Magnetic friction: From Stokes to Coulomb behavior

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We demonstrate that in a ferromagnetic substrate which is continuously driven out of equilibrium by a field moving with constant velocity v, at least two types of friction may occur when v goes to zero. The substrate may feel a friction force proportional to v (Stokes friction) if the field changes on a time scale which is larger than the intrinsic relaxation time. On the other hand, the friction force may become independent of v in the opposite case (Coulomb friction). These observations are analogous to, e.g., solid friction. The effect is demonstrated in both the Ising (one spin dimension) and the Heisenberg (three spin dimensions) models, irrespective of which kind of dynamics (Metropolis spin-flip dynamics or Landau-Lifshitz-Gilbert precessional dynamics) is used. For both models the limiting case of Coulomb friction can be treated analytically. Furthermore we present an empiric expression reflecting the correct Stokes behavior and therefore yielding the correct crossover velocity and dissipation.

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Friction phenomena, despite their huge importance in everyday life, are still not fully understood. Different friction mechanisms are possible, leading to different dependencies of the friction forces on the driving velocity. Microscopically, one often assumes Stokes-like friction, i.e., a linear velocity dependence. However, this atomistic view is in conflict with Coulomb friction at the interface between solids, because it approaches a nonzero absolute value in the limit of small velocities, independent of the materials and their surface conditions.[1–6] A possible solution was offered by the simple model developed by Prandtl and Tomlinson, in which a stick-slip instability was responsible for Coulomb friction.[7,8] They suggested a surface atom to be coupled by a spring of stiffness k to a slider which moves with constant velocity v. The atom interacts with the surface via a periodic potential and experiences a viscous friction force proportional to its velocity $\dot{x}$. If k is sufficiently small with respect to the potential height, the atom first gets stuck in the potential minima and slips when the tension gets large enough. The slip motion $\dot{x}$ does not depend on the slider's velocity v, and one observes Coulomb friction. However, when k is large with respect to the potential height, the atom moves with the slider's velocity and the friction force is Stokesian. The crossover from one regime to the other has been studied recently.[9]

What remains a puzzle, however, is that Coulomb friction is a far more general phenomenon than one might expect from the Prandtl-Tomlinson model, which is formulated in terms of elastic forces in a periodic potential. For example, Stokes as well as Coulomb behavior has also been observed for magnetic friction, where elastic forces are absent. Being guided by a detailed investigation of the crossover between both types of magnetic friction, a unifying principle can be formulated that applies to the magnetic as well as to the elastic case.

So far, magnetic friction has been studied in two different types of models. Ising models with single-spin-flip dynamics, where two half spaces move with respect to each other, yield Coulomb friction.[10–12] Analogous results have been obtained in the Potts model.[13] On the other hand, a magnetic dipole scanning a Heisenberg surface showed Stokesian friction[14–17] (always provided the velocity is not too large).

Recently a work has been published in which a point-like magnetic perturbation moves through an Ising model.[18,19] The authors claim to have observed Stokes friction, which is in conflict with our results for similar models.[10,11] Here we present an explanation of this discrepancy and clarify under what conditions either Stokes or Coulomb friction occurs.

The systems studied in Refs. 10–13 have in common that the motion occurs in a discretized way: The system is at rest for a certain number a/v of Monte Carlo sweeps (MCS), after which one half space is moved by one lattice constant a. Accordingly we have a periodic excitation and relaxation procedure, where excitation is fast (happens in between two subsequent spin-flip attempts), whereas relaxation extends over a/v MCS. By contrast, in Refs. 14–17 excitation is slow, because due to the dipole-dipole interaction, a substrate spin feels the approaching tip a long time in advance.

