resistance is equal to the quantum of resistance: $R=R_q$. In that case, the TLL parameter is $K=1/2$ [see Eq. (1)], and Eq. (3) is exactly solvable through a refermionization procedure,[19,36,50–53] thus one can perform a nonperturbative analysis of transport properties. The refermionization introduces new independent chiral fermions with a frequency-dependent transmission amplitude:

$$t(\omega)=\frac{\omega}{\omega+ieV_B/2\hbar}, \tag{5}$$

thus a transmission coefficient

$$\mathcal{T}(\omega)=|t(\omega)|^2=\frac{4\hbar^2\omega^2}{4\hbar^2\omega^2+e^2V_B^2}=\frac{\tau^2\omega^2}{\tau^2\omega^2+(1-\tau)^2\omega_c^2}, \tag{6}$$

where $eV_B=2\hbar\omega_c v_B^2$ is the energy crossover between the WBS and SBS regimes. The profile of $\mathcal{T}(\omega)$ is shown in the lower panel of Fig. 1. Obviously, $\mathcal{T}(\omega)$ is identical to 1 (perfect effective transmission) when $\tau=1$, whatever the frequency is.

As we will show later on, $\mathcal{T}(\omega=eV/\hbar)$ yields the nonlinear differential conductance $G(V,\omega=0)$ at zero temperature. In accordance with the features concerning the crossover from WBS to SBS, one sees that as soon as τ deviates from one, $\mathcal{T}(\omega)$ decreases quickly to zero at low frequency compared to $eV_B/\hbar$. From Eq. (6), notice that $\mathcal{T}(\omega=v_B\omega_c)$ coincides with the effective transmission τ.

III. RESULTS

In this section, we present the formal results for the dc current, the nonsymmetrized noise, and the differential conductance. We first calculate the dc current, defined as the average of the time derivative of the charge which is transferred through the conductor: $I(V)=\langle\hat{I}(t)\rangle=\langle\dot{Q}(t)\rangle$, where $\hat{I}$ is the current operator. Since we consider a dc applied voltage, $I(V)$ is time independent. The details of the calculation are presented in Appendix A. We obtain[54]

$$I(V)=\frac{e}{4\pi}\int_{-\infty}^{\infty}d\omega\mathcal{T}(\omega)[f(\hbar\omega-eV/2)-f(\hbar\omega+eV/2)], \tag{7}$$

which corresponds to the Landauer formulation of the current, where the Fermi-Dirac distribution function is given by $f(\hbar\omega)=[1+\exp(\hbar\omega/k_BT)]^{-1}$. The density of states multiplied by the velocity is a constant, equal to $1/2\pi$, because the energy spectrum of the new fermions is linear too. Equation (7) describes the behavior of the dc current over all voltage and temperature ranges, starting from the WBS regime down to the SBS regime. We have to recall that the model describes a strongly correlated system where interactions can not be treated by any mean-field approach, and that the transmission amplitude incorporates their nontrivial effects. Notice that in Eq. (7) we have made the choice of extending the limits of integration to plus and minus infinity. Strictly speaking, these limits should be $-\omega_c$ and ω_c, however, we have checked that the correction terms are negligible. In the following, a similar choice will be made in all integrals over frequencies.

Next, we calculate the FF nonsymmetrized noise defined as the Fourier transform of the current fluctuations:

$$S(V,\omega)=\int_{-\infty}^{\infty}dt\,e^{i\omega t}\langle\delta\hat{I}(0)\delta\hat{I}(t)\rangle, \tag{8}$$

where $\delta\hat{I}(t)=\hat{I}(t)-\langle\hat{I}\rangle$. We obtain the following result (see Appendix B for details):

$$S(V,\omega)=\frac{e^2}{4\pi}\sum_{\pm}\int_{-\infty}^{\infty}d\omega'([\mathcal{T}(\omega')\mathcal{T}(\omega+\omega')+|t(\omega')-t(\omega+\omega')|^2/4][1-f(\hbar\omega'\pm eV/2)]f(\hbar\omega'+\hbar\omega\pm eV/2)$$
$$+[\mathcal{T}(\omega')-\mathcal{T}(\omega')\mathcal{T}(\omega+\omega')-|t(\omega')-t(\omega+\omega')|^2/4][1-f(\hbar\omega'\pm eV/2)]f(\hbar\omega'+\hbar\omega\mp eV/2)). \tag{9}$$

A crucial and surprising observation is that this expression differs from that obtained within the scattering approach for a one-channel conductor with an energy-dependent transmission[55–58] (see Appendix C for more details on that comparison), even when applied to chiral fermions.[59]

It is interesting to cast Eq. (9) under the alternative form

$$\begin{aligned} S(V,\omega) = {}& \frac{e}{2}\sum_{\pm} N(\hbar\omega \pm eV)[I(\pm V) + I(2\hbar\omega/e \pm V)] \\ &+ \frac{e^2}{4\pi}\sum_{\pm}[N(\hbar\omega \pm eV) - N(\hbar\omega)]\int_{-\infty}^{\infty} d\omega' \\ &\times \left[\mathcal{T}(\omega')\mathcal{T}(\omega+\omega') + \frac{|t(\omega') - t(\omega+\omega')|^2}{4}\right] \\ &\times [f(\hbar\omega + \hbar\omega' \pm eV/2) - f(\hbar\omega' \mp eV/2)], \end{aligned} \quad (10)$$

where $N(\hbar\omega) = [\exp(\hbar\omega/k_BT) - 1]^{-1}$ is the Bose-Einstein distribution function. Notice that in the trivial limit $\tau = \tau_0 = 1$ (i.e., perfect transmission), the voltage drop across the mesoscopic conductor is $V/2$ and the FF noise reduces to $S(V,\omega) = \hbar\omega N(\hbar\omega)G_q$ whatever the temperature is, where $G_q = e^2/h$ is the quantum of conductance. An interesting point is that, in the right-hand side of Eq. (10), both distribution functions f for fermions and N for bosons (of the electromagnetic environment, thus electron-hole excitations) are involved, which explains the fact that the effective voltage is not the same in their arguments: one has $\hbar\omega \pm eV/2$ for the function f, and $\hbar\omega \pm eV$ for the function N.

From Eq. (9), we deduce immediately the ZF noise

$$\begin{aligned} S(V,0) = {}& \frac{e^2}{4\pi}\sum_{\pm}\int_{-\infty}^{\infty} d\omega'(\mathcal{T}^2(\omega')[1 - f(\hbar\omega' \pm eV/2)] \\ &\times f(\hbar\omega' \pm eV/2) + \mathcal{T}(\omega')[1 - \mathcal{T}(\omega')] \\ &\times [1 - f(\hbar\omega' \pm eV/2)]f(\hbar\omega' \mp eV/2)). \end{aligned} \quad (11)$$

Contrary to what occurs for the FF noise, one recovers, in the ZF limit, an expression in terms of the transmission coefficient $\mathcal{T}$ similar to that within the scattering theory.

