Electromagnetic cloaking of cylindrical objects by multilayer or uniform dielectric claddings

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We show that dielectric or even perfectly conducting cylinders can be cloaked by a uniform or a layered dielectric cladding, without the need of any exotic or magnetic material parameters. In particular, we start by presenting a simple analytical concept that can accurately describe the cloaking effect obtained with conical silver plates in the visible spectrum. The modeled structure has been originally presented in Tretyakov *et al.* [Phys. Rev. Lett. **103**, 103905 (2009)], where its operation as a cloak in the optical frequencies was studied only numerically. We model rigorously this configuration as a multilayer dielectric cover surrounding the cloaked object, with excellent agreement to the simulation results of the actual device. The concept of using uniform or multilayer dielectric covers, with relative permittivities larger than unity, is then successfully extended to cloaking of impenetrable objects such as conducting cylinders.

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I. INTRODUCTION

An electromagnetic cloak is a device that minimizes or even nullifies the effects of scattering from various objects by rendering them electromagnetically invisible to a detector. One well-known cloaking technique is the so-called "transformation-optics" method[1–4] which uses strongly inhomogeneous and highly anisotropic materials to perform complex transformations for the incident wave. Another cloaking technique is based on the cancellation of the scattering effects of a magnetodielectric object, with a suitable metamaterial or plasmonic cladding,[5,6] which neutralizes the polarization current of the primary scatterer. At this point, one should make clear that the major advantage of the first method ("transformation optics") against the second one ("scattering cancellation") and almost any other realizable cloaking concept is that it works regardless of the electrical size of the cloaked region. On the contrary, most of the rest techniques are applicable to volumes with moderate electrical dimensions. This is particularly the case for the "scattering cancellation" cloaking which simply cancels the effect of the dipolar (zeroth) term from the scattering series solution. Unfortunately, the actual construction of an electromagnetic cloaking device, capable of hiding an object in free space is very difficult with both the aforementioned approaches. Here care should be taken, not to confuse an electromagnetic free-space cloak with so-called ground-plane cloaks or acoustic cloaks. The two latter are less demanding when it comes to the complexity of material parameters, and therefore have been shown to be possible to realize even with simple dielectric materials.[7–11]

Suggestions to overcome the drawback of complexity in free-space cloaks includes, for example, the use of transmission-line networks or other waveguiding structures instead of complex materials with exotic properties.[12] With transmission-line networks, the object to be electromagnetically hidden is strongly limited in size and geometry,[13] but cloaking of impenetrable (e.g., perfectly metallic) objects has been shown to be possible with a set of conical conducting plates located around a cylindrical cloaked region.[14] This metal-plate cloak has been practically realized in microwaves and numerically proven to be functional for a wide band of optical frequencies.[15] However, the underlying theory and the physical principles that govern the related phenomena have not been yet thoroughly understood.

In this work, we introduce a simple nonmagnetic free-space electromagnetic cloak which is comprised only of single or multilayer isotropic dielectric claddings. The idea originates from an analytical model of the previously reported metal-plate cloak operating in the visible range.[15] The used model is a simple mathematical concept comprising multiple concentric cylinders of different dielectric permittivities. The choice of the permittivities has been made by considering the tapered conical lines, under the related plane-wave excitation, as a series connection of capacitors. The analytically evaluated response of the described model is shown to have a remarkable coincidence with the corresponding results obtained from full-wave simulations of the actual device for various values of input parameters.

Since the aforementioned simple model appears to resemble so successfully the real-world metal-plate cloaking structure, we tried to adopt an even simpler concept to cloak an impenetrable, perfectly electrically conducting (PEC) volume. This is a clearly more challenging case since one can hide practically anything in a PEC volume by avoiding to interact with the environment, which does not happen for penetrable objects. We introduce uniform or layered dielectric claddings which are optimized by sweeping the relative permittivity values of these covers. The scattering reduction achieved by the proposed devices is quite high, given their simple structure. Moreover, the demonstrated cloaking bandwidths can be considered as wide, a property attributed to the fact that the materials required in the claddings are simple dielectrics with relative permittivities larger than unity.