Now we present a simple one-dimensional model that interpolates between both cases. We consider a position-dependent field $h_z(r')$, which is moved continuously with constant velocity v. r is given in units of a, and $v = \dot{r}$. Then the discrete motion can be modeled as a step function, as shown in Fig. 1 as solid line. For a certain time $1/v$, exactly one spin is exposed to the field with constant amplitude until the field reaches the next spin. Additionally the amplitude of the dipole field used in Refs. 14–17 is plotted. From Ref. 15 we know that for this case the adjustment of the spins with respect to the moved field happens in an adiabatic way, or in other words, the time scale of relaxation is below that of the excitation. To generalize these setups, we consider a field with steepness $\delta r \ll 1$,

$$h_z(r') = \frac{h}{\left(e^{-\frac{r'}{\delta r}} + 1\right)\left(e^{-\frac{1-r'}{\delta r}} + 1\right)}, \tag{1}$$

which may be tuned from the step-like field [$\delta r = 0$, now called limiting case (i)] to a slowly varying field [$\delta r \approx 0.1$, case (ii)]. By shifting this field according to $r' = r - vt$,[20] we can directly influence the time scale at which the excitation at a fixed position r occurs, $\tau_{\text{switch}} \propto \delta r/v$.

We first consider a chain of classical, normalized Heisenberg spins ($|\mathbf{S}_r| = 1$) of length L with lattice spacing a,

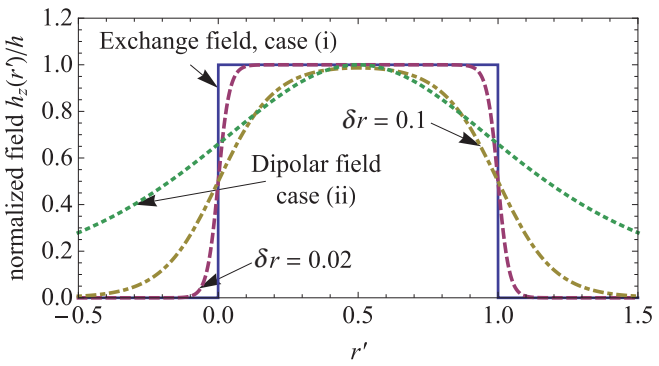

FIG. 1. (Color online) The dynamics in the different studies can be mapped on a time-dependent field with amplitude $h_z(r')$ (here normalized by its maximal value h) interacting with the spins positioned at integer sites. The discrete motion in the Ising and Potts model then corresponds to a step function, which may be treated as a fixed spin interacting via exchange with one partner on the chain. The amplitude of a dipole field is sketched for comparison. The field used in this work may be tuned by adjusting the parameter δr from one limiting case to the other.

which interact with the field defined above. The corresponding time-dependent Hamiltonian is

$$\mathcal{H}(t) = -\sum_{r=1}^{L} J\ \mathbf{S}_r \cdot \mathbf{S}_{r+1} + d_x S_{r,x}^2 + h_z(r - vt) S_{r,z}, \qquad (2)$$

with the exchange constant J. To get a well-defined ground state, we use an easy axis anisotropy ($d_x > 0$) and antiperiodic boundary conditions $\mathbf{S}_{r+L} = -\mathbf{S}_r$. The spins perform Landau-Lifshitz-Gilbert dynamics,[21,22]

$$\frac{\mu_s(1+\alpha^2)}{\gamma}\frac{\partial \mathbf{S}_r}{\partial t} = \mathbf{S}_r \times \frac{\partial \mathcal{H}}{\partial \mathbf{S}_r} + \alpha \mathbf{S}_r \times \left(\mathbf{S}_r \times \frac{\partial \mathcal{H}}{\partial \mathbf{S}_r}\right), \qquad (3)$$

consisting of a precessional motion with a frequency proportional to γ/μ_s, and a damping with the damping constant α. For simplicity, we neglect temperature here, and the dynamic parameters yield a spin relaxation time τ_{rel}. The friction force F can be either calculated from the dissipated power P_{diss} or the pumping power P_{pump}, which are equal in the stationary state due to energy conservation and therefore we subsequently use $F = \langle P \rangle / v$ after time averaging. The two cases can be described by

$$P(v) \propto v^{\phi}, \qquad (4)$$

with the dissipation exponent $\phi = 1$ ($\phi = 2$) for the Coulomb (Stokes) case. P can be extracted from the energy terms by

$$P(t) = \frac{\partial \mathcal{H}}{\partial t} = \sum_{r=1}^{L} \frac{\partial h_z(r - vt)}{\partial t} S_{r,z}, \qquad (5)$$

which represents the power pumped into the system by the motion. In our simulations [see Fig. 3(a)], we found $\phi = 2$ for large τ_{switch}, which corresponds to the results in Refs. 14–17. For sufficiently small τ_{switch} we get $\phi = 1$, which was known from simulations in the Ising model and was now reproduced in the Heisenberg model.