Let us now calculate the FF conductance, which is related to the nonsymmetrized noise through the exact relation[27,29]

$$\text{Re}[G(V,\omega)] = \frac{S(V,-\omega) - S(V,\omega)}{2\hbar\omega}. \quad (12)$$

Reporting Eq. (9) in Eq. (12), all the terms that contain a product of transmission coefficients vanish, and we find simply

$$\begin{aligned} \text{Re}[G(V,\omega)] = {}& \frac{e^2}{4h\omega}\sum_{\pm}\int_{-\infty}^{\infty} d\omega'\mathcal{T}(\omega')[f(\hbar\omega' \pm eV/2) \\ &- f(\hbar\omega + \hbar\omega' \pm eV/2)], \end{aligned} \quad (13)$$

which obeys unexpectedly the exact relation[60]

$$\text{Re}[G(V,\omega)] = \frac{e}{4\hbar\omega}\left[I(V + 2\hbar\omega/e) - I(V - 2\hbar\omega/e)\right]. \quad (14)$$

This is a central result of our paper: even though our computation is nonperturbative, one recovers a similar relation to that obtained within the perturbative Tien-Gordon theory, with a renormalized charge $e/2$.[32,33] Indeed, such a relation has been recently shown in Ref. 35 to be generally valid for all strongly correlated systems at arbitrary dimensions, with possible coupling to an arbitrary electromagnetic environment, provided one requires the tunneling regime. In one-dimensional systems, the same work has shown the universality of the relation for one or many weak impurities in the WBS regime aside from its validity in the SBS regime or for tunneling barriers. Here, surprisingly, for a TLL with an impurity and $K = 1/2$, or a conductor coupled to a quantum resistance, Eq. (14) is valid for all voltage, temperature, and frequency ranges (below $\hbar\omega_c$) with a renormalized charge $e/2$.

It is interesting as well to specify to frequencies much larger than the applied voltage V, where the FF conductance becomes voltage independent, thus reaches a linear regime. In that case, Eq. (14) can be approximated simply, as in Ref. 35, by

$$\text{Re}[G(V \ll \hbar\omega/e,\omega)] \approx \frac{eI(2\hbar\omega/e)}{\hbar\omega}. \quad (15)$$

In the two following sections, starting from the expressions derived here, we first recall some known results of the stationary regime (Sec. IV), and next we discuss in details the new results of the time-dependent regime (Sec. V).

IV. DIFFERENTIAL CONDUCTANCE AND ZF NOISE

In this section, we explicit in more details the dc transport in all the temperature and voltage ranges, starting from the low-temperature regime ($k_BT \ll eV$) and ending with the high-temperature regime ($k_BT \gg eV$).

Low-temperature behavior. In the limit of strictly zero temperature, the integral over frequency in Eq. (7) can be performed analytically and the dc current reads as[36–38]

$$I(V) = \frac{G_qV_B}{2}\left[\frac{V}{V_B} - \arctan\left(\frac{V}{V_B}\right)\right]. \quad (16)$$

The dc current is thus given by a product of V_B multiplied by a function of V/V_B. This result has been used successfully to describe recent experimental data.[61] For perfect effective transmission $\tau = 1$ (i.e., $V_B = 0$), we recover Ohm's law: $I(V) = V/(2R_q)$, whereas at $\tau < 1$, the dc current is reduced; DCB persists even though one starts from a good effective transmission. Even more, one can check, as shown in Ref. 7, that in the SBS regime, at $V \ll V_B$, one recovers a power-law behavior controlled by the exponent $1 + 2R/R_q = 3$ as in the $P(E)$ theory, and obtained rather for a weakly transmitting conductor.[1] For that, one expands Eq. (16) with respect to V/V_B:

$$I(V \ll V_B) = \frac{G_qV^3}{6V_B^2}. \quad (17)$$

In the opposite limit of the WBS regime, i.e., at $V \gg V_B$, the reduction to the perfect current, called the backscattering current

$$I_B(V) = \frac{G_qV}{2} - I(V), \quad (18)$$

can be expanded with respect to V_B/V:

$$I_B(V \gg V_B) \simeq \frac{G_qV_B}{2}\left(\frac{\pi}{2} - \frac{V_B}{V}\right). \quad (19)$$

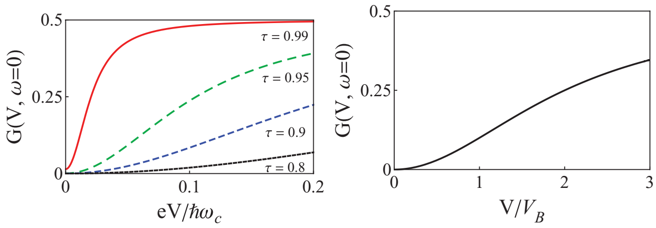

FIG. 2. (Color online) Left panel: Differential conductance, in units of e^2/h, as a function of $eV/\hbar\omega$, for different values of τ, at $k_BT/\hbar\omega_c = 0.001$. Since we have $eV_B/\hbar\omega_c = 2(1-\tau)/\tau$, the corresponding values of V_B are $eV_B/\hbar\omega_c = 0.02$ (red solid line), $eV_B/\hbar\omega_c = 0.1$ (green dashed line), $eV_B/\hbar\omega_c = 0.22$ (blue short dashed line), and $eV_B/\hbar\omega_c = 0.5$ (black dotted line). Right panel: All the curves of the left graph scale to a single one when one considers the variation with V/V_B. We take $k_BT/eV_B = 0.001$.

Notice that the first term on the right-hand side corresponds to that obtained within the perturbative computation[38] $I_B(V) \sim V^{2K-1}$, which becomes voltage independent at $K = 1/2$.

The differential conductance can be obtained either by letting $\omega \to 0$ in Eq. (13), or by differentiating the dc current of Eq. (16). At zero temperature, it reads as

$$G(V,\omega = 0) = \frac{dI(V)}{dV} = \frac{G_q}{2}\left[1 - \frac{V_B^2}{V^2 + V_B^2}\right], \qquad (20)$$

and obeys the relation

$$G(V,\omega = 0) = \frac{G_q}{2}\mathcal{T}\left(\frac{eV}{2\hbar}\right), \qquad (21)$$

where the behavior of the transmission coefficient $\mathcal{T}$ is shown on the lower panel of Fig. 1. Notice that, at $\tau = 1$, the differential conductance becomes constant (linear): it is equal to $G_q/2$ as the ballistic conductor is in series with a resistance $R = R_q$. At $\tau < 1$, $G(V,\omega = 0)$ can formally reach $G_q/2$ for $V \to \infty$, nevertheless, it stays below since eV is limited by the cutoff $\hbar\omega_c$.

The left panel of Fig. 2 gives the differential conductance at low temperature, which shows a ZBA that is more pronounced when τ is reduced. A surprising result is its large sensitivity with respect to small variations of τ in the vicinity of 1 (see the red and green curves in Fig. 2 for example): it is due both to the rapid variations of $\mathcal{T}(\omega')$ at frequencies low compared to ω_c (when τ is close to 1, see the lower panel of Fig. 1), which are precisely those that give the dominant contribution to the integral over ω' in Eq. (13).[62] We can see that the crossover V_B varies rapidly in the vicinity of τ close to one, thus, the SBS regime is reached much more rapidly when τ decreases. It is important to recall that all the curves scale to the same one when one scales the voltage by V_B, as shown in the right panel of Fig. 2.

