

# Pressure-induced phonon softenings and the structural and magnetic transitions in CrO<sub>2</sub>

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To investigate the pressure-induced structural transitions of chromium dioxide (CrO<sub>2</sub>), phonon dispersions and total-energy band structures are calculated as a function of pressure. The observed structural transition has been theoretically reproduced at  $P \approx 10$  GPa from the ground-state tetragonal CrO<sub>2</sub> (t-CrO<sub>2</sub>) of the rutile type to orthorhombic CrO<sub>2</sub> (o-CrO<sub>2</sub>) of the CaCl<sub>2</sub> type. The half-metallic property is found to be preserved in o-CrO<sub>2</sub>. The softening of the Raman-active B<sub>1g</sub> phonon mode, which is responsible for this structural transition, is demonstrated. The second structural transition is found to occur for  $P \geq 61.1$  GPa from ferromagnetic (FM) o-CrO<sub>2</sub> to nonmagnetic monoclinic CrO<sub>2</sub> (m-CrO<sub>2</sub>) of the MoO<sub>2</sub> type, which is related to the softening mode at  $\mathbf{q} = R(\frac{1}{2}, 0, \frac{1}{2})$ . The third structural transition has been identified at  $P = 88.8$  GPa from m-CrO<sub>2</sub> to cubic CrO<sub>2</sub> of the CaF<sub>2</sub> type that is a FM insulator.

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## I. INTRODUCTION

Chromium dioxide (CrO<sub>2</sub>), which crystallizes in the tetragonal structure of the rutile type, is a well-known material because of its half-metallic nature with  $T_c = 390$  K.<sup>1</sup> The origin of the ferromagnetic (FM) and half-metallic property of CrO<sub>2</sub> was explained in terms of the double-exchange model.<sup>2,3</sup> Due to the crystal field of distorted (flattened) Cr-O<sub>6</sub> octahedra, Cr  $t_{2g}$  states are split into lower  $d_{xy}$  and higher  $d_{xz}/d_{yz}$  states. Out of two  $d$  electrons of Cr<sup>4+</sup>, one occupies the lower  $d_{xy}$  that is localized, while the other occupies the higher  $d_{xz}/d_{yz}$  that are delocalized near the Fermi level ( $E_F$ ) due to the hybridization with O  $p$  states. Then the double-exchange interaction arises from the Hund coupling between localized  $d_{xy}$  and delocalized half-filled  $d_{xz}/d_{yz}$  states, so as to produce the FM and half-metallic properties.

In contrast to numerous reports on electronic and magnetic properties of CrO<sub>2</sub>, there has been a relatively small number of studies on structural and lattice dynamical properties of CrO<sub>2</sub>. In particular, there is no experimental or theoretical report on the phonon-dispersion curve for CrO<sub>2</sub>, except for a few Raman studies.<sup>4-6</sup> Under pressure, CrO<sub>2</sub> is known to undergo the structural transition from the ground-state tetragonal CrO<sub>2</sub> (t-CrO<sub>2</sub>) to the orthorhombic CrO<sub>2</sub> (o-CrO<sub>2</sub>) of the CaCl<sub>2</sub> type at  $P = 12-17$  GPa.<sup>4,7</sup> The question that follows is whether there will be additional structural transitions from o-CrO<sub>2</sub> at higher pressure. In fact, this question is not just for CrO<sub>2</sub> but is also relevant to the structural stability issue of transition-metal (TM) dioxides (TMO<sub>2</sub>). Note that TMO<sub>2</sub> shows diverse structures depending on the TM elements.<sup>8-11</sup> Furthermore, the magnetic properties of CrO<sub>2</sub> under pressure are intriguing, such as (i) whether the half-metallic nature is preserved, and (ii) when CrO<sub>2</sub> becomes nonmagnetic.

