PHYSICAL REVIEW B **85**, 100503(R) (2012)

Anharmonic Josephson current in junctions with an interface pair breaking

Yu. S. Barash
Institute of Solid State Physics, Russian Academy of Sciences, Chernogolovka, Moscow District, 142432 Russia
(Received 14 November 2011; published 5 March 2012)

Planar superconducting junctions with a large effective Josephson coupling constant and a pronounced interface pair breaking are shown to represent weak links with small critical currents and strongly anharmonic current-phase relations. The supercurrent near T_c is described taking into account the interface pair breaking as well as the current depairing and the Josephson coupling-induced pair breaking of arbitrary strengths. An analytical expression for the anharmonic supercurrent, which is in excellent agreement with the numerical data presented, is obtained. In junctions with a large effective Josephson coupling constant and a pronounced interface pair breaking, the current-induced depairing is substantially enhanced in the vicinity of the interface thus having a crucial influence on the current-phase relation despite a small depairing in the bulk.

DOI: 10.1103/PhysRevB.85.100503 PACS number(s): 74.50.+r, 74.20.De

The Josephson current is one of the remarkable manifestations of quantum coherence on the macroscopic scale in condensed matter physics. The supercurrent depends on the phase difference of the order parameters across the junction interface. The study of the current-phase relation (CPR) in the junctions makes it possible to identify physical processes, which form supercurrents under diverse conditions. It is also beneficial for junction applications. The problem attracted much attention while studying both highly transparent junctions with strongly anharmonic CPRs and tunnel junctions, where the second harmonic of the supercurrent comes into play due to the suppression of the first one.[1–10] The latter takes place in the junctions involving unconventional superconductors with special interface-to-crystal orientations and at 0-π transitions.

One of the earlier theoretical results, clarifying a variety of aspects of the problem, is the anharmonic CPR for the superconducting point contacts.[11–15] Due to the negligibly small current-induced pair breaking at any transparency value, the theory of point contacts is simplified. The depairing plays an important role in forming anharmonic CPRs in highly transparent planar junctions, unlike its negligible role in point contacts. Since the critical current j_c of usual planar junctions becomes, with increasing transparency, comparable with the depairing current $j_{\rm dp}$ in the bulk, the junctions do not represent weak links. In other words, in the junctions, the current-induced depairing brings about a pronounced anharmonicity only in the crossover from the tunnel Josephson current to the bulk superflow.[1,16–18]

There are at least three types of pair breaking processes taking place in charge transport in the superconducting junctions: the pair breaking produced by the phase-dependent Josephson coupling, by the current and by the interface itself. These are the very same effects which can lead to a noticeably anharmonic CPR not only at low or intermediate temperatures, but also near T_c. Here, I show that planar junctions with a large effective Josephson coupling constant and a pronounced interface pair breaking can possess strongly anharmonic CPRs and small critical currents satisfying the condition $j_c \ll j_{\rm dp}$. An enhancement of the current-induced depairing near the interface will be identified. The anharmonic supercurrent near T_c will be obtained within the Ginzburg-Landau (GL) theory in the presence of all three types of the pair breaking processes of arbitrary strengths. Along with the numerical solution based on GL equations, an analytical CPR will be derived and shown to be in excellent agreement with the numerical data in a wide range of parameters. For tunnel junctions, the obtained results present a description of higher harmonics of the supercurrent and extend the known expressions for the first and second harmonics to include the effects of interface pair breaking.

The CPRs obtained earlier near T_c with the microscopic boundary conditions for standard dirty s-wave junctions,[16,19] have been considered in literature solely as the particular properties of the specific systems.[1] The anharmonic CPR obtained in this paper, and influenced by the interface pair breaking, is of general form inherent in the GL theory, and is applicable to a variety of planar junctions including those containing $d_{x^2-y^2}$-wave superconductors and/or magnetic interlayers.