In the following we calculate the velocity $v_\times$ at which a crossover from one regime to the other occurs. For case (i) only two spins contribute to the sum in Eq. (5) at the discrete times $vt \in \mathbb{Z}$ (at all other times and positions the field remains constant), and we can calculate the averaged pumping power by discretizing $\partial_t h_z$,

$$P_{\mathrm{C}} = -hv(\langle S_{1',z}\rangle - \langle S_{0',z}\rangle). \qquad (6)$$

For the time τ_{ca}, corresponding to the time at which the amplitude of the field stays nearly constant, no pumping or excitation occurs. We consider $\tau_{\mathrm{ca}} > \tau_{\mathrm{rel}}$, i.e., the system always relaxes to equilibrium after a pumping event. Since the equilibrium configuration does not depend on the dynamics, Eq. (6) tells us that here $\phi = 1$. The equilibrium configuration for our choice of boundary conditions is a domain wall (DW) state, where the out-of-axis component is determined by the field and thus points in the z direction. As the field interacts mainly with only one spin, the shape of the DW is not influenced by h and we may use the continuum limit profile ($a \to 0$),[23]

$$\mathbf{m}^{\mathrm{H}}(r') = (\tanh(r'/\ell),\ 0,\ \mathrm{sech}(r'/\ell)), \qquad (7)$$

with the DW width $\ell = \sqrt{J/(2d_x)}$, which can be calculated from minimizing the free energy.[24] By inserting $\langle \mathbf{S}_{0'} \rangle = \mathbf{m}^{\mathrm{H}}(0)$ and $\langle \mathbf{S}_{1'} \rangle = \mathbf{m}^{\mathrm{H}}(1)$ into Eq. (6) we now can calculate the power which is pumped into the system during each switching event. This quantity can be visualized in a potential plot. We again assume that h does not influence the shape of the DW but rather its center r_{dw}. Because in limiting case (ii) the system is always near equilibrium and in limiting case (i) it always reaches the ground state before being excited out of equilibrium, this assumption is justified and we can describe the whole configuration with r_{dw}. We look at one cycle at which the field's peak moves from 0 to 1, corresponding to the times $-1/2 \leqslant vt \leqslant 1/2$. For given t we can calculate the system's total energy as a function of r_{dw} (see the potential lines in Fig. 2). If the system evolved quasistatically, it would always be in the current potential minimum. In this picture P_{C}/v corresponds to the energy difference between the energy at $r_{\mathrm{dw}} = 0$, $vt = -1/2$ (the equilibrium state) and $vt = 1/2$ (the state which is present when the peak of the field has moved to the next spin while the DW is still at the same site). Results from simulations (plotted as squares in Fig. 2) confirm this: At $vt = 0$ the system is excited to the upper state in a short time, and relaxes to the new ground state by adjusting r_{dw} slowly afterward, until it reaches the new ground-state configuration with $r_{\mathrm{dw}} = 1$. Simulations of the second limiting case (circles in Fig. 2) confirm that the system is always near equilibrium, thus the DW slightly lags behind the ground state.

From our simulations, we found the pumping power

$$P_{\mathrm{S}}^{\mathrm{H}} = \frac{d_x \mu_s}{J\gamma\delta r}\alpha v^2. \qquad (8)$$

The factor $d_x v/(\delta r J)$ originates in the synchronization with the field, which changes at the time scale τ_{switch}. The factor $\alpha v \mu_s/\gamma$ emerges from spin dynamics, yielding a retardation of the DW as derived in Ref. 15. Setting

$$P_{\mathrm{C}}^{\mathrm{H}}(v) \stackrel{!}{=} P_{\mathrm{S}}^{\mathrm{H}}(v) := P_{\times}^{\mathrm{H}}(v_{\times}^{\mathrm{H}}) \qquad (9)$$

