

# Magneto-optical probe of quantum Hall states in a wide parabolic well modulated by random potential

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Polarized photoluminescence from weakly coupled random multiple well quasi-three-dimensional electron system is studied in the regime of the integer quantum Hall effect. Two quantum Hall ferromagnetic ground states assigned to the uncorrelated miniband quantum Hall state and to the spontaneous interwell phase coherent dimer quantum Hall state are observed. Photoluminescence associated with these states exhibits features caused by finite-size skyrmions: dramatic reduction of the electron spin polarization when the magnetic field is increased past the filling factor  $\nu = 1$ . The effective skyrmion size is larger than in two-dimensional electron systems.

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## I. INTRODUCTION

A variety of quantum phases due to the strong interlayer electronic correlations were predicted in electron multiple quantum wells at the filling factor  $\nu = 1$ .<sup>1</sup> Depending on the ratio  $d/l_B$ , where  $d$  is the interlayer separation and  $l_B$  is the magnetic length, different ground states may be realized. These states are hard to observe experimentally; they require high electron mobility, which is difficult to achieve in multiple quantum wells because of impurity scattering. This problem may be resolved in multiple quantum wells embedded in a remotely doped wide parabolic well where high mobilities were demonstrated.<sup>2,3</sup> The high mobility multiple quantum wells were shown to respond very differently to those with low mobilities. In a strong magnetic field, the low mobility multiple quantum wells behave as a stack of independent layers connected in parallel.<sup>4,21</sup> Thus observed quantum Hall states are the uncorrelated ground states, while a manifestation of a new correlated quantum phase determined by the magnetic field induced redistribution of the electrons over the quantum wells was found in high mobility multiple layers.<sup>5</sup> In this work, in order to study low energy excitations of quantum Hall ferromagnetic in the vicinity of  $\nu = 1$ , the same samples are examined by magnetophotoluminescence. The lowest energy charged excitations of quantum Hall ferromagnetic, skyrmions,<sup>6,7</sup> were observed in single GaAs quantum well by optically pumped nuclear magnetic resonance,<sup>8</sup> by polarized optical absorption<sup>9–11</sup> and recently, by polarized photoluminescence (PL).<sup>12</sup> The data consistent with the presence of the skyrmions were obtained by means of heat capacity measured as a function of the temperature in Ref. 13. Magnetotransport measurements were also employed to study the skyrmions, and somewhat controversial results were obtained in Refs. 14–17. A small  $g$  factor of GaAs makes the Zeeman energy much smaller than the Coulomb energy. This provides dominance of the exchange energy responsible for creation of the skyrmions. Moreover, in order to favor skyrmions, the  $g$  factor was tuned by variation of the Al content in AlGaAs alloy<sup>18</sup> and applying hydrostatic pressure.<sup>19</sup> To increase the sensitivity of standard nuclear magnetic resonance, the GaAs/AlGaAs isolated multiple quantum wells were studied in Ref. 20.

However, no skyrmions were detected to date in coupled multiple quantum wells (superlattices).

In this paper, we report on investigation of quantum Hall ferromagnetic states formed in weakly coupled multiple quantum wells at the filling factor  $\nu = 1$ . In addition to the expected uncorrelated miniband quantum Hall state, first observed in Ref. 21, we found the spontaneous interwell phase coherent dimer quantum Hall state predicted in Ref. 22. Moreover, we demonstrate experimental evidence of skyrmion excitations in weakly coupled intentionally disordered quasi-three-dimensional multiple quantum wells, and the role of the interwell interaction is shown.

## II. EXPERIMENTAL

The structure studied here consists of a 240-nm-wide AlGaAs parabolic well modulated by a sequence of  $N = 10$  weakly coupled (the interlayer miniband width is  $W \simeq 1.5$  meV) quantum wells with random widths separated by 4.2 nm barriers. The largest (34 nm) GaAs quantum well (QW<sub>0</sub>) is in the center of the parabolic well. In order to separate emissions from individual quantum wells, they are randomized by a deliberate random variation of their thickness  $n$  about the nominal value  $n = 65$  ML. The disorder energy  $\Delta$  was defined as the width of the Gaussian distribution of the single-electron energies corresponding to individual quantum wells. The strength of the disorder was characterized by the ratio  $\delta = \frac{\Delta}{\alpha W}$ , where  $\alpha \simeq 6$  is the screening factor.<sup>4</sup> In this paper, we focus on the data obtained in the multiple quantum well structure with the interlayer disorder parameter  $\delta = 1.4$ . Similar results are found in the structure with  $\delta = 2.3$ . The electron concentration and mobility at  $T = 4.2$  K in the parabolic well are  $3.7 \times 10^{11} \text{ cm}^{-2}$  and  $2 \times 10^5 \text{ cm}^2/\text{Vs}$ , respectively.