The free energy functional for Josephson junctions near T_c results in the GL equations and the boundary conditions (BC) for them.[20–24] Consider symmetric junctions with a spatially constant width, which is much less than the Josephson penetration length, and with a plane interlayer at $x = 0$ of zero length within the GL approach. Assume the usual form of the GL free energy, which applies, for example, to s-wave and $d_{x^2-y^2}$-wave junctions. If the Josephson coupling $g_J|\Psi_+ - \Psi_-|^2$ is strong, not only this term but all the interface and bulk contributions to the free energy generally participate in the formation of CPRs as a consequence of the dependence of absolute values of the order parameters at the interface on the phase difference. This concerns, in particular, the gradient bulk term $K|\nabla\Psi|^2$ and the interface contribution of the form $g(|\Psi_+|^2 + |\Psi_-|^2)$.

Moving on to the order parameter $f(x)e^{i\chi(x)}$ normalized to $f = 1$ in the bulk without superflow, one gets the first integral of the GL equation in the presence of the supercurrent[25] in the form of

$$\left(\frac{df}{d\tilde{x}}\right)^2 + f^2 - \frac{1}{2}f^4 + \frac{4\tilde{j}^2}{27f^2} = 2f_\infty^2 - \frac{3}{2}f_\infty^4. \qquad (1)$$

Here, $\tilde{x} = x/\xi$, $\xi = \xi(T)$ is the superconducting coherence length, $\tilde{j}$ is the spatially constant normalized current density $\tilde{j} = j/j_{\rm dp} = -(3\sqrt{3}/2)(d\chi/d\tilde{x})f^2$, and f_∞ is the asymptotic value of f in the depth of the superconducting leads.

The BC introduce in the GL theory at least two characteristic lengths $\ell = K/g_J$ and $\delta = K/g$. The effective dimensionless Josephson $g_\ell = g_J\xi(T)/K$ and interface $g_\delta = g\xi(T)/K$ coupling constants, associated with these lengths, will be used below. For symmetric junctions with f continuous through the interface, the BC for f as well as the expression for the Josephson current via f_0 and the phase difference $\chi = \chi_- - \chi_+$ at the interface, are obtained from the BC for complex order parameters:

$$\left(\frac{df}{d\tilde{x}}\right)_\pm = \pm\left(g_\delta + 2g_\ell \sin^2\frac{\chi}{2}\right) f_0, \qquad \tilde{j} = \frac{3\sqrt{3}}{2} g_\ell f_0^2 \sin\chi. \tag{2}$$

Here, the effective phase-dependent extrapolation length $b(\chi) = [\delta^{-1} + 2\ell^{-1}\sin^2(\frac{\chi}{2})]^{-1}$ controls the pair breaking produced by the phase difference and by the interface. Let's denote $g_b(\chi) = (g_\delta + 2g_\ell \sin^2\frac{\chi}{2})$.

Since the material parameters in the normal state g_J and g are not assumed to depend here on T near T_c, one should have $|g_\ell| \gg 1$ and/or $|g_\delta| \gg 1$ quite close to T_c due to large values of $\xi(T)$. However, the coupling constants g_J and g can themselves be very small and the temperature range with large g_ℓ and/or g_δ be too narrow, as it occurs in standard tunnel junctions. Due to a very small surface pair breaking in conventional s-wave junctions, one parameter g_ℓ is usually assumed to describe the interfaces in Eq. (2) rather than both g_ℓ and g_δ as is in the regular case. At $\chi = 0$, such symmetric junctions contain no pair breaking at all, and the BC (2) is reduced to $df(0)/dx = 0$.

If g_J and/or g were very small, one would need to introduce into Eq. (2) the terms of the next order of smallness, in particular, in powers of the order parameter. Such terms could be of importance and bring about additional phase dependence and material-dependent parameters to the problem. Here, only the simplest conditions will be assumed, when Eq. (2) applies to a wide range of values of g_ℓ and g_δ. This agrees with the microscopic model results[16,19,20,26–29] and, for instance, takes place within the GL approach for sufficiently large values of g_ℓ and g_δ, which is the particular focus of this paper.