In addition to the well-known scattering cancellation method,[5,6] several examples of designs of nonmagnetic cloaks using isotropic materials have been presented in the recent literature.[16–19] Some of these designs rely on multilayer claddings made of various isotropic dielectrics with which one can realize an *effectively* anisotropic permittivity or permeability.[16,17] Other concepts utilize radially positioned ferroelectric or glass particles (also creating an effectively

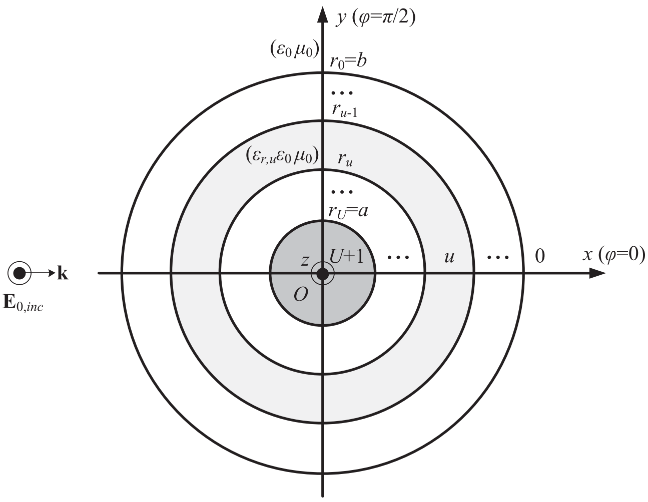

FIG. 1. The geometry of the generic model comprising several concentric cylindrical layers around the cloaked object. The cloaked object is situated in region $(U+1)$.

anisotropic material),[18,19] and another exploits Fabry-Perot resonances in a high-index dielectric.[20] In contrast to these previous works, in this paper we show that multilayer *as well as* single layer isotropic dielectrics with a relative permittivity larger than unity and of moderate value (e.g., $1 < \epsilon_r < 10$) can be used to reduce the total scattering from PEC cylinders under TE incidence (i.e., electric field is parallel to the cylinder's axis). This is an important point in the view of practical realization, since such dielectrics are low-loss and readily available. It should also be noted that the single-layer dielectric covers, commonly employed in scattering cancellation cloaks (achieving to hide dielectric objects instead of PEC ones), require a relative permittivity smaller than unity when cloaking objects in free space. These substances cannot be straightforwardly realized for frequencies where plasmonic materials are not readily available.[21,22]

Although the method of cloaking PEC objects with said simple dielectrics is limited to cylinders with electrically small diameter, we do not employ the dipolar approximation commonly used in the design of "scattering cancellation" cloaks, but instead we take into account all the contributing scattering terms. In the example showed later in this paper (diameter of the cloaked cylinder being one-fifth of the free space wavelength), the first order scattering term (the quadrupole term) is shown to be significant enough to affect the total scattering from the cloaked object. It is also evident that such strong cloaking effects as are theoretically possible with "transformation-optics" cloaks are not possible with the proposed approach, unless the hidden object is electrically extremely small. However, due to the use of simple dielectrics, the proposed approach can be comparable to "transformation-optics" cloaks that have so far been realized and measured.[23] For example, in our example of a PEC cylinder with diameter one-fifth of the free space wavelength, about 60% and 75% reductions in the total scattering width are achieved when using a single-layer and a multilayer dielectric cover, respectively.

II. GENERIC MODEL OF CONCENTRIC CYLINDERS

The proposed model to mimic the physical mechanism of wave propagation in metal-plate devices is fairly simple and its configuration is shown in Fig. 1, where the used cylindrical coordinate system (ρ,ϕ,z) is also defined. There are just U infinitely long cylindrical layers each of which is assigned to an integer number $u = 1,\ldots,U$, occupying the region $r_{u-1} < \rho < r_u$ and being filled with magnetically inert dielectric materials of relative permittivities $\epsilon_{r,u}$. One can clearly note that $r_0 = b$ and $r_U = a$, while the background medium is vacuum (with ϵ_0,μ_0 intrinsic electromagnetic parameters) and is taken as the region 0 $(\rho > b)$ of our configuration with $\epsilon_{r,0} = 1$. The internal cylinder, which corresponds to the region $(U+1)$ $(\rho < a)$, can be either penetrable with relative dielectric constant $\epsilon_{r,(U+1)}$ or PEC. The assumed harmonic time dependence is of the form $e^{+j2\pi f t}$ (f is the operating frequency) and is suppressed throughout the analysis.

We consider a plane-wave excitation of electric field:[24]

$$\mathbf{E}_{0,inc} = \mathbf{z}e^{-jk_0\rho\cos\phi} = \mathbf{z}\sum_{n=-\infty}^{+\infty} j^{-n}J_n(k_0\rho)e^{jn\phi}, \quad (1)$$

where $k_0 = 2\pi f\sqrt{\epsilon_0\mu_0}$ is the free-space wave number and J_n is the nth ordered Bessel function. The total field in each layer $u = 0,\ldots,(U+1)$ possesses only a z component E_u whose expression is given by

$$E_u = \sum_{n=-\infty}^{+\infty}[A_{n,u}J_n(k_u\rho) + B_{n,u}H_n(k_u\rho)]e^{jn\phi}, \quad (2)$$

where $k_u = k_0\sqrt{\epsilon_{r,u}}$, H_n is the nth-ordered Hankel function of the second type, and $A_{n,u}, B_{n,u}$ are sequences of unknown complex coefficients. After imposing the necessary boundary conditions at $\rho = r_u, u = 0,\ldots,(U-1)$, one obtains the following relations connecting the coefficients of two adjacent layers:

$$\begin{bmatrix} A_{n,u} \\ B_{n,u} \end{bmatrix} = \mathbf{T}_{n,u}\cdot\begin{bmatrix} A_{n,(u+1)} \\ B_{n,(u+1)} \end{bmatrix}, \quad (3)$$

for integer n. The explicit form of the transfer matrix[25,26] $\mathbf{T}_{n,u}$ is shown in the two-column Eq. (4).