At zero temperature, the ZF noise of Eq. (11) becomes

$$S(V,\omega = 0) = \frac{G_q eV_B}{4}\left|\arctan\left(\frac{V}{V_B}\right) - \frac{VV_B}{V^2 + V_B^2}\right|. \qquad (22)$$

As for the dc current, the ZF noise is given by a product of V_B multiplied by a function of V/V_B. One can check that the ZF noise obeys Eq. (4), with $R = R_q$, which confirms that the ZBA is exactly linked to the ZF shot noise in the presence of the electromagnetic environment.

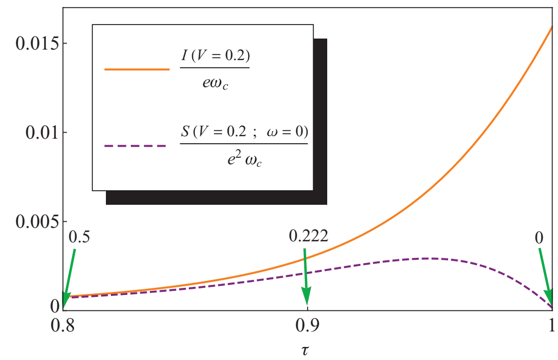

FIG. 3. (Color online) dc current and ZF noise as a function of the effective transmission τ, for $eV/\hbar\omega_c = 0.2$, at a low temperature: $k_BT/\hbar\omega_c = 0.001$. The arrows indicate the associated values of $eV_B/\hbar\omega_c$. With a frequency cutoff of about 1 THz, the value 0.01 $e\omega_c$ on the axis corresponds to a current of about 10 nA.

Figure 3 shows the dc current and the ZF noise as functions of τ, at low temperatures compared to voltage $k_BT \ll eV$: the current shows an increasing behavior, whereas the ZF noise is nonmonotonous. This is due to the fact that the current is expressed as the integral of the transmission coefficient [see Eq. (7)], and that the integral expression of the ZF noise of Eq. (11) contains a term proportional to $\mathcal{T}(\omega')[1 - \mathcal{T}(\omega')]$. For $V_B > V$ (i.e., $\tau < 0.9$ using the parameters of Fig. 3), the current and ZF noise curves converge to the same value. Thus, the Fano factor [given by the ratio $S(V,\omega = 0)/|I(V)|$] tends to e, the value obtained in the Poissonian regime with independent transfer events of charge e through the mesoscopic conductor. For $V_B \ll V$, the transfer events are no more independent, but the noise is still close to zero because of the $[1 - \mathcal{T}(\omega')]$ factor. In addition, the Fano factor [defined this time by the ratio $S(V,\omega = 0)/|I_B(V)|$, where I_B is the backscattering current given by Eq. (18)] is equal to $e^* = e/2$. In the FQHE at $\nu = K = 1/(2n+1)$, the Fano factor would be given by fractional charge $e^* = Ke$. In the DCB context, this renormalization is related to the dc conductance $G_q/2$ without backscattering.

Intermediate-temperature behavior. The left panel of Fig. 4 shows the differential conductance at zero voltage as a function of temperature for various values of τ. When the temperature increases, the ZBA is suppressed, a behavior similar to what is obtained within the P(E) theory.[1,63] Again, all these curves coincide when one considers their variation with respect to

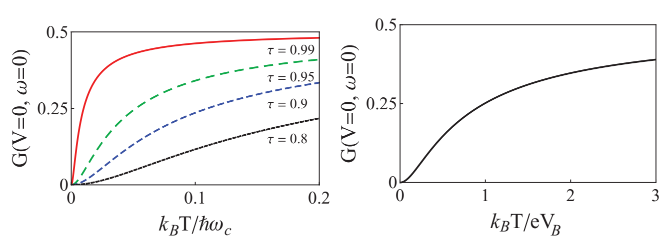

FIG. 4. (Color online) Left panel: Differential conductance, in units of e^2/h, as a function of temperature T, in units of $\hbar\omega_c/k_B$, for different values of τ, at $V = 0$. Right panel: All the curves of the left graph scale to a single one when one considers the variation with respect to k_BT/eV_B.

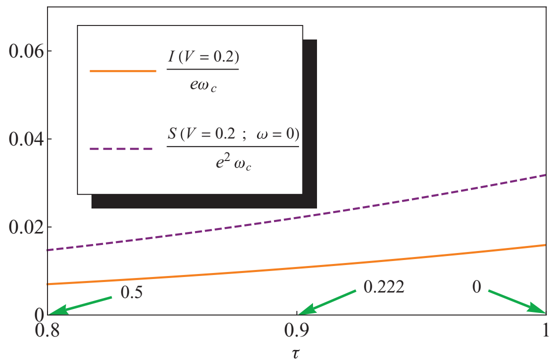

FIG. 5. (Color online) dc current and ZF noise as a function of the effective transmission τ, for $eV/\hbar\omega_c = 0.2$ and $k_BT/\hbar\omega_c = 0.2$. The arrows indicate the associated values of $eV_B/\hbar\omega_c$.

k_BT/eV_B (see right panel of Fig. 4). Figure 5 shows that the current and the ZF noise increase both monotonously with τ, and that the noise is larger than the current due to thermal fluctuations. In the intermediate-temperature regime $k_BT \approx eV$, the ZF noise has a totally different dependence on τ in comparison to its behavior at low temperature due again to an increasing contribution of the thermal noise. However, the total noise is not a direct superposition of shot noise and thermal noise, but results from a more complicated interplay between them.[16,23,27]

High-temperature behavior. In the equilibrium regime (i.e., for $k_BT \gg eV$), the dc current of Eq. (7) becomes strictly linear in V, and the linear conductance takes the value[36–39]

$$\begin{aligned} G &= G(V=0,\omega=0) \\ &= \frac{G_q}{2}\left[1 - \frac{eV_B}{4\pi k_BT}\Psi'\left(\frac{1}{2}+\frac{eV_B}{4\pi k_BT}\right)\right], \end{aligned} \quad (23)$$

where $\Psi(x) = \Gamma'(x)/\Gamma(x)$, and Γ is the gamma function. In particular, in the SBS, at $k_BT \ll eV_B$, one recovers the power law

