

Long-scale dynamics of crystalline membranesV. V. Lebedev¹ and E. I. Kats^{1,2}¹*Landau Institute for Theoretical Physics, RAS, Moscow, Russia*²*Laue-Langevin Institute, Grenoble, France*

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We investigate effects of long-wavelength dynamic fluctuations in free-standing crystalline membranes by means of the renormalization group method. Thermal fluctuations lead to power dispersion laws for two acoustic in-plane sound modes (longitudinal and transversal) and one out-of-plane bending mode governed by the renormalized static elastic and bending modules. There is no logarithmic contribution to the attenuation of the modes in the marginal membrane dimension $D = 4$; therefore, in the dimensionality $D - \epsilon$, the attenuations are slave, following the same scaling laws as the dispersion laws. We anticipate that our results can be relevant for a better understanding of the graphene (and other crystalline films) dynamics.

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I. INTRODUCTION

It is well known that physics of solid bodies depends on their effective dimensionality. Accordingly, low-dimensional structures have always attracted a great deal of interest, and many quasi-two-dimensional systems (including various electronic heterostructures and also membranes, Langmuir monolayers, smectic liquid-crystalline films, and so on) have been investigated during the last decades. Particularly, an impressive technological progress in the graphene film's preparation¹⁻⁷ gives recently possibilities for experimental investigations of free-standing crystalline sheets of atomic thickness. In fact, similar technology^{8,9} can be used to produce free-standing crystalline films from many other materials (both conducting like graphene or insulating like boron nitride) possessing layer structure in bulk. Quite differently, another class of crystalline membrane surfaces (red blood cell is a famous example) is possible in fixed connectivity (tethered or polymerized) lipid membranes when covalent cross links are replaced by weaker Van der Waals forces, tending to crystallize the lipid bilayers at diminishing temperature.¹⁰ This experimental emergence of stable truly two-dimensional systems makes theoretical investigations of their vibrational properties an important direction of researches not driven by pure curiosity. Indeed, beyond the graphene, remarkable electronic transport, and optic properties, it becomes clear that graphene mechanical features are also remarkable [e.g., thermal conductivity of a graphene sheet is much higher than that even for a few layers of graphite¹¹ (see, also, Ref. 12)]. This and other nonelectronic graphene properties make it a promising candidate for various nanoelectromechanical device applications, e.g., vibrational mass detectors.¹³

Theoretical studies of thermodynamic (static) properties of crystalline membranes started actually long before the "graphene era"¹⁴⁻¹⁸ and are still an active area of research nowadays¹⁹⁻²⁷ (these papers will be partially commented in what follows, and much more publications could be added to this list). Surprisingly, to our knowledge, no work was devoted to dynamics of crystalline membranes. The main purpose of our paper is to fill this gap and to study theoretically fluctuational dynamics of a freely suspended crystalline membrane. Having in mind elastic properties of the membranes, we will not touch here their electronic properties

and the electron-phonon interaction. Note, however, that our findings can be applied for the conducting graphene sheets. First of all, graphene is semimetal, and the gap vanishes only at the K points, where the two massless bands cross. Therefore, electronic excitations can often be neglected, and phonon dispersions provide all the information that is needed to calculate various thermodynamic and dynamic properties.²⁸ Furthermore, a characteristic time scale of the electrons is determined by the Fermi velocity $v_F \simeq 10^8$ cm/s, whereas elastic deformations are changed on a longer time scale dictated by the speed of sound $v_s \simeq (10^5-10^6)$ cm/s. Thus, to treat the graphene mechanical properties, one can use the adiabatic approach integrating out fast electronic degrees of freedom. However, certainly vibrational modes due to electron-phonon coupling affect essentially electronic properties, e.g., temperature-dependent resistivity. It is worth to note works^{19,20} where anomalous temperature dependence of resistivity has been rationalized in terms of one-loop renormalization of the flexural phonon mode in free-standing graphene. Resistivity due to flexural modes in bilayer graphene has been investigated in Ref. 21, and, very recently, molecular dynamics simulations have been applied to compute frequency and lifetime of all phonon modes in graphene and nanotubes.²²