According to the calculations, eight quantum wells are occupied. At appropriate excitation intensity, they emit eight corresponding PL lines. We would like to emphasize that the bending shape of the valence band profile provides an internal spectrometer which separates PL emissions from the quantum wells.<sup>23</sup> Therefore, individual responses of each quantum well can be studied.

The circularly polarized PL measurements have been performed at base temperature  $T \simeq 300$  mK at magnetic fields up to 18 T. A single 550- $\mu\text{m}$ -diameter optical fiber delivers the laser excitation (532 nm,  $\approx 1$  mW/cm<sup>2</sup>) to the sample in a <sup>3</sup>He cryostat and brings back the collected PL signal to a spectrometer. A linear polarizer and a quarter-wave plate are installed in front of the sample. The circularly polarized PL components are measured by reversing the polarity of the magnetic field. The experiments were performed with the magnetic field perpendicular to the sample surface ( $\theta = 0$ ) and tilted out to  $\theta = 70^\circ$ . A standard low-frequency (10 Hz) lock-in technique is used to measure the magnetoresistance of the Van der Pauw sample.

### III. RESULTS AND DISCUSSION

The plot of the  $\sigma^+$  polarized magneto-PL is shown in Fig. 1. Three strong PL lines are observed in the range of the emission from the modulated parabolic well at low excitation intensity used in the experiment. According to the calculation of the transition energies in each quantum well, these lines are assigned to the central quantum well QW<sub>0</sub>, and to the nearest neighbor QW<sub>-1</sub> and next-neighbor QW<sub>-2</sub> quantum wells. The magnetic field induced changes in the PL intensity correlate with the quantum Hall (QH) states measured in the magnetotransport [Fig. 2(a)]. The integrated intensities of the differently polarized PL lines from the central quantum well and from the neighboring quantum wells are shown in Figs. 2(b) and 2(c), respectively. The PL features associated with the fractional QH states formed in the central quantum well are found as well. QH states with the filling factors  $\nu_{SL} = 1$ ,  $\nu_D = 1$ , and  $\nu_0 = 1$  related to different quantum phases are observed. We denote the filling factors of the low-field multiple layer phases as  $\nu_{SL}$  and  $\nu_D$ . They will be discussed further. The filling factor  $\nu_0$  corresponds to the single-well high-field phase found in Ref. 5.

Following Ref. 5, at  $\nu_0 = 1$  the magnetic field induces a quantum transition separating two phases with different distributions of the electron density over the quantum wells. In the low-field phase, the electrons are nearly uniformly distributed

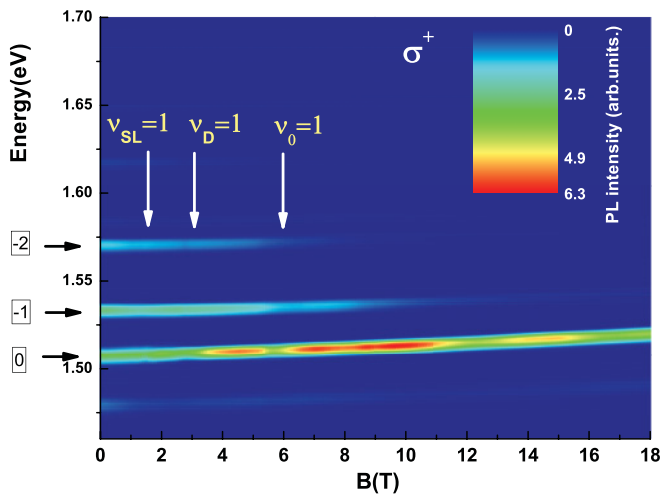


FIG. 1. (Color online)  $\sigma^+$  polarized magnetophotoluminescence of the superlattice with the disorder parameter  $\delta = 1.4$ .