$$G = \frac{2\pi^2 G_q k_B^2 T^2}{3e^2 V_B^2}. \quad (24)$$

This result is in accordance with the generic behavior of the TLL in the SBS regime,[38,43] where one has $G \sim T^{2/K-2}$, as $K = 1/2$ here. It is also similar to that obtained within the $P(E)$ theory in the tunneling regime, even though the conductor is well transmitting here.[7] Notice that this dependence on temperatures $k_BT \gg eV_B$ in Eq. (24) is similar to that of the nonlinear conductance on voltages $V \ll V_B$, as one can see from differentiating Eq. (17). This confirms, as generally expected, that temperature and voltage play symmetric roles. More precisely, the differential conductance at both finite temperatures and voltages obeys a scaling law: $G \sim T^\alpha F(V/T)$, where $F(x \ll 1) \simeq x^\alpha$ and $F(x \gg 1) \to$ constant value. This leads in particular to the same power-law behavior with respect to $\max\{k_BT, eV\}$. While this is valid in the SBS regime as we have just shown, with $\alpha = 2$, it turns out that this scaling behavior is violated in the opposite limit of WBS as we discuss in detail now. More precisely, one has to consider the backscattering conductance, defined as the differential of the backscattering current in Eq. (18): $G_B = G_q/2 - G$. Using Eq. (23) at $k_BT \gg eV_B$ yields

$$G_B = \frac{\pi G_q eV_B}{16 k_BT}, \quad (25)$$

in accordance with the behavior V_BT^{2K-2} at arbitrary K. We see, however, that this dependence on temperature is different from that on voltage obtained by expanding Eq. (20) in the WBS regime ($V \gg V_B$) [see also Eq. (19)]:

$$G_B = \frac{G_q V_B^2}{2V^2}. \quad (26)$$

As we have already commented concerning Eq. (19), for generic values of K, the lowest-order expansion of the backscattering current is given by $I_B \sim V_BV^{2K-1}$, leading to a differential conductance proportional to $(2K-1)V_BV^{2K-2}$, which cancels at $K = 1/2$. This can be seen as well by differentiating the constant term on the right-hand side of Eq. (19): thus, one needs the next term, yielding to the first nonzero contribution in Eq. (26).

V. FF CONDUCTANCE AND FF NOISE

In this section, we discuss the dynamical properties of our system by performing plot analysis of the formal expressions of the FF conductance $\mathrm{Re}[G(V,\omega)]$ and the FF nonsymmetrized noise $S(V,\omega)$ obtained in Sec. III. Both quantities depend not only of the voltage V and frequency ω, but also on temperature. More precisely, they can be cast into a scaling form, being a function of V/V_B, $\hbar\omega/eV_B$, and k_BT/eV_B. All the dependencies can be obtained exactly, provided these energies are below $\hbar\omega_c$, but we have to make restrictions on the number of curves presented. Here, we will focus first on low temperatures compared to voltage and frequency, $k_BT \ll \{\hbar\omega, eV\}$, thus both out-of-equilibrium and quantum regime allowing for $\hbar\omega$ to be of the order or greater than eV. Then, we will consider intermediate and high temperatures compared to the voltage.

Low-temperature behavior. At $T = 0$, using Eqs. (14) and (16), the FF conductance reads as

$$\mathrm{Re}[G(V,\omega)] = \frac{G_q}{2}\left[1 - \frac{eV_B}{4\hbar\omega}\sum_{\pm}\arctan\left(\frac{2\hbar\omega \pm eV}{eV_B}\right)\right]. \quad (27)$$

For $\tau = 1$, it reduces to the perfect conductance (equal to $G_q/2$), whereas for $\tau < 1$, it acquires a frequency dependence as shown on Fig. 6. Since $\mathrm{Re}[G(V,\omega)]$ is an even function of ω, it is plotted at positive frequencies only. This result is interesting in the sense that the FF conductance has a nonmonotonous behavior. It exhibits a minimum (see the left panel of Fig. 6) at a frequency depending on both V and V_B, which is fixed by the cancellation of the derivative $\partial_\omega\mathrm{Re}[G(V,\omega)]$. When τ decreases, this nonmonotonous behavior disappears, and the FF conductance increases regularly with frequency. All these curves scale to the same one when one considers their variation with respect to $\hbar\omega/eV_B$, at fixed V/V_B and k_BT/eV_B (see right panel of Fig. 6). Notice that because of the finite value of the voltage, the FF conductance acquires a nonzero value, even though small, at zero frequency. This has been checked by zooming around $\omega = 0$ (not shown).

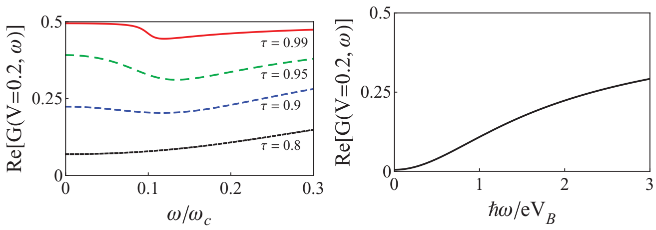

FIG. 6. (Color online) Left panel: FF conductance, in units of e^2/h, as a function of ω/ω_c, for different values of τ, at $eV/\hbar\omega_c = 0.2$ and $k_BT/\hbar\omega_c = 0.001$. Right panel: All the curves of the left graphic scale to a single one when one considers their variation with $\hbar\omega/eV_B$. We take $V/V_B = 0.2$ and $k_BT/eV_B = 0.001$.

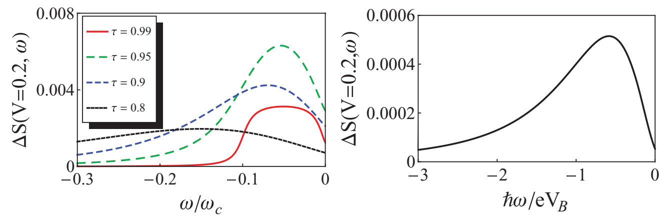

FIG. 7. (Color online) Left panel: Nonsymmetrized excess noise at negative frequency, in units of $e^2\omega_c$, as a function of ω/ω_c, for different values of τ, at $eV/\hbar\omega_c = 0.2$ and $k_BT/\hbar\omega_c = 0.001$. Right panel: Nonsymmetrized excess noise at negative frequency, in units of $e^2V_B/\hbar$, as a function of $\hbar\omega/eV_B$, at $V/V_B = 0.2$ and $k_BT/eV_B = 0.001$.

We turn now our attention to the FF noise. At $T = 0$, the integral in Eq. (9) can be performed analytically and the FF nonsymmetrized noise reads as

$$S(V,\omega) = G_q eV_B \mathcal{F}\left(\frac{V}{V_B},\frac{\hbar\omega}{eV_B}\right), \tag{28}$$

where the dimensionless function $\mathcal{F}$ is given by

$$\begin{aligned}\mathcal{F}(\tilde{V},\tilde{\omega}) = & -\tilde{\omega}\Theta(-\tilde{\omega}) + \frac{1}{8}\sum_{\pm}[\pm\Theta(-\tilde{\omega}\pm\tilde{V})\arctan(\tilde{V})\\ & +[3\Theta(-\tilde{\omega}) - \Theta(-\tilde{\omega}\mp\tilde{V})]\arctan(2\tilde{\omega}\pm\tilde{V})]\\ & +\frac{1}{8\tilde{\omega}}\sum_{\pm}[-\Theta(-\tilde{\omega})+\Theta(-\tilde{\omega}\pm\tilde{V})]\\ & \times[\ln(1+\tilde{V}^2) - \ln(1+(2\tilde{\omega}\mp\tilde{V})^2)].\end{aligned} \tag{29}$$