In this work, we investigate effects of long-wavelength dynamic fluctuations in crystalline membranes. Fluctuations around the flat phase are analyzed by means of the renormalization group (RG) method. A key technical component of our theoretical study is an evaluation of the dynamic scaling exponents for the $(D = 4 - \epsilon)$ -dimensional crystalline membrane embedded into the $(d = 5 - \epsilon)$ -dimensional space in terms of the dynamic effective action. We show that thermal fluctuations lead to scaling relations for the dispersion laws of two acoustic in-plane sound modes (longitudinal and transversal) and one out-of-plane bending mode. The scaling is determined by nonlinear terms in the dynamic equations (mode coupling), and the dynamics of the modes follows renormalization of the static elastic and bending modules. The fluctuational mode attenuation (imaginary part of the dispersion law) has the same order over small hydrodynamic parameter k as the real part of the dispersion law. There is no logarithmic contribution to the attenuation of the modes in the marginal membrane dimension $D = 4$; therefore, in the dimensionality $D - \epsilon$, the attenuations are

slave, following the same scaling relations as the dispersion laws.

The structure of our paper is as follows. In Sec. II, we introduce the basic description of the elasticity of crystalline membranes to be employed in the rest of the paper. For a nearly flat membrane, deformations are parametrized by the in-plane distortion field u_α and the out-of-plane height displacement h . In the harmonic approximation, the in-plane and bending modes are decoupled, and their interaction renormalizes the elastic energy. In Sec. III, we describe the elements of the diagram technique needed to study dynamic effects induced by thermal fluctuations. This technique is further exploited in Sec. IV to estimate long-wavelength eigenmode dispersion laws. In Sec. V, we give a brief summary of our results and discuss also some possible applications and open questions.

II. BASIC EQUATIONS

It is worth to note that thin free-standing films at scales larger than the film thickness can be treated as two-dimensional systems embedded into three-dimensional space. An extra (with respect to the genuine $D = 2$ flat films) degree of freedom (displacement in the direction orthogonal to the film) leads to the additional (with respect to the in-plane sound modes) soft long-wavelength mode, the so-called bending mode. We assume that in equilibrium the membrane is flat and parallel to the X - Y plane and characterize its bending deformations by the displacement $h(x, y)$ of the membrane along the Z axis. Any deviation from the flat shape costs bending energy, which reads in the main approximation²⁹ as

$$E_b = \int dx dy \frac{\kappa}{2} (\nabla^2 h)^2, \quad (1)$$

where κ is the bending modulus [for the graphene film, $\kappa \simeq 1.1$ eV (Ref. 4)].

The elasticity of the crystalline membrane can be described in terms of the elastic energy³⁰

$$E_{\text{el}} = \int dS \left\{ \frac{\lambda}{2} (w_{11} + w_{22})^2 + \mu w_{\mu\nu}^2 \right\}, \quad (2)$$

where $dS = \sqrt{1 + (\nabla h)^2} dx dy$, μ and λ are elastic moduli (Lame coefficients), the integration in Eq. (2) is performed along the membrane, and dS is an element of its area. The energy (2) with two elastic moduli is written for the sake of simplicity for an isotropic crystal (say, for the hexagonal crystal). For the graphene films, $\mu \simeq 9$ eV/Å² and $\lambda \simeq 2$ eV/Å².⁴ It is worth noting that any deformation acting on graphene leads to wrinkling because of the nearly negligible threshold for buckling instabilities in thin plates (the bending rigidity scales with the cube of the thickness so that a thin membrane can not support even arbitrarily small shear or compression without wrinkling on scales large compared to thickness). However, for all its bending softness, graphene exhibits the largest in-plane Young's modulus and, although easy to bend, is extremely hard to stretch.

The distortion tensor $w_{\mu\nu}$ characterizes membrane deformations with respect to the equilibrium.³⁰ For relatively small

elastic and bending deformations, the distortion tensor can be written as^{31,32}

$$2w_{\mu\nu} = \partial_\mu u_\nu + \partial_\nu u_\mu + \partial_\mu h \partial_\nu h, \quad (3)$$

where \mathbf{u} is the membrane in-plane displacement with respect to the equilibrium position. Note that the structure of the distortion tensor (3) reflects the fact that the bending deformation inevitably produces some membrane distortion. Bending energy (1) is invariant under translations (a) along the z direction and rotations around any axis $h \rightarrow h + c_1 a + c_2 \mathbf{r} \times \theta$ (where c_1 and c_2 are constants, and θ is the rotation angle along a given axis). Furthermore, the distortion tensor (3) is invariant under transformation $\delta u_\nu = -\theta_\nu h$, $\delta h = \theta_\nu u_\nu$ (θ_ν is a rotation angle) that reflects rotational invariance of the film.

