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# Band gap in graphene induced by vacuum fluctuations

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The electrons in undoped graphene behave as massless Dirac fermions. Therefore, graphene can serve as a unique condensed-matter laboratory for the study of various relativistic effects, including quantum electrodynamics (QED) phenomena. Although theoretical models describing electronic properties of graphene have been elaborated in details, the QED effects were usually neglected. In this paper, we demonstrate theoretically that QED can drastically modify electronic properties of graphene. We predict the following QED effect: the opening of the band gap in a graphene monolayer placed inside a planar microcavity filled with an optically active media. We show that this phenomenon occurs due to the vacuum fluctuations of the electromagnetic field and is similar to such a well-known phenomenon as a vacuum-induