Deterministic chaos can act as a decoherence suppressor

Jing Zhang,[1,2,*] Yu-xi Liu,[2,3] Wei-Min Zhang,[4] Lian-Ao Wu,[5] Re-Bing Wu,[1,2] and Tzyh-Jong Tarn[2,6]
[1]*Department of Automation, Tsinghua University, Beijing 100084, People's Republic of China*
[2]*Center for Quantum Information Science and Technology, Tsinghua National Laboratory for Information Science and Technology, Beijing 100084, People's Republic of China*
[3]*Institute of Microelectronics, Tsinghua University, Beijing 100084, People's Republic of China*
[4]*Department of Physics and Center for Quantum Information Science, National Cheng Kung University, Tainan 70101, Taiwan*
[5]*Department of Theoretical Physics and History of Science, The Basque Country University (EHU/UPV) and IKERBASQUE, Basque Foundation for Science, E-48011, Bilbao, Spain*
[6]*Department of Electrical and Systems Engineering, Washington University, St. Louis, Missouri 63130, USA*
(Received 26 October 2011; published 7 December 2011)

We propose a strategy to suppress non-Markovian decoherence of a qubit by coupling the qubit to a deterministic chaotic setup with the broad power distribution in particular in the high-frequency domain. Although it is widely believed that chaos induces decoherence, we find, surprisingly, that the unitary dynamics induced by the chaotic setup can be helpful for the decoherence suppression. Compared with the existing decoherence control methods such as the usual dynamical decoupling, we do not need to impose high-frequency controls, because the high-frequency components in our method are generated by the chaotic setup, and the design of complex optimized control pulses used in the modified dynamical decoupling approaches is also not necessary. Using superconducting quantum circuits as an example, we demonstrate how to realize our general method. We find that various noises in a wide frequency domain, including low-frequency $1/f$, high-frequency ohmic, subohmic, and superohmic noises, can be efficiently suppressed by coupling the qubit to a Duffing oscillator acting as the chaotic setup. Significantly, the decoherence time of the qubit is prolonged approximately 100 times in magnitude.

DOI: 10.1103/PhysRevB.84.214304 PACS number(s): 03.67.Pp, 03.65.Yz, 05.45.Gg, 05.45.Mt

I. INTRODUCTION

Solid-state quantum information processing, especially superconducting quantum computation,[1–3] develops rapidly in recent years. One of the basic features that makes quantum information unique is the quantum parallelism resulted from quantum coherence and entanglement. However, the inevitable interaction between the qubit and its environment leads to qubit-environment entanglement that deteriorates quantum coherence of the qubit. In solid-state systems, the decoherence process is mainly caused by the non-Markovian noises induced, e.g., by the two-level fluctuators in the substrate and the charge and flux noises in the circuits.[4–15]

There have been numbers of proposals for suppressing non-Markovian noises in solid-state systems. Most of them suppress noises in a narrow frequency domain, e.g., low-frequency noises.[4–16] Among the proposed strategies, the dynamical decoupling control (DDC)[17] is relatively successful in suppressing non-Markovian noises in a broad frequency domain and has recently been demonstrated in solid-state systems experimentally.[18–20] The main idea of the DDC is to utilize high-frequency pulses to flip states of the qubit rapidly, averaging out the qubit-environment coupling. The higher the frequency of the control pulse is, the better the decoherence suppression effects are. Efforts have been made to optimize the control pulses[21–29] in the DDC; however, the requirements of generating extremely high-frequency control pulses or complex optimized pulses limit its application in solid-state quantum information system.

In this paper, we propose a method to extend the decoherence time of the qubit by coupling it to a deterministic chaotic setup.[30–34] It is well known that the chaotic dynamics, deterministic or nondeterministic, induces inherent decoherence.[35–44] This inherent decoherence can be studied by a physically realizable quantity called quantum Loschmidt echo,[38–42] which is defined as the overlap of the chaotic dynamics and a slightly perturbed dynamics. With the increase of the strength of perturbation, four different decoherence regimes appear, which include perturbative, Fermi golden rule, Lyapunov, and the strong semiclassical regimes.[43] Thus, chaos is always looked as a source of decoherence in the literature. However, we find, surprisingly, that the frequency shift of the qubit induced by a deterministic chaotic setup, which has not drawn enough attention in the literature, can help to suppress decoherence of the qubit. The main merits of this method are: (i) the high-frequency components, which contribute to the suppression of the non-Markovian noises, are generated by the chaotic setup even driven by a low-frequency field and, thus, (ii) generating deterministic but complex optimal control pulses is not necessary.

The paper is organized as follows: In Sec. II, the mechanism of the decoherence suppression by chaotic signals is presented. In Sec. III, we show how to generate the chaotic signal by nonlinear Duffing oscillator and its application to suppress $1/f$ noises. Using superconducting quantum circuits as an example, we show how to realize our proposal in Sec. IV. The conclusions and forecast of the future work are presented in Sec. V.

II. DECOHERENCE SUPPRESSION BY CHAOTIC SIGNALS

Let us consider the coupling between a two-level system (or saying a qubit) and its environment. Here we assume that the environment is modelled by a set of two-level systems. The system-environment Hamiltonian then can be expressed as the following Hamiltonian:[15]

$$\hat{H}_{qE} = \frac{(\omega_q + \delta_q)}{2}\hat{S}_z + \sum_i \frac{\omega_i}{2}\hat{\sigma}_z^{(i)} + \sum_i (g_i \hat{\sigma}_+^{(i)} \hat{S}_- + \text{H.c.}), \tag{1}$$

where ω_q (ω_i), $\hat{S}_z$ $(\hat{\sigma}_z^{(i)})$, and $\hat{S}_\pm$ $(\hat{\sigma}_\pm^{(i)})$ are the angular frequency, the z-axis Pauli operator, and the ladder operators of the qubit (the i-th two-level fluctuator in the environment) and g_i is the coupling constant between the qubit and the i-th two-level fluctuator. The time-dependent parameter

$$\delta_q \equiv \delta_q(t) = \sum_\alpha A_\alpha \cos(\omega_{d\alpha} t + \phi_\alpha)$$

in Eq. (1) is an angular frequency shift induced by chaotic signals and can be expressed as a linear combination of sinusoidal components with small amplitudes and high frequencies, i.e.,

$$\delta_q(t) = \sum_\alpha A_\alpha \cos(\omega_{d\alpha} t + \phi_\alpha),$$

which satisfies that

(i) the high-frequency condition:

$$\omega_{d\alpha} \gg |\omega_{c2} - \omega_q|, |\omega_q - \omega_{c1}|;$$

(ii) the small amplitude condition:

$$A_{d\alpha}/\omega_{d\alpha} \ll 1,$$

where ω_{c1} and ω_{c2} are cutoff frequencies that will be specified below. Using the Fourier-Bessel series identity:[45]

$$e^{ix\sin y} = \sum_n J_n(x) e^{iny}$$

with $J_n(x)$ as the n-th Bessel function of the first kind and the approximation $J_0(x) \approx 1 - x^2/4$ for $x \ll 1$, we obtain the following effective system-environment Hamiltonian (see the derivations in Appendix A):

$$\hat{H}_{\text{eff}} = \frac{\omega_q}{2}\hat{S}_z + \sum_i \frac{\omega_i}{2}\hat{\sigma}_z^{(i)} + \sqrt{F}\sum_i (g_i \hat{\sigma}_+^{(i)} \hat{S}_- + \text{H.c.}), \tag{2}$$

with the renormalized coupling constant factor

$$F = \exp\left[-\pi \int_{\omega_{cd}}^{\infty} \frac{S_{\delta_q}(\omega)}{\omega^2}\right] d\omega.$$

$S_{\delta_q}(\omega)$ is the power spectrum density of the signal $\delta_q(t)$. ω_{cd} is the lower bound of the frequency of $\delta_q(t)$ and is assumed to satisfy the condition

$$\omega_{cd} \gg |\omega_{c2} - \omega_q|, |\omega_q - \omega_{c1}|.$$

The correction factor F may become extremely small when $\delta_q(t)$ is induced by a deterministic chaotic signal which has a broadband frequency spectrum, in particular, in the high-frequency domain. The qubit decouples from its environment when $F \to 0$.

In order to show the suppression of the decoherence effects of $\delta_q(t)$, let us consider the decay rate of the nondiagonal entries of the qubit state,

$$\Gamma_q(t) = -\frac{d}{dt}\log\left|\langle 0|\text{tr}_E\left\{\vec{T}\exp\left[-i\int_0^t \hat{H}_{qE}(\tau)\,d\tau\right] \times \hat{\rho}_{qE}(0)\,\vec{T}\exp\left[i\int_0^t \hat{H}_{qE}(\tau)\,d\tau\right]\right\}|1\rangle\right|, \tag{3}$$

where $\vec{T}$ is the time-ordering operator, $\hat{\rho}_{qE}(0)$ is the initial state of the qubit and its environment, and $\text{tr}_E(\cdot)$ represents the partial trace operation of the degrees of freedom of the environment. States $|0\rangle$ and $|1\rangle$ are the two eigenstates of the qubit. It is shown in Appendix A that $\Gamma_q \to 0$ when $F \to 0$.

If the initial state is separable, $\hat{\rho}_{qE}(0) = \hat{\rho}_{q0} \otimes \hat{\rho}_{E0}$, where we consider the environment $\hat{\rho}_{E0}$ at the zero temperature, we can obtain an analytical expression of $\Gamma_q(t)$ under the second-order approximation[46] (see the derivations in Appendix A):

$$\Gamma_q(t) = F\int_{\omega_{c1}}^{\omega_{c2}} d\omega \frac{J(\omega)\sin(\omega_q - \omega)t}{\omega_q - \omega} = F\Gamma_{q0}, \tag{4}$$

where $J(\omega) = \sum_i g_i^2 \delta(\omega - \omega_i)$ is the spectral density of the environment and Γ_{q0} is the damping rate of the qubit when $\delta_q(t) = 0$. Since the frequencies of the fluctuators distribute in a finite domain, Γ_q is restricted to be integrated in the finite-frequency domain $[\omega_{c1},\omega_{c2}]$. As analyzed in Eq. (4), the decay rate Γ_{q0} of the qubit is suppressed by the chaotic signal $\delta_q(t)$ when the correction factor F becomes small.

III. GENERATION OF CHAOTIC SIGNALS AND SUPPRESSION OF 1/f NOISES

We now show the validity of our method. As an example, we consider the suppression of the $1/f$ noises of a qubit with free Hamiltonian $\hat{H}_q = \omega_q \hat{S}_z$ by coupling it to a driven Duffing oscillator[47] with Hamiltonian

$$\hat{H}_{\text{Duff}} = \omega_o \hat{a}^\dagger \hat{a} - \frac{\mu}{4}(\hat{a} + \hat{a}^\dagger)^4 - I(t)\frac{1}{\sqrt{2}}(\hat{a} + \hat{a}^\dagger), \tag{5}$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators of the nonlinear Duffing oscillator, $\omega_o/2\pi$ stands for the frequency of the fundamental mode of the oscillator, and μ is the nonlinear constant. $I(t) = I_0\cos(\omega_d t)$ denotes the classical driving field with the amplitude I_0 and frequency $\omega_d/2\pi$. We employ the interaction between the qubit and the Duffing oscillator with the Hamiltonian $\hat{H}_{qo} = g_{qo}\hat{S}_z \hat{a}^\dagger \hat{a}$, which can be obtained, e.g., by the Jaynes-Cummings model under the large detuning regime.[48] Here, g_{qo} is the coupling strength between the qubit and the oscillator.

By tracing out the degrees of freedom of the Duffing oscillator, we find that the effective Hamiltonian of the qubit-environment system becomes (see the analysis in Appendix B):

$$\hat{H}_{qE} = \frac{[\omega_q + \delta_q(t) + \xi(t)]}{2}\hat{S}_z + \sum_i \frac{\omega_i}{2}\hat{\sigma}_z^{(i)} + \sum_i (g_i \hat{\sigma}_+^{(i)} \hat{S}_- + \text{H.c.}). \tag{6}$$

There are two aspects of effects induced by the Duffing oscillator. The deterministic angular frequency shift $\delta_q(t)$ can be used to suppress the decoherence of the qubit as analyzed. However, the effective classical stochastic signal $\xi(t)$ leads to additional (pure-dephasing) decoherence effects of the qubit. The decoherence effects induced by the chaotic dynamics, e.g., the quantum Loschmidt echo,[36–42] have been well studied in the literature.[35–44] However, the unitary dynamics induced by $\delta_q(t)$ has not drawn enough attention of researchers. Our decoherence suppression strategy uses the unexplored dynamics, under condition that the decoherence suppression induced by $\delta_q(t)$ is predominant in comparison with the opposite decoherence acceleration process induced by $\xi(t)$.

In this section, the noise spectrum of the environment is specified as a $1/f$ spectrum, which can be expressed as

$$J(\omega) = \frac{A}{|\omega|}, \tag{7}$$

where A is the strength of the spectrum. Such a $1/f$ noise spectrum has been widely used to describe dynamical phenomena in classical circuit such as transport in electronic devices. In recent years, the extensive studies in nanodevices, especially in solid-state quantum computation systems, show that $1/f$ noise is a crucial source of decoherence in the low-frequency regime in such systems, which may be induced by, e.g., the two-level fluctuators in the substrate and the flux or charge noises in the circuit (see, e.g., Refs. 4–14).