The expression (3) being substituted into Eq. (2) leads to the following harmonic (of the second order) and anharmonic (of the third and fourth orders) contributions into the elastic energy:

$$E_{\text{el}}^{(2)} = \int d^2 r \left[\frac{\mu + \lambda}{2} (\partial_\alpha u_\alpha)^2 + \frac{\mu}{2} \partial_\alpha u_\beta \partial_\alpha u_\beta \right], \quad (4)$$

$$E_{\text{el}}^{(3)} = \int d^2 r \left[\frac{\lambda}{2} \partial_\alpha u_\alpha (\nabla h)^2 + \mu \partial_\alpha u_\beta \partial_\alpha h \partial_\beta h \right], \quad (5)$$

$$E_{\text{el}}^{(4)} = \int d^2 r \frac{\lambda + 2\mu}{8} (\nabla h)^4. \quad (6)$$

The energies [Eqs. (1) and (4)–(6)] govern thermal fluctuations of a crystalline membrane that lead to logarithmic renormalization of the modules κ , μ , λ in the marginal (upper critical dimension) $D = 4$ (which is a dimensionality of the membrane). One can derive by straightforward calculations the following one-loop RG equations^{14–17} (cf. also with the results of more involved calculations based on the so-called self-consistent screening approximation, which is exact in the limit $d \rightarrow \infty$, presented in Refs. 18, 23, and 24 or on the nonperturbative RG approach²⁵):

$$\begin{aligned} \frac{d\mu}{d\xi} &= -\frac{T\mu^2}{96\pi^2\kappa^2}, \\ \frac{d\lambda}{d\xi} &= -\frac{T}{16\pi^2\kappa^2} (\lambda^2 + \mu\lambda + \mu^2/6), \\ \frac{d\kappa}{d\xi} &= \frac{T}{16\pi^2\kappa} \frac{4\mu^2 + 5\mu\lambda}{2\mu + \lambda}. \end{aligned} \quad (7)$$

Here, $\xi = \ln(\Lambda r)$ is the RG logarithmic factor and Λ is the ultraviolet (microscopic) cutoff wave vector of the order of inverse membrane thickness. A stable fixed point of the RG equations in $D = 4$ [Eq. (7)] is $\lambda = -\mu/3$. In fact, the RG flow equations have four fixed points found quite some time ago in Ref. 15. For $D = 4$ membranes, only the fixed point $\lambda = -\mu/3$ describes stable solid membranes [for instance, another fixed point of the Eqs. (7), $\lambda = -\mu/2$, would lead in $D = 4$ to zero bulk elastic modulus $B = \lambda + (2\mu/D)$]. Since we are not interested in calculating the values of the exponents extrapolating $D = 4$ results to $D = 2$ physical membranes, we restrict ourselves to only this stable in $D = 4$ fixed point $\lambda = -\mu/3$. Universal scaling relations for static and dynamic correlation functions hold for any stable fixed point. In the

point $\lambda = -\mu/3$, the RG equations are reduced to a single equation $\partial_\xi g = -g^2$ for the invariant charge

$$g = \frac{89}{480\pi^2} \frac{T\mu}{\kappa^2}. \quad (8)$$

Then, one finds that both elastic moduli have for $D = 4$ the same scaling

$$\mu, \lambda \propto \xi^{-5/89}, \quad (9)$$

whereas the bending module κ increases at large scales as

$$\kappa \propto \xi^{42/89}. \quad (10)$$

One can extend the equations for the membrane dimensionality $D = 4 - \epsilon$ and to estimate the λ , μ , and κ scalings for the physical dimensionality $D = 2$ (as it has been done in Refs. 14–18 and 23–25). Unfortunately, such a wild extrapolation to $\epsilon = 2$ of the results obtained at $\epsilon \ll 1$ is not a very reliable procedure. The actual values of the exponents can not be found by such an approach. One has to rely on numerical simulations²⁶ or experimental electromechanical resonator data.^{33,35} The RG results may be used for qualitative predictions only, and Eqs. (9) and (10) tell us that due to thermal fluctuations, elastic moduli μ and λ decrease in the long-wavelength limit, while the bending modulus κ increases in this limit as some powers. Moreover, the scaling is characterized by a single scaling exponent. It is usually introduced for the bending module $\kappa \propto k^{-\zeta}$, then

$$\mu, \lambda \propto \kappa^2 k^\epsilon \rightarrow k^{2-2\zeta} \quad (11)$$

for the physical value $\epsilon = 2$. The above scaling arguments are quite robust. With all said in mind, we stop here reproducing known^{14–18,23–25} static results and will follow the same strategy to investigate dynamic fluctuation effects.

