Kondo effect in interacting nanoscopic systems: Keldysh field integral theory

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Kondo physics in nonequilibrium interacting nanoscale devices is an attractive fundamental many-particle phenomenon with a rich potential for applications. Due to enormous complexity its clear and flexible theory is still highly desirable. We develop a physically transparent analytical theory capable of correctly describing the Kondo effect in strongly interacting systems at temperatures close to and above the Kondo temperature. We derive a nonequilibrium Keldysh field theory valid for a system with any finite electron-electron interaction which is much stronger than the coupling of the system to contacts. Finite electron-electron interactions are treated involving as many slave-boson degrees of freedom as one needs for a concrete many-body system. In a small vicinity of the zero slave-bosonic field configuration weak slave-bosonic oscillations, induced by the dot-contacts tunneling, are described by an effective Keldysh action quadratic in the slave-bosonic fields. For clarity the theory is presented for the single impurity Anderson model but the construction of the Keldysh field integral is universal and applicable to systems with more complex many-body spectra.

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I. INTRODUCTION

With the emergence of technological feasibility in manufacturing small artificial systems with few electrons and using single molecules as electronic devices, quantum nonequilibrium has taken a new wind lasting already for more than two decades. On one side the diminutiveness of these nanoscale systems facilitates their fast passage to a nonequilibrium state already at tiny external voltages which calls for more comprehensive nonequilibrium theories beyond linear response. On the other side at these scales electron-electron correlations start playing a significant role leading to phenomena which are fundamentally absent in the noninteracting counterparts. A remarkable representative of such nonequilibrium interacting phenomena is the Kondo effect[1,2] revealing itself in the differential conductance as a zero-bias anomaly having a universal temperature dependence[3,4] with the energy scale known as the Kondo temperature T_K.

The Kondo physics turns out to be highly complex. In particular, in its temperature dependence it has two regimes, weak and strong coupling ones, with the crossover taking place in the vicinity of the Kondo temperature. This has given rise to the development of two types of analytical theories valid either deep below or much above T_K. In the construction of such theories the concept of a slave boson has played an extremely fruitful role and it has been used both for the first[5–7] and second[8] types of analytical theories. The Kondo effect is governed by infrared physics. A powerful method capturing this behavior is the renormalization group method applied to the Kondo model, both in its analytical[9] and numerical[10] implementations. The analytical RG theory has been generalized[11,12] to nonequilibrium and provides a controlled theory at bias voltages $eV > kT_K$.

Numerical approaches have certain advantages. For example, the numerical renormalization group theory[13] can cover both the weak and strong coupling regimes. Unfortunately, its generalization to nonequilibrium is still problematic. Another numerical method, the noncrossing approximation (NCA),[14,15] is able to deal with nonequilibrium situations but it gets less reliable below T_K. However, the main disadvantage of all numerical techniques is that they do not provide a clear picture of a physical phenomenon and, therefore, analytical theories have higher fundamental priority.

The aim of the present study is to demonstrate that the field-theoretic approach[16] based on the Keldysh field integral in the slave-bosonic representation is a powerful tool in the development of analytical theories for the Kondo effect in nanoscale systems and that the derivation of the effective Keldysh action represents a universal, straightforward, and physically clear way to possible theoretical generalizations inspired by modern experiments. In addition to the technical flexibility this field-theoretic framework in its original simplest formulation is already close to the crossover between the weak and strong coupling regimes: It is valid for temperatures $T \gtrsim T_K$. Therefore, after a proper generalization it may penetrate the strong coupling regime and become an analytical theory of both the first and second type.

To be specific, here we will develop a generalization to a particularly complicated and at the same time extremely important aspect related to a strong but finite electron-electron interaction in a quantum dot (QD). Up to now many analytical theories (which are mainly of the first type) have only been able to describe the Kondo physics in QDs with either rather weak[17] or infinitely strong[6–8] electron-electron interactions. Thus, an analytical theory which could be placed between these two extreme cases is in great demand, especially, if this theory is of the second type which is more interesting for applications. It is our goal here to present such a theory.