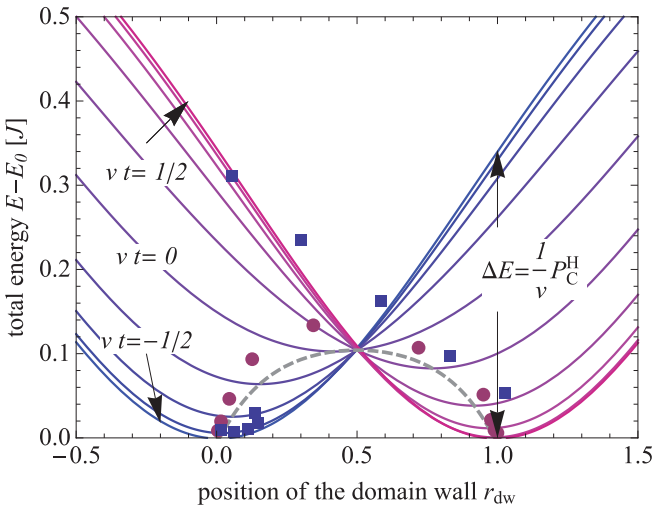

FIG. 2. (Color online) The system can be parametrized by the center of the DW r_{dw}, thus for different times the total energy of the system can be calculated. This is done for field parameter $\delta r = 0.1$ for 10 equidistant times (blue to purple curves). If the system evolved quasistatically, it would follow the configuration of minimal energy, marked by the curve. Simulation results show both cases, the points indicate energy vs r_{dw} for 10 time steps: (i) ■ The energy P_C^H/v is periodically pumped into the system, which relaxes independent from τ_{switch} afterward. As the switching occurs at $vt = 0$, we first see the relaxation from the preceding excitation in the left minimum, and at $vt = 1/2$ the relaxation in the right minimum is not finished. (ii) • The system follows with a lag, but stays near equilibrium, slightly above the minimal energy configuration.

yields the crossover velocity[25]

$$v_\times^H = \frac{h\delta r}{\alpha d_x}\frac{\gamma J}{\mu_s}[1 - \mathrm{sech}(1/\ell)], \tag{10}$$

where the system performs a crossover from the Stokes-friction state to the Coulomb-friction state. In Fig. 3(c) these crossover quantities have been calculated and the simulation results have been rescaled appropriately. The simulation data fit excellently over several magnitudes with the derived crossover quantities; the remaining deviations are discussed below. We performed also simulations of the isotropic Ising model with the same field and periodic boundary conditions ($S_{r+N} = S_r$). The Ising spins undergo spin-flip dynamics with Metropolis probability.[26] Randomly chosen spins are flipped with the probability $p_f = \min[1, \exp(-\beta\Delta E)]$, where β is the inverse temperature and ΔE the energy difference between the flipped and the not flipped state.

For (i) we again find a behavior $\phi = 1$. We assume that for (i) the spins relax after each switching event to the ground-state profile which can be calculated via transfer matrix methods:[11]

$$m^I(r') = \tanh(\beta h)[\tanh(\beta J)]^{|r'|}. \tag{11}$$

With $\langle S_{0',z}\rangle = m^I(0)$ and $\langle S_{1',z}\rangle = m^I(1)$ in Eq. (6), we get P_C^I. $\phi = 2$ is observed for (ii), and we fitted