We now come to show numerical results, using system parameters:

$$(\omega_o, g_{qo}, \mu) = (\omega_q, 0.03\omega_q, 0.25\omega_q). \tag{8}$$

The strength of the $1/f$ environmental noise A is assumed to be $A/\omega_q = 0.1$.

The evolution of the coherence $C_{xy} = \langle \hat{S}_x \rangle^2 + \langle \hat{S}_y \rangle^2$ of the qubit and the spectrum analysis of the angular frequency shift $\delta_q(t)$ are presented in Fig. 1. As shown in Figs. 1(b) and 1(c), if the amplitude I_0 of the sinusoidal driving field $I(t)$ is tuned such that $I_0/\omega_q = 5$ and 30, the signals $\delta_q(t)$ exhibit periodic and chaotic behaviors. As shown in Fig. 1(a), in the periodic regime, the decoherence of the qubit is almost unaffected by the Duffing oscillator. The trajectory in the periodic case (green curve with plus signs) coincides with that of natural decoherence (black triangle curve), as in Fig. 1(a). In the chaotic regime, the decoherence of the qubit is efficiently slowed down [see the blue solid curve in Fig. 1(a) representing the trajectory in the chaotic case]. This demonstrates that, with the increase of the distribution of the spectral energy in the high-frequency domain, the decoherence effects are suppressed as explained in the last section.

To calculate the damping rates of the nondiagonal entries of the qubit, we, first, calculate

$$f_{01} = \langle \alpha | e^{it(\hat{H}_{\mathrm{Duff}} + g_{qo}\hat{a}^\dagger \hat{a})} e^{-it(\hat{H}_{\mathrm{Duff}} - g_{qo}\hat{a}^\dagger \hat{a})} | \alpha \rangle, \tag{9}$$

where $|\alpha\rangle$ is the initial state of the Duffing oscillator. We take $|\alpha\rangle$ as the vacuum state. Since the Hamiltonian of the Duffing oscillator is nonlinear, we cannot obtain the analytical expression of $f_{01}(t)$. Here, we use the quantum optics toolbox "QotoolboxV015"[49] for MATLAB to numerically solve it. $f_{01}(t)$ induces two effects: (i) the angular frequency shift of the qubit denoted by $\delta_q(t)$ and (ii) the quantum Loschmidt echo,[36–42] i.e., the pure dephasing decoherence of the qubit with damping rate Γ_{Duff}. It can be shown that $\delta_q(t)$ and $\Gamma_{\mathrm{Duff}}(t)$ can be given as [see Eq. (B2) in Appendix B]:

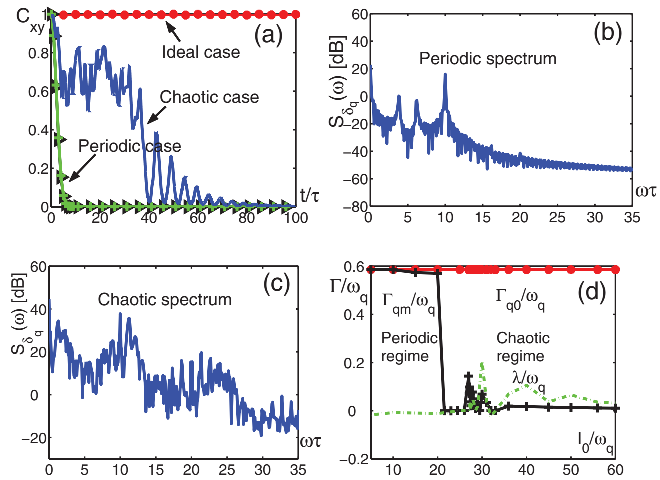

FIG. 1. (Color online) Decoherence suppression by the auxiliary chaotic setup. (a) The evolution of the coherence $C_{xy} = \langle \hat{S}_x \rangle^2 + \langle \hat{S}_y \rangle^2$ of the state of the qubit, where the red asterisk curve and the black triangle curve represent the ideal trajectory without any decoherence and the trajectory under natural decoherence and without corrections and the green curve with plus signs and the blue solid curve denote the trajectories with $I_0/\omega_q = 5$ and 30. With these parameters, the dynamics of the Duffing oscillator exhibits periodic and chaotic behaviors. $\tau = 2\pi/\omega_q$ is a normalized time scale. [(b) and (c)] The energy spectra of $\delta_q(t)$ with $I_0/\omega_q = 5$ (the periodic case) and 30 (the chaotic case). The energy spectrum $S_{\delta_q}(\omega)$ is in the unit of decibels (dB). (d) The normalized decoherence rates Γ/ω_q versus the normalized driving strength I_0/ω_q.

$$\delta_q(t) = \frac{d}{dt}\mathrm{Im}[\log f_{01}(t)], \tag{10}$$

$$\Gamma_{\mathrm{Duff}} = \frac{d}{dt}\mathrm{Re}[\log f_{01}(t)]. \tag{11}$$

The natural decoherence rate Γ_{q0} of the qubit induced by the environmental noises can be calculated by

$$\Gamma_{q0} = \int_{\omega_{c1}}^{\omega_{c2}} d\omega J(\omega) \frac{\sin[(\omega_q - \omega)t]}{\omega_q - \omega}.$$

The modified environment-induced decoherence rate Γ_q of the qubit can be numerically solved by

$$\Gamma_q = \int_{\omega_{c1}}^{\omega_{c2}} d\omega J(\omega) \mathrm{Re} \int_0^t e^{i(\omega_q - \omega)(t-t') + i\int_{t'}^{t} \delta_q dt''} dt',$$

where $\delta_q(t)$ has been given by Eq. (10). The total decoherence rate Γ_{qm} of the qubit can be calculated by

$$\Gamma_{qm} = \Gamma_q + \Gamma_{\mathrm{Duff}}.$$

We compare in Fig. 1(d) the natural decoherence rate Γ_{q0} with the modified decoherence rate Γ_{qm} [*both include the environment-induced decoherence modified by* $\delta_q(t)$ *and the decoherence induced by* $\xi(t)$] of the qubit versus different strengths I_0 of the driving field. Figure 1(d) shows that the decoherence process is efficiently slowed down when the strength I_0 of the driving field increases. It is interesting to note that there seems to exist a phase transition around $I_0/\omega_q = 20$, i.e., a sudden change of the modified decoherence rate Γ_{qm} (see the black solid curve with plus signs). It is noticeable that, around this point, the dynamics of the Duffing oscillator enters the chaotic regime which is indicated by a positive Lyapunov exponent λ [see the green dash-dotted curve in Fig. 1(d)]. The modified decoherence rate Γ_{qm} changes dramatically in the parameter regime $20 \leqslant I_0/\omega_q \leqslant 35$, which is the soft-chaos regime of the Duffing oscillator. When the dynamics of the Duffing oscillator enters the hard-chaos regime at $I_0/\omega_q \approx 35$, the modified decoherence rate Γ_{qm} is stabilized at a value much smaller than the natural decoherence rate Γ_{q0}. The simulation shows that the decoherence of the qubit is efficiently suppressed by chaotic signal, even if there exists an additional decoherence introduced by the auxiliary chaotic setup. Further calculations show that the modified decoherence rate Γ_{qm} in the chaotic regime is roughly 100 times smaller than the unmodified decoherence rate Γ_{q0}, meaning that the decoherence time of the qubit can be prolonged 100 times using the chaotic signal.