In this work, to investigate the pressure-induced structural transitions of CrO<sub>2</sub>, we have studied phonon dispersions and total energies of relevant CrO<sub>2</sub> structures as a function of pressure. Based on the calculated phonon dispersions and the total energies, we have found three possible structural transitions with increasing pressure. The first transition is consistent with the known transition from t-CrO<sub>2</sub> to o-CrO<sub>2</sub>. At this transition, FM and half-metallic properties are preserved, in agreement with previous reports in the literature.<sup>4,7</sup>

The second transition is from o-CrO<sub>2</sub> to monoclinic CrO<sub>2</sub> (m-CrO<sub>2</sub>) of the MoO<sub>2</sub> type, which is nonmagnetic (NM). The third transition is identified from m-CrO<sub>2</sub> to cubic CrO<sub>2</sub> (c-CrO<sub>2</sub>) of the CaF<sub>2</sub> type. Interestingly, c-CrO<sub>2</sub> is a FM insulator even at the high pressure of  $P \geq 88.8$  GPa. Note that the second and third structural transitions are our findings for CrO<sub>2</sub> under the high pressure.

## II. COMPUTATIONAL DETAILS

Band structures and phonon dispersions of CrO<sub>2</sub> were obtained by employing the pseudopotential band method and the linear response method, respectively, implemented in the QUANTUM ESPRESSO code.<sup>12,13</sup> The generalized gradient approximation (GGA) is utilized for the exchange-correlation potential. Self-consistent electron and phonon band calculations were carried out after the full relaxation of internal atomic positions and lattice parameters. We have also employed the all-electron full-potential linearized augmented plane-wave (FLAPW) band method,<sup>14</sup> implemented in the WIEN2K package,<sup>15</sup> to check the results from the pseudopotential band method.

We have considered various structures of CrO<sub>2</sub>. At the ambient pressure, the stable phase is t-CrO<sub>2</sub> of the rutile type ( $P4_2/mnm$ ), in which Cr atoms are positioned at (0,0,0) and  $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ , while O atoms are positioned at  $\pm(u, u, 0)$  and  $\pm(\frac{1}{2} + u, \frac{1}{2} - u, \frac{1}{2})$ . Initial lattice constants and atomic positions adopted before the full relaxation are  $a = b = 4.421$  Å,  $c = 2.916$  Å, and  $u = 0.3043$ .<sup>16</sup> For the high-pressure phase of o-CrO<sub>2</sub> of the CaCl<sub>2</sub> type ( $Pnmm$ ), we have adopted  $a = 4.3874$  Å,  $b = 4.2818$  Å,  $c = 2.8779$  Å,  $u_x = 0.299$ , and  $u_y = 0.272$ .<sup>4</sup> For candidate structural phases at the higher pressure, we considered m-CrO<sub>2</sub> of the MoO<sub>2</sub> type ( $P2_1/c$ )<sup>10,11</sup> and c-CrO<sub>2</sub> of the CaF<sub>2</sub> type ( $Fm\bar{3}m$ ).<sup>17</sup> In the latter, a Cr atom is positioned at (0, 0, 0), and O atoms at (0.25, 0.25, 0.25), (0.25, 0.25, 0.75).

## III. RESULTS

Figure 1 shows the phonon dispersions of t-CrO<sub>2</sub> and o-CrO<sub>2</sub> at the ambient and high pressures. As shown in Fig. 1(a), t-CrO<sub>2</sub> at the ambient pressure has regular phonon