There is no need to solve differential equation (1) in order to find f_0, and, consequently, to find $\tilde{j}$ via Eq. (2). One puts $x = 0$ in Eq. (1) and, using Eq. (2), eliminates the current and the first derivative of the order parameter. This results in a biquadratic relation between the self-consistent values of f_0^2 and f_∞^2. The second relation between them follows from the current conservation and the asymptotic formulas in the bulk. The current-induced depairing in the bulk is conveniently described via the superfluid velocity $\tilde{j} = (3\sqrt{3}/2)\tilde{v}_s(1 - \tilde{v}_s^2)$, $f_\infty^2 = 1 - \tilde{v}_s^2$.[30,31] Equating the asymptotic expression for the current to that in Eq. (2) with $f_0^2 = (1 - \tilde{v}_s^2)\alpha$, one obtains $\tilde{v}_s = \alpha g_\ell \sin\chi$. Considering that both quantities f_0 and f_∞ as well as the current itself are now expressed via the only variable α, the fourth-order polinomial equation for α follows from the biquadratic relation between f_0 and f_∞:

$$2g_b^2(\chi)\alpha - (1-\alpha)^2\left[1 - \alpha(\alpha+2)g_\ell^2 \sin^2\chi\right] = 0. \tag{3}$$

Equation (3) is exact within the conventional GL approach with BC (2). In the particular case of standard s-wave junctions, $g_b(\chi) = 2g_\ell \sin^2(\chi/2)$. Then Eq. (3) is reduced to Eq. (8) of Ref. 16, if one corrects a misprint $\Gamma_B \to \Gamma_B^2$ in Eq. (8) and identifies the parameter of the GL theory $g_\ell^{-1} = \ell/\xi$ with the model parameter Γ_B entering the microscopic BC for dirty s-wave superconductors.

An analytical solution of the problem can be obtained assuming a small depairing in the bulk $\tilde{j}^2 \ll 1$ that allows to use $f_\infty^2 \approx 1 - (4/27)\tilde{j}^2$ and to disregard the smaller terms on the right-hand side of Eq. (1). Then, one gets from Eqs. (1) and (2) a biquadratic equation for f_0 that results in the analytical solution for the CPR:

$$\tilde{j}(g_\ell, g_\delta, \chi) = \frac{3\sqrt{3}g_\ell \sin\chi}{2\left(1 + 2g_\ell^2\sin^2\chi\right)}\Big[1 + g_b^2(\chi) + g_\ell^2\sin^2\chi - \sqrt{\left(g_b^2(\chi) + g_\ell^2\sin^2\chi\right)^2 + 2g_b^2(\chi)}\Big]. \tag{4}$$

Since only higher-order terms begining with $\propto \tilde{j}^4$ have been neglected in its derivation, the CPR (4) turns out to describe the current behavior almost perfectly if $\tilde{j} < 0.7$. For $\tilde{j} > 0.7$, it gives a good interpolation of the numerical solution based on Eq. (3), resulting in the deviations not exceeding 10%.

As seen in Eqs. (3) and (4), the anharmonic Josephson current $\tilde{j}$ depends, in general, on the two dimensionless effective coupling constants g_ℓ and g_δ and the phase difference χ. According to the simple physical arguments as well as the microscopic results,[26–29] a variation of tunneling parameters principally modifies g_ℓ, while the surface pair breaking mostly contributes to g_δ. This signifies that the junction transparency D enters the combination of microscopic parameters representing g_ℓ. The last statement agrees with the microscopic results for s-wave junctions with nonmagnetic interfaces,[16,19,26–28] where the corresponding combination is sometimes identified as the effective transparency.[32,33] The microscopic estimations of the effective Josephson coupling constant g_ℓ directly follow from those results. In the s-wave tunnel junctions ($D \ll 1$), one gets $g_\ell \sim D\xi(T)(l^{-1} + \xi_0^{-1})$, where l is the mean-free path. In dirty superconductors, the ratio $\xi(T)/l$ can easily reach 100 even at low temperatures. Hence, for small and moderate transparencies, the quantity $g_\ell \sim D\xi(T)/l$ can vary from vanishingly small values in the tunneling limit to those well exceeding 100 near T_c. In highly transparent junctions $[(1 - D) \ll 1]$, the parameter $g_\ell \propto (1 - D)^{-1}$ can be arbitrary large.[34] The quantity g_ℓ can also take on negative values, which correspond to π junctions, as seen in Eq. (4).