$$\mathbf{T}_{n,u} = \begin{bmatrix} \frac{J_n'(k_{u+1}r_u)H_n(k_ur_u)k_{u+1} - H_n'(k_ur_u)J_n(k_{u+1}r_u)k_u}{J_n'(k_ur_u)H_n(k_ur_u)k_u - H_n'(k_ur_u)J_n(k_ur_u)k_u} & \frac{H_n'(k_{u+1}r_u)H_n(k_ur_u)k_{u+1} - H_n'(k_ur_u)H_n(k_{u+1}r_u)k_u}{J_n'(k_ur_u)H_n(k_ur_u)k_u - H_n'(k_ur_u)J_n(k_ur_u)k_u} \\ \frac{J_n'(k_ur_u)J_n(k_{u+1}r_u)k_u - J_n'(k_{u+1}r_u)J_n(k_ur_u)k_{u+1}}{J_n'(k_ur_u)H_n(k_ur_u)k_u - H_n'(k_ur_u)J_n(k_ur_u)k_u} & \frac{J_n'(k_ur_u)H_n(k_{u+1}r_u)k_u - H_n'(k_{u+1}r_u)J_n(k_ur_u)k_{u+1}}{J_n'(k_ur_u)H_n(k_ur_u)k_u - H_n'(k_ur_u)J_n(k_ur_u)k_u} \end{bmatrix}. \quad (4)$$

By successive application of (3) for $u = 0,\ldots,(U-1)$, one arrives at

$$\begin{bmatrix} A_{n,0} \\ B_{n,0} \end{bmatrix} = \mathbf{T}_{n,0} \cdot \mathbf{T}_{n,1} \cdots \mathbf{T}_{n,(U-1)} \cdot \begin{bmatrix} A_{n,U} \\ B_{n,U} \end{bmatrix} = \begin{bmatrix} M_{11}(n) & M_{12}(n) \\ M_{21}(n) & M_{22}(n) \end{bmatrix} \cdot \begin{bmatrix} A_{n,U} \\ B_{n,U} \end{bmatrix}. \quad (5)$$

Depending on what is the filling material of the core region $(U+1)$, namely dielectric (diel.) with permittivity $\epsilon_{r,(U+1)}$ or PEC, the following expression for the coefficients of the U region (boundary conditions at $\rho = r_U = a$) is formulated:

$$\begin{bmatrix} A_{n,U} \\ B_{n,U} \end{bmatrix} = \begin{cases} \mathbf{T}_{n,U} \cdot \begin{bmatrix} A_{n,(U+1)} \\ B_{n,(U+1)} \end{bmatrix}, & \text{diel.} \\ \begin{bmatrix} 1 \\ -\frac{J_n(k_U r_U)}{H_n(k_U r_U)} \end{bmatrix} A_{n,U}, & \text{PEC} \end{cases}. \quad (6)$$

By inspection of (1), it is directly obtained that $A_{n,0} = j^{-n}$, while the physical demand for the bounded field into the cloaked region, is translated into $B_{n,(U+1)} = 0$. Therefore, the scattering field by the device into the vacuum background area can be readily derived through the related coefficient $B_{n,0}$ by combining (5), (6) in each case:

$$B_{n,0} = \begin{cases} j^{-n} \frac{M_{21}(n)[\mathbf{T}_{n,U}]_{11} + M_{22}(n)[\mathbf{T}_{n,U}]_{21}}{M_{11}(n)[\mathbf{T}_{n,U}]_{11} + M_{12}(n)[\mathbf{T}_{n,U}]_{21}}, & \text{diel.} \\ j^{-n} \frac{M_{21}(n) - M_{22}(n)\frac{J_n(k_U r_U)}{H_n(k_U r_U)}}{M_{11}(n) - M_{12}(n)\frac{J_n(k_U r_U)}{H_n(k_U r_U)}}, & \text{PEC} \end{cases}. \quad (7)$$

The notation $[\mathbf{D}]_{vw}$ is used for the (v,w) element of the matrix $\mathbf{D}$.