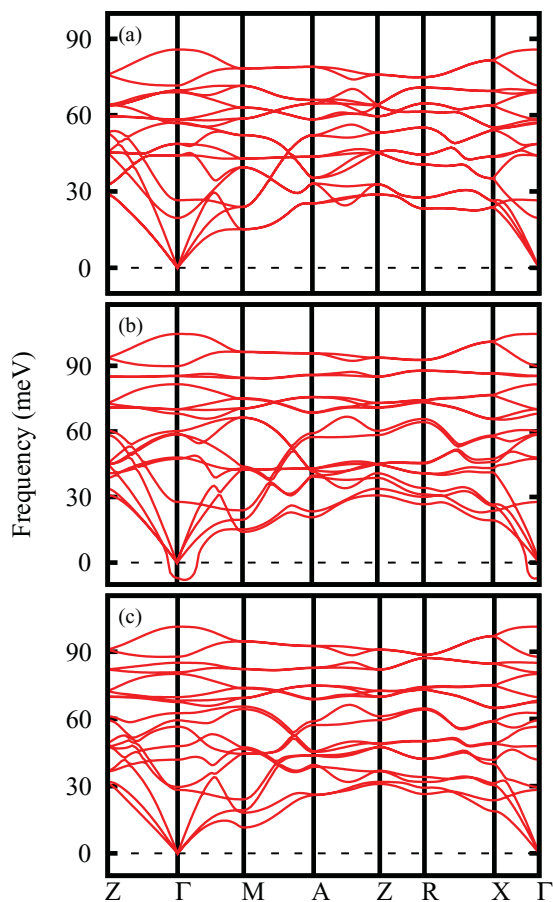


FIG. 1. (Color online) The phonon-dispersion curves of CrO<sub>2</sub>. (a) FM t-CrO<sub>2</sub> of the rutile type at the ambient pressure. (b) FM t-CrO<sub>2</sub> at  $P = 34.0$  GPa. Notice the phonon softening at  $\Gamma$ , which corresponds to the B<sub>1g</sub> mode. The negative frequency here represents the imaginary part of the phonon frequency. (c) FM o-CrO<sub>2</sub> of the CaCl<sub>2</sub> type at  $P = 34.1$  GPa.

dispersions, reflecting the stable phase of t-CrO<sub>2</sub> at the ambient pressure. In contrast, t-CrO<sub>2</sub> at  $P = 34.0$  GPa in Fig. 1(b) has a softening phonon mode at  $\mathbf{q} = \Gamma$ , indicating the structural instability of t-CrO<sub>2</sub> at this pressure. The softened mode corresponds to the B<sub>1g</sub> mode that is Raman active. As shown in Fig. 2, the B<sub>1g</sub> mode generates the rotating motions of oxygen ions. The resulting lattice displacements induce the structural transformation from t-CrO<sub>2</sub> of the rutile type to o-CrO<sub>2</sub> of the CaCl<sub>2</sub> type. Figure 1(c) provides the phonon dispersion of o-CrO<sub>2</sub> at  $P = 34.1$  GPa. The phonon dispersion is regular, implying that o-CrO<sub>2</sub> is stable at this pressure. Therefore, Fig. 1 clearly demonstrates that there is a structural transition from t-CrO<sub>2</sub> to o-CrO<sub>2</sub> at the pressure of  $P \leq 34$  GPa.

In Fig. 3(a), we plotted the calculated Raman-active phonons of t-CrO<sub>2</sub> and o-CrO<sub>2</sub> as a function of pressure. There are four Raman modes (B<sub>1g</sub>, E<sub>g</sub>, A<sub>1g</sub>, B<sub>2g</sub>) for t-CrO<sub>2</sub>, and six Raman modes (A<sub>g</sub>, B<sub>1g</sub>, B<sub>2g</sub>, B<sub>3g</sub>, A<sub>g</sub>, B<sub>1g</sub>) for o-CrO<sub>2</sub>.<sup>4,6,18,19</sup> Our data are consistent with experimental data up to  $P \approx 40$  GPa.<sup>4</sup> With an increase in pressure, one can clearly see the softening of the B<sub>1g</sub> mode of t-CrO<sub>2</sub>, which indicates the structural instability of t-CrO<sub>2</sub>. One can also notice two transition points. The first one corresponds to the