Here, Θ is the Heaviside function, $\tilde{V} = V/V_B$, and $\tilde{\omega} = \hbar\omega/eV_B$. Notice that the FF noise is an even function of V. Because of this parity, we will specify only to positive values of the voltage. We recall that the FF nonsymmetrized noise at positive (negative) frequencies corresponds to emission (absorption) noise.[64]

An important fact we observe is that the emitted noise vanishes exactly above eV. In the perturbative regime, one usually expects that the emitted noise vanishes above eKV.[23] Indeed, it has been shown universally for a weak tunneling junction (between strongly correlated systems in arbitrary dimensions and with possible coupling to an environment) that the FF noise associated to tunneling vanishes above qV, where q is the transferred charge.[29] Nevertheless, when higher-order processes with respect to tunneling are taken into account, one expects the implication of a many-body complicated process, and energy conservation at the level of one particle can not be used anymore. In strongly correlated systems or in a conductor coupled to an environment, we are not aware of any nonperturbative statement about the existence of a value of the frequency ω over which the emitted noise vanishes. However, we obtain an exact result here: the emitted noise vanishes strictly at frequencies $\hbar\omega > eV$, whatever the values of V and V_B are. This is due to the Bose-Einstein distribution functions of Eq. (10), which reduce, in the zero-temperature limit, to the Heaviside functions of Eq. (29). Their presence is related to the photon exchange between the conductor and the environment corresponding to electron-hole–type excitations. One can think of two possible scenarios: (i) The new fermions are independent. For noninteracting electrons, scattering theory predicts that the emitted noise vanishes above eV. Even though the FF noise in our case does not obey the same relation with respect to the transmission amplitude $t(\omega)$, one can imagine that a similar fact holds. (ii) The second scheme is that the system can not emit at higher frequencies compared to what the generator can afford, and not compared to the voltage across the conductor $V/2$ if it was perfect. If this is plausible, it opens the question as to whether such a result is universal for the TLL with an arbitrary value of the parameter K, thus for other values of the resistance R. Equivalently, for $\hbar\omega < -eV$, we have $S(V,-\omega) = 0$, and Eq. (12) reduces to[29]

$$S(V,\omega < -eV/\hbar) = -2\hbar\omega\,\mathrm{Re}[G(V,\omega)], \tag{30}$$

which means that at negative frequencies below $-eV/\hbar$ and zero temperature, the absorption FF noise is governed by the FF conductance.

It is also interesting to express the FF nonsymmetrized excess noise, which is often measured experimentally, both because zero-point fluctuations are not easy to measure (apart from a pioneering work in Ref. 65), and in order to subtract undesirable sources of noise. It is given by

$$\Delta S(V,\omega) = S(V,\omega) - S(V=0,\omega). \tag{31}$$

At zero temperature, it reads as

$$\Delta S(V,\omega) = G_q eV_B[\mathcal{F}(\tilde{V},\tilde{\omega}) - \mathcal{F}(0,\tilde{\omega})], \tag{32}$$

where the function $\mathcal{F}$ is given by Eq. (29).

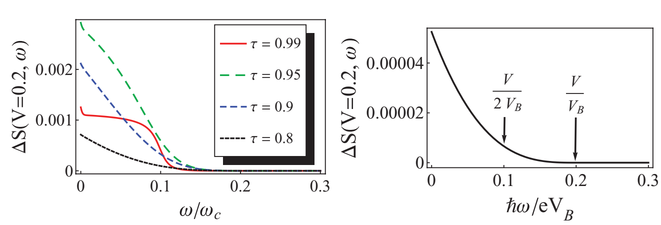

FIG. 8. (Color online) Left panel: Nonsymmetrized excess noise at positive frequency, in units of $e^2\omega_c$, as a function of ω/ω_c, for different values of τ, at $eV/\hbar\omega_c = 0.2$ and $k_BT/\hbar\omega_c = 0.001$. Right panel: Nonsymmetrized excess noise at positive frequency, in units of $e^2V_B/\hbar$, as a function of $\hbar\omega/eV_B$, at $V/V_B = 0.2$ and $k_BT/eV_B = 0.001$.

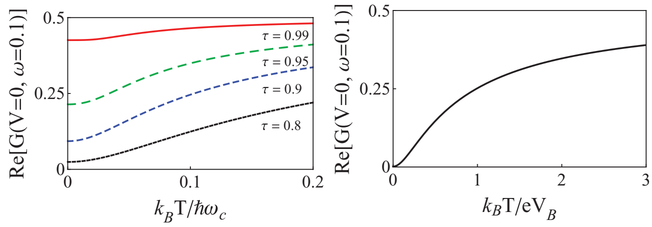

FIG. 9. (Color online) Left panel: FF conductance, in units of e^2/h, as a function of $k_BT/\hbar\omega_c$, for different values of τ, at $V = 0$ and $\omega/\omega_c = 0.1$. Right panel: All the curves of the left graphic scale to a single one when one considers their variations with respect to k_BT/eV_B. We take $V = 0$ and $\hbar\omega/eV_B = 0.1$.

In the left panels of Figs. 7 and 8 is plotted the excess noise at a very low temperature[66] at positive and negative frequencies, respectively. The red curve corresponds to a value of $V_B \ll V$, i.e., to the WBS regime. In that case, one sees, according to Eq. (29), that the dependence close to $\hbar\omega = \pm e^*V = \pm eV/2$ is due mainly to the arctan function: this explains the step whose width is controlled by V_B. The step is smoothed out at higher V_B (i.e., lower τ) in the green and blue dashed curves of Fig. 7. However, the asymmetry between absorbed and emitted excess noise is always present, whatever the value of the effective transmission is. This asymmetry has been shown to be related, in a universal way, to nonlinearity, using Eq. (12).[29] Here, the nonlinearity is induced by the electromagnetic environment. All these curves at different τ scale to a unique one as it is shown in the right panels of Figs. 7 and 8, where ΔS is plotted as a function of $\hbar\omega/eV_B$ when the ratios V/V_B and k_BT/eV_B are fixed. We have to draw attention to the fact that this requires us to change simultaneously all energy scales while V_B changes.

Intermediate-temperature behavior. Next, we look at the effect of the temperature on the FF conductance and the FF noise. The left panel of Fig. 9 shows that the FF conductance increases monotonously with temperature. Again, all the curves scale to a single one when one considers the variation with respect to k_BT/eV_B at fixed $\hbar\omega/eV_B$ (see the right panel of Fig. 9). The FF excess noise at intermediate temperatures is plotted on the left panel of Fig. 10. However, the asymmetry of the FF excess noise is still visible in that regime, and we observe its enhancement when the effective transmission decreases. Again, all the curves scale to a single one when one considers the variation with $\hbar\omega/eV_B$ at fixed values of V/V_B and k_BT/eV_B (see the right panel of Fig. 10).