The paper is organized as follows. In Sec. II we formulate the problem and transform it into a slave-bosonic representation. The effective Keldysh action is derived in Sec. III. It is used to derive the tunneling density of states in Sec. IV. With Sec. V we conclude.

II. HAMILTONIAN AND SLAVE BOSONS

We will show a nonperturbative, both in the QD-contacts coupling and electron-electron interaction, field-theoretic

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construction for the case of the single impurity Anderson model (SIAM) of an interacting QD,

$$\hat{H}_{\rm d} = \sum_{\sigma} \epsilon_{\rm d} \hat{n}_{{\rm d},\sigma} + U \hat{n}_{{\rm d},\uparrow} \hat{n}_{{\rm d},\downarrow}, \tag{1}$$

where $\epsilon_{\rm d}$ is the single-particle electronic ($\sigma = \uparrow, \downarrow$) energy level of the QD, U is the strength of the electron-electron interaction, and $\hat{n}_{{\rm d},\sigma} = d_\sigma^\dagger d_\sigma$ with $d_\sigma^\dagger$, d_σ being the QD electronic creation and annihilation operators. The Hamiltonian given by Eq. (1) is sufficient to show the basic steps of the theory without technical complications playing a principal role in the formalism. The generalization to more complicated models is straightforward as will become clear from the derivation below.

To complete the formulation of the problem one introduces the tunneling Hamiltonian,

$$\hat{H}_{\rm T} = \sum_{a\sigma} (c_a^\dagger T_{a\sigma} d_\sigma + d_\sigma^\dagger T_{a\sigma}^* c_a), \tag{2}$$

and the contacts Hamiltonian,

$$\hat{H}_{\rm C} = \sum_{a} \epsilon_a c_a^\dagger c_a. \tag{3}$$

In Eqs. (2) and (3) $c_a^\dagger$ (c_a) are the contacts electronic creation (annihilation) operators with respect to the state a, which is a quantum electronic state in the left (L) or the right (R) contact. The quantity $T_{a\sigma}$ in Eq. (2) is the probability amplitude for the tunneling event from the QD state σ to the contact state a, and ϵ_a in Eq. (3) is the contact electronic single-particle energy corresponding to the state a.

Before constructing the Keldysh field theory we rewrite the QD Hamiltonian, Eq. (1), and the tunneling Hamiltonian, Eq. (2), using slave bosons and new fermions replacing the original electrons of the QD. To do this we will employ the approach developed for magnetic impurities by Barnes.[18] It was also used later for superconducting compounds by Zou and Anderson.[19] This approach represents a natural extension of the infinite-U slave-boson approach.[20] An alternative finite-U slave-boson method was proposed by Kotliar and Ruckenstein.[21] The essential difference between the two methods is that the latter one, instead of new fermions, uses magnetic bosons. However, as mentioned in Ref. 21, it does not matter how one introduces slave particles if the corresponding constraints are taken into account exactly: The final results must be independent of the slave-particle scheme.

To apply the Barnes and Zou and Anderson technique one exploits the knowledge of the many-particle spectrum of the SIAM Hamiltonian, Eq. (1): $|0\rangle$, $\epsilon_0 = 0$; $|\uparrow\rangle$, $|\downarrow\rangle$, $\epsilon_\uparrow = \epsilon_\downarrow = \epsilon_{\rm d}$; $|\uparrow\downarrow\rangle$, $\epsilon_{\uparrow\downarrow} = 2\epsilon_{\rm d} + U$. Introducing the slave-bosonic creation and annihilation operators $e^\dagger$, e for the empty ($|0\rangle$), $d^\dagger$, d for the doubly occupied ($|\uparrow\downarrow\rangle$) QD states ($e$-state and d-state for later reference) and new fermionic creation and annihilation operators $p_\sigma^\dagger$, p_σ for the singly occupied ($|\uparrow\rangle$, $|\downarrow\rangle$) QD states, so that $d_\sigma^\dagger = e p_\sigma^\dagger + \sigma d^\dagger p_{-\sigma}$, $d_\sigma = e^\dagger p_\sigma + \sigma d p_{-\sigma}^\dagger$, Eqs. (1) and (2) can be rewritten as