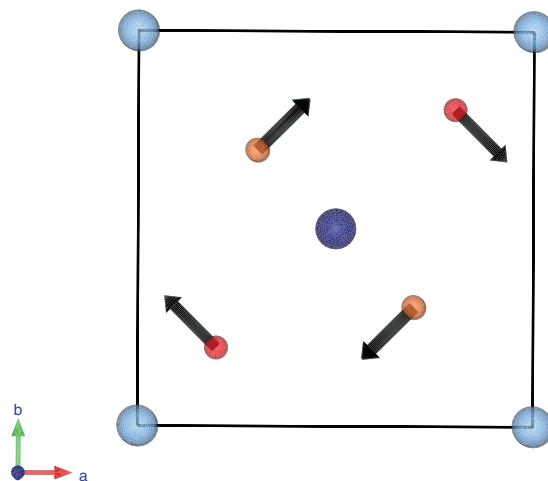


FIG. 2. (Color online) The normal mode of the B<sub>1g</sub> soft phonon at  $\Gamma$  for  $P = 34.0$  GPa. The blue and light-blue circles represent Cr ions, while the orange and red circles (with black arrows) represent oxygen ions. Blue and orange ions are located at  $z = \frac{1}{2}$ . Only the oxygen ions move in this mode.

transition from t-CrO<sub>2</sub> to o-CrO<sub>2</sub> at  $P = 9.8$  GPa. At this transition, CrO<sub>2</sub> keeps its FM and half-metallic properties.<sup>4,7,17</sup> The second one corresponds to the transition at  $P = 76.0$  GPa. The stable structure for  $P \geq 76$  GPa has not been identified yet.

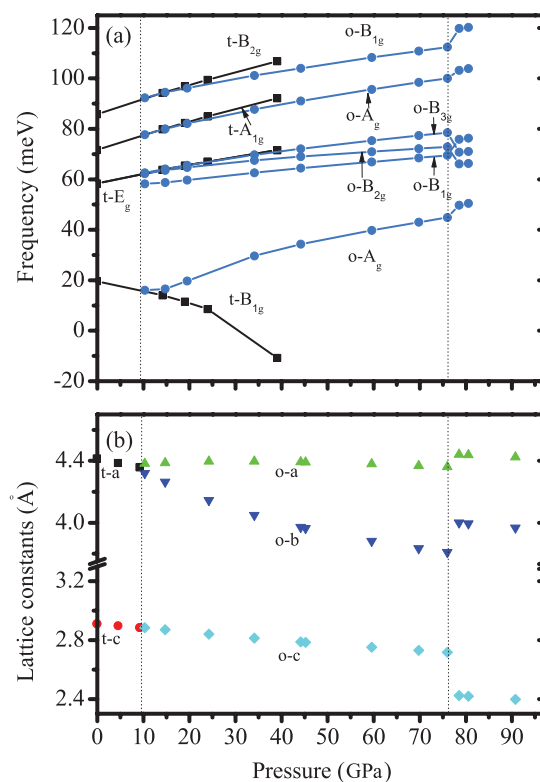


FIG. 3. (Color online) (a) Calculated Raman-active phonon frequencies of t-CrO<sub>2</sub> and o-CrO<sub>2</sub> vs pressure. t- and o- stand for t-CrO<sub>2</sub> and o-CrO<sub>2</sub>, respectively. The lines connecting the data are a guide for the eyes. Two phase transitions are noticed at  $P = 9.8$  and  $P = 76.0$  GPa, which are marked by vertical lines. (b) Calculated equilibrium lattice constants of t-CrO<sub>2</sub> and o-CrO<sub>2</sub> vs pressure.

The phonon anomalies at the two transition points are also revealed in the variation of lattice constants of CrO<sub>2</sub> under pressure. In Fig. 3(b), the lattice constants are plotted as a function of pressure. One can see the anomalous behaviors of the lattice constants at the two transition points that are coincident with those in Fig. 3(a).

To investigate the second structural transition in more detail, we have examined the behavior of the magnetic moment. Srivastava *et al.*<sup>17</sup> once reported that there would be a magnetic transition in t-CrO<sub>2</sub> from half metallic to NM at  $P \approx 65$  GPa. However, as discussed above, there occurs a structural transition from t-CrO<sub>2</sub> to o-CrO<sub>2</sub> at the low pressure of about  $P = 9.8$  GPa. Hence, in Fig. 4(a), we have examined the magnetic-moment behavior for o-CrO<sub>2</sub>. It is seen that there is a FM to NM transition at  $P = 76.0$  GPa, which is close to the second structural transition point. The magnetic moment of o-CrO<sub>2</sub> suddenly drops at this transition point ( $P = 76.0$  GPa). The half-metallic property persists up to this pressure. This magnetic transition was also observed by Kuznetsov *et al.*,<sup>7</sup> who obtained the transition pressure of  $P \approx 53$  GPa by using the pseudopotential band method implemented in the VASP code. It is thus tempting to conjecture that the second transition observed in Fig. 3 corresponds to the magnetic transition in o-CrO<sub>2</sub>. However, the phonon-dispersion curve in Fig. 4(b) for NM o-CrO<sub>2</sub> at  $P = 80.5$  GPa shows the strong phonon softenings, indicating that even NM o-CrO<sub>2</sub> is unstable at the pressure of  $P > 76.0$  GPa. Therefore, it is not possible that FM o-CrO<sub>2</sub> changes into NM o-CrO<sub>2</sub> with an increase in pressure. There might be an additional structural transition in this pressure range.

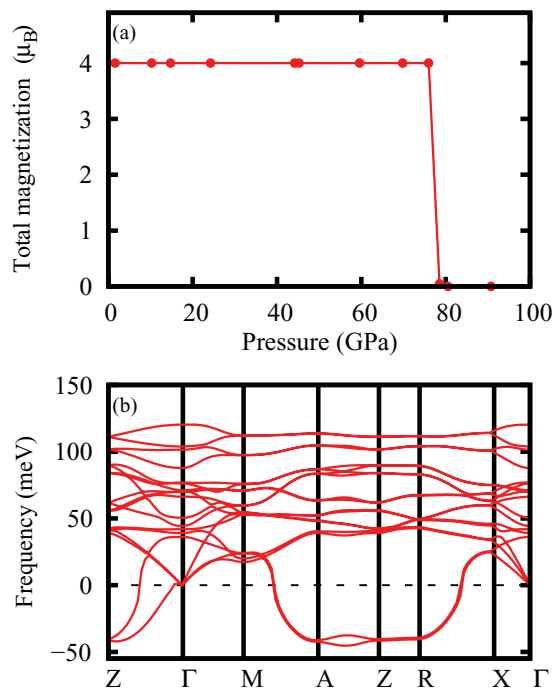


FIG. 4. (Color online) (a) Calculated magnetic moment of o-CrO<sub>2</sub> vs pressure. Magnetic transition from the FM to the NM phase occurs at  $P = 76.0$  GPa. (b) The phonon-dispersion curve of NM o-CrO<sub>2</sub> at  $P = 80.5$  GPa.

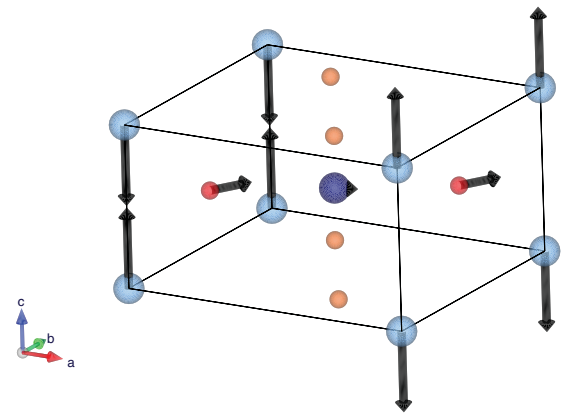


FIG. 5. (Color online) The normal mode of the softened phonon at  $\mathbf{q} = R$  for  $P = 80.5$  GPa. The blue and light-blue circles represent Cr ions, while the (smaller) orange and red circles represent oxygen ions.