$$P_S^I \propto v^2/\delta r. \tag{12}$$

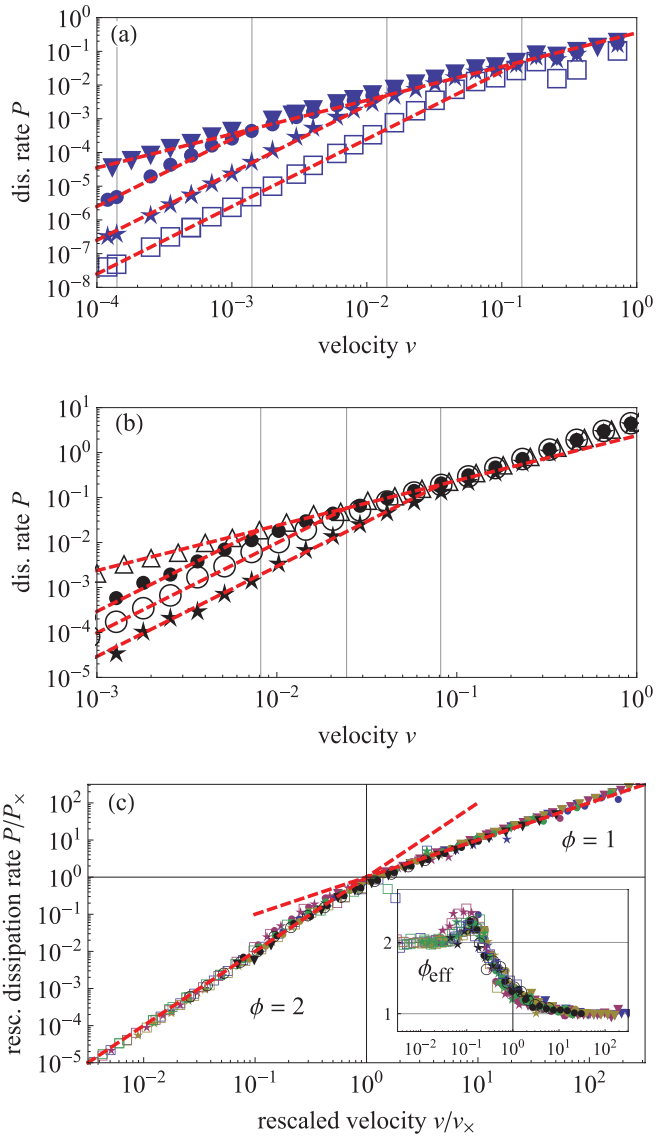

FIG. 3. (Color online) Dissipated power vs velocity (in natural units) for the (a) Heisenberg ($d_x = 0.5J, \alpha = 0.5, h = J$, blue or gray) and the (b) Ising ($\beta = 1/J, h = 10J$, black) models. The simulated δr are $\delta r = \infty$ (△), 10^{-4} (▼), 10^{-3} (•), 3×10^{-3}(○), 10^{-2} (★), and 10^{-1} (□). The grid lines mark the corresponding $v_\times$, the dashed lines display the calculated $P_C(v)$ from Eq. (6) and fitted $P_S(v)$. For (c) we calculated explicitly the crossover quantities $P_\times$ and $v_\times$ for both models from Eqs. (9) and (10), and plot the data again rescaled. Additionally we varied $\alpha = 0.3$ (purple), $h = 2J$ (green), and $d_x = 0.25J$ (yellow) for the same δr set. In the inset we plot an effective exponent $\phi_{eff} = \partial \log P/\partial \log v$, and get a universal crossover from 2 to 1.

We calculated again the crossover velocity $v_\times^I$, which is additionally plotted in Fig. 3(b), and rescaled all data points for the crossover plot, Fig. 3(c).

Comparing Figs. 3(a) and 3(b), we come to the main result of our investigation, namely, the coincidence concerning the crossover between both models, despite the substantial remaining differences such as the dynamics of the models. The present deviations from the crossover curve are discussed below. The slight increase of P, observed in the regime $v > 0.1$

for all δr in the Ising model, is due to the fact that the system has not enough time to relax back to equilibrium before the next shift takes place and $m^{\mathrm{I}}(1)$ becomes significantly smaller than its equilibrium value. As the Heisenberg model contains spin wave excitations, we observed the generation of spin waves above a threshold velocity.[16] In the crossover plot these spin waves cause a kink above $v/v_{\times} = 0.1$ for $\alpha = 0.3$ (and a higher peak in the effective exponent plot). For very high velocities we observe a lowering of the power, which is due to a segregation of the peak of the field and the DW, leading to a reduced $m_z^{\mathrm{H}}(0) < 1$. This state with lowered dissipation has already been observed and reported.[17]

In conclusion, we presented a new model which shows for the case of magnetic friction a transition from Stokes to Coulomb behavior, analogous to the Prandtl-Tomlinson model for solid friction. Whereas there the elastic stiffness of the slider is the crucial parameter, it is the switching time of the magnetic field in our case. The comparison of both models sheds new light on the universal origin of Coulomb behavior, which is based on a separation of the relaxation time from the much larger time scale on which the system gets excited. Our findings are in accordance to field theoretical results by Demery *et al.*,[18,19] who also found Stokes-like friction because their model does not contain discrete sites and thus $\tau_{\mathrm{ca}} = 0$, i.e., the field is continuously driving the system. However, their simulation results are not correct, because they simulated an Ising model with a discontinuous motion of a field, which is known to show Coulomb friction. This discrepancy stems from an incorrect definition of the friction force [Eq. (50) in Ref. 19]; a correct definition has been presented in Ref. 10.

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