$$\hat{H}_{\rm d} = \sum_{\sigma} \epsilon_{\rm d} p_\sigma^\dagger p_\sigma + (2\epsilon_{\rm d} + U) d^\dagger d, \tag{4}$$

$$\hat{H}_{\rm T} = \sum_{a\sigma} [T_{a\sigma} c_a^\dagger (e^\dagger p_\sigma + \sigma d p_{-\sigma}^\dagger) + T_{a\sigma}^* (e p_\sigma^\dagger + \sigma d^\dagger p_{-\sigma}) c_a]. \tag{5}$$

The full Hamiltonian is $\hat{H} = \hat{H}_{\rm d} + \hat{H}_{\rm C} + \hat{H}_{\rm T}$. Since the physical subspace, the QD Fock space, is generated by the four states, $|0\rangle$, $|\uparrow\rangle$, $|\downarrow\rangle$, $|\uparrow\downarrow\rangle$, the total number of the slave bosons and new fermions, $\hat{Q} \equiv e^\dagger e + \sum_\sigma p_\sigma^\dagger p_\sigma + d^\dagger d$, must be equal to one, $\hat{Q} = \hat{I}$. One takes this constraint into account by replacing the total Hamiltonian with a different one, $\hat{H} \to \hat{H} + \mu \hat{Q}$, and taking the limit $\mu \to \infty$ at the end of all calculations. In comparison with mean field theories this projection onto the physical subspace is exact. Therefore, it is also applicable for high temperatures and is suitable for a development of both the first- and second-type analytical theories.

At this point we would like to mention that, as is obvious from above, the applicability of the Barnes and Zou and Anderson approach is not restricted only by SIAM, Eq. (1). It is much more universal and applicable to an arbitrary many-particle system for which its many-body spectrum has been found. Then one can, in full analogy with above, introduce slave bosons for the states with the even particle numbers and new fermions for the states with the odd particle numbers.

III. EFFECTIVE KELDYSH ACTION

Using Eqs. (3)–(5) we develop a slave-bosonic Keldysh field theory. To this end we construct the Keldysh field integral.[22] The field theory is obtained performing the functional integration over all the fermionic fields and eventually is formulated through the effective Keldysh action. In comparison with Ref. 16 this action depends now not only on the e-state slave-bosonic field, which we denote as $\chi(t)$, but also on the d-state slave-bosonic field, which we denote as $\xi(t)$:

$$\begin{aligned} S_{\rm eff}[\chi^{\rm cl}(t), \chi^{\rm q}(t); \xi^{\rm cl}(t), \xi^{\rm q}(t)] &= S_{{\rm B}e}^{(0)}[\chi^{\rm cl}(t), \chi^{\rm q}(t)] + S_{{\rm B}d}^{(0)}[\xi^{\rm cl}(t), \xi^{\rm q}(t)] \\ &\quad + S_{\rm tun}[\chi^{\rm cl}(t), \chi^{\rm q}(t); \xi^{\rm cl}(t), \xi^{\rm q}(t)], \end{aligned} \tag{6}$$

where $\chi^{\rm cl(q)}(t)$, $\xi^{\rm cl(q)}(t)$ are the classical (quantum) components of the slave-bosonic fields, the actions $S_{{\rm B}e}^{(0)}$ and $S_{{\rm B}d}^{(0)}$ describe the free dynamics of the e- and d-states, respectively, and the action $S_{\rm tun}$ embodies the complex tunneling dynamics of both the e- and d-states.