IV. EXPERIMENTAL FEASIBILITY IN SUPERCONDUCTING CIRCUITS

Our proposal can be demonstrated using the solid-state quantum devices, e.g., the superconducting qubit system, as schematically shown in Fig. 2. It is similar to the widely used qubit readout circuit[50] but works in a quite different parameter regime. In this superconducting circuit, a single Cooper pair box (SCB) is coupled to a dc superconducting quantum interference device (SQUID) consisting of two Josephson junctions with capacitances $\tilde{C}_J$ and Josephson energies $\tilde{E}_J$ and a paralleled current source. The SCB is composed of two Josephson junctions with capacitances C_J and Josephson energies E_J. The difference between the circuit in Fig. 2 and the readout circuit in Ref. 50 is that the rf-biased Josephson junction is replaced by a dc-SQUID—the chaotic setup.

The Hamiltonian of the circuit in Fig. 2 is written as

$$\hat{H}_{\mathrm{SCB-SQUID}} = E_C(\hat{n} - n_g)^2 - 2E_J \cos\frac{\hat{\phi}}{2}\cos\hat{\theta} + \tilde{E}_C\hat{\tilde{n}}^2 - 2\tilde{E}_J\cos\frac{\phi_e}{2}\cos\hat{\phi} - \phi_0 I_e\hat{\phi}, \tag{12}$$

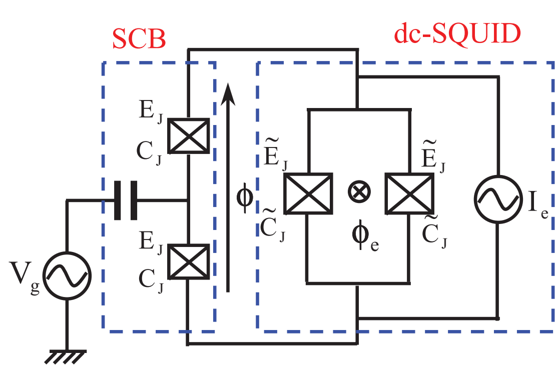

FIG. 2. (Color online) Schematic diagram of the decoherence suppression superconducting circuit in which a SCB is coupled to a current-biased dc-SQUID.

where $E_C = 2e^2/(C_g + 2C_J)$ is the charging energy of SCB with C_g as the gate capacitance; $n_g = -C_g V_g/2e$ is the reduced charge number, in units of the Cooper pairs, with V_g as the gate voltage; $\hat{n}$ is the number of Cooper pairs on the island electrode of SCB with $\hat{\theta}$ as its conjugate operator; $\tilde{E}_C = e^2/\tilde{C}_J$ is the charging energy, $\hat{\tilde{n}}$ is the charge operator of the dc-SQUID, and $\hat{\phi}$ the conjugate operator; and ϕ_e and I_e are the external flux threading the loop of the dc-SQUID and the external bias current of the dc-SQUID. Here, we consider a zero external flux threading the loop of the coupled SCB–dc-SQUID system. In this case, the phase drop across the SCB is equal to the phase drop across the dc-SQUID $\tilde{\phi}$.

Expanding the Hamiltonian of the SCB in the Hilbert space of its two lowest energy states and leaving the lowest nonlinear terms of ϕ, we can obtain the following Hamiltonian:[51]

$$\hat{H}_{\mathrm{SCB-SQUID}} = E_J\hat{S}_z + \frac{C_g V_g}{2e}E_C\hat{S}_x + (\omega_o + g_{qo}\hat{S}_z)\hat{a}^\dagger\hat{a} - \frac{\mu}{4}(\hat{a}+\hat{a}^\dagger)^4 - \beta I_e(t)(\hat{a}+\hat{a}^\dagger), \tag{13}$$

where $\hat{S}_{x,y,z}$ are the Pauli operators of the SCB; the parameters in the Hamiltonian are given by

$$\omega_o = \sqrt{16\tilde{E}_C\tilde{E}_J\cos\frac{\phi_e}{2}}, \quad g_{qo} = \sqrt{\frac{4E_J^2\tilde{E}_C}{\tilde{E}_J}\left|\cos\frac{\phi_e}{2}\right|},$$

$$\mu = \frac{1}{3}\tilde{E}_C, \quad \beta = \phi_0\left(\frac{\tilde{E}_C}{\tilde{E}_J\left|\cos\frac{\phi_e}{2}\right|}\right)^{1/4};$$

the annihilation operator $\hat{a}$ is defined by $\hat{a} = (\hat{x} + i\hat{p})/\sqrt{2}$, where

$$\hat{x} = \frac{1}{\sqrt{2}}\left(\frac{\tilde{E}_J|\cos\phi_e/2|}{\tilde{E}_C}\right)^{1/4}\hat{\phi}$$

is the normalized position operator of the dc-SQUID with $\hat{p}$ as its conjugate momentum operator. By introducing the ac gate voltage $V_g = V_{g0}\cos(\omega_g t)$ with amplitude V_{g0} and frequency ω_g, the Hamiltonian in Eq. (13) can be reexpressed in the rotating frame as

$$\hat{H}_{\mathrm{SCB-SQUID}} = \omega_q\hat{S}_z + \frac{C_g V_{g0}}{2e}E_C\hat{S}_x - \frac{\mu}{4}(\hat{a}+\hat{a}^\dagger)^4 + (\omega_o + g_{qo}\hat{S}_z)\hat{a}^\dagger\hat{a} - \beta I_e(t)(\hat{a}+\hat{a}^\dagger),$$

where $\omega_q = E_J - \omega_g$.