The range of variation of the interface coupling g_δ can likewise be quite wide. For s-wave superconductor-insulator interfaces, the Josephson coupling vanishes and the extrapolation length b is reduced to δ. The microscopic estimations of δ in such cases show it to be very large usually resulting in a negligibly small contribution to the BC, unlike the superconductor-normal metal interfaces.[20] The length δ can vary widely for d-wave superconductor-insulator flat surfaces, where it substantially depends on surface-to-crystal orientations.[35–37] Although in this case δ is strongly influenced by the surface roughness, in particular, by faceting.[8]

A regular situation is characterized by a local suppression of the order parameter at the interface. For this condition to hold, the effective extrapolation length $b(\chi)$ should be

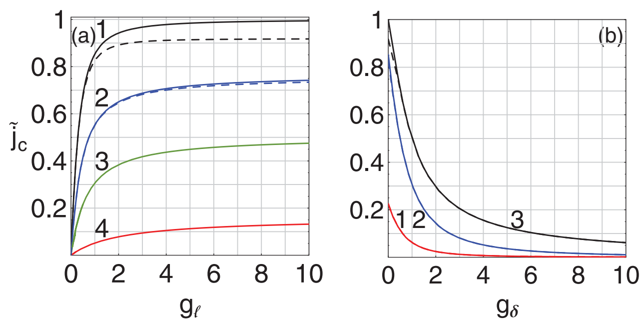

FIG. 1. (Color online) (a) $\tilde{j}_c$ as a function of g_ℓ, taken for various g_δ: (1) $g_\delta = 0$, (2) $g_\delta = 0.4$, (3) $g_\delta = 1$, and (4) $g_\delta = 4$. (b) $\tilde{j}_c$ as a function of g_δ, taken for various g_ℓ: (1) $g_\ell = 0.1$, (2) $g_\ell = 1$, and (3) $g_\ell = 100$.

positive at any phase difference and, hence, $g_\delta,(g_\delta + 2g_\ell) > 0$. A superconducting state occurs locally near the interface above the bulk T_c under the opposite condition $b(\chi) < 0$ with χ ensuring the free energy minimum.[38] Only the simplest conditions $g_{\delta,\ell} > 0$ will be analyzed in detail in this paper, although the main results obtained here apply to substantially more general circumstances. Other conditions, including magnetic field effects and/or negative $g_{\delta,\ell}$, will be studied elsewhere.

Figures 1(a) and 1(b) show the critical current $\tilde{j}_c$ as a function of coupling constants g_ℓ and g_δ. Solid curves have been calculated based on Eq. (3). Dashed curves correspond to the analytical expression (4). Only for a small interface pair breaking ($g_\delta \lesssim 1$) and for $g_\ell \gtrsim 1$, the current $\tilde{j}_c$ becomes comparable with 1, i.e., with the deparing current in the bulk. Thus the condition $g_\ell \gtrsim 1$ is the hallmark of a strong Josephson coupling. Comparatively small deviations of dashed curves from the solid ones are discernible only when the current exceeds about 0.7. With increasing g_δ, the growing interface pair breaking suppresses the critical current. For $g_\delta \gtrsim 4$, the critical current remains quite small $\tilde{j}_c \ll 1$ at any g_ℓ, which would normally occur in conventional tunnel junctions with small effective transparencies. In other words, in the regime of strong interface pair breaking $g_\delta > 4$, the junctions represent weak links at any g_ℓ, including $g_\ell \gtrsim 1$.