III. OPTICAL CLOAK MADE OF CONICAL PLATES

In this section, we are going to test the model analyzed above on how accurately it describes the operation of an actual device. It has been shown that the so-called metal-plate cloak configuration, already constructed for radio frequencies, can work in the optical band as well.[15] The optical device comprises periodically stacked, solid, conical, silver plates (of outer diameter $2b$) positioned around the cylindrical cloaked region (of diameter $2a$) with a small air gap (of thickness g) in between. One period of this structure is shown in Fig. 2 and it is a (hollow) waveguide, with linearly decreasing height from H (at $\rho = b$) to h (at $\rho = a$), leading the z-polarized fields around the cloaked object. In a specific case,[15] the constructing material of the plates is silver, the (optimal) operating frequency $f_0 \cong 590$ THz, while the physical dimensions are given by $b = 113$ nm, $a = 50$ nm, $g = 15$ nm, $H = 13$ nm, and $h = 2.5$ nm. It should be stressed that the cloaked region is filled with silver, too. In the following, both in the analytical model and in the numerical simulations, we use the data for the complex frequency-dependent permittivity of silver given by Johnson and Christy.[27] Note that the silver's dielectric constant is negative close to $f = f_0$, namely, $\text{Re}[\epsilon_{r,\text{silver}}(f_0)] \cong -10$.

With reference to the model analyzed above, we make use of the formulas corresponding to a dielectric core since the region $(U+1)$ is solid silver; therefore, $\epsilon_{r,(U+1)} = \epsilon_{r,\text{silver}}$. In addition, the Uth layer is the air gap; thus, $\epsilon_{r,U} = 1$ and $r_{U-1} = a + g$. It is apparent that our model does not take into account the periodic dimension variation of the actual configuration with respect to z, since the stack of conical plates is replaced by a structure of homogeneous concentric cylinders. However, the radial inhomogeneity (contrary to the axial one) of the cloaking device is possible to be imitated by choosing properly the dielectric permittivities of the cylindrical layers. But how can one compute $\epsilon_{r,u}$ for $u = 1,\ldots,(U-1)$ in a correct and reliable way?

In Fig. 3, one observes the side view of half the biconical cell as the shape possesses cylindrical symmetry. Assume that the cylindrical layer in the vicinity of the representative surface $\rho = r_u$ (containing both the corresponding vacuum aperture and the related solid silver plates), would be replaced by a (locally) homogeneous dielectric, with the same external dimensions, of relative permittivity $\epsilon_{r,u}$, whose value should be estimated. From the similarity of triangles, the height ζ_u of the representative vacuum aperture is found equal to

$$\zeta_u = H\frac{r_u - a - g}{b - a - g} + h\frac{b - r_u}{b - a - g}. \quad (8)$$

If the inclination of the tapered plates is relatively low (which is the case for the specific choice of b,a,g,H,h), then the z-polarized electric field will be almost normal to the sloping boundaries separating the silver from the vacuum. As a result, the arbitrary cross section at $\rho = r_u$ can be considered as a series connection of three capacitors: two with the height $(H - \zeta_u)/2$ filled with silver and one (placed in between) with height

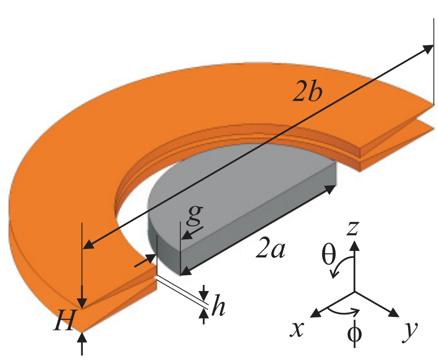

FIG. 2. (Color online) The physical configuration of a single conical metal-plate cell. One-half (cut along the xz plane) of a single cell is shown.

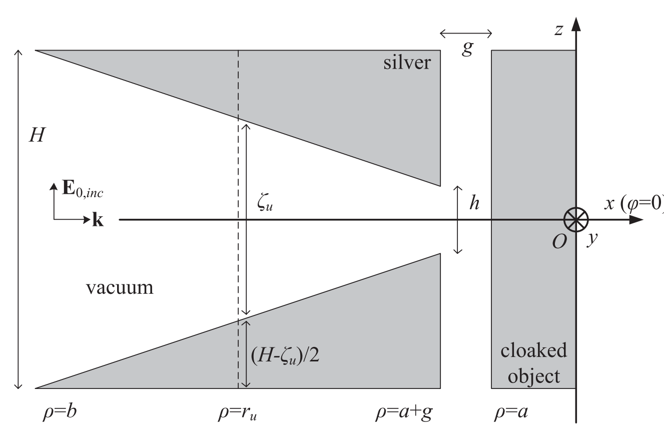

FIG. 3. The side view of a single conical metal-plate cell.