Under the condition that $C_g V_{g0} E_C/2e \ll \omega_q = E_J - \omega_g$, the SCB works near the optimal point. Thus, we only need to consider the relaxation of the SCB. Under this condition, we can write down the full Hamiltonian of the total system composed of the SCB, the dc-SQUID, and the environment as

$$\hat{H}_{\mathrm{tot}} = \omega_q\hat{S}_z + g_{qo}\hat{S}_z\hat{a}^\dagger\hat{a} + \left[-\frac{\mu}{4}(\hat{a}+\hat{a}^\dagger)^4 + \omega_o\hat{a}^\dagger\hat{a} - \beta I_e(t)(\hat{a}+\hat{a}^\dagger)\right] + \sum_i\frac{\omega_i}{2}\hat{\sigma}_z^{(i)} + \sum_i(g_i\hat{\sigma}_+^{(i)}\hat{S}_- + \mathrm{H.c.}),$$

TABLE I. Decoherence suppression against various noises for experimentally accessible parameters: $E_J/2\pi = 5$ GHz, $\omega_g/2\pi = 4.999$ GHz, $\tilde{E}_C/2\pi = 0.188$ MHz, and $\tilde{E}_J \cos\frac{\phi_e}{2}/2\pi = 12.032$ MHz.

Type of noises	Frequency domain	$\bar{\Gamma}_{q0}$	$\bar{\Gamma}_{qm}$	$T_1 = T_2$
$1/f$ noise	[10 kHz, 1 MHz]	0.58 MHz	5.4 kHz	187 μs
Ohmic	$[2\omega_q/3, 3\omega_q/2]$	0.35 MHz	5.4 kHz	187 μs
Subohmic	$[2\omega_q/3, 3\omega_q/2]$	0.35 MHz	5.4 kHz	187 μs
Superohmic	$[2\omega_q/3, 3\omega_q/2]$	0.36 MHz	5.4 kHz	187 μs

which is just the Hamiltonian of the total system composed of the qubit, the Duffing oscillator, and the environment which can be reduced to the effective Hamiltonian given in Eq. (6) by averaging out the degrees of freedom of the Duffing oscillator (see the derivation in Appendix B).

As has been analyzed, the dc-SQUID, acting as the auxiliary Duffing oscillator, can be used to suppress low-frequency $1/f$ noises of the qubit. Using the experimentally accessible parameters as shown in the title to Table I, we show the decoherence suppression effects for low-frequency $1/f$, high-frequency ohmic $[J(\omega) = \omega e^{-\omega/5\omega_q}]$, subohmic $[J(\omega) = \omega^{1/2} e^{-\omega/5\omega_q}]$, and superohmic $[J(\omega) = \omega^2 e^{-\omega/5\omega_q}]$ noises. All simulations are summarized in Table I. It is found that our method works equally well for different types of noises. The numerical simulations manifest that our strategy is independent of the sources and frequency domains of the noises. The final modified decoherence rates for these different noises are almost the same because the decoherence effects induced by the environmental noises are all greatly suppressed. The modified decoherence rates of the qubit are mainly caused by the auxiliary chaotic setup, i.e., the dc-SQUID. It is also shown in Table I that the modified decoherence rate $\Gamma_{qm}/2\pi$ of the qubit can be reduced to 5 kHz. This low decoherence rate corresponds to a long decoherence time $T_1 = T_2 \approx 200\,\mu$s. The magnitude is one order longer than the decoherence time of the superconducting qubits realized in experiments (see, e.g., Refs. 4–10).

V. CONCLUSIONS

In conclusion, we have proposed a strategy to increase the decoherence time of a qubit by coupling it to a deterministic chaotic setup. The broad power distribution of the auxiliary chaotic setup, in particular, in the high-frequency domain, helps us to suppress various non-Markovian noises, e.g., low-frequency $1/f$ noise, high-frequency ohmic, subohmic, and superohmic noises, and, thus, freeze the state of the qubit even if we consider the additional decoherence induced by the chaotic setup. As an example, we apply our method to a coupled SCB-SQUID system. We find that the decoherence time of the qubit can be efficiently prolonged approximately 100 times in magnitude in such a system.

Our method may be applied to other systems which have the Hamiltonian as shown in Eq. (1), e.g., atom-optical systems or quantum dot systems, in which chaotic dynamics can be induced by a nonlinear optical cavity. Additionally, our discussions also give a new perspective for the reversibility and irreversibility induced by nonlinearity.

ACKNOWLEDGMENTS

J. Zhang would like to thank H. T. Tan and M. H. Wu for helpful discussions. J. Zhang and R. B. Wu are supported by the National Natural Science Foundation of China under Grants No. 61174084, No. 61134008, and No. 60904034. Y.-X. Liu is supported by the National Natural Science Foundation of China under Grants No. 10975080, No. 61025022, and No. 60836001. T. J. Tarn would also like to acknowledge partial support from the US Army Research Office under Grant No. W911NF-04-1-0386. L. A. Wu has been supported by the Ikerbasque Foundation Start-up, the Basque Government (Grant No. IT472-10), and the Spanish MEC (Project No. FIS2009-12773-C02-02). W. M. Zhang is supported by the National Science Council (NSC) of ROC under Contract No. NSC-99-2112-006-008-MY3.

APPENDIX A: DECOHERENCE SUPPRESSION BY THE CHAOTIC SIGNAL $\delta_q(t)$

(i) The Hamiltonian of the total system composed of the system and the environment can be written as

$$\hat{H}_{qE} = \frac{(\omega_q + \delta_q)}{2}\hat{S}_z + \sum_i \frac{\omega_i}{2}\hat{\sigma}_z^{(i)} + \sum_i (g_i \hat{\sigma}_+^{(i)} \hat{S}_- + \text{H.c.}), \tag{A1}$$

which can be expressed in the interaction picture as

$$\hat{\tilde{H}}_{qE} = \sum_i [\tilde{g}_i(t)\hat{\sigma}_+^{(i)}\hat{S}_- + \text{H.c.}], \tag{A2}$$

where

$$\tilde{g}_i(t) = g_i \exp\left[-i(\omega_q - \omega_i)t - i\int_0^t \delta_q(t')dt'\right].$$