Though Eq. (4) is a combined result of all depairing effects, the origin of its characteristic anharmonic features is traced back unambiguously. The whole of the phase-dependence in Eq. (4), except for that contained in $g_b(\chi)$, is generated by the current via f_∞ on the right hand side or by the last term on the left hand side in Eq. (1). Such dependence would retain the CPR (4) unchanged under the transformation $\chi \to \pi - \chi$. The symmetry is destroyed by the phase dependence of $g_b^2(\chi)$, which originates from the BC (2) and can become pronounced, if $|g_\delta| \lesssim 2|g_\ell|$.

Whereas the CPR (4) is derived by assuming small depairing effects in the bulk, the depairing can be of crucial importance in Eq. (4) within its domain of applicability. This is the case in the presence of a pronounced interface pair breaking, where an enhancement of the current-induced depairing, unlike the bulk, occurs near interfaces of junctions with $g_\ell \gg 1$. In particular, the phase-dependent term in the denominator in Eq. (4), which is directly induced by the depairing, plays a key role in the case $g_\ell \gg 1$ in restricting the normalized current value. The bracketed expression in the denominator originates from the coefficient before f_0^4 in the biquadratic equation for f_0. The relative depairing correction coming from the bulk is $(8/27)\tilde{j}^2 = 2g_\ell^2 f_0^4 \sin^2\chi$ and its smallness signifies $2g_\ell^2 \sin^2\chi f_0^4 \ll 1$. As seen, the term $2g_\ell^2 \sin^2\chi$ in the denominator is allowed to exceed the unit considerably, when the condition $2g_\ell^2 \sin^2\chi f_0^4 \ll 1$ holds at the expense of a strongly suppressed order parameter at the interface $f_0^4 \ll 1$. Numerical results corroborate that, if $g_\delta \gtrsim 4$, the condition is satisfied at any g_ℓ including $g_\ell \gg 1$ [see also Figs. 1(a) and 1(b)]. This validates keeping Eq. (4) without its expanding in powers of $g_\ell^2 \sin^2\chi$ and explains the quantitative applicability of Eq. (4) to junctions with the pronounced interface pair breaking at arbitrary g_ℓ.

A number of specific CPRs follow from Eq. (4) under a variety of particular conditions. Consider here two basic examples. The tunneling limit shows up in Eq. (4) under the condition $|g_\ell| \ll 1$. Developing Eq. (4) as series in g_ℓ at any value of g_δ, one obtains numerous harmonics whose weight is determined by g_ℓ and g_δ rather than by the transparency itself. The first- and the second-order terms result in

$$\tilde{j} \approx \tilde{j}_{c1}^{(1)} \left[\sin\chi - \frac{2g_\ell \mathrm{sgn}(g_\delta)}{\sqrt{2+g_\delta^2}} \left(\sin\chi - \frac{1}{2}\sin 2\chi \right) \right]. \quad (5)$$