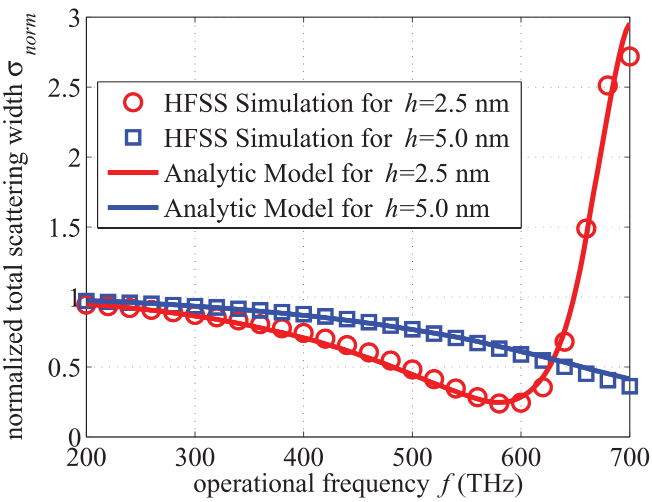

FIG. 4. (Color online) The total scattering width of the metal-plate structure normalized by the corresponding quantity of the uncloaked object as a function of the operating frequency. The HFSS simulations for the actual device are compared with the results obtained through the analytical model implementation. Plot parameters are as follows: $b = 113$ nm, $a = 50$ nm, $g = 15$ nm, $H = 13$ nm, $U = 8$.

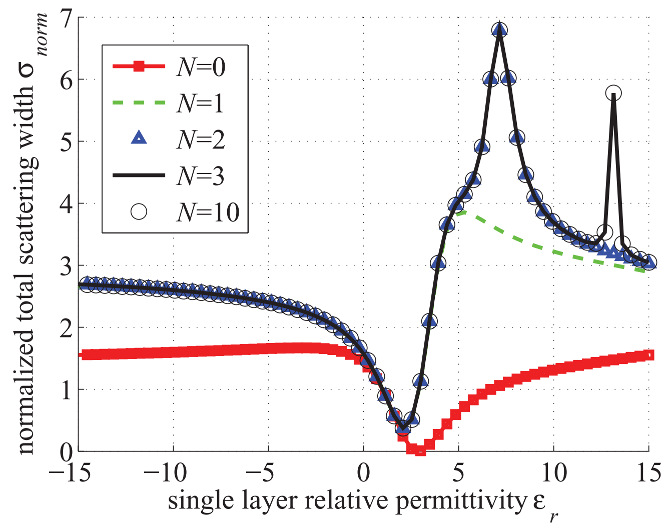

FIG. 5. (Color online) The normalized total scattering width of a silver rod surrounded by a homogeneous cladding as function of the relative permittivity of the cladding. Plot parameters are as follows: $b = 113$ nm, $a = 50$ nm, $f_0 = 590$ THz, $U = 1$.

ζ_u filled with vacuum. Accordingly, this serial cluster can be replaced by a single capacitor with a dielectric material of relative permittivity $\epsilon_{r,u}$, based on the well-known equivalent capacitance formula: $\frac{H}{\epsilon_{r,u}} = \frac{(H-\zeta_u)/2}{\epsilon_{r,\text{silver}}} + \frac{\zeta_u}{1} + \frac{(H-\zeta_u)/2}{\epsilon_{r,\text{silver}}}$. In this sense, the rigorous expression for (effective) $\epsilon_{r,u}$ is hyperbolic with respect to both r_u, ζ_u and is given as follows:

$$\epsilon_{r,u} = \frac{H\epsilon_{r,\text{silver}}}{H + (\epsilon_{r,\text{silver}} - 1)\zeta_u}. \tag{9}$$

We use layers of the same thickness (due to the constant inclination of the silver conical plates), namely, $r_u = b - \frac{u}{U-1}(b - a - g)$ for $u = 1,\ldots,(U-1)$. After having defined all the parameters of the theoretical structure $(\epsilon_{r,u}, r_u)$ based on the actual quantities of the real configuration $(b,a,g,H,h,\epsilon_{r,\text{silver}})$, we can develop the corresponding model and quantify its efficiency. It is remarkable that (for the given ranges of the input parameters) all the relative permittivities $\epsilon_{r,u}$ are positive and greater than unity.

A crucial quantity for the operation and the performance of a cloaking device is the total scattering width of the whole cylindrical structure normalized by the corresponding width of the uncloaked one. The smaller the magnitude, the better the device serves its purpose. The quantity is defined by

$$\sigma_{\text{norm}} = \frac{\int_0^{2\pi} \left| \sum_{n=-N}^{N} B_{n,0} j^n e^{jn\phi} \right|^2 d\phi}{\int_0^{2\pi} \left| \sum_{n=-N}^{N} B'_{n,0} j^n e^{jn\phi} \right|^2 d\phi}, \tag{10}$$

where N is a sufficiently large integer to achieve convergence for the series and $B'_{n,0}$ denotes the coefficients of the scattering field for the uncloaked cylinder. In Fig. 4 we show the variation of σ_{norm} with respect to the operating frequency for two alternative heights $h = 2.5$ and $h = 5$ nm. In each case, we perform a numerical simulation of the actual metal-plate device with ANSYS HFSS full-wave software.[28] The obtained numerical results are compared to those derived through implementation of the corresponding multilayer analytical model, in the way described above.