III. MEMBRANE DYNAMICS

To examine the membrane dynamics, one has to take into account its kinetic energy

$$\mathcal{K} = \int dx dy \frac{j^2}{2\rho}. \quad (12)$$

In Eq. (12), ρ is the mass density of the membrane (for graphene, $\rho = 7.6 \times 10^{-8}$ g/cm²), and in what follows, we use units with $\rho = 1$. We assume that the membrane environment is tenuous and neglect its influence to the membrane. In dynamics (like in statics), all physical characteristics [including kinetic energy (12)] are defined for physical dimension $D = 2$ in $d = 3$, and only the RG procedure is realized for a marginal dimension $D = 4$ in $d = 5$.

The reactive (nondissipative) membrane dynamic equations can be derived on the basis of the Poisson bracket method.^{31,32} Since we are interested in only in-plane sound and out-of-plane bending modes, we need only the Poisson brackets for the momentum densities j_α , j_z (j_α are the momentum density components projected into the membrane plane, and the Z axis is orthogonal to that plane) and the variables u_μ and h . The explicit formulas for Poisson brackets, involving membrane hydrodynamic variables (momentum density j_α , in-plane crystalline lattice displacements u_β , and membrane

displacement h along the normal), can be found in chapter 6 of Ref. 32 (see also original papers, Refs. 31 and 36). Applying these formulas for the crystalline membrane, we end up with only two nonzero Poisson brackets

$$\{j_\alpha(x_1, y_1), u_\mu(x_2, y_2)\} = \delta_{\alpha\mu} \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad (13)$$

$$\{j_z(x_1, y_1), h(x_2, y_2)\} = \delta(\mathbf{r}_1 - \mathbf{r}_2). \quad (14)$$

The Poisson brackets together with the energy contributions (1), (2), and (12) lead to the following dissipationless dynamic equations for the elastic (sound) and bending modes:

$$\partial_t^2 h + \kappa \nabla^4 h - \partial_\alpha (\sigma_{\alpha\beta} \partial_\beta h) = 0, \quad (15)$$

$$\partial_t^2 u_\alpha - \partial_\beta \sigma_{\alpha\beta} = 0, \quad (16)$$

where all nonlinear effects are lumped into the stress tensor

$$\sigma_{\alpha\beta} = \frac{\delta E_{\text{el}}}{\delta w_{\alpha\beta}} = \lambda \delta_{\alpha\beta} w_{\mu\mu} + 2\mu w_{\alpha\beta}, \quad (17)$$

where E_{el} is defined in Eq. (2). Fluctuation effects in the long-wavelength dynamics can be described in terms of a diagrammatic technique first developed by Wyld,³⁷ who has considered hydrodynamic turbulence. Further, the technique was generalized and formulated in terms of functional integrals in Refs. 38–40. Here, we follow the version of this technique described in Ref. 32.

Correlation functions of our membrane can be obtained as functional integrals with the weight $\exp(i\mathcal{J})$, where the effective action \mathcal{J} is constructed in accordance with Eqs. (15) and (16):

$$\mathcal{J} = \int dt d\mathbf{r} (p \partial_t^2 h + \kappa p \nabla^4 h + \partial_\alpha p \sigma_{\alpha\beta} \partial_\beta h + p_\alpha \partial_t^2 u_\alpha + \partial_\beta p_\alpha \sigma_{\alpha\beta}). \quad (18)$$