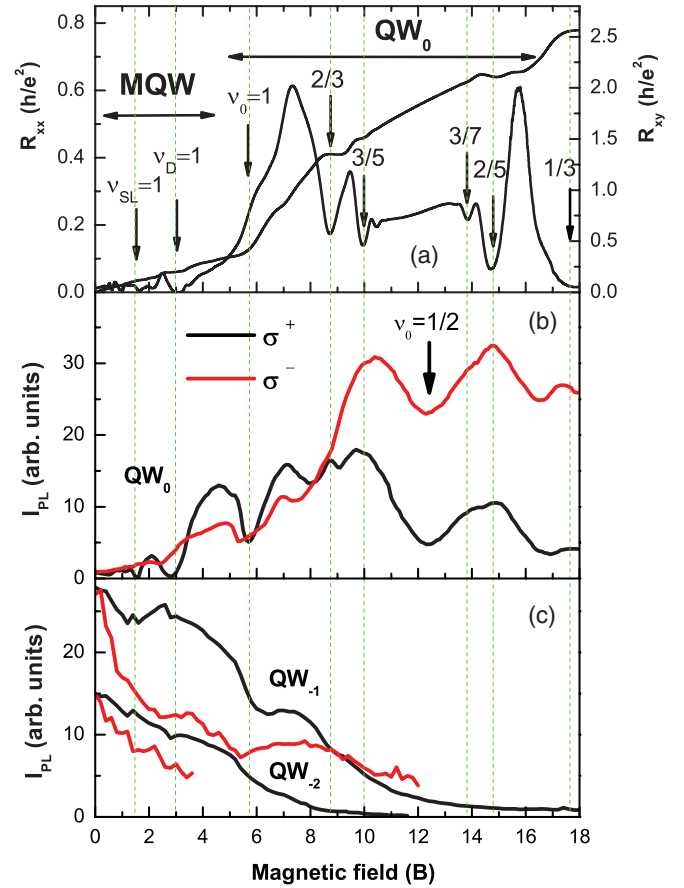


FIG. 2. (Color online) Magnetoresistance and Hall resistance (a),  $\sigma^+$  and  $\sigma^-$  polarized integrated photoluminescence intensities from the central quantum well (b) and  $\sigma^+$  and  $\sigma^-$  polarized integrated photoluminescence intensities from the lateral quantum wells (c) measured in the superlattice with  $\delta = 1.4$  as functions of the magnetic field.

over the quantum wells, and, therefore, all the quantum wells connected in parallel contribute to the conductivity of the sample. In this case, each of eight occupied quantum wells contains about 12% of the total electron concentration. In the high-field phase, the electrons in the lateral AlGaAs quantum wells are localized, while the central GaAs quantum well with considerably enhanced electron density determines the in-plane magnetoresistance of the sample revealing well developed fractional QH states in the range of high magnetic field. The Hall resistance data show that in high-field phase roughly half of the total number of electrons is placed in the central quantum well. Consequently, in the central quantum well, in the high-field phase the electron concentration increases. Consistent with the magnetoresistance data are the results obtained by magnetophotoluminescence. It ought to be mentioned that the magnetic field driven change in the integrated PL intensity cannot be directly attributed to the corresponding change in the occupied density of electron states, because the optical matrix element may depend on the magnetic field. Nevertheless, the integrated PL intensities shown in Figs. 2(b) and 2(c) qualitatively demonstrate the increasing electron density in the central quantum well and the decreasing densities in the neighboring quantum wells.

Hence, with the increasing magnetic field, the electrons move from the neighboring quantum wells to the central quantum well.

The filling factor  $\nu_{SL} = 1$  related to the zero resistance  $R_{xx}$  at the magnetic field  $B = 1.5$  T is attributed to the miniband QH state. The magnitude at the plateau of the Hall resistance  $R_{xy} = h/\nu_{SL}e^2N$  corresponding to this state points to the contribution of  $N = 8$  occupied quantum wells connected in parallel, which is in agreement with the number of occupied quantum wells obtained by the calculations and with the number of the PL lines emitted by the structure at the appropriate (large enough) excitation intensity. In this case, the multiple quantum well system behaves as a stack of independent quantum wells. The zero resistance  $R_{xx}$  observed at the magnetic field  $B = 3$  T corresponds to the miniband QH state with the filling factor  $\nu_{SL} = 1/2$ . In the following, we demonstrate that this QH state is likely the paired dimer state with the total filling factor  $\nu_D = 2\nu_{SL} = 1$  suggested in the QH multiple quantum wells as a ground state with the lowest energy.<sup>22</sup>

In order to reveal a nature of the QH state at  $B = 3$  T, we determine the degree of the circular polarization according to the formula:

$$P_\sigma = \frac{I^- - I^+}{I^- + I^+}, \quad (1)$$

where  $I^+$ ,  $I^-$  are the integrated PL peak intensities measured in the  $\sigma^+$  and  $\sigma^-$  polarizations, respectively. The circular polarization of the PL emission is associated with the polarization of the excitonic emission with effective  $g$  factor:  $g_{ex} = g_e + g_h$ , where  $g_e$  and  $g_h$  are for the electrons and for the holes, respectively. Consequently, the measured excitonic polarization has two contributions due to the polarizations of the electrons and of the holes.<sup>24</sup> As demonstrated in Ref. 25, in GaAs quantum wells the polarization of the holes monotonically increases with the magnetic field, at the same time as the electron polarization exhibits sharp features attributed to the QH states.

The energies of the polarized PL peak positions from the central quantum well ( $E_0$ ), together with the difference between them ( $\Delta_s$ ) associated with the spin-splitting energy, are shown as functions of the filling factor in Fig. 3(a). In a strong magnetic field, the PL emission in  $\sigma^+$  polarization is associated with the transition between the heavy-hole state with  $m_j = -3/2$  to the upper electron spin state  $m_j = -1/2$ , while the heavy-hole state with  $m_j = +3/2$  and the lower electron spin state  $m_j = +1/2$  are involved in the  $\sigma^-$  polarization. The red shifts observed in both polarizations in vicinities of the filling factors  $\nu_{SL} = 1$  and  $\nu_D = 1$  are due to changes in the binding energy of the interband exciton ( $E_X$ ) and in the binding energy of the spin wave ( $E_{SW}$ ), which are defined according to Ref. 26 as:  $\Delta E = E_{SW} - E_X$ . Similar red shifts of the polarized PL peak energies were also observed in single GaAs/AlGaAs quantum wells in Ref. 27.

Differently polarized integrated intensities of the PL emission from the central quantum well are shown as functions of the filling factor in Fig. 3(b). The zeros of the  $\sigma^+$  polarized intensity are associated with the zeros of the resistance observed at the magnetic fields  $B = 1.5$  T and  $B = 3$  T, and they point out the complete polarization of these QH states.