The excellent agreement between the two sets of data is directly demonstrated. Such a coincidence could be anticipated since the comparison is made between the response of an actual device and that of a model trying to imitate its behavior. In other words, the aforementioned approximation of the effective medium is verified and found suitable for modeling analytically the metal-plate cloak. When it comes to the results themselves, there are large frequency bands where the magnitude σ_{norm} takes values much smaller than unity. In addition, the metal-plate cloaking for $h = 5$ nm is functional at larger frequencies than the device with $h = 2.5$ nm does.

In the considered numerical example, the electrical dimension of the structure is relatively small which could make one think that just the omnidirectional ($N = 0$) term of the sums in (10), is sufficient to evaluate the quantity σ_{norm}. If such an argument was correct, then the cloaking behavior demonstrated above would be attributed to the well-known "scattering cancellation" phenomenon.[5,6] We believe that this is not the case since the electrical size of the structure is not so small. To support this statement, we assume the corresponding "scattering cancellation" device for the silver cloaked object of radius a. This is a homogeneous cladding of relative permittivity ϵ_r and radius b without an air gap. In Fig. 5, we evaluate the normalized total scattering width σ_{norm} from (10) as function of ϵ_r for several truncation limits N. We are mainly interested in the interval $0 < \epsilon_r < 5$ where smaller σ_{norm} are observed and it is clear that for $N \geqslant 2$, the result does not change significantly. However, the first harmonic ($N = 1$) is necessary to be included in the computation since the zeroth term is minimized (and more specifically nullified) at larger ϵ_r. The nullification of the omnidirectional term is the "scattering cancellation" solution and it is different from the observed one. We do not claim that perfect cloaking beyond the dipolar limit is possible with the use of so simple equipment (single layer, isotropic material). In fact, we conclude that even for

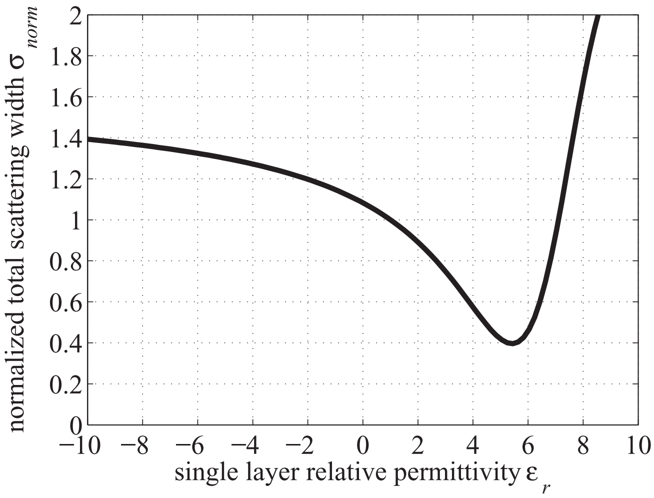

FIG. 6. The normalized total scattering width of a PEC cylinder surrounded by a homogeneous cladding, as a function of the relative permittivity of the cladding. Sweep of ϵ_r is made to find the optimal value for cloaking at the frequency f_0 with fixed values of a and b. The uniform ($U = 1$) cloak has a thickness $(b - a)$ (with $b = 2a = \lambda_0/5$) and is made of a dielectric material with ϵ_r.

scatterers with small electrical size, the contribution of the higher order terms ($N \geqslant 1$) could be significant in finding the optimal antiscattering cladding. Thus, considering only the zeroth term, could lead to erroneous results.

IV. CLOAKING OF CONDUCTING CYLINDERS

As was shown in the previous section, a multilayer as well as a single-layer dielectric cover can be used to reduce the scattering from a cylinder composed of an ϵ_r-negative material. The same can be of course achieved by using the well-known "scattering cancellation method." However, this technique has not been used so far to hide a conducting object by a layered or a uniform cover made of dielectrics with $\epsilon_r > 1$. Here we show, by using the analytical model described in Sec. II that such a case is possible at least for conducting cylinders of moderate electrical size.

The electromagnetic fields weakly penetrate the cloaked silver cylinder studied in the previous section, so it is expected that also conducting cylinders (i.e., impenetrable objects) could also be partially cloaked with simple multilayer or even uniform covers. However, a clear distinction should be made between the conical structure of Fig. 2 (including its analytical model) studied in the previous section, and the cloaking of perfectly conducting cylinders studied in this section. The previous was a device operating in the visible whereas the latter is a much more general case, only restricted by the fact that, for example, metals are not well conducting at very high frequencies. Thus, the results presented in this section apply mostly to lower frequencies, such as microwaves (where metals are very good conductors). This also implies that dielectrics with the relative permittivities larger than unity with moderate losses are readily available. In the following, to present a generalized case, we normalize the dimensions and the results to the free-space wavelength λ_0.