Here, the auxiliary fields p and p_α are conjugated to h and u_α , respectively. We examine pair correlation functions

$$\begin{aligned} \langle h(t, \mathbf{r}) h(0, 0) \rangle &= \int \frac{d\omega d\mathbf{q}}{(2\pi)^{d+1}} \exp(-i\omega t + i\mathbf{q}\mathbf{r}) \Phi(\omega, \mathbf{q}), \\ \langle u_\alpha(t, \mathbf{r}) u_\beta(0, 0) \rangle &= \int \frac{d\omega d\mathbf{q}}{(2\pi)^{d+1}} \exp(-i\omega t + i\mathbf{q}\mathbf{r}) F_{\alpha\beta}(\omega, \mathbf{q}), \end{aligned} \quad (19)$$

as well as the corresponding pair response functions $\langle h(t, \mathbf{r}) p(0, 0) \rangle = \Theta(t, \mathbf{r})$ and $\langle u_\alpha(t, \mathbf{r}) p_\beta(0, 0) \rangle = G_{\alpha\beta}(t, \mathbf{r})$. They are related by the fluctuation-dissipation theorem, which reads as

$$\begin{aligned} \Phi(\omega, \mathbf{q}) &= -\frac{2T}{\omega} \text{Im} \Theta(\omega, \mathbf{q}), \\ F_{\alpha\beta}(\omega, \mathbf{q}) &= -\frac{2T}{\omega} \text{Im} G_{\alpha\beta}(\omega, \mathbf{q}). \end{aligned} \quad (20)$$

It is worth noting that similar to Eq. (20), expressions can be derived for higher-order correlation and response functions. All these relations based on the fluctuation-dissipation theorem are exact and independent of an approximation used for the calculation of the response and correlation functions.

Similarly to the energy, the effective action (18) can be separated into the harmonic (quadratic in p, h, p_α, u_α) part and third- and fourth-order interaction terms

$$\mathcal{J}^{(2)} = \int dt d\mathbf{r} [p \partial_t^2 h + \kappa p \nabla^4 h + p_\alpha \partial_t^2 u_\alpha + (\mu + \lambda) \partial_\alpha p_\alpha \partial_\beta u_\beta + \mu \partial_\beta p_\alpha \partial_\beta u_\alpha], \quad (21)$$

$$\mathcal{J}^{(3)} = \int dt d\mathbf{r} \left[\frac{1}{2} (\lambda \nabla \mathbf{p} \delta_{\alpha\beta} + \mu \partial_\alpha p_\beta + \mu \partial_\beta p_\alpha) \partial_\alpha h \partial_\beta h + \partial_\alpha p \partial_\beta h (\lambda \delta_{\alpha\beta} \nabla \mathbf{u} + \mu \partial_\alpha u_\beta + \mu \partial_\beta u_\alpha) \right], \quad (22)$$

$$\mathcal{J}^{(4)} = \frac{1}{2} \int dt d\mathbf{r} (2\mu + \lambda) (\nabla p \nabla h) (\nabla h)^2. \quad (23)$$

The expressions (21)–(23) can be used as a starting point for the perturbation theory. Bare correlation and response functions can be found easily by the Gaussian integration with the quadratic part (21) of the effective action:

$$\Phi = \frac{2\pi T}{\sqrt{\kappa} q^2} \delta(\omega^2 - \kappa q^4), \quad (24)$$

$$F_{\alpha\beta} = \frac{2\pi T}{\sqrt{2\mu + \lambda} q} \delta[\omega^2 - (2\mu + \lambda) q^2] \frac{q_\alpha q_\beta}{q^2} + \frac{2\pi T}{\sqrt{\mu} q} \delta(\omega^2 - \mu q^2) \left(\delta_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2} \right). \quad (25)$$

The arguments of the δ functions in Eq. (25) reflect the well-known bare dispersion laws³⁰ (see, also, more recent results on the harmonic dispersion laws⁴¹ and also with some perturbative anharmonic corrections included^{19,20,42}). In metallic systems such as doped graphene, phonon damping due to electron-phonon coupling^{43,44} has to be taken into consideration. Therefore, strictly speaking, our work (neglecting electrons at all) holds only for insulating crystalline membranes. Note, however, that electron-phonon contributions lead to regular dependences on wave vectors,⁴³ whereas fluctuation contributions, which we are discussing here, have singular dependence on wave vectors. Our hope is that small but singular effects can be disentangled from even larger but regular effects. Aside from this, electron-phonon coupling is not expected to change dramatically in multilayer graphene films, whereas fluctuation effects are suppressed at the scales on the order of the film thickness. From Eqs. (24) and (25), we get in the harmonic approximation for the bending mode $\omega = \pm \sqrt{\kappa} q^2$ and for the longitudinal $\omega = \pm \sqrt{2\mu + \lambda} q$ and the transverse $\omega = \pm \sqrt{\mu} q$ sound modes (remind that we put $\rho = 1$).