Two candidate structures for  $P > 76.0$  GPa are m-CrO<sub>2</sub> of the MoO<sub>2</sub> type and c-CrO<sub>2</sub> of the CaF<sub>2</sub> type. The monoclinic structure of the MoO<sub>2</sub> type is chosen from the expectation that with increasing pressure, two *d* electrons of Cr become itinerant and the local environment becomes similar to that of MoO<sub>2</sub>. Also, TiO<sub>2</sub> and NbO<sub>2</sub> were reported to undergo the structural transitions under high pressure to the monoclinic structure of the baddeleyite type.<sup>20–22</sup> Concerning another candidate, namely, c-CrO<sub>2</sub> of the CaF<sub>2</sub> type, there were previous reports predicting the structural transitions from the CaCl<sub>2</sub> type to CaF<sub>2</sub> type for CrO<sub>2</sub><sup>17</sup> and RuO<sub>2</sub>.<sup>23</sup> In fact, the CaF<sub>2</sub> type is a typical structure of TMO<sub>2</sub>. For example, ZrO<sub>2</sub> and HfO<sub>2</sub>, which have relatively large cations, crystallize in a CaF<sub>2</sub>-type structure at high temperature.<sup>24,25</sup>

In VO<sub>2</sub>, a softening of phonon frequency was observed at  $\mathbf{q} = R(\frac{1}{2}, 0, \frac{1}{2})$ , which corresponds to the atomic movements from the tetragonal structure of the rutile type to the monoclinic structure.<sup>26</sup> Indeed, the softening mode at  $\mathbf{q} = R$  in Fig. 4(b) is related to this structural transition because the orthorhombic structure of the CaCl<sub>2</sub> type is nothing but the distorted

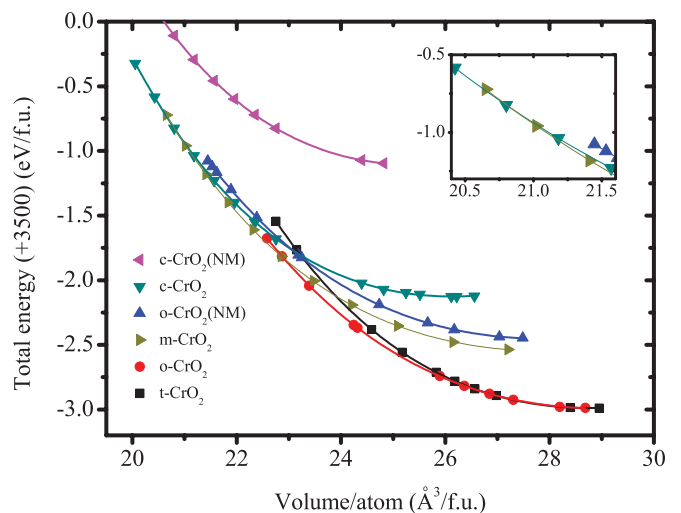


FIG. 6. (Color online) Total energies of various CrO<sub>2</sub> structures vs volume. Data are fitted by the Birch-Murnaghan equation of state.

TABLE I. The fitting parameters of the Birch-Murnaghan equation of state.  $B_0$  is the bulk modulus (units in GPa) and  $B'_0$  is its pressure derivative.

	t-CrO <sub>2</sub>	o-CrO <sub>2</sub>	o-CrO <sub>2</sub> (NM)	c-CrO <sub>2</sub>	c-CrO <sub>2</sub> (NM)	m-CrO <sub>2</sub>
$B_0$	236.9	196.9	196.0	255.3	256.6	169.9
$B'_0$	4.5	3.9	3.8	4.3	4.8	4.8

rutile-type structure. Figure 5 depicts the normal mode of the softened phonon mode at  $\mathbf{q} = R$ . The displacements generate Cr-Cr dimerization along the  $c$  axis, which is consistent with the main distortions of the transition from the rutile type to the monoclinic structure. It is thus reasonable to expect the second transition to be from o-CrO<sub>2</sub> of the CaCl<sub>2</sub> type to m-CrO<sub>2</sub> of the MoO<sub>2</sub> type.