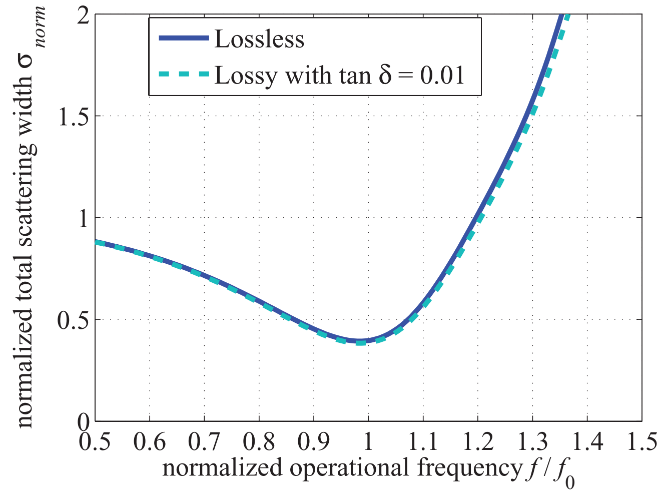

FIG. 7. (Color online) Normalized total scattering width as a function of the normalized frequency. The uniform ($U = 1$) cloak has a thickness $(b - a)$ (with $b = 2a = \lambda_0/5$) and is made of a dielectric material with $\epsilon_r = 5.42$.

We study an electrically thin, PEC cylinder with radius $a = \lambda_0/10$ (the electrical size of this cylinder is of the same order as the cloaked object in the previous section). We assign a dielectric cover around this cylinder and consider three cases: (i) a uniform cover with constant ϵ_r, (ii) a multilayer cover with linearly varying $\epsilon_{r,u}$, (iii) a multilayer cover with hyperbolically varying $\epsilon_{r,u}$ [as in (9)]. For simplicity, we choose $b = 2a$ in all the studied cases. However, the choice of b and a can be made freely; the resulting value of (optimal) $\epsilon_{r,U}$ of the cover material simply changes if the values of a and b are changed.

In the first case, we assign a constant ϵ_r to the material surrounding the PEC cylinder. The analytical model of Sec. II is used to optimize the value of ϵ_r (see Fig. 6). It is evident that for these values of a and b, the optimal value is $\epsilon_r = 5.42$ and with that, the normalized total scattering width of the cloaked PEC cylinder is slightly less than 0.4, that is, the cloak

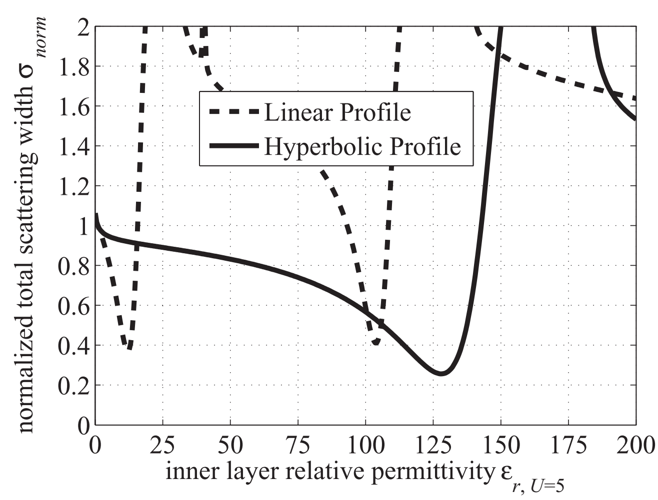

FIG. 8. Sweep of $\epsilon_{r,U=5}$ to find the optimal value for cloaking at the frequency f_0 with fixed values of a and $b = 2a = \lambda_0/5$. Linear and hyperbolic profiles ($U = 5$) are considered.

TABLE I. Values of relative permittivities with linearly changing ϵ_r.

u	1	2	3	4	5
$\epsilon_{r,u}$	3.22	5.44	7.66	9.88	12.1

TABLE II. Values of relative permittivities with hyperbolically changing ϵ_r.

u	1	2	3	4	5
$\epsilon_{r,u}$	1.25	1.66	2.47	4.85	128

reduces the total scattering width of the PEC cylinder by more than 60%.

The frequency dependence of the normalized total scattering width is illustrated in Fig. 7, demonstrating a reasonably broadband cloaking effect:The relative bandwidth where the total scattering width of the PEC cylinder is reduced to half or less is about 21%. It is quite interesting to note that moderate losses do not deteriorate the cloaking effect; with a loss tangent of 0.01, which is very typical to commercially available dielectrics in the microwave region, the cloaking effect is actually slightly improved compared to the lossless case.