The solution of the Schrödinger equation with the Hamiltonian given in Eq. (A2) can be written as

$$\begin{aligned}|\tilde{\psi}(t)\rangle &= \vec{T}\exp\left[-i\int_0^t \hat{\tilde{H}}_{qE}(\tau)d\tau\right]|\psi_0\rangle \\ &= \left[1 + \sum_{n=1}^{\infty}(-i)^n \int_0^t \hat{\tilde{H}}_{qE}(t_1)\,dt_1 \cdots\right]|\psi_0\rangle \\ &= \left[1 + \sum_{n=1}^{\infty}(-i)^n \int_0^t \sum_i (\tilde{g}_i\hat{\sigma}_+^{(i)}\hat{S}_- + \text{H.c.})dt_1 \cdots\right]|\psi_0\rangle,\end{aligned} \tag{A3}$$

where $\vec{T}$ is the time-ordering operator. Using the Fourier-Bessel series identity,[45] $e^{ix\sin y} = \sum_n J_n(x)e^{iny}$ with $J_n(x)$ as the n-th Bessel function of the first kind, it can be calculated that

$$\begin{aligned}\int_0^t \tilde{g}_i(t')dt' &= g_i \int_0^t \exp\left[-i(\omega_q-\omega_i)t' - i\int_0^{t'} \delta_q(t'')dt''\right]dt' \\ &= g_i \int_0^t \exp\left[-i(\omega_q-\omega_i)t' - i\int_0^{t'} \sum_\alpha A_{d\alpha}\cos(\omega_{d\alpha}t''+\phi_\alpha)dt''\right]dt' \\ &= g_i \int_0^t \exp\left[-i(\omega_q-\omega_i)t' - i\sum_\alpha \frac{A_{d\alpha}}{\omega_{d\alpha}}\sin(\omega_{d\alpha}t'+\phi_\alpha)\right]dt' \\ &= g_i \int_0^t \exp[-i(\omega_q-\omega_i)t'] \prod_\alpha \sum_{n_\alpha} J_{n_\alpha}\left(\frac{A_{d\alpha}}{\omega_{d\alpha}}\right)\exp[-in_\alpha\omega_{d\alpha}t' - in_\alpha\phi_\alpha]dt' \\ &= g_i \sum_{n_\alpha}\prod_\alpha J_{n_\alpha}\left(\frac{A_{d\alpha}}{\omega_{d\alpha}}\right)\int_0^t \exp\left[-i\left(\omega_q-\omega_i+\sum_\alpha n_\alpha\omega_{d\alpha}\right)t' - in_\alpha\phi_\alpha\right]dt' \\ &= g_i \sum_{n_\alpha}\prod_\alpha J_{n_\alpha}\left(\frac{A_{d\alpha}}{\omega_{d\alpha}}\right)e^{-i\sum_{n_\alpha} n_\alpha\phi_\alpha}\frac{1}{-i\left(\omega_q-\omega_i+\sum_\alpha n_\alpha\omega_{d\alpha}\right)}[e^{-i(\omega_q-\omega_i+\sum_\alpha n_\alpha\omega_{d\alpha})t}-1]. \end{aligned} \tag{A4}$$

Since $\omega_{d\alpha} \gg |\omega_i - \omega_q|$, we have

$$\frac{1}{-i\left(\omega_q-\omega_i+\sum_\alpha n_\alpha\omega_{d\alpha}\right)} \ll \frac{1}{-i(\omega_q-\omega_i)},$$

if any $n_\alpha \neq 0$. We then can obtain the approximation expression:

$$\begin{aligned}\int_0^t \tilde{g}_i(t')\,dt' &= \prod_\alpha J_0\left(\frac{A_{d\alpha}}{\omega_{d\alpha}}\right) g_i \frac{[e^{-i(\omega_q-\omega_i)t}-1]}{-i(\omega_q-\omega_i)} \\ &= \prod_\alpha J_0\left(\frac{A_{d\alpha}}{\omega_{d\alpha}}\right)\int_0^t g_i e^{-i(\omega_q-\omega_i)t'}dt'. \end{aligned} \tag{A5}$$

Furthermore, from the assumption $A_{d\alpha} \ll \omega_{d\alpha}$, by taking the approximations $J_0(x) \approx 1-(x^2/4)$ and $\log(1+x)\approx x$ for $x \ll 1$, $\prod_\alpha J_0(A_{d\alpha}/\omega_{d\alpha})$ can be calculated as

$$\begin{aligned}\prod_\alpha J_0\left(\frac{A_{d\alpha}}{\omega_{d\alpha}}\right) &= \exp\left[\sum_\alpha \log J_0\left(\frac{A_{d\alpha}}{\omega_{d\alpha}}\right)\right] \\ &= \exp\left(-\frac{1}{4}\sum_\alpha \frac{A_{d\alpha}^2}{\omega_{d\alpha}^2}\right) \\ &= \exp\left[-\frac{\pi}{2}\int_{\omega_{cd}}^\infty \frac{S_{\delta_q}(\omega)}{\omega^2}d\omega\right] = \sqrt{F}, \end{aligned} \tag{A6}$$

where

$$F = \exp\left[-\pi\int_{\omega_{cd}}^\infty \frac{S_{\delta_q}(\omega)}{\omega^2}d\omega\right].$$

Thus, we have

$$\begin{aligned}\int_0^t \tilde{g}_i(t')dt' &= \sqrt{F}\int_0^t g_i e^{-i(\omega_q-\omega_i)t'}dt' \\ &\equiv \sqrt{F}\int_0^t \tilde{\tilde{g}}_i(t')dt'. \end{aligned} \tag{A7}$$

Substituting Eq. (A7) into Eq. (A3), we can obtain

$$\begin{aligned}|\tilde{\psi}(t)\rangle &= \left[1+\sum_{n=1}^\infty(-i\sqrt{F})^n\int_0^t\sum_i(\tilde{\tilde{g}}_i\hat{\sigma}_+^{(i)}\hat{S}_- + \text{H.c.})dt_1\cdots\right]|\psi_0\rangle \\ &= \vec{T}\exp\left[-i\sqrt{F}\int_0^t \hat{\tilde{\tilde{H}}}_{qE}(\tau)d\tau\right]|\psi_0\rangle, \end{aligned}$$

where

$$\hat{\tilde{\tilde{H}}}_{qE} = \sum_i (g_i e^{-i(\omega_q-\omega_i)t}\hat{\sigma}_+^{(i)}\hat{S}_- + \text{H.c.}).$$

From the above analysis, we can obtain the following effective Hamiltonian of the total system in the Schrödinger picture:

$$\begin{aligned}\hat{H}_{\text{eff}} &= \frac{\omega_q}{2}\hat{S}_z + \sum_i \frac{\omega_i}{2}\hat{\sigma}_z^{(i)} + \sqrt{F}\sum_i(g_i\hat{\sigma}^{(i)}\hat{S}_- + \text{H.c.}) \\ &= \hat{H}_0 + \sqrt{F}\hat{H}_I. \end{aligned}$$