Dissipative terms are not included in the above expressions. The reason is that the fluctuational attenuation resulting from nonlinear mode coupling appears to be much stronger than the bare attenuation. In the long-wavelength limit, these fluctuational dissipative terms are parametrically larger than the bare dissipative terms. The fluctuational mode attenuation (imaginary part of the dispersion law) has the same order over small hydrodynamic parameter k as the real part of the dispersion law. It is worth remarking that the regular

viscouslike ($\propto k^2$) dissipation in the bending mode is strictly zero due to rotational symmetry of the system (which holds in the long-wavelength limit for crystalline membranes similarly as for free-standing liquid-crystalline films³⁶).

IV. RENORMALIZATION

We start with in-plane sound modes. As in statics, we consider a $D = 4$ membrane embedded into $d = 5$ space, where we deal with a marginal logarithmic renormalizations. The sound modes' attenuations occur as a result of the coupling between sound and bending modes. The coupling governs both the real and imaginary parts of the dispersion law's fluctuational renormalizations.

According to the third-order effective action term $\mathcal{J}^{(3)}$ [Eq. (22)], the dynamic renormalization of the in-plane phonon modes is determined by the one-loop self-energy function

$$\Sigma_{\alpha\beta\mu\nu}(\omega, \mathbf{k}) = \int \frac{d\omega' d^4 q}{(2\pi)^5} (q_\alpha q_\beta - k_\alpha k_\beta / 4) (q_\mu q_\nu - k_\mu k_\nu / 4) \times \Phi(\omega', \mathbf{q} - \mathbf{k}/2) \Theta(\omega' + \omega, \mathbf{q} + \mathbf{k}/2). \quad (26)$$

In the main logarithmic approximation, $\omega \ll \omega'$ and $k \ll q$. Then, for $\omega \rightarrow 0$ and $k \rightarrow 0$, the integral (26) is reduced to a residue in the pole $\omega' = 0$, and the real part of the self-energy function $\hat{\Sigma}$ reads as

$$\Sigma_{\alpha\beta\mu\nu} = -\frac{T}{384\pi^2 \kappa^2} (\delta_{\alpha\beta} \delta_{\mu\nu} + \delta_{\alpha\mu} \delta_{\beta\nu} + \delta_{\alpha\nu} \delta_{\beta\mu}) \xi. \quad (27)$$

As it should be, the expression (27) reproduces the static renormalization of the Lamé coefficients λ and μ [Eq. (9)].

To find the imaginary part of the self-energy function (26), i.e., fluctuational attenuation for the in-plane sound modes, we have to keep in that [Eq. (26)] $\omega \neq 0$. Since the sound modes (even after fluctuational renormalization) are still harder than the renormalized bending mode [cf. Eq. (11)], the wave vector k in Eq. (26) can be safely neglected. Then, we find

$$\text{Im} \Sigma_{\alpha\beta\mu\nu} = -\frac{T \text{sign} \omega}{3072\pi \kappa^2} (\delta_{\alpha\beta} \delta_{\mu\nu} + \delta_{\alpha\mu} \delta_{\beta\nu} + \delta_{\alpha\nu} \delta_{\beta\mu}). \quad (28)$$

We see that there are no logarithmic factors in the expression. In fact, it is a general property, and in a spirit of basically exact fluctuation-dissipation theorem, the imaginary part of the self-energy function (unlike its real part) does not contain logarithmic factors in any order of the perturbation theory. Indeed, each diagram in the perturbation theory for the $\text{Im} \Sigma$ [Eq. (26)] contains the lines $\text{Re} \Theta$ and $\text{Re} G_{\alpha\beta}$ defined accordingly to Eq. (20) (in contrast to the lines $\text{Im} \Theta$ and $\text{Im} G_{\alpha\beta}$ in the diagrams for $\text{Re} \Sigma$). All other blocks in the diagrams for $\text{Im} \Sigma$ can be collected in the statically driven contribution (which certainly contains the logarithmic factor yielding to nontrivial scaling). However, the additional integration related to the lines $\text{Re} \Theta$ and $\text{Re} G_{\alpha\beta}$ does not produce a new logarithmic factor. Thus, we anticipate that in $4 - \epsilon$ dimensionality where nontrivial scaling has to be observed the imaginary part of $\Sigma_{\alpha\beta\mu\nu}$ (i.e., in-plane sound-mode attenuations) is of the order of its real part.