In the second scenario, we model the case of Fig. 1 for $U = 5$ and the value of $\epsilon_{r,u}$ depending on the layer number u linearly. To find the optimal values of $\epsilon_{r,u}$ for cloaking at $f = f_0$, we plot σ_{norm} as a function of $\epsilon_{r,5}$ (i.e., the maximum value of ϵ_r), while $\epsilon_{r,u}$ (for $u = 1,2,3,4$) is linearly decreasing from $\epsilon_{r,5}$ to $\epsilon_{r,0} = 1$. The result shown in Fig. 8 demonstrates that for the chosen dimensions a and b, the lowest positive value of $\epsilon_{r,5}$, corresponding to a minimum in the normalized total scattering width, is $\epsilon_{r,5} = 12.1$. In that case, the relative permittivities of the five-layer cloak are as shown in Table I.

With the values of Table I, the normalized total scattering width as a function of the frequency looks as shown in Fig. 9. In the same graph, the curve for the constant-permittivity cladding is depicted for comparison and also the corresponding numerical results obtained with ANSYS HFSS (showing good agreement with our analytical findings) have been attached.

Finally, we study the third scenario (i.e., the case with ϵ_r depending hyperbolically on the layer number $u = 0,\ldots,U$), where again $U = 5$. The variation of the normalized total

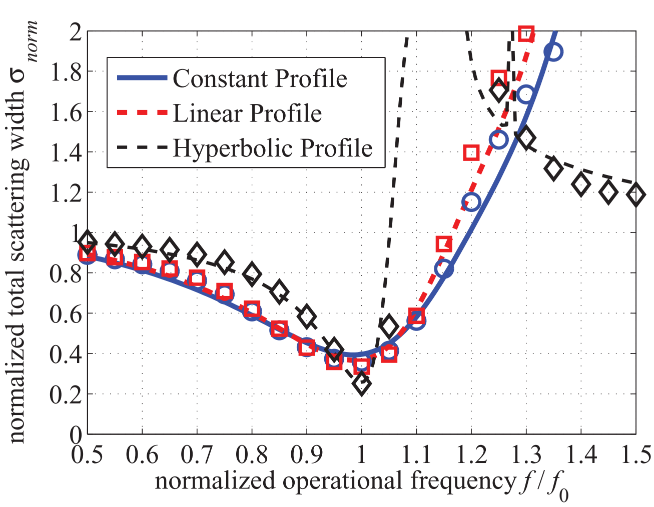

FIG. 9. (Color online) Normalized total scattering width as a function of the normalized frequency for constant, linear, and hyperbolic profile ($U = 5$). The circles, squares, and diamonds denote the numerical results of each of the previous cases, respectively.

scattering width σ_{norm} with respect to $\epsilon_{r,5}$, is shown in Fig. 8, resulting in the value $\epsilon_{r,5} = 128$ for optimal operation at f_0. With this value, the relative permittivities of the five-layer cloak are as shown in Table II. The frequency dependence of the normalized total scattering width is illustrated in Fig. 9. Again, we verify our analytical evaluations by plotting the simulation results in the same figure.

Comparing the three curves of Fig. 9, we can conclude that changing the profile of $\epsilon_{r,u}$ from constant to linear and hyperbolic, improves the obtained scattering reduction at the frequency f_0, but at the same time, the bandwidth of efficient cloaking decreases. Concerning practical issues, the constant and linear profiles are easily realizable (the required values of $\epsilon_{r,u}$ are feasible), whereas the hyperbolic profile is far from practical. However, the cloaking performance with the constant profile is not much different from the linear profile, so it may not be worth the increased complexity to use even the linear one.

It is clear that cloaking of PEC objects with simple dielectric covers is far from ideal cloaking such that is in theory possible with, for example, "transformation-optics." However, it is evident that the cloaking efficiencies presented in this work, are comparable to the experimental and numerical results obtained with various other cloaking techniques that have been realized with composite metamaterials.[21,23,29,30]

V. CONCLUSIONS

We have presented a very simple analytical concept, based on transfer matrices at multilayered cylindrical structures. It has been found that this model describes accurately the previously reported cloaking effect obtained with conical silver plates in the visible frequency range. The effectiveness of the analytical model is verified by comparing the results of the normalized total scattering widths originating from it, to results obtained by numerical simulation software. The fidelity of the proposed concept allows it to be used in device design and to save computational resources due to its simplicity. The same analytical model is also used to demonstrate that, surprisingly, cloaking of impenetrable (perfectly conducting) objects is possible with simple dielectric covers whose relative permittivity surpasses unity. Such a property renders this type of electromagnetic configurations easily realizable, unlike most other cloaking devices reported in the literature.

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