Any QD observable $\hat{O} = \mathcal{F}(d_\sigma^\dagger, d_\sigma)$ admits the Keldysh field integral representation in terms of the effective Keldysh action Eq. (6),

$$\begin{aligned} \langle \hat{O} \rangle(t) = \frac{1}{\mathcal{N}_0} \lim_{\mu\to\infty} e^{\beta\mu} &\times \int \mathcal{D}[\chi(t), \xi(t)] e^{\frac{i}{\hbar} S_{\rm eff}[\chi^{\rm cl}(t), \chi^{\rm q}(t); \xi^{\rm cl}(t), \xi^{\rm q}(t)]} \\ &\times \mathcal{F}[\chi^{\rm cl}(t), \chi^{\rm q}(t); \xi^{\rm cl}(t), \xi^{\rm q}(t)], \end{aligned} \tag{7}$$

where $\mathcal{N}_0$ is the normalization factor (see Ref. 16) and β is the inverse temperature.

The general structure of $S_{\rm tun}$ is the same as in Ref. 16, $S_{\rm tun}[\chi^{\rm cl}(t),\chi^{\rm q}(t);\xi^{\rm cl}(t),\xi^{\rm q}(t)] = -i\hbar\,{\rm tr}\ln[I+\mathcal{T}G^{(0)}]$. However, the matrices $G^{(0)}$ and $\mathcal{T}$ are now different. The free Green's function matrix $G^{(0)}$ has now an explicit spin dependence,

$$G^{(0)} = \begin{pmatrix} \sigma\, G^{(0)}_{\rm d}(\sigma t|\sigma' t') & 0 \\ 0 & \sigma\, G^{(0)}_{\rm C}(\bar{a}\sigma t|\bar{a}'\sigma' t') \end{pmatrix}, \quad (8)$$

where $G^{(0)}_{\rm d,C}$ are the fermionic Keldysh Green's function matrices for the isolated QD and contacts. These matrices are diagonal in the spin space. In Eq. (8) we have picked out the spin index from the state a, that is, $a \equiv \{\bar{a},\sigma\}$ and $\bar{a}$ is the state a but the spin. This explicit spin dependence appears because the mapping $\{d^\dagger_\sigma, d_\sigma\} \to \{e^\dagger, e; d^\dagger, d; p^\dagger_\sigma, p_\sigma\}$ has an explicit spin dependence. What is more crucial is that now, due to this spin-dependent mapping, the tunneling matrix $\mathcal{T}$ is not diagonal anymore in the spin space. It has the same block structure as in Ref. 16 but its blocks have a different architecture in the spin space:

$$M_{\rm T}(\bar{a}\sigma t|\sigma' t') = \frac{1}{\hbar}\delta(t-t')\begin{pmatrix} T_{\bar{a}\uparrow\uparrow}(t) & \tilde{T}_{\bar{a}\uparrow\downarrow}(t) \\ -\tilde{T}^*_{\bar{a}\downarrow\uparrow}(t) & -T^*_{\bar{a}\downarrow\downarrow}(t) \end{pmatrix}, \quad (9)$$

where $T_{\bar{a}\sigma\sigma'}(t) \equiv T_{\bar{a}\sigma\sigma'}\hat{\chi}^\dagger(t)$, $\tilde{T}_{\bar{a}\sigma\sigma'}(t) \equiv -\sigma' T_{\bar{a}\sigma\,-\sigma'}\hat{\xi}(t)$. Here $T_{\bar{a}\sigma\sigma'} = \delta_{\sigma\sigma'}T$ which assumes a symmetric coupling to the contacts. The matrix $\hat{\chi}(t)$ is the same as in Ref. 16 and the new matrix $\hat{\xi}(t)$ is

$$\hat{\xi}(t) \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} \xi^{\rm q}(t) & \xi^{\rm cl}(t) \\ \xi^{\rm cl}(t) & \xi^{\rm q}(t) \end{pmatrix}. \quad (10)$$

One can see from Eqs. (9) and (10) that the off-diagonal blocks in the spin space have an anomalous matrix structure in the Keldysh space: the classical components are the off-diagonal elements while the quantum components are on the main diagonal. Physically this nondiagonal spin structure and the anomalous structure in the Keldysh space are the consequence of the QD population fluctuations between one and two electrons. These fluctuations are now allowed due to the finite value U of the electron-electron interaction.