One can say that dynamical fluctuations slave, following static (thermodynamic) fluctuations. It is worth noting that such a rather unusual situation (for instance, for standard second-order phase transitions, critical dynamics

is totally independent of the thermodynamic fluctuational renormalization⁴⁵) is not completely unexpected. Similar behavior holds for free-standing smectic films³⁶ (although modes and their couplings in the both cases are very different, driven by elasticity for crystalline membranes and by surface tension for smectics).

The attenuation of the bending mode is harder to calculate. The fact is that the one-loop contribution into the bending-

mode attenuation (corresponding self-energy function) exactly vanishes. It can be seen by a direct inspection of all one-loop diagrams constructed according with the effective actions (21)–(23). Therefore, we have to estimate two-loop terms. Since the effective action is quadratic over elastic degrees of freedom u_α , they can be integrated out, and we end up with the following fourth-order term in the integrand of the effective action for the bending mode:

$$\Gamma = \partial_\alpha p \sigma_{\alpha\beta} \partial_\beta h = \partial_\alpha p \left\{ \frac{2\mu(\mu + \lambda)}{2\mu + \lambda} (\nabla h)^2 \partial_\alpha h + \frac{2\mu(\mu + \lambda)}{2\mu + \lambda} \partial_\beta h \frac{\partial_\alpha \partial_\beta}{\nabla^2} \frac{\partial_\mu \partial_\nu}{\nabla^2} (\partial_\mu h \partial_\nu h) - \frac{\mu\lambda}{2\mu + \lambda} \partial_\beta h \frac{\partial_\alpha \partial_\beta}{\nabla^2} (\nabla h)^2 - \frac{\mu\lambda}{2\mu + \lambda} \partial_\alpha h \frac{\partial_\mu \partial_\nu}{\nabla^2} (\partial_\mu h \partial_\nu h) - \mu \partial_\beta h \frac{\partial_\mu \partial_\beta}{\nabla^2} (\partial_\alpha h \partial_\mu h) - \mu \partial_\beta h \frac{\partial_\mu \partial_\alpha}{\nabla^2} (\partial_\beta h \partial_\mu h) \right\}, \quad (29)$$

where the nonlocal ($\propto 1/\nabla^2$) terms are the price to pay for integrating out the sound modes. By counting of ω and q powers, we estimate that these two-loop contributions yield to the nonzero bending-mode attenuation $\propto g^2$ with the universal invariant RG charge $g = 89T\mu/(480\pi^2\kappa^2)$, the same as in statics (8). Thus, for the bending mode, we arrive at the same conclusion as for the in-plane sound modes. Namely, the imaginary and real parts of the bending dispersion law have the same scaling with a single exponent, related universally with the exponent for the sound modes.

A number of remarks are in order here. First, by a simple inspection of the integrand Γ (reshuffling the six terms and changing the order of derivatives), we check that $\Gamma \rightarrow 0$ at $k \rightarrow 0$ (as it should be due to rotational symmetry). The main contributions to Γ come from the range when all three wave vectors entering the integral (external k and two independent internal q_1, q_2) are of the same order $k \simeq q_1 \simeq q_2$. There is not any specific symmetry requirement in this case. Therefore, we do believe that by the definition (29), non-negative quantity Γ is not zero for not too small wave vector k and corresponding bending-mode frequency $\omega \propto k^2$. Second, as a note of caution, we should mention also another danger. The integral of Γ could be in principle divergent for the bare dissipationless correlation functions. The source of this potential danger is a resonant denominator occurring from the energy conservation laws (corresponding δ functions). However, in $D = 4$, the quadratic over q denominators may not lead to the integral divergence.

V. DISCUSSION AND OPEN QUESTIONS

In recent years, the discovery of graphene offers a high interest from scientists belonging to different communities. At the beginning of the “graphene era,” much attention has been devoted to electronic properties. However, suspended two-dimensional films allow the unique possibility of exploring other nonelectronic properties. The free-standing films manifest at once features of hard and soft condensed matter physics. Very little remains known about graphene nonelectronic

properties. The situation is now rapidly changing and this brings an interesting new direction in graphene (and other free-standing films) researches.