(ii) From the definition of $\Gamma_q(t)$, it can be calculated as

$$\begin{aligned}\Gamma_q(t) &= -\frac{d}{dt}\log\left|\langle 0|\text{tr}_E\left\{\vec{T}\exp\left[-i\int_0^t \hat{H}_{qE}(\tau)\,d\tau\right]\hat{\rho}_{qE}(0)\,\vec{T}\exp\left[i\int_0^t \hat{H}_{qE}(\tau)\,d\tau\right]\right\}|1\rangle\right| \\ &= -\frac{d}{dt}\log|\langle 0|\text{tr}_E[\exp(-i\hat{H}_{\text{eff}}t)\hat{\rho}_{qE}(0)\exp(i\hat{H}_{\text{eff}}t)]|1\rangle| \\ &= -\frac{d}{dt}\log\left|e^{-i\omega_q t}\langle 0|\text{tr}_E\left\{\vec{T}\exp\left[-i\sqrt{F}\int_0^t \hat{\tilde{\tilde{H}}}_{qE}(\tau)d\tau\right]\hat{\rho}_{qE}(0)\vec{T}\exp\left[i\sqrt{F}\int_0^t \hat{\tilde{\tilde{H}}}_{qE}(\tau)\,d\tau\right]\right\}|1\rangle\right| \\ &= -\frac{d}{dt}\log\left|\langle 0|\text{tr}_E\left\{\vec{T}\exp\left[-i\sqrt{F}\int_0^t \hat{\tilde{\tilde{H}}}_{qE}(\tau)d\tau\right]\hat{\rho}_{qE}(0)\vec{T}\exp\left[i\sqrt{F}\int_0^t \hat{\tilde{\tilde{H}}}_{qE}(\tau)\,d\tau\right]\right\}|1\rangle\right|. \end{aligned} \tag{A8}$$

When $F \to 0$, we have

$$\Gamma_q \to -\frac{d}{dt}\log|\langle 0|\mathrm{tr}_E[\hat{\rho}_{qE}(0)]|1\rangle| = 0. \tag{A9}$$

(iii) If the initial state of the total system composed of the qubit and the environment is a separable state $\hat{\rho}_{qE}(0) = \hat{\rho}_{q0}\otimes\hat{\rho}_{E0}$, we can obtain the following master equation under the second-order approximation:[46]

$$\begin{aligned}\dot{\hat{\rho}}_s =& -i[(\omega_q+\delta_q)\hat{S}_z,\hat{\rho}_s] + \sum_i g_i^2\int_0^t d\tau \mathrm{tr}_E\{[\hat{S}_+\hat{\sigma}_-^{(i)}(t),\\ &[\hat{S}_-(\tau - t)\hat{\sigma}_+^{(i)}(\tau),\hat{\rho}_s(t)\otimes\hat{\rho}_{E0}]]\}\\ &+\sum_i g_i^2\int_0^t d\tau \mathrm{tr}_E\{[\hat{S}_-\hat{\sigma}_+^{(i)}(t),\\ &[\hat{S}_+(\tau - t)\hat{\sigma}_-^{(i)}(\tau),\hat{\rho}_s(t)\otimes\hat{\rho}_{E0}]]\},\end{aligned}$$

where

$$\hat{S}_\pm(t) = \hat{S}_\pm \exp\left\{\pm i\left[\omega_q t + \int_0^t \delta_q(t')dt'\right]\right\}$$

and

$$\hat{\sigma}_\pm^{(i)}(t) = \hat{\sigma}_\pm^{(i)}\exp(\pm i\omega_i t)$$

are the ladder operators of the qubit and the i-th two-level fluctuator in the interaction picture; and

$$\begin{aligned}\hat{\rho}_s =& \mathrm{tr}_E\vec{T}\exp\left[-i\int_0^t \hat{H}_{qE}(\tau)\,d\tau\right]\hat{\rho}_{qE}(0)\\ &\vec{T}\exp\left[i\int_0^t \hat{H}_{qE}(\tau)\,d\tau\right]\end{aligned} \tag{A10}$$

is the reduced density operator of the qubit.

Consider a zero-temperature environment, the above master equation can be rewritten as

$$\dot{\hat{\rho}}_s = -i\left[\tfrac{1}{2}(\omega_q+\delta_q+\Delta\omega_q)\hat{S}_z,\hat{\rho}_s\right] + 2\tilde{\Gamma}_q\mathcal{D}[\hat{S}_-]\hat{\rho}_s, \tag{A11}$$

where

$$\Delta\omega_q = \int_{\omega_{c_1}}^{\omega_{c_2}} d\omega J(\omega)\,\mathrm{Im}\int_0^t e^{i(\omega_q-\omega)(t-t')+i\int_{t'}^t \delta_q dt''}dt' \tag{A12}$$

and

$$\tilde{\Gamma}_q = \int_{\omega_{c_1}}^{\omega_{c_2}} d\omega J(\omega)\mathrm{Re}\int_0^t e^{i(\omega_q-\omega)(t-t')+i\int_{t'}^t \delta_q dt''}dt'; \tag{A13}$$

the superoperator $\mathcal{D}[\hat{S}_-]\hat{\rho}_s$ is defined as

$$\mathcal{D}[\hat{S}_-]\hat{\rho}_s = \hat{S}_-\hat{\rho}_s\hat{S}_+ - \tfrac{1}{2}\hat{S}_+\hat{S}_-\hat{\rho}_s - \tfrac{1}{2}\hat{\rho}_s\hat{S}_+\hat{S}_-;$$

and $J(\omega) = \sum_i g_i^2\delta(\omega-\omega_i)$ is the spectral density of the environment.

It can be checked from Eq. (A11)

$$\langle\hat{S}_-\rangle(t) = e^{\int_0^t[-\tilde{\Gamma}_q + i(\omega_q+\delta_q+\Delta\omega_q)]d\tau}\langle\hat{S}_-\rangle(0),$$

where $\langle\hat{S}_-\rangle(t) = \mathrm{tr}(\hat{S}_-\hat{\rho}_s(t))$. Thus, from the definition of Γ_q, we have

$$\begin{aligned}\Gamma_q =& -\frac{d}{dt}\log\left|\langle 0|\mathrm{tr}_E\left\{\vec{T}\exp\left[-i\int_0^t \hat{H}_{qE}(\tau)\,d\tau\right]\hat{\rho}_{qE}(0)\right.\right.\\ &\left.\left.\times\vec{T}\exp\left[i\int_0^t \hat{H}_{qE}(\tau)\,d\tau\right]\right\}|1\rangle\right|\\ =& -\frac{d}{dt}\log|\langle 0|\hat{\rho}_s(t)|1\rangle| = -\frac{d}{dt}\log|\langle\hat{S}_-(t)\rangle|\\ =& -\frac{d}{dt}\log|e^{-\int_0^t\tilde{\Gamma}_q(\tau)d\tau}|\cdot|\langle\hat{S}_-\rangle(0)| = \tilde{\Gamma}_q.\end{aligned} \tag{A14}$$