Here, $\tilde{j}_{c1}^{(1)} = (3\sqrt{3}/4)g_\ell(\sqrt{2+g_\delta^2} - |g_\delta|)^2$ is the main contribution to the first harmonic $\tilde{j}_1 = \tilde{j}_{c1}\sin\chi$ that is applicable at any g_δ. Under the condition $|g_\delta|,|g_\ell| \ll 1$ it is reduced to the well-known result for tunnel junctions $\tilde{j}_{c0} \equiv (3\sqrt{3}/2)g_\ell$, which is only justified when disregarding the interface pair breaking. In the opposite limit $g_\delta^2 \gg 1$, the pair breaking strongly suppresses the current and $\tilde{j}_{c1}^{(1)} \approx \tilde{j}_{c0}/(2g_\delta^2) \ll \tilde{j}_{c0}$, as is also known.[4,20–22,24,35,39] In particular, the original current $j_{c1}^{(1)} = \tilde{j}_{c1}^{(1)} j_{\mathrm{dp}} \propto (T_c - T)$ for $|g_\delta| \ll 1$ and $j_{c1}^{(1)} \propto (T_c - T)^2$ for $g_\delta^2 \gg 1$ near T_c. The second-order terms in g_ℓ bring about the main contribution to the second harmonic $\tilde{j}_2 = \tilde{j}_{c2}\sin 2\chi$ as well as corrections to the first one. The relative weight of the second harmonic in Eq. (5) diminishes with increasing g_δ^2. The sign of $\tilde{j}_{c1}$ coincides with the sign of g_ℓ, while the sign of j_{c2} is determined by the sign of g_δ. For small pair breaking $0 < g_\delta \ll 1$, the second-order term $\propto g_\ell^2$ is simplified to the following correction to the current $-\sqrt{2}j_{c0}g_\ell[\sin\chi - (1/2)\sin 2\chi]$, in agreement with the corresponding microscopic results[16,40] for dirty and pure s-wave junctions. Note that the phase dependence generated by the current depairing shows up in Eq. (4) beginning with the third order terms in g_ℓ.

The second example reveals the strongly anharmonic features contained in Eq. (4). Consider junctions with the strong interface pair breaking $g_\delta^2 \gg 1$. Then a comparatively simple approximate expression follows from Eq. (4):

$$\tilde{j} \approx \frac{3\sqrt{3}g_\ell \sin\chi}{4\left[g_\delta^2 + 4(g_\delta + g_\ell)g_\ell \sin^2\frac{\chi}{2}\right]}. \quad (6)$$

The corresponding critical current $\tilde{j}_c = 3\sqrt{3}g_\ell/4|g_\delta(g_\delta + 2g_\ell)| \ll 1$ is always small. The associated phase difference is determined by the relation $\sin\chi_c = |g_\delta(g_\delta + 2g_\ell)|/[(g_\delta + g_\ell)^2 + g_\ell^2]$. It varies widely; χ_c is small $\approx (g_\delta/g_\ell)$, if $g_\ell \gg g_\delta$,

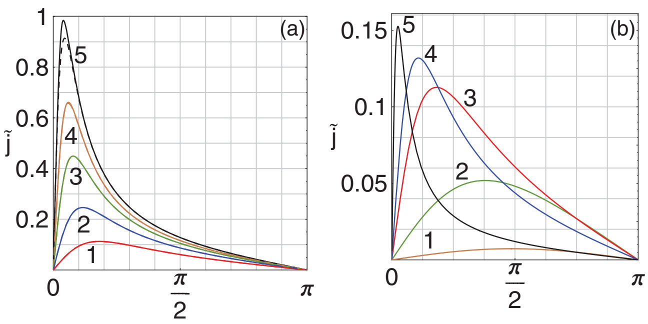

FIG. 2. (Color online) (a) CPRs $\tilde{j}(\chi)$ for $g_\ell = 5$ and various g_δ: (1) $g_\delta = 4$, (2) $g_\delta = 2$, (3) $g_\delta = 1$, (4) $g_\delta = 0.5$, and (5) $g_\delta = 0$. (b) CPRs for $g_\delta = 4$ taken for various g_ℓ: (1) $g_\ell = 0.1$, (2) $g_\ell = 1$, (3) $g_\ell = 5$, (4)$g_\ell = 10$, and (5)$g_\ell = 50$.

and approaches $\pi/2$ in the opposite limit $g_\delta \gg g_\ell$. Strongly anharmonic CPRs show up in Eq. (6) under the conditions $g_\ell^2 \gg g_\delta^2 \gg 1$. Also one has $j_c \propto (T_c - T)^2$. Thus, at finite g, the temperature dependence $j_c(T)$ is quadratic quite close to T_c, where $g_\delta \gg 1$. With increasing $T_c - T$, a crossover to the linear dependence on the temperature takes place in the region $T_c - T \ll T_c$, for sufficiently small g.