We present here a calculation of vibrational mode fluctuational renormalization for fixed connectivity or crystalline membranes. We have formulated a RG approach to study dynamical fluctuations in atomically thin freely suspended crystalline films. Our RG analysis clearly illustrates the long-wavelength scaling behavior of the dispersion laws for all gapless vibrational modes. In fact, for long-scale dynamics, it is not essential whether the film is in a crystal or glasslike state. All the nontrivial scaling features that we have studied in this paper are based on a coupling between in-plane and out-of-plane modes related to a nonzero shear modulus $\mu \neq 0$. This made all gapless modes anomalous.

Our main message in this paper is a nontrivial qualitative statement about unique scaling behavior (with a single exponent) of all dynamical characteristics in the crystalline free-standing membranes. We have proved it only for $D = 4 - \epsilon$ membrane embedded into $d = 5 - \epsilon$ space (with $\epsilon \ll 1$). As it is usual in the RG procedure to describe the physical cases $D = 2$ and $d = 3$, we should rely on a rather dangerous extrapolation of the results. However, the nature is on our side, and if we are not interested in the values of the exponents, the fact of universal scaling behavior [single exponent reproducing in the spirit of fluctuation-dissipation theorem (20) static behavior, e.g., Eq. (11)] can be checked by rather cumbersome but straightforward analysis of higher-order diagrams renormalizing the actions (21)–(23). Static (thermodynamic) nonlinearity is so strong that it governs dynamical renormalizations. This fact is based on the exact fluctuation-dissipation theorem [see more details in the previous section, where we have presented the arguments on the validity of Eq. (28)].

We show that both the reactive (real part of the dispersion laws) and dissipative (imaginary part of the dispersion laws) terms are equally renormalized by fluctuations. Therefore, dynamics fluctuations are governed by the static ones. The fluctuational mode attenuation (imaginary part of the dispersion

law) has the same order over small hydrodynamic parameter k as the real part of the dispersion law. For both (longitudinal and transversal) in-plane sound modes, we predict $\text{Re } \omega \propto \text{Im } \omega \propto k^{2-\zeta}$, and for the bending mode $\text{Re } \omega \propto \text{Im } \omega \propto k^{2-\zeta/2}$ [cf. with the static laws (11)]. To avoid confusion, we should say that, in the framework of our approach, we are dealing with the mode broadening (attenuation). It occurs as a result of nonlinear mode coupling between in-plane sound and out-of-plane bending modes. We are not interested in entropy production (dissipation), which holds due to short-scale (large wave vectors) viscous or hyperviscous microscopic processes. If external strains are applied to a membrane, anharmonic effects would be highly suppressed (see Ref. 48) and only short-scale mode broadening survives.

It seems fair to state that we realized after accomplishing this work that things have turned out to be considerably more complicated (and thus more interesting) than expected. Understanding all of its limitations, we nevertheless hope that our theory captures the essential physics of crystalline membrane vibrations. Besides, it is a mandatory first step to rationalize some remaining (in this field) at least partially unexplained (especially for graphene and other carbon allotropes) phenomena such as, e.g., the following:

(i) Why are graphene free-standing films compressed under heating? Indeed, the mode Grüneisen parameters are usually positive since phonon frequencies decrease when the solid expands. However, some negative-mode Grüneisen parameters for low-frequency acoustic modes can arise and

sometimes²⁸ compete with positive ones, giving a negative thermal expansion. But, in “conventional” weakly nonlinear materials, it could happen only at low temperatures, when only the lowest acoustic modes can be excited.

(ii) Why can graphene sheets be easily folded and simultaneously fractured as a glass (which were usually considered as contradicting properties)?

(iii) Why are graphene films impenetrable even for very small noble atoms such as He?

(iv) How do graphene films melt (see discussion in Ref. 49)? Note finally that our theoretical approach can be generalized for biologically relevant membranes of fixed connectivity (such as red blood cell membranes) immersed into a liquid. At the time of writing, the experimental significance of vibrational fluctuations investigated in this paper, for various properties of graphene (or other crystalline or polymerized glasslike) films, is far from clear. However, it seems certain to necessitate a revision of some of our present concepts regarding dynamic phenomena in thin free-standing crystalline films. Our hope is that our paper is provocative enough to stimulate additional research efforts. Besides, with recent technological progress in atomic thickness, free-standing crystalline-film preparation,⁸ acoustic resonance method investigations,^{33–35} and efficient atomistic²⁶ or molecular dynamics⁴⁶ simulations, one might hope to get more favorable conditions for observation of fluctuational phenomena (all the more that high-quality scattering measurements have become available⁴⁷).

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