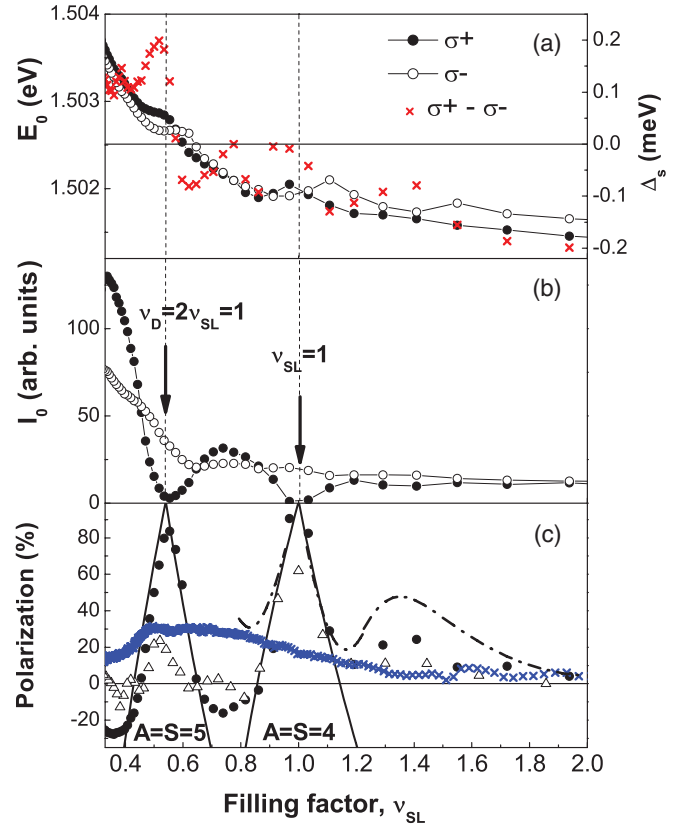


FIG. 3. (Color online) Energy (a) and integrated intensity (b) of the differently polarized photoluminescence emission from the central quantum well. Crosses in (a) are the splittings between differently polarized PL lines. Panel (c) shows polarization of the photoluminescence emission from the central quantum well of the superlattices with  $\delta = 1.4$  (closed circles for  $\theta = 0$  and crosses for  $\theta = 70^\circ$  as a function of the magnetic field component perpendicular to the sample surface) and with  $\delta = 2.3$  (open triangles for  $\theta = 0$ ) measured in the range of the miniband quantum Hall states as functions of the filling factor. Solid lines were calculated according to Eq. (2), while the dash-dotted line shows the spin polarization for a Skyrme crystal square lattice.<sup>29</sup>

Full polarization of the QH state observed at  $B = 3$  T does not correspond to a half-filled filling factor, and it allows us to attribute this state to the paired dimer state with the total filling factor  $\nu_D = 1$ . This ground state emerges in multiple quantum well electron systems due to a spontaneous interwell phase coherence, when the phase coherence is established only within individual pairs of neighboring quantum wells. As shown in Ref. 22, in a system with less than ten quantum wells the dimer ground state is stable at any ratio  $d/l_B$ , and the energy of the dimer state is smaller than that of a state in which phase coherence arises between all quantum wells.<sup>1</sup> Thus, two different quantum Hall ferromagnetic ground states of the multiple quantum well electron system are found: the uncorrelated miniband state and the correlated dimer state.

For noninteracting electrons, the PL in  $\sigma^+$  polarization must vanish at the filling factor  $\nu < 1$  because of depopulation of the upper electron spin state. On the contrary, the intensity of the PL observed in  $\sigma^+$  polarization enhanced as the magnetic field was increased past the filling factors  $\nu_{SL} = 1$  and  $\nu_D = 1$ .

As explained below, this indicates a magnetic field induced depolarization of the quantum Hall ferromagnetic state.

The polarization of the emission from the central quantum well determined according to Eq. (1) is shown in Fig. 3(c). According to Ref. 24, in wide GaAs quantum wells  $g$  factors of electrons and holes have opposite signs. Therefore, we assign the negative polarization to the contribution of the holes, which achieves about 20% at the filling factor  $\nu_D = 1$ . Two closely full polarized peaks, associated with the spin polarization of the electrons, are observed at the filling factors corresponding to the miniband QH state and to the dimer QH state. In the case of noninteracting electrons, their spins must be completely aligned as the magnetic field is increased over the filling factor  $\nu = 1$ . A rapid depolarization of the electron system observed with the decreasing filling factor is considered as a manifestation of the charged spin-texture skyrmion excitations of the corresponding quantum Hall ferromagnetic states.<sup>8,9</sup> It ought to be mentioned that according to the data presented in Figs. 3(a) and 3(c), the peaks in the PL polarization associated with the skyrmions are found in the magnetic field ranges where the spin-splitting energy closely achieves its smallest value. In agreement with the theory, vanishing spin-splitting energy favors large-size skyrmions.

In order to determine basic characteristics of the observed skyrmions, we applied the simple model proposed in Ref. 8. In this model, the spin polarization of the QH state with the filling factor  $\nu = 1$  is defined as:

$$S_z = \frac{A}{2} \left\{ \theta(1 - \nu) \left[ \frac{2}{\nu} (1 - A) - (1 - 2A) \right] + \theta(\nu - 1) \left[ \frac{2S}{\nu} + (1 - 2S) \right] \right\}, \quad (2)$$

with  $\theta(x) = 1$  and 0 for  $x > 0$  and  $x < 0$ , respectively.  $S$  and  $A$  give the numbers of spin flips above and below  $\nu = 1$  and due to electron-hole symmetry  $S = A$ . Noninteracting electrons imply the effective skyrmion spin  $S = A = 1$ , while for a two-dimensional electron system formed in GaAs the theory predicts a skyrmion spin of 3.5.<sup>28</sup> This value was found in excellent agreement with the experimental data presented in Refs. 8 and 9.