Some of the CPRs $\tilde{j}(\chi)$ are shown in Figs. 2(a) and 2(b). Except for the first curve in Fig. 2(b), the strongly anharmonic CPRs in junctions with large Josephson couplings are displayed. As seen in Fig. 2(a), the heights of the anharmonic peaks diminish considerably and the peak positions change weakly, when the interface pair breaking goes up. Although the anharmonicity can be well pronounced even in the presence of quite a large pair breaking. This concerns, in particular, the curve 1 in Fig. 2(a), which is identical to the curve 3 in Fig. 2(b) shown there in a different scale. Equation (4) describes the CPRs almost perfectly and the corresponding dashed curves can be distinguished from the exact solid ones only near the high peak of curve 5 in Fig. 2(a). All curves in Fig. 2(b) are also well approximated by a simple formula (6) with deviations (not shown) approaching only several percent. However, in contrast to Eq. (4), Eq. (6) does not apply to describing upper three curves in Fig. 2(a). The CPR similar to Eq. (6) was found earlier within the microscopic description of the dirty s-wave junctions with metallic interlayers.[19] The strong pair breaking can take place in those junctions, if the interlayer conductivity considerably exceeds the normal conductivity of the superconducting metal.

In conclusion, the paper reveals the qualitative features and develops the quantitative description of the anharmonic Josephson current near T_c. The interface pair breaking as well as the current depairing and the Josephson coupling-induced pair breaking have been taken into account and shown to play an important part in forming the CPR. The results obtained, in particular, concern the junctions involving d-wave superconductors and/or magnetic interlayers.

The support of RFBR grant No. 11-02-00398 is acknowledged.