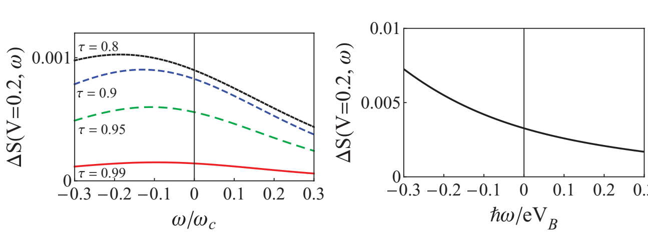

FIG. 10. (Color online) Left panel: Nonsymmetrized excess noise, in units of $e^2\omega_c$, as a function of ω/ω_c, for different values of τ. We use the values $eV/\hbar\omega_c = 0.2$ and $k_BT/\hbar\omega_c = 0.2$. Right panel: Nonsymmetrized excess noise, in units of $e^2V_B/\hbar$, as a function of $\hbar\omega/eV_B$, at $V/V_B = 0.2$ and $k_BT/eV_B = 0.2$.

High-temperature behavior. At high temperature compared to voltage, $k_BT \gg eV$, we have checked, using Eqs. (10) and (14), that the FF nonsymmetrized noise is related to the FF conductance and becomes voltage independent in that limit according to the fluctuation-dissipation theorem

$$S(V \ll k_BT/e,\omega) = 2\hbar\omega N(\hbar\omega)\mathrm{Re}[G(V \ll k_BT/e,\omega)]. \tag{33}$$

VI. CONCLUSION

In this paper, we have studied both stationary and time-dependent transport properties of a well-transmitting one-channel conductor embedded in an Ohmic environment with a quantum of resistance $R_q = h/e^2$. We have taken advantage of the mapping of this problem to a TLL with a particular value of the interaction parameter $K = 1/2$. This has allowed us to obtain results which are nonperturbative with respect to the resistance of the environment R [here being equal to R_q, see Eq. (1)], and which describes all regimes of voltages, frequencies, and temperatures below the cutoff ω_c. The results are controlled by a unique scaling and nonuniversal parameter V_B, which depends on both the properties of the conductor and the environment. While the dc properties can as well be derived exactly for other values of R (analytically at zero temperature, otherwise one would need numerical methods), using the Bethe-ansatz solution for the impurity problem in the TLL, the particular choice of $R = R_q$ has allowed us to obtain the first nonperturbative results for both the FF noise and the FF conductance. For the latter, we have used its universal relation to the asymmetry of the emission and absorption noise via an out-of-equilibrium fluctuation-dissipation theorem type formula [see Eq. (12)] derived in Ref. 31.

We have first recalled the results for the dc transport obtained in a TLL with parameter $K = 1/2$ in order to apply them to our present problem and to discuss them in more details. The key procedure is based on refermionization, i.e., casting the nontrivial strong correlation effects into new chiral fermions with an energy-dependent transmission coefficient $\mathcal{T}(\omega)$. This can not be obtained by any "adiabatic" evolution from the effective transmission τ (which explains our choice for a different notation). The dc current through the mesoscopic conductor takes a form similar to that within the scattering theory, applied to the new fermions: it is expressed as the integral over energy of $\mathcal{T}(\omega)$ times the difference of the Fermi-Dirac distribution functions of the left and right reservoirs. Even for a good effective transmission τ, one recovers the DCB at energies below the voltage V_B. The reduction to the dc current has a power law with different exponents when voltages are high or low compared to V_B, corresponding to the WBS and SBS regimes. In particular, in the latter limit, one recovers the behavior predicted within the $P(E)$ theory for $V \ll V_B$ for a weakly transmitting conductor, even though we consider a well transmitting one here. In these two opposite limits, the ZF noise becomes Poissonian, with a Fano factor respectively given by $e/2$ or e. The former renormalization is related to the conductance of the conductor in the perfect limit, being in series with the environmental quantum resistance. The ZF noise obeys as well the same expression in terms of $\mathcal{T}(\omega)$ as within the scattering approach. Nevertheless, this does not hold for the FF noise: even though it can be expressed in terms of the transmission amplitude $t(\omega)$: this is a surprising and crucial result of our paper. Another interesting fact is that the emitted FF noise vanishes strictly above eV at zero temperature, a fact common to noninteracting electrons, which is not obvious to expect within the underlying strongly correlated system. We have shown that the FF conductance does not obey the scattering approach formulation for the new fermions. Rather, it obeys a simple relation that involves dc currents. This kind of relation was previously shown for tunneling barriers within the Tien-Gordon theory, or in the FQHE at simple fillings.[23] Recently, it has been extended to arbitrary filling factors, and even more has been shown to be universal either for tunneling barriers in arbitrary dimensions, as well as weak barriers in one dimension.[35] It is quite remarkable that such a relation extends to arbitrary regimes within the TLL model at $K = 1/2$. It turns out that the FF conductance has a nonmonotonous behavior with respect to frequency, having a minimum at a frequency that depends both on the applied voltage and the scaling voltage V_B, and the value of which is reduced when the effective transmission τ decreases.

We have shown that both the FF conductance and the FF noise have a universal behavior at different τ when voltages, temperatures, and frequencies are all divided by the same scaling voltage V_B. This extends the result obtained for the differential conductance, valid for arbitrary values of the parameter K (thus of the resistance R in our problem) where we expect to get the same scaling behavior for time-dependent transport as well.

The coherent conductor connected to an Ohmic environment offers a unique framework to realize a TLL with a tunable parameter K, and a unique possibility to realize $K = 1/2$. The present experiments on that direction are very promising.[11] Beyond this issue, our work provides benchmark results for time-dependent transport in various fundamental problems: the DCB phenomena, other strongly correlated systems where the special value $K = 1/2$ has been studied fully,[48] and more generally those where one solves the interacting problem in terms of new independent particles, such as within the Bethe-ansatz methods for the impurity problem in TLL.[13] We have highlighted counterintuitive facts, in particular, we have shown that one can not systematically apply the scattering approach to those new independent particles.

ACKNOWLEDGMENTS

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APPENDIX A: CALCULATION OF THE CURRENT

The mapping of Ref. 7 applies to the effective action once degrees of freedom apart from those at $x = 0$ are integrated out. It is more convenient, for the present computation, to consider the extended TLL Hamiltonian over space coordinate x, with an impurity located at $x = 0$ (see Fig. 11):

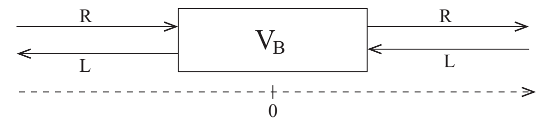

FIG. 11. Schematic representation of the left and right propagating channels in the reservoirs. $eV_B = 2\hbar\omega_F v_B^2$ is the energy scale that characterizes backscattering of electrons at the conductor position $x = 0$.

$$\mathcal{H} = \frac{\hbar v_F}{4\pi}\int dx\{[\partial_x\phi(x,t)]^2 + [\partial_x\tilde{\phi}(x,t)]^2\} + \sqrt{\frac{\hbar\omega_F eV_B}{32\pi^3}}e^{i\phi(0,t)-ieVt/2\hbar} + \text{H.c.}, \tag{A1}$$

where the bosonic fields ϕ and $\tilde{\phi}$ are related to the initial bosonic fields describing right (R) and left (L) movers through

$$\phi(x,t) = \frac{1}{\sqrt{2}}[\phi_R(x,t) + \phi_L(x,t)], \tag{A2}$$

$$\tilde{\phi}(x,t) = \frac{1}{\sqrt{2}}[\phi_R(x,t) - \phi_L(x,t)], \tag{A3}$$

where $\psi_{L,R} = \eta_{L,R}e^{i\phi_{L,R}(x,t)}/\sqrt{2\pi a}$ are the initial fermionic operators. a is the distance cutoff of the TLL theory. The Klein factors $\eta_{L,R}$ allow us to have the proper commutation relation for the fermionic operators. Notice that in Eq. (3), the bosonic field refers to $\phi(t) \equiv \phi(0,t)$.