To identify the additional structural transition at the higher pressure, we have compared the total energies of the candidate structures. From the total energy vs volume curves in Fig. 6, which are fitted by the Birch-Murnaghan equation of state (see Table I), one can identify three structural phase transitions. The first one is from FM t-CrO<sub>2</sub> to FM o-CrO<sub>2</sub> at the estimated pressure of  $P = 12.2$  GPa, which is consistent with the phonon calculation in Fig. 3. For  $P \geq 61.1$  GPa, NM m-CrO<sub>2</sub> becomes the most stable, which corresponds to the second transition from o-CrO<sub>2</sub> to m-CrO<sub>2</sub>, as discussed in the phonon study of Fig. 4. For  $P \geq 88.8$  GPa, c-CrO<sub>2</sub>, which is a FM insulator, becomes the most stable. The more stable FM and insulating phase, compared to the NM metallic phase, of c-CrO<sub>2</sub> at this high pressure is extraordinary. The magnetic moment of c-CrO<sub>2</sub> amounts to  $\sim 2\mu_B/\text{Cr}$ , which is close to those in t-CrO<sub>2</sub> and o-CrO<sub>2</sub>. Note, however, that c-CrO<sub>2</sub> is an insulator, not a half metal. We have confirmed this result by the FLAPW band method too. The present result is different from that by Srivastava *et al.*,<sup>17</sup> who obtained the stable NM metallic phase of c-CrO<sub>2</sub> for  $P > 90$  GPa. The different result is likely to come from their use of a simple tight-binding linearized muffin-tin orbital (TB-LMTO) band method.

The transition from m-CrO<sub>2</sub> to c-CrO<sub>2</sub> is thought to originate from the increasing packing ratio. There are six and eight oxygens around Cr in m-CrO<sub>2</sub> and c-CrO<sub>2</sub>, respectively. Haines *et al.*<sup>23</sup> proposed several possible paths of structural

transition from a rutile- to CaF<sub>2</sub>-type structure in TMO<sub>2</sub>. Interestingly, one of the paths is the same as the present structural transition path: rutile-type ( $P4_2/mnm$ )  $\rightarrow$  CaCl<sub>2</sub>-type ( $Pnmm$ )  $\rightarrow$  MoO<sub>2</sub>-type ( $P2_1/c$ )  $\rightarrow$  CaF<sub>2</sub>-type ( $Fm\bar{3}m$ ). But they did not take into account the magnetic state.

#### IV. CONCLUSION

We have studied the pressure effect on the structural properties of CrO<sub>2</sub> by performing the phonon-dispersion and total-energy band structure calculations. By combining two analysis methods, we have found that there are three structural transitions with increasing pressure up to 100 GPa. The first one is the transition from t-CrO<sub>2</sub> of the rutile type to o-CrO<sub>2</sub> of the CaCl<sub>2</sub> type at  $P \approx 10$  GPa (9.8 GPa from phonon-dispersion analysis, while 12.2 GPa from the total-energy study). The FM and half-metallic properties of CrO<sub>2</sub> are preserved at this transition. The second structural transition is from FM o-CrO<sub>2</sub> to NM m-CrO<sub>2</sub>, which corresponds to the lattice displacement of the phonon softening at  $\mathbf{q} = R$  in o-CrO<sub>2</sub>. The transition pressure is  $P = 76.0$  GPa from the phonon-dispersion analysis, whereas it is  $P = 61.1$  GPa from the total-energy study. The third structural transition is from NM m-CrO<sub>2</sub> to FM c-CrO<sub>2</sub> at  $P = 88.8$  GPa, which is accompanied by the metal-to-insulator transition.

#### ACKNOWLEDGMENTS

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