The spin polarizations calculated in this way in the ranges of the filling factors  $\nu_{SL} = 1$  and  $\nu_D = 1$  are depicted in Fig. 3(c) by solid lines. According to the best fits of the calculated polarizations to the experimental data, slightly different effective skyrmion spins were obtained for the miniband and dimer QH states,  $S = A = 4$  and 5, respectively. Both are found larger than the spin of the skyrmion in a single GaAs quantum well. This means a larger size of the skyrmion in multiple quantum well electron systems. Dash-dotted line in Fig. 3(c) demonstrates the spin polarization calculated near the  $\nu = 1$  ferromagnetic ground state of a two-dimensional electron gas for a Skyrme crystal with square lattice,<sup>29</sup> which well accounts for the low-field experimental data. It ought to be mentioned that as follows from Figs. 2(b) and 2(c), the observed skyrmions are formed mainly in the central GaAs quantum well with the highest mobility, while only weak minima were found at the corresponding filling factors in the  $\sigma^+$  polarized PL from the neighboring AlGaAs quantum wells. This shows a crucial importance of high mobility for

development of the skyrmions. Moreover, according to the data presented in Fig. 2(b), a narrow minimum of the  $\sigma^+$  polarized PL intensity observed at the magnetic field  $B = 6$  T points to a formation of skyrmions in the  $\nu_0 = 1$  QH state of the central quantum well. This state was observed in the magnetic field considerably higher than those corresponding to the miniband and dimer quantum Hall states. Consequently, a larger Zeeman energy resulted in a not complete polarization of the  $\nu_0 = 1$  state.

In a magnetic field tilted out of the direction perpendicular to the sample surface, the ratio  $\tilde{g}$  between the Zeeman and Coulomb energy increases causing the skyrmions to disappear at  $\tilde{g} > 0.02$ .<sup>28</sup> We examined the magneto-PL measured at the tilted angle  $\theta = 70^\circ$ . Assuming in GaAs  $\tilde{g} = 0.01$ ,<sup>14</sup> the angle tilting out to  $70^\circ$  will result in  $\tilde{g} = 0.017$ , which is close to the theoretical limit. Indeed, as shown in Fig. 3(c), both polarization peaks disappear at such a tilted magnetic field.

Moreover, the increasing temperature destroys skyrmions. In Refs. 8–10, the skyrmions were observed in the temperature range  $T = 0.5 - 1.6$  K, while skyrmions vanishing already at  $T = 0.5$  K were found in Ref. 11. In our samples, the polarization peaks attributed to skyrmions are observed at  $T = 0.3$  K, and they completely disappear at the temperature  $T = 1.6$  K.

Similar results were obtained in the multiple quantum well structure with the larger interlayer disorder parameter  $\delta = 2.3$ . The relevant polarization of the PL intensity is depicted as a function of the filling factor in Fig. 3(c) by open triangles. These data show that the skyrmions related to the miniband and dimer QH states vanish with the increasing interlayer disorder. Moreover, the polarization of the dimer state disappears faster than that of the miniband state. Indeed, in the structure with  $\delta = 2.3$ , the obtained polarization of the dimer state (about 30%) well corresponds to the expected polarization of the half-filled quantum Hall state ( $\nu = 1/2$ ).<sup>30</sup> Therefore, in this sample the dimer state is thought to be destroyed by the interlayer disorder. Hence, our data show that the interwell interaction essentially influences both the spontaneous interwell phase coherence and the formation of the observed skyrmions. The interwell interaction may explain a larger size of the skyrmions observed in the multiple quantum wells embedded in a wide parabolic well as compared to the skyrmions found in isolated quantum wells. According to Ref. 31, the size quantization considerably reduces the electron  $g$  factor when the quantum well thickness approaches 7 nm, which is close to the width of the AlGaAs quantum wells neighboring the central GaAs one. Moreover, a presence of Al in the neighboring quantum wells also decreases their  $g$  factor. The interwell interaction may cause the effect of decreasing  $g$  factor to be essential even in the central quantum well favoring large size skyrmions.

#### IV. CONCLUSION

In conclusion, we have found evidence for two QH ferromagnetic ground states in high mobility weakly coupled multiple layer quasi-three-dimensional electron systems: One state is the uncorrelated miniband state and another is the spontaneous interwell phase coherent dimer state. Polarized PL emissions associated with both QH states reveal features



attributed to the finite-size skyrmions. In accordance with the theory, these features vanish when the magnetic field is tilted. The effective macroscopic spins of the skyrmions were found to be larger than that of the skyrmion spins reported in a single quantum well.

## ACKNOWLEDGMENTS

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