The average current through the conductor is given by[42]

$$I = ev_F\langle\hat{\rho}_R - \hat{\rho}_L\rangle, \tag{A4}$$

where v_F is the Fermi velocity. The operators $\hat{\rho}_R$ and $\hat{\rho}_L$ refer to the densities of right- and left-moving electrons in TLL reservoirs (see Fig. 11). They are related to the new fermionic operators introduced in the refermionization procedure through the relations

$$\hat{\rho}_R(x,t) = \frac{\tilde{\psi}^\dagger(x,t)\tilde{\psi}(x,t) + \psi^\dagger(x,t)\psi(x,t)}{2}, \tag{A5}$$

$$\hat{\rho}_L(x,t) = \frac{\tilde{\psi}^\dagger(-x,t)\tilde{\psi}(-x,t) - \psi^\dagger(-x,t)\psi(-x,t)}{2}, \tag{A6}$$

where $\psi(x,t) = \eta e^{i\phi(x,t)}/\sqrt{2\pi a}$ and $\tilde{\psi}(x,t) = \tilde{\eta}e^{i\tilde{\phi}(x,t)}/\sqrt{2\pi a}$ are the new fermionic fields associated to the bosonic fields ϕ and $\tilde{\phi}$: $\psi(x,t)$ is affected by backscattering and by the applied voltage [see Eq. (A1)], whereas $\tilde{\psi}(x,t)$ is neither affected by backscattering nor by the applied voltage. In Eq. (A4), the average value is defined as the average over the scattering state[19] minus the average at equilibrium (i.e., $V = 0$) since we have to take the normal order[52] in the products of operators that appear in Eqs. (A5) and (A6). Thus, we immediately conclude that $\langle\tilde{\psi}^\dagger(x,t)\tilde{\psi}(x,t)\rangle = 0$. The Klein factors η and $\tilde{\eta}$ allow us to have the proper commutation relation for the new fermionic operators.

From Eqs. (A5) and (A6), we understand that $\hat{\rho}_R$ and $\hat{\rho}_L$ depend on the position x, however, the average of the difference $\langle\hat{\rho}_R - \hat{\rho}_L\rangle$, which appears in the current, does not depend on the position since the average of the total current is a conserved quantity (it is the reason why we do not keep the x dependency in the current).

Using this Hamiltonian, one can write the equations of motion for the fields. The solution for the fermionic field $\tilde{\psi}$, associated with the free bosonic field, is that of free propagating electrons, whereas the solution for ψ is given by[19] (from now, we take $x > 0$)

$$\psi(-x,t) = \frac{1}{\sqrt{2\pi a\omega_F}}\int_{-\infty}^{\infty} a_\omega e^{-i(\omega+\frac{eV}{2\hbar})\frac{x}{v_F}-i\omega t}d\omega, \quad (A7)$$

$$\psi(x,t) = \frac{1}{\sqrt{2\pi a\omega_F}}\int_{-\infty}^{\infty} b_\omega e^{i(\omega+\frac{eV}{2\hbar})\frac{x}{v_F}-i\omega t}d\omega, \quad (A8)$$

where the operator $b_\omega = t(\omega)a_\omega + r(\omega)a^\dagger_{-\omega}$ is a combination of the annihilation and creation operators $a^\dagger_\omega$ and a_ω, which obey the commutation relation $\{a_\omega, a^\dagger_{\omega'}\} = \delta_{\omega,\omega'}$. The transmission and reflection amplitudes read as

$$t(\omega) = \frac{2\hbar\omega}{2\hbar\omega + ieV_B}, \quad (A9)$$

$$r(\omega) = \frac{ieV_B}{2\hbar\omega + ieV_B}. \quad (A10)$$

With the help of the solutions given by Eqs. (A7) and (A8), one can calculate the average over the product of the two fermionic fields:

$$\langle\psi^\dagger(-x,t)\psi(-x,t)\rangle = \frac{1}{2\pi v_F}\int_{-\infty}^{\infty} d\omega[f(\hbar\omega - eV/2) - f(\hbar\omega)] \quad (A11)$$

and

$$\langle\psi^\dagger(x,t)\psi(x,t)\rangle = \frac{1}{2\pi v_F}\int_{-\infty}^{\infty} d\omega[2\mathcal{T}(\omega) - 1] \times [f(\hbar\omega - eV/2) - f(\hbar\omega)], \quad (A12)$$

where $\mathcal{T} = t^* t$. We have used the following average values: $\langle a_\omega a_{\omega'}\rangle = 0$ and $\langle a^\dagger_\omega a_{\omega'}\rangle = f(\omega - eV/2)\delta_{\omega,\omega'}$, with f the Fermi-Dirac distribution function. Reporting Eqs. (A11) and (A12) into Eqs. (A4) and (A5), we finally obtain Eq. (7).

APPENDIX B: NOISE CALCULATION

To calculate the FF nonsymmetrized noise, we need to evaluate the following correlators ($x > 0$):

$$S_1 = \int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle\psi^\dagger(-x,0)\psi(-x,0)\psi^\dagger(-x,t)\psi(-x,t)\rangle,$$
$$S_2 = \int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle\psi^\dagger(x,0)\psi(x,0)\psi^\dagger(-x,t)\psi(-x,t)\rangle,$$
$$S_3 = \int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle\psi^\dagger(-x,0)\psi(-x,0)\psi^\dagger(x,t)\psi(x,t)\rangle, \quad (B1)$$
$$S_4 = \int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle\psi^\dagger(x,0)\psi(x,0)\psi^\dagger(x,t)\psi(x,t)\rangle.$$

The determination of S_1 is rather simple since it involves contributions that are not affected by the backscattering:

$$\langle\psi^\dagger(-x,0)\psi(-x,0)\psi^\dagger(-x,t)\psi(-x,t)\rangle = \frac{1}{4\pi^2a^2\omega_F^4}\int d\omega_1\int d\omega_2\int d\omega_3\int d\omega_4\langle a^\dagger_{\omega_1}a_{\omega_2}a^\dagger_{\omega_3}a_{\omega_4}\rangle \times e^{i(-\omega_1+\omega_2-\omega_3+\omega_4)\frac{x}{v_F}+i(\omega_3-\omega_4)t}. \quad (B2)$$

