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Microwave photoresistance in a two-dimensional electron gas with separated Landau levels

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Theories of microwave-induced resistance oscillations in high-mobility two-dimensional electron gas predict that, with decreasing oscillation order n or with increasing frequency ω, the photoresistance maxima should appear closer to the cyclotron resonance harmonics due to increased Landau-level separation. In this experimental study we demonstrate that, while for a given ω the peaks do move toward the harmonics with decreasing n, there is no corresponding movement with increasing ω for a given n. These findings show that the positions of the photoresistance maxima cannot be directly linked to the Landau-level separation, challenging our current understanding of the phenomenon.

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Magnetotransport in high Landau levels of two-dimensional electron systems (2DESs) exhibits a variety of phenomena, such as microwave-induced resistance oscillations (MIROs),[1–6] phonon-induced resistance oscillations,[7,8] Hall field-induced resistance oscillations,[9,10] and several classes of combined oscillations.[11–13] Experimentally, all these effects often extend into the regime of separated Landau levels where even more phenomena, such as radiation-induced zero-resistance states,[14,15] dc field-induced zero-differential resistance states,[16] and a sharp photoresistivity peak near the second harmonic of the cyclotron resonance,[17] emerge. On the other hand, the majority of the theoretical proposals[5,6,8,10,13] focus on the overlapping Landau-level regime and, as a result, their direct applicability to many experiments remains uncertain.

Theoretically, two mechanisms are usually discussed in relation to MIRO, *displacement*[5,18] and *inelastic*.[6,19] The displacement contribution[5,18] originates from the modification of impurity scattering by microwave radiation, while the inelastic mechanism[6,19] owes to the microwave-induced nonequilibrium distribution of electrons. In both cases, MIROs are understood in terms of optical transitions between the disorder-broadened Landau levels. In the regime linear in microwave intensity and overlapping Landau levels, the oscillatory photoresistivity can be described by[19]

$$\frac{\delta\rho_\omega(\epsilon)}{\rho_0} \simeq -\eta\mathcal{P}_\omega\lambda^2\epsilon\sin 2\pi\epsilon. \tag{1}$$

Here, ρ_0 is the resistivity at $B=0$, $\epsilon=\omega/\omega_c$, $\omega=2\pi f$ is the microwave frequency, $\omega_c=eB/m^*$ is the cyclotron frequency of an electron with an effective mass m^*, $\lambda=\exp(-\pi\alpha_\omega\epsilon)$ is the Dingle factor, $\alpha_\omega=(\omega\tau_q)^{-1}$, τ_q is the quantum lifetime, η is a scattering parameter,[20] and $\mathcal{P}_\omega$ is the dimensionless parameter proportional to the microwave power which, for circular polarization, is given by[12]

$$\mathcal{P}_\omega(\epsilon)=\frac{\mathcal{P}^0_\omega}{(1-\epsilon^{-1})^2+\beta^2_\omega},\quad \mathcal{P}^0_\omega=\frac{e^2\mathcal{E}^2_{ac}v^2_F}{\varepsilon_{eff}\hbar^2\omega^4}, \tag{2}$$

where $\beta_\omega\equiv(\omega\tau_{em})^{-1}$, $\tau^{-1}_{em}=n_e e^2/2\sqrt{\varepsilon_{eff}}\epsilon_0 m^* c$, $2\sqrt{\varepsilon_{eff}}=\sqrt{\varepsilon}+1$, $\varepsilon=12.8$ is the dielectric constant of GaAs, v_F is the Fermi velocity, and $\mathcal{E}_{ac}$ is the external (unscreened) microwave electric field.

The photoresistance vanishes at the harmonics of the cyclotron resonance $\epsilon=n=1,2,3,\ldots$, and the positions of the MIRO maxima (ϵ^+) and minima (ϵ^-) are given by

$$\epsilon^\pm_n=n\mp\delta_n, \tag{3}$$

where $\delta_n\equiv|\epsilon^\pm_n-n|$ is usually called the *phase*. In a typical high-mobility 2DES, $\tau_q\sim\tau_{em}\sim10^{-11}$ s and $\alpha_\omega\sim\beta_\omega\ll1$ at $f\sim10^{11}$ Hz. As a result, for all $n\neq1$, Eq. (1) predicts $\delta_n\simeq1/4$. However, close to the cyclotron resonance, the phase can become significantly smaller at higher ω due to enhancement of $\mathcal{P}_\omega$ near the cyclotron resonance.

In the regime of separated Landau levels, Eq. (1) is no longer valid and the phase δ_n will be governed by the ratio of the Landau level width Γ to the cyclotron energy

$$\delta_n\simeq\frac{\kappa\Gamma}{\hbar\omega_c}\simeq\frac{\kappa\Gamma}{\hbar\omega}n. \tag{4}$$

Here, $\kappa\sim1$ (Ref. 21) and the last approximation in Eq. (4) is justified at $\Gamma\ll\hbar\omega_c$. To illustrate the origin of Eq. (4), we consider, as an example, the leading part of the displacement contribution[22,23]

$$\delta\rho_\omega(\epsilon)\propto\partial_\omega\langle\nu_\varepsilon\nu_{\varepsilon+\hbar\omega}\rangle_\varepsilon, \tag{5}$$

where ν_ε is the density of states at energy ε and $\langle\cdots\rangle_\varepsilon$ denotes averaging over the cyclotron energy $\hbar\omega_c$.[24] At $\Gamma<\hbar\omega_c$, the photoresistivity $\delta\rho_\omega(\epsilon)$ will be substantial only when the initial (ν_ε) and the final ($\nu_{\varepsilon+\hbar\omega}$) densities of states overlap. As a result, the detuning from the closest cyclotron resonance harmonic must be close to Γ, $\hbar|\omega-n\omega_c|\sim\Gamma$, i.e., the condition equivalent to Eq. (4).

Equation (4) predicts that in the regime of separated Landau levels the phase δ_n should decrease when one lowers the oscillation order n or raises the microwave frequency ω. It also suggests that the evolution of the phase with the magnetic field should yield direct information on the B dependence of Γ, which is *not* readily available from conventional transport measurements. However, as we show below, our understanding of the phenomenon needs to be further improved before one attempts to extract Γ from Eq. (4).

In this Rapid Communication we systematically examine the phase of MIRO over a wide range of microwave

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frequencies, covering both the overlapping and separated Landau-level regimes. We find that for a given frequency ω the phase of high-order ($n \gtrsim 3$) MIRO is close to 1/4, in agreement with Eq. (1), and is significantly smaller for lower orders, in agreement with Eq. (4) and previous studies.[2–4] However, we observe *no* decrease of δ_n with increasing ω within the accuracy of our measurements; for any given n, the phase remains constant over the whole range of frequencies studied. This finding contradicts Eq. (4), indicating that the phase reduction commonly observed at lower-order MIRO[2–4] cannot be explained by existing theories of microwave photoconductivity.

Our sample is a Hall bar (width $w = 100$ μm) fabricated from a GaAs/$Al_{0.24}Ga_{0.76}As$ 300-Å-wide quantum well grown by molecular beam epitaxy. The density n_e and the mobility μ were 3.6×10^{11} cm^{-2} and $\simeq 1.0 \times 10^7$ cm^2/V s, respectively. Microwave radiation of frequency f (60–180 GHz), generated by Gunn and backward wave oscillators, was delivered to the sample via either a WR-28 waveguide or a 1/4-in.-diam light pipe. The microwave intensity was kept sufficiently low to ensure that all measurements were performed in the regime linear in microwave power.[27] The resistivity ρ_ω was measured at $T \approx 0.5$ K under continuous microwave irradiation using a standard low-frequency lock-in technique.

In Figs. 1(a)–1(c) we present magnetoresistivity $\rho_\omega(B)$ under microwave irradiation of (a) $f = 76$ GHz, (b) $f = 105$ GHz, and (c) $f = 150$ GHz. All three data sets exhibit pronounced MIRO extending over a progressively wider range of the magnetic fields with increasing microwave frequency, as prescribed by Eq. (1). The data obtained at $f = 105$ GHz also show that the photoresistivity near the second harmonic of the cyclotron resonance $\omega/\omega_c = 2$ clearly reveals a double-peak structure. We attribute the sharper, lower B feature [cf. ↓ in Fig. 1(b)], to the recently reported so-called $\mathcal{X}_2$ peak.[17] As a result of its characteristic frequency dependence,[17] this peak is not observed in our 2DES at lower frequencies [cf. Fig. 1(a)] and becomes dominant at higher frequencies [cf. ↓ in Fig. 1(c)]. In what follows, we systematically investigate the positions of *all* the photoresistivity maxima, including the $\mathcal{X}_2$ peak.

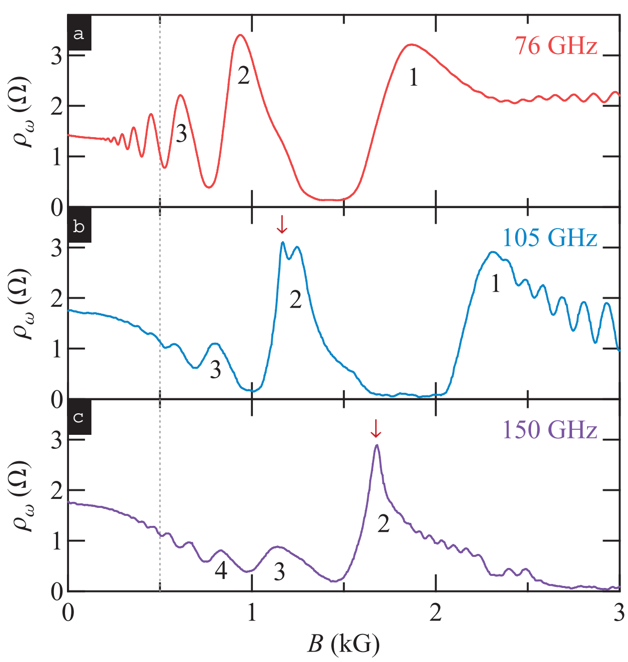

FIG. 1. (Color online) Magnetoresistivity $\rho_\omega(B)$ under microwave irradiation of (a) $f = 76$ GHz, (b) $f = 105$ GHz, and (c) $f = 150$ GHz. Photoresistance peaks are marked by integers, indicating the closest cyclotron resonance harmonic. The vertical line marks the transition from overlapping to separated Landau levels estimated from $\omega_c\tau_q = \pi/2$ (Refs. 25 and 26).

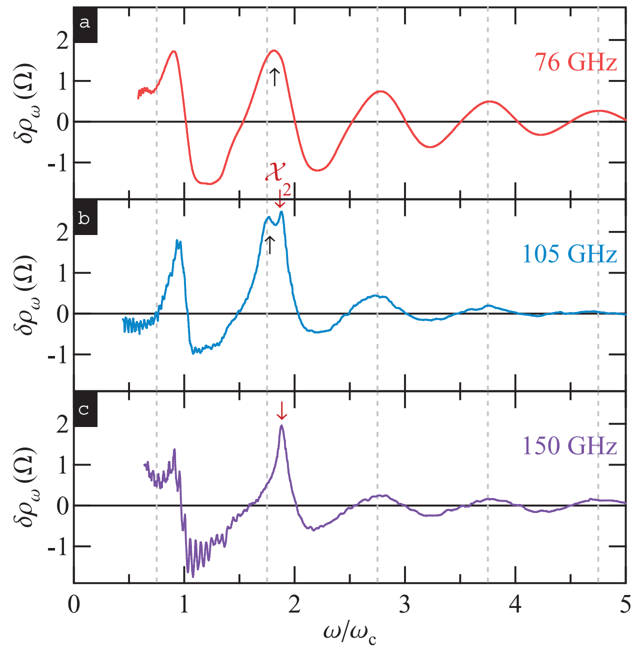

FIG. 2. (Color online) Microwave photoresistivity $\delta\rho_\omega$ vs ω/ω_c for (a) $f = 76$ GHz, (b) $f = 105$ GHz, and (c) $f = 150$ GHz. The solid vertical lines correspond to $\omega/\omega_c = n - 1/4$.

We start by extracting the oscillatory part of the resistivity $\delta\rho_\omega$ from the data in Fig. 1 and presenting the result as a function of $\omega/\omega_c \propto 1/B$ in Fig. 2. Plotted in such a way, the data readily reveal for *all* microwave frequencies that higher-order ($n \geqslant 3$) MIRO peaks are well described by $\omega/\omega_c = n - \delta_n$, $\delta_n = 1/4$ (cf. vertical lines), in agreement with Eq. (1). This observation is in contrast to the lower-order ($n = 1,2$) peaks which exhibit considerably reduced phase values. As discussed above, the phase reduction is anticipated in the regime of separated Landau levels, regardless of the physical mechanism or the shape of the Landau level. More specifically, Eq. (4) predicts that for a given (high enough) frequency ω, the phase δ_n should decrease with decreasing n. This result is consistent with our observations, as $\delta_1 < \delta_2 < \delta_3$. At the same time, Eq. (4) also prescribes that for a given (low enough) cyclotron resonance harmonic n, the phase should monotonically *decrease* with increasing microwave frequency ω, as $\delta_n \propto \Gamma/\omega$. However, as we show next, our experimental findings fail to confirm this expectation.

Using the photoresistivity data such as that shown in Fig. 2, we extract the peak positions for all frequencies studied. The results are presented in Figs. 3(a)–3(d), displaying the phase δ_n versus microwave frequency f for (a) $n = 1$, (b) $n = 2$, (c) $n = 3$, and (d) $n = 4$. For $n = 3$ and $n = 4$ [cf. Figs. 3(c) and 3(d), respectively] we observe that the phase shows very little variation with microwave frequency and is close to 1/4

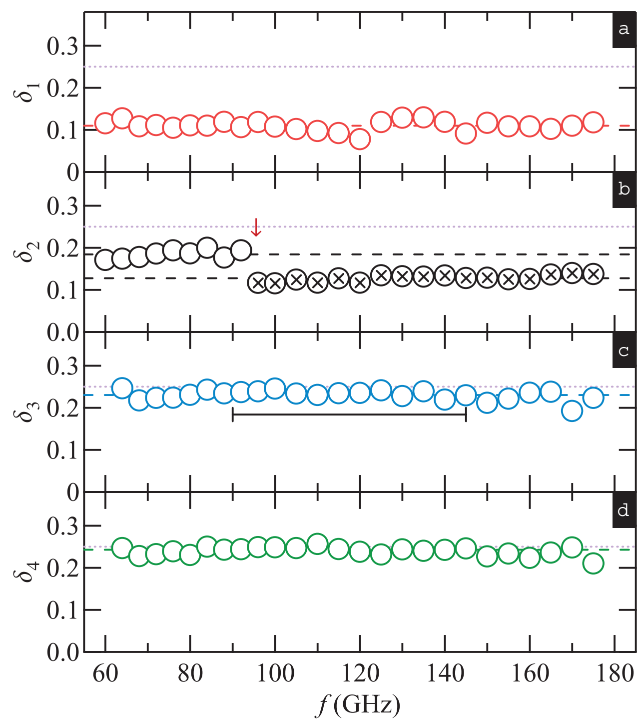

FIG. 3. (Color online) Phase δ_n of the $\delta\rho_\omega$ maxima vs f for (a) $n = 1$, (b) $n = 2$, (c) $n = 3$, and (d) $n = 4$. The dashed lines represent average values $\langle\delta_n\rangle$: (a) 0.110, (b) 0.184 and 0.128, (c) 0.230, and (d) 0.243. The horizontal line in (c) is drawn at $\langle\delta_2\rangle = 0.184$ over the range of B to the left-hand side of $\downarrow$ in (b).

(cf. solid lines), a theoretical value expected in the regime of overlapping Landau levels—see Eq. (1).[28] Indeed, the dashed lines drawn at average phase values $\langle\delta_3\rangle = 0.230$ and $\langle\delta_4\rangle = 0.243$ show no sign of decrease at higher frequencies. On the other hand, for the peak near $n = 2$ [cf. Fig. 3(b)] the phase clearly shows a jump occurring near 100 GHz in our 2DES. This jump marks the appearance of the $\mathcal{X}_2$ peak,[17] which is absent at lower frequencies but dominates the response at higher frequencies. Apart from this jump, the phase again shows little change with increasing frequency both for the MIRO peak ($f \leqslant 92$ GHz) and for the $\mathcal{X}_2$ peak ($f \geqslant 96$ GHz).

The average value of the phase at the $n = 2$ MIRO peak is reduced considerably compared to $n \geqslant 3$ MIRO peaks averaging at $\langle\delta_2\rangle = 0.184$ (cf. higher dashed line) and the phase of the $\mathcal{X}_2$ peak is even lower averaging at 0.128 (cf. lower dashed line). Examination of the phase of the fundamental ($n = 1$) MIRO peak in Fig. 3(a) reveals a further reduced phase $\langle\delta_1\rangle = 0.110$ (cf. dashed line), which again is almost *independent* of the microwave frequency. Somewhat larger fluctuations of δ_1 about the average value can be attributed to the overlap with the Shubnikov–de Haas oscillations and to the unavoidable variations in the incident microwave power which might affect the phase through the enhancement of $\mathcal{P}_\omega$ near the cyclotron resonance—see Eq. (2).[29]

The ratio of the magnetic fields at the second and at the third MIRO maxima can be estimated as $B_2/B_3 \simeq (3 - \langle\delta_3\rangle)/(2 - \langle\delta_2\rangle) \simeq 1.5$. This value is a factor of 2 lower than the variation in microwave frequency in our experiment and, therefore, the phases δ_2 and δ_3 can be directly compared over the same range of magnetic fields. Indeed, the lower-frequency range (below the onset of the $\mathcal{X}_2$ peak), where the phase of the second MIRO peak can be reliably determined, can be mapped to a frequency range for the third MIRO peak. This range is represented in Fig. 3(c) by a horizontal line drawn between 90 and 145 GHz. It is clear that not only within this range but also at higher frequencies the phase of the third peak $\langle\delta_3\rangle = 0.230$ remains higher than $\langle\delta_2\rangle = 0.184$.

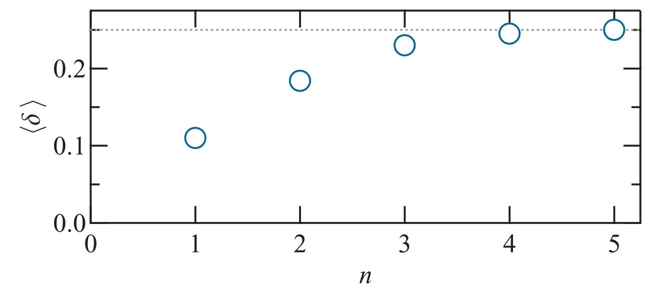

FIG. 4. (Color online) Phase values obtained by averaging over all microwave frequencies vs the peak order n.

Taken together, our findings bring us to the conclusion that in our 2DES the phase of MIRO is determined primarily by the oscillation order n. This result is summarized in Fig. 4, showing average phase values as a function of n. We notice that a very similar dependence was previously observed for $f = 57$ GHz.[2] In the present Rapid Communication, we demonstrate that this dependence appears to be universal, i.e., the phase values are not influenced by the microwave frequency.

In summary, we have studied the microwave-induced resistance oscillations and the $\mathcal{X}_2$ peak in a high-mobility 2DES over a wide range of microwave frequencies. For each microwave frequency, we have found that the phase of the lower-order MIRO becomes smaller with decreasing order n, consistent with earlier experiments.[2–4] However, aside from an abrupt phase change near the second harmonic of the cyclotron resonance associated with the appearance of the $\mathcal{X}_2$ peak,[17] the phase of *all* photoresistance maxima, including the $\mathcal{X}_2$ peak, is found to be *independent* of the microwave frequency and, thus, of the magnetic field. These findings contradict the generally accepted view that in the regime of separated Landau levels the phase value directly reflects the ratio of the Landau level width to the cyclotron energy and therefore should decrease with the magnetic field.

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[28] Observation of a $1/4$ phase at $n \gtrsim 3$ confirms that for all frequencies studied our measurements are performed in the regime linear in microwave intensity.

[29] At high enough microwave power, the MIRO phase can be reduced due to, e.g. multiphoton processes, see e.g. A. T. Hatke, M. A. Zudov, M. Khodas, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **84**, 241302(R) (2011).