From the derivation of Eq. (A4), we have

$$\int_0^t e^{i2\omega_-(t-t')+i\int_{t'}^t\delta_q(t'')dt''}dt' \approx Fe^{i\omega_- t}\left(\frac{\sin\omega_- t}{\omega_-}\right), \tag{A15}$$

where $\omega_- = (\omega_q - \omega)/2$. Then, from Eqs. (A12), (A14), and (A15), it can be shown

$$\Gamma_q = \tilde{\Gamma}_q = F\int_{\omega_{c_1}}^{\omega_{c_2}} d\omega\frac{J(\omega)\sin(\omega_q-\omega)t}{\omega_q-\omega} = F\Gamma_{q0}. \tag{A16}$$

APPENDIX B: DYNAMICS OF THE QUBIT INDUCED BY THE DUFFING OSCILLATOR

The total Hamiltonian of the qubit, the Duffing oscillator, and the environment can be written as

$$\begin{aligned}\hat{H}_{\mathrm{tot}} =& \frac{\omega_q}{2}\hat{S}_z + g_{qo}\hat{S}_z\hat{a}^\dagger\hat{a} + \hat{H}_{\mathrm{Duff}}\\ &+\sum_i\frac{\omega_i}{2}\hat{\sigma}_z^{(i)} + \sum_i(g_i\hat{\sigma}_+^{(i)}\hat{S}_- + \mathrm{H.c.}),\end{aligned} \tag{B1}$$

from which the system state of the total system can be written as

$$\hat{\rho}_{\mathrm{tot}} = e^{-i\hat{H}_{\mathrm{tot}}t}[\hat{\rho}_{qE}(0)\otimes|\alpha\rangle\langle\alpha|]e^{i\hat{H}_{\mathrm{tot}}t},$$

where $\hat{\rho}_{qE}(0)$ and $|\alpha\rangle\langle\alpha|$ are the initial states of the qubit-environment system and the Duffing oscillator.

It can be calculated that

$$\begin{aligned}\langle\hat{S}_-\rangle =& \mathrm{tr}\hat{S}_-\hat{\rho}_{\mathrm{tot}}(t) = \mathrm{tr}_E\mathrm{tr}_{\mathrm{Duff}}\langle 0|e^{-i\hat{H}_{\mathrm{tot}}t}[\hat{\rho}_{qE}(0)\otimes|\alpha\rangle\langle\alpha|]e^{i\hat{H}_{\mathrm{tot}}t}|1\rangle\\ =& \mathrm{tr}_E\mathrm{tr}_{\mathrm{Duff}}[e^{-it(\hat{H}_{\mathrm{Duff}}-g_{qo}\hat{a}^\dagger\hat{a})}|\alpha\rangle\langle\alpha|e^{it(\hat{H}_{\mathrm{Duff}}+g_{qo}\hat{a}^\dagger\hat{a})}\langle 0|e^{-i(\hat{H}_0+\hat{H}_I)t}\hat{\rho}_{qE}(0)e^{i(\hat{H}_0+\hat{H}_I)t}|1\rangle]\\ =& f_{01}(t)\langle 0|\mathrm{tr}_E[e^{-i(\hat{H}_0+\hat{H}_I)t}\hat{\rho}_{qE}(0)e^{i(\hat{H}_0+\hat{H}_I)t}]|1\rangle\\ \equiv& e^{-\int_0^t\Gamma_{\mathrm{Duff}}(t')dt'+i\int_0^t\delta_q(t')dt'}\langle 0|\mathrm{tr}_E[e^{-i(\hat{H}_0+\hat{H}_I)t}\hat{\rho}_{qE}(0)e^{i(\hat{H}_0+\hat{H}_I)t}]|1\rangle,\\ \langle\hat{S}_z\rangle =& \mathrm{tr}[\hat{S}_z\hat{\rho}_{\mathrm{tot}}(t)] = \mathrm{tr}[\hat{S}_z e^{-i(\hat{H}_0+\hat{H}_I)t}\hat{\rho}_{qE}(0)e^{i(\hat{H}_0+\hat{H}_I)t}],\end{aligned} \tag{B2}$$

where

$$f_{01} = \langle\alpha|e^{it(\hat{H}_{\rm Duff}+g_{qo}\hat{a}^\dagger\hat{a})}e^{-it(\hat{H}_{\rm Duff}-g_{qo}\hat{a}^\dagger\hat{a})}|\alpha\rangle = e^{-\int_0^t \Gamma_{\rm Duff}(t')dt'+i\int_0^t \delta_q(t')dt'},$$

$$\hat{H}_0 = \frac{\omega_q}{2}\hat{S}_z + \sum_i \frac{\omega_i}{2}\sigma_z^{(i)},$$

$$\hat{H}_I = \sum_i (g_i\hat{\sigma}_+^{(i)}\hat{S}_- + \text{H.c.});$$

and $\text{tr}_E(\cdot)$ and $\text{tr}_{\rm Duff}(\cdot)$ are the partial trace operations about the degrees of freedom of the environment and the Duffing oscillator. From Eq. (B2), the Duffing oscillator leads to two aspects of effects: (i) an angular frequency shift $\delta_q(t)$ along the $\hat{S}_z$ axis of the qubit which suppresses the environment-induced decoherence and (ii) a pure dephasing channel with damping rate $\Gamma_{\rm Duff}$ which deteriorates the quantum coherence of the qubit state. The dephasing process induced by Duffing oscillator can be equivalently expressed by a classical Gaussian noise $\xi(t)$ along the $\hat{S}_z$ axis of the qubit, which satisfies that

$$E[\xi(t)] = 0, \quad E\left\{\left[\int_0^t \xi(t')dt'\right]^2\right\} = 2\int_0^t \Gamma_{\rm Duff}(t')dt', \tag{B3}$$

where $E(\cdot)$ is the average about the classical stochastic noise $\xi(t)$ (see, e.g., Ref. 15). In fact, a Hamiltonian term $\xi(t)\hat{S}_z$ leads to a damping factor $E(e^{-i\int_0^t \xi(t')dt'})$ for $\langle\hat{S}_-\rangle$. Since $\xi(t)$ is a Gaussian noise, we have

$$E(e^{-i\int_0^t \xi(t')dt'}) = e^{-E\{[\int_0^t \xi(t')dt']^2\}/2} = e^{-\int_0^t \Gamma_{\rm Duff}(t')dt'}.$$

It means that the classical stochastic noise $\xi(t)$ along the $\hat{S}_z$ axis of the qubit leads to the same damping effect given by Eq. (B2). With the above analysis, the Hamiltonian of the qubit-environment system by averaging out the degrees of freedom of the Duffing oscillator can be expressed as

$$\hat{H}_{qE} = \frac{[\omega_q + \delta_q(t) + \xi(t)]}{2}\hat{S}_z + \sum_i \frac{\omega_i}{2}\hat{\sigma}_z^{(i)} + \sum_i (g_i\hat{\sigma}_+^{(i)}\hat{S}_- + \text{H.c.}).$$

*jing-zhang@mail.tsinghua.edu.cn

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