[1] A. A. Golubov, M. Y. Kupriyanov, and E. Il'ichev, Rev. Mod. Phys. **76**, 411 (2004).
[2] C. C. Tsuei and J. R. Kirtley, Rev. Mod. Phys. **72**, 969 (2000).
[3] A. I. Buzdin, Rev. Mod. Phys. **77**, 935 (2005).
[4] V. B. Geshkenbein and A. I. Larkin, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 306 (1986) [JETP Lett. **43**, 395 (1986)].
[5] Y. Tanaka, Phys. Rev. Lett. **72**, 3871 (1994).
[6] S. Yip, Phys. Rev. B **52**, 3087 (1995).
[7] E. Il'ichev, V. Zakosarenko, R. P. J. IJsselsteijn, H. E. Hoenig, V. Schultze, H.-G. Meyer, M. Grajcar, and R. Hlubina, Phys. Rev. B **60**, 3096 (1999).
[8] H. Hilgenkamp and J. Mannhart, Rev. Mod. Phys. **74**, 485 (2002).
[9] T. Lindström, S. A. Charlebois, A. Y. Tzalenchuk, Z. Ivanov, M. H. S. Amin, and A. M. Zagoskin, Phys. Rev. Lett. **90**, 117002 (2003).
[10] T. Lindström, J. Johansson, T. Bauch, E. Stepantsov, F. Lombardi, and S. A. Charlebois, Phys. Rev. B **74**, 014503 (2006).
[11] I. O. Kulik and A. N. Omelyanchuk, Pis'ma Zh. Eksp. Teor. Fiz. **21**, 216 (1975) [JETP Lett. **21**, 96 (1975)].
[12] I. O. Kulik and A. N. Omelyanchuk, Fiz. Nizk. Temp. **3**, 945 (1977) [Sov. J. Low Temp. Phys. **3**, 459 (1977)].
[13] W. Habekorn, H. Knauer, and J. Richter, Phys. Status Solidi A **47**, K161 (1978).
[14] K. K. Likharev, Rev. Mod. Phys. **51**, 101 (1979).
[15] A. V. Zaitsev, Zh. Eksp. Teor. Fiz. **86**, 1742 (1984) [Sov. Phys. JETP **59**, 1015 (1984)].
[16] M. Y. Kupriyanov, Pis'ma Zh. Eksp. Teor. Fiz. **56**, 414 (1992) [JETP Lett. **56**, 399 (1992)].
[17] F. Sols and J. Ferrer, Phys. Rev. B **49**, 15913 (1994).
[18] J. K. Freericks, B. K. Nikolić, and P. Miller, Int. J. Mod. Phys. B **16**, 531 (2002).
[19] Z. G. Ivanov, M. Y. Kupriyanov, K. K. Likharev, S. V. Meriakri, and O. V. Snigirev, Fiz. Nizk. Temp. **7**, 560 (1981) [Sov. J. Low Temp. Phys. **7**, 274 (1981)].
[20] P. G. de Gennes, *Superconductivity of Metals and Alloys* (Addison Wesley, Reading, MA, 1966).
[21] S.-K. Yip, O. F. De Alcantara Bonfim, and P. Kumar, Phys. Rev. B **41**, 11214 (1990).
[22] M. Sigrist and K. Ueda, Rev. Mod. Phys. **63**, 239 (1991).
[23] E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics II: Theory of the Condensed State* (Butterworth-Heinemann, Oxford, 1995).
[24] V. P. Mineev and K. Samokhin, *Introduction to Unconventional Superconductivity* (Gordon & Breach Science, New York, 1999).
[25] J. S. Langer and V. Ambegaokar, Phys. Rev. **164**, 498 (1967).
[26] V. P. Galaiko, A. V. Svidzinskii, and V. A. Slyusarev, Zh. Eksp. Teor. Fiz. **56**, 835 (1969) [Sov. Phys. JETP **29**, 454 (1969)].
[27] E. N. Bratus' and A. V. Svidzinskii, Teor. Mat. Fiz. **30**, 239 (1977) [Theor. Math. Phys. **30**, 153 (1977)].
[28] A. V. Svidzinskii, *Spatially Innhomogeneous Problems in the Theory of Superconductivity* (Nauka, Moscow, 1982).
[29] V. B. Geshkenbein, Zh. Eksp. Teor. Fiz. **94**, 368 (1988) [Sov. Phys. JETP **67**, 2166 (1988)].
[30] J. Bardeen, Rev. Mod. Phys. **34**, 667 (1962).

[31] M. Tinkham, *Introduction to Superconductivity*, 2nd ed. (McGraw-Hill, New York, 1996).

[32] E. V. Bezuglyi, E. N. Bratus', and V. P. Galaiko, Low Temp. Phys. **25**, 167 (1999).

[33] E. V. Bezuglyi, A. S. Vasenko, V. S. Shumeiko, and G. Wendin, Phys. Rev. B **72**, 014501 (2005).

[34] The solutions based on (3) or (4) satisfy the relation $|g_b|f_0 \lesssim 1$, in particular, at large values of $|g_\ell|$ and/or $|g_\delta|$. In view of (2), this agrees with the condition that a strong suppression of the order parameter on each side of the interface takes place on a scale comparable with $\xi(T)$.

[35] Y. S. Barash, A. V. Galaktionov, and A. D. Zaikin, Phys. Rev. B **52**, 665 (1995).

[36] M. Alber, B. Bäuml, R. Ernst, D. Kienle, A. Kopf, and M. Rouchal, Phys. Rev. B **53**, 5863 (1996).

[37] D. F. Agterberg, J. Phys. Condens. Matter **9**, 7435 (1997).

[38] A. F. Andreev, Pis'ma Zh. Eksp. Teor. Fiz. **46**, 463 (1987) [JETP Lett. **46**, 584 (1987)].

[39] G. Deutscher and K. A. Müller, Phys. Rev. Lett. **59**, 1745 (1987).

[40] T. N. Antsygina and A. V. Svidzinskii, Teor. Mat. Fiz. **14**, 412 (1973) [Theor. Math. Phys. **14**, 306 (1973)].