At this point it is important to mention that in our field theory there is no need to scan over a huge family of diagrams in order to find diagrams recovering the symmetry between virtual transitions into the empty and doubly occupied states. Violation of this symmetry represents the main problem in extensions[15] of NCA for finite-U systems. In contrast, in our field theory on one side one does not have to battle with the diagrammatic combinatorics and on the other side one does not need to care about the symmetry violation since both virtual transitions are equally treated by keeping in the effective Keldysh action the same powers of both the empty and double occupancy slave-bosonic fields. Below we keep terms only up to the second order in these fields and it is obvious that the same can be done at any order. From the diagrammatic perspective the theory could be close in spirit to the equilibrium imaginary-time approach[23] for the infinite-U Anderson model.

As we are going to construct an analytical theory of the second type, we consider a QD in the Kondo regime formed by weak (see Ref. 16) slave-bosonic oscillations excited by the electronic tunneling between the QD and contacts. In this case one can expand $S_{\rm tun}$ around the zero configurations of the slave-bosonic fields $\chi(t)$ and $\xi(t)$ up to the second order:

$$\begin{aligned} &S_{\rm tun}[\chi^{\rm cl}(t),\chi^{\rm q}(t);\xi^{\rm cl}(t),\xi^{\rm q}(t)] \\ &= \frac{\pi\Gamma}{2\hbar}\int dt\int dt'(\bar{\chi}^{\rm cl}(t)\quad \bar{\chi}^{\rm q}(t)) \\ &\quad\times\begin{pmatrix} 0 & \Sigma^-_{{\rm B}e}(t-t') \\ \Sigma^+_{{\rm B}e}(t-t') & \Sigma^{\rm K}_{{\rm B}e}(t-t') \end{pmatrix}\begin{pmatrix} \chi^{\rm cl}(t') \\ \chi^{\rm q}(t') \end{pmatrix} \\ &\quad-\frac{\pi\Gamma}{2\hbar}\int dt\int dt'(\bar{\xi}^{\rm cl}(t)\quad \bar{\xi}^{\rm q}(t)) \\ &\quad\times\begin{pmatrix} 0 & \Sigma^-_{{\rm B}d}(t-t') \\ \Sigma^+_{{\rm B}d}(t-t') & \Sigma^{\rm K}_{{\rm B}d}(t-t') \end{pmatrix}\begin{pmatrix} \xi^{\rm cl}(t') \\ \xi^{\rm q}(t') \end{pmatrix}, \end{aligned} \quad (11)$$

where $\Sigma^{+,-,{\rm K}}_{{\rm B}e}$ and $\Sigma^{+,-,{\rm K}}_{{\rm B}d}$ are the slave-bosonic self-energies of the e- and d-states. The self-energies $\Sigma^{+,-,{\rm K}}_{{\rm B}e}$ have been derived in Ref. 16 and the new objects, $\Sigma^{+,-,{\rm K}}_{{\rm B}d}$, are

$$\begin{aligned} \Sigma^{\pm}_{{\rm B}d}(t-t') &\equiv \frac{i}{2}\sum_x\big[g^{\rm K}_x(t-t')g^{\pm}_{\rm d}(t-t') \\ &\quad + g^{\pm}(t-t')g^{\rm K}_{\rm d}(t-t')\big], \\ \Sigma^{\rm K}_{{\rm B}d}(t-t') &\equiv \frac{i}{2}\sum_x\big\{g^{\rm K}_x(t-t')g^{\rm K}_{\rm d}(t-t') \\ &\quad + [g^+_{\rm d}(t-t') - g^-_{\rm d}(t-t')] \\ &\quad \times [g^+(t-t') - g^-(t-t')]\big\}, \end{aligned} \quad (12)$$

where $x = {\rm L,R}$ and the functions $g^{+,-,{\rm K}}_{\rm d}$, $g^{\rm K}_x$, $g^{\pm}$ are the same as in Ref. 16. We use the Lorentzian density of states for the contacts, $\nu_{\rm C}(\epsilon) = \nu_{\rm C}W^2/(\epsilon^2+W^2)$, and define $\Gamma \equiv 4\pi\nu_{\rm C}|T|^2$.