We use Wick's theorem to calculate the correlator $\langle a^\dagger_{\omega_1}a_{\omega_2}a^\dagger_{\omega_3}a_{\omega_4}\rangle$. After successive integrations over frequencies and times, we obtain

$$S_1 = \frac{1}{2\pi v_F^2}\int_{-\infty}^{\infty} d\omega' f(\hbar\omega + \hbar\omega' - eV/2) \times [1 - f(\hbar\omega' - eV/2)]. \quad (B3)$$

The calculation of S_2 and S_3 mix a_ω and b_ω operators since

$$\langle\psi^\dagger(x,0)\psi(x,0)\psi^\dagger(-x,t)\psi(-x,t)\rangle = \frac{1}{4\pi^2a^2\omega_F^4}\int d\omega_1\int d\omega_2\int d\omega_3\int d\omega_4\langle b^\dagger_{\omega_1}b_{\omega_2}a^\dagger_{\omega_3}a_{\omega_4}\rangle \times e^{i(-\omega_1+\omega_2-\omega_3+\omega_4)\frac{x}{v_F}+i(\omega_3-\omega_4)t} \quad (B4)$$

and

$$\langle\psi^\dagger(-x,0)\psi(-x,0)\psi^\dagger(x,t)\psi(x,t)\rangle = \frac{1}{4\pi^2a^2\omega_F^4}\int d\omega_1\int d\omega_2\int d\omega_3\int d\omega_4\langle a^\dagger_{\omega_1}a_{\omega_2}b^\dagger_{\omega_3}b_{\omega_4}\rangle \times e^{i(-\omega_1+\omega_2-\omega_3+\omega_4)\frac{x}{v_F}+i(\omega_3-\omega_4)t}. \quad (B5)$$

We obtain $S_2 = S_3$ with

$$S_2 = \frac{1}{2\pi v_F^2}\int_{-\infty}^{\infty} d\omega' f(\hbar\omega + \hbar\omega' - eV/2) \times [1 - f(\hbar\omega' - eV/2)][t(\omega')t^*(\omega')t(\omega+\omega') \times t^*(\omega+\omega') - r(\omega')r^*(\omega')r(\omega+\omega')r^*(\omega+\omega')]. \quad (B6)$$

Next, we calculate S_4, which involves b_ω and $b^\dagger_\omega$ operators, since

$$\langle\psi^\dagger(x,0)\psi(x,0)\psi^\dagger(x,t)\psi(x,t)\rangle = \frac{1}{4\pi^2a^2\omega_F^4}\int d\omega_1\int d\omega_2\int d\omega_3\int d\omega_4\langle b^\dagger_{\omega_1}b_{\omega_2}b^\dagger_{\omega_3}b_{\omega_4}\rangle \times e^{i(-\omega_1+\omega_2-\omega_3+\omega_4)\frac{x}{v_F}+i(\omega_3-\omega_4)t}. \quad (B7)$$

Applying Wick's theorem, we obtain

$$S_4 = \frac{1}{2\pi v_F^2}\int_{-\infty}^{\infty} d\omega'\Big\{f(\hbar\omega + \hbar\omega' - eV/2) \times [1 - f(\hbar\omega' - eV/2)][|t(\omega')|^2|t(\omega+\omega')|^2 + |r(\omega')|^2|r(\omega+\omega')|^2 - 2t(\omega')t^*(\omega+\omega')r^*(\omega') \times r(\omega+\omega')] + \sum_\pm f(\hbar\omega + \hbar\omega' \pm eV/2) \times [1 - f(\hbar\omega' \mp eV/2)][|t(\omega')|^2|r(\omega+\omega')|^2 + t(\omega')t^*(\omega+\omega')r^*(\omega')r(\omega+\omega')]\Big\}. \quad (B8)$$

Finally, the FF nonsymmetrized noise of Eq. (9) is obtained from $S(V,\omega)=e^2v_F^2[S_1+S_2+S_3+S_4]/4$, where we replace, everywhere it appears, $r(\omega)$ by $1-t(\omega)$.

APPENDIX C: COMPARISON TO THE RESULTS OBTAINED WITHIN THE SCATTERING THEORY

In this appendix, we compare the expressions of current, FF conductance, and noise that we have established to those obtained in the framework of the scattering theory. It has been already noted that the expressions of the dc current given by Eq. (7) and the ZF noise are formally identical to those derived from the scattering approach (i.e., they are expressed as integrals of the transmission coefficient times Fermi-Dirac distribution functions), even though the nontrivial many-body effects are encoded into this transmission. We show here that the FF conductance and the FF noise, even though expressed in terms of the transmission and reflection amplitudes, do not obey the relation derived within the scattering approach.

To make a comparison with the FF noise given in the literature,[55–58] we need to symmetrize the noise given by Eq. (9):

$$S(V,\omega)+S(V,-\omega)=\frac{e^2}{4\pi}\int_{-\infty}^{\infty}d\omega'([\mathcal{T}(\omega')\mathcal{T}(\omega+\omega')+|t(\omega')-t(\omega+\omega')|^2/4][F_{++}(\omega,\omega')+F_{--}(\omega,\omega')]+[\mathcal{T}(\omega')-\mathcal{T}(\omega')\mathcal{T}(\omega+\omega')-|t(\omega')-t(\omega+\omega')|^2/4][F_{+-}(\omega,\omega')+F_{-+}(\omega,\omega')]), \tag{C1}$$

where $F_{ss'}(\omega,\omega')=\sum_{\pm}f(\hbar\omega'\pm\hbar\omega\pm seV/2)[1-f(\hbar\omega'\pm s'eV/2)]$. Using our notations, the symmetrized noise of a coherent one-channel coherent conductor with an energy-dependent transmission $\mathcal{T}$ can be obtained within the scattering approach[57]

$$S_{\text{scattering}}(V,\omega)+S_{\text{scattering}}(V,-\omega)=\frac{e^2}{4\pi}\int_{-\infty}^{\infty}d\omega'([\mathcal{T}(\omega')\mathcal{T}(\omega+\omega')+|t(\omega')-t(\omega+\omega')|^2]F_{++}(\omega,\omega')+\mathcal{T}(\omega')\mathcal{T}(\omega+\omega')\times F_{--}(\omega,\omega')+\mathcal{T}(\omega')[1-\mathcal{T}(\omega+\omega')]F_{+-}(\omega,\omega')+\mathcal{T}(\omega+\omega')[1-\mathcal{T}(\omega')]F_{-+}(\omega,\omega')). \tag{C2}$$

The main difference between Eqs. (C1) and (C2) resides into the factors in front of F_{++} and F_{--}, which are identical in Eq. (C1) but not in Eq. (C2). The same remark applies to the factors in front of F_{+-} and F_{-+}. If one had a transmission coefficient $\mathcal{T}$ independent on energy, both expressions would have been identical.

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