The advantage of this quadratic model is that it is analytically solvable. However, its applicability is limited by temperatures $T \gtrsim T_{\rm K}$ and QD chemical potentials $\Gamma \lesssim \mu_0 - \epsilon_{\rm d} \lesssim U - \Gamma$ which assumes that the theory is valid for $U \gg \Gamma$. The latter is exactly our primary goal.

Let us say a few words about the structure of the tunneling action. As one can see from Eq. (11), it does not contain terms mixing $\chi(t)$ and $\xi(t)$. In the second order such terms just do not appear. The physical picture behind this is that the charge excitations corresponding to the single and double occupancies do not interact. This is valid for large U. However, even for large U the excitations can interact at low temperatures through the Kondo resonance for μ_0 in the vicinity of the symmetric point, $\epsilon_{\rm d} + U/2$. Therefore, one expects that in this vicinity the quadratic theory is not valid at low temperatures.

IV. TUNNELING DENSITY OF STATES AND RESULTS

It is now a straightforward task to calculate observables using Eq. (11). The observable storing the whole universe of the Kondo physics in QDs is the tunneling density of states (TDOS), $\nu_\sigma(\epsilon) \equiv -(1/\hbar\pi){\rm Im}[G^+_{{\rm d}\sigma\sigma}(\epsilon)]$, where $G^+_{{\rm d}\sigma\sigma'}(\epsilon)$ is the retarded QD Green's function. Using the functional integral

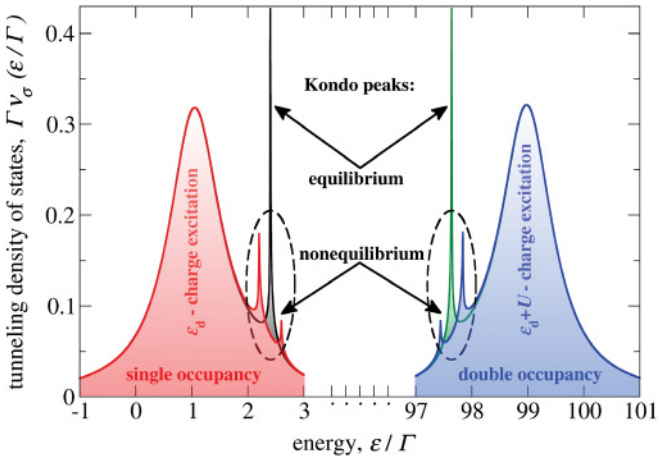

FIG. 1. (Color online) The analytical result, Eq. (13), for the QD TDOS. Here $kT = 0.0035\Gamma$, $\epsilon_d = 0$, $\mu_0 - \epsilon_d = 2.4\Gamma$ and 97.6Γ, $U = 100\Gamma$, $W = 1000\Gamma$. For nonequilibrium $eV = 0.4\Gamma$.

representation, Eq. (7), and performing the Gaussian integral we obtain

$$\nu_\sigma(\epsilon) = \mathcal{Z}(\epsilon)\left\{\frac{1}{\left[\epsilon_d - \epsilon + g\Sigma_e^R(\epsilon)\right]^2 + \left[g\Sigma_e^I(\epsilon)\right]^2} + \frac{1}{\left[\epsilon_d + U - \epsilon + g\Sigma_d^R(\epsilon)\right]^2 + \left[g\Sigma_d^I(\epsilon)\right]^2}\right\}, \quad (13)$$

where $g \equiv \pi\Gamma/2$. In Eq. (13) $\Sigma_e^{R(I)}$ is the real (imaginary) part of the e-state retarded slave-bosonic self-energy (see Ref. 16) and $\Sigma_d^{R(I)}$ is the real (imaginary) part of the d-state retarded slave-bosonic self-energy, $\Sigma_d^R(\epsilon) = W\epsilon/[\pi(\epsilon^2 + W^2)] - \Sigma_e^R(\epsilon)$, $\Sigma_d^I(\epsilon) = -W^2[2 - n_L(\epsilon) - n_R(\epsilon)]/[2\pi(\epsilon^2 + W^2)]$, where n_L, n_R are the Fermi-Dirac distributions of the contacts electrons, $n_{L(R)}(\epsilon) = \{\exp[\beta(\epsilon - \mu_0 \pm eV/2)] + 1\}^{-1}$, characterized by an external voltage V. The function $\mathcal{Z}$ in Eq. (13) is

$$\mathcal{Z}(\epsilon) \equiv \frac{\Gamma}{4\pi}\frac{W^2}{\epsilon^2 + W^2}\frac{n(\epsilon_d)[1 - n_{Bd}(\epsilon_d)]}{1 - n_L(\epsilon_d)n_R(\epsilon_d) - n_{Bd}(\epsilon_d)n(\epsilon_d)}, \quad (14)$$

where $n(\epsilon) \equiv n_L(\epsilon) + n_R(\epsilon) - 2n_L(\epsilon)n_R(\epsilon)$, $n_{Bd}(\epsilon) \equiv \{\exp[\beta(2\epsilon + U - 2\mu_0)] + 1\}^{-1}$.

The QD TDOS, Eq. (13), is shown in Fig. 1. Since U is finite, the TDOS has two charge excitations, the single and double occupancy excitations close to ϵ_d and $\epsilon_d + U$, respectively. For a QD in which the strength U of the electron-electron interaction is finite the behavior of the Kondo resonance as a function of $\mu_0 - \epsilon_d$ is quite different from the limiting case $U = \infty$. For a finite value of U the Kondo peak first decreases when $\mu_0 - \epsilon_d$ increases up to the symmetric point $U/2$ and after this point increases again when $\mu_0 - \epsilon_d$ approaches U, so that two symmetrically located Kondo peaks, close to ϵ_d and $\epsilon_d + U$, have the same height as demonstrated in Fig. 1. This means that the Kondo temperature T_K as a function of $\mu_0 - \epsilon_d$ decreases up to $U/2$ and then increases again. Our Keldysh field theory correctly predicts this behavior of T_K obtained from the scaling in the universal temperature dependence of the conductance as shown in Fig. 2. This result agrees with the general expression (see, e.g., Ref. 3), $kT_K/\Gamma \sim (W/\Gamma)\exp\{-2\pi(\mu_0 - \epsilon_d)[U - (\mu_0 - \epsilon_d)]/\Gamma U\}$. In Fig. 2 we also compare our theory with the numerical renormalization group theory[3,4,13]. This comparison proves that, indeed, our theory is of the second type and it is reliable for temperatures $T \geqslant 2T_K$.

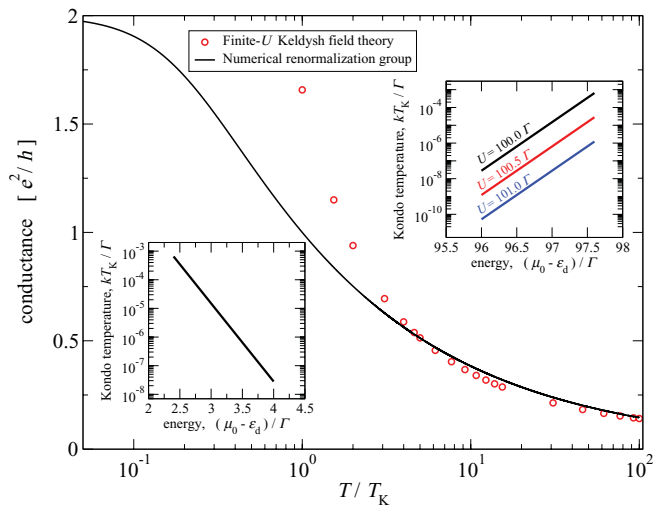

FIG. 2. (Color online) The universal temperature dependence of the conductance. Insets show the two qualitatively different regimes in the Kondo temperature dependence on $\mu_0 - \epsilon_d$.

V. CONCLUSION

In summary, we have developed an analytical nonequilibrium Keldysh field theory for the Kondo effect in QDs with finite electron-electron interactions (U) which are much stronger than the QD-contacts coupling (Γ). The theory is nonperturbative in both U and Γ and valid for temperatures $T \geqslant 2T_K$. Although, for clarity, it has been presented for the case of SIAM with normal contacts, the construction of the Keldysh field integral has a universal theoretic scheme applicable to setups with ferromagnetic or superconducting contacts coupled to any interacting nanoscopic system after its many-particle spectrum has been found.

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[1] D. C. Ralph and R. A. Buhrman, Phys. Rev. Lett. **72**, 3401 (1994).
[2] D. Goldhaber-Gordon, H. Shtrikman, D. Mahalu, D. Abusch-Magder, U. Meirav, and M. A. Kastner, Nature (London) **391**, 156 (1998).
[3] D. Goldhaber-Gordon, J. Göres, M. A. Kastner, H. Shtrikman, D. Mahalu, and U. Meirav, Phys. Rev. Lett. **81**, 5225 (1998).
[4] M. Grobis, I. G. Rau, R. M. Potok, H. Shtrikman, and D. Goldhaber-Gordon, Phys. Rev. Lett. **100**, 246601 (2008).
[5] R. Aguado and D. C. Langreth, Phys. Rev. Lett. **85**, 1946 (2000).

[6]R. López and D. Sánchez, Phys. Rev. Lett. **90**, 116602 (2003).
[7]Z. Ratiani and A. Mitra, Phys. Rev. B **79**, 245111 (2009).
[8]N. Sivan and N. S. Wingreen, Phys. Rev. B **54**, 11622 (1996).
[9]P. W. Anderson, J. Phys. C **3**, 2436 (1970).
[10]K. G. Wilson, Rev. Mod. Phys. **47**, 773 (1975).
[11]A. Rosch, J. Paaske, J. Kroha, and P. Wölfle, Phys. Rev. Lett. **90**, 076804 (2003).
[12]J. Paaske, A. Rosch, and P. Wölfle, Phys. Rev. B **69**, 155330 (2004).
[13]T. A. Costi and A. C. Hewson, J. Phys. Condens. Matter **6**, 2519 (1994).
[14]N. S. Wingreen and Y. Meir, Phys. Rev. B **49**, 11040 (1994).
[15]K. Haule, S. Kirchner, J. Kroha, and P. Wölfle, Phys. Rev. B **64**, 155111 (2001).
[16]S. Smirnov and M. Grifoni, Phys. Rev. B **84**, 125303 (2011).
[17]L. Mühlbacher, D. F. Urban, and A. Komnik, Phys. Rev. B **83**, 075107 (2011).
[18]S. E. Barnes, J. Phys. F: Metal Phys. **6**, 1375 (1976).
[19]Z. Zou and P. W. Anderson, Phys. Rev. B **37**, 627 (1988).
[20]P. Coleman, Phys. Rev. B **29**, 3035 (1984).
[21]G. Kotliar and A. E. Ruckenstein, Phys. Rev. Lett. **57**, 1362 (1986).
[22]A. Altland and B. Simons, *Condensed Matter Field Theory*, 2nd ed. (Cambridge University Press, Cambridge, 2010).
[23]B. Jin and Y. Kuroda, J. Phys. Soc. Jpn. **57**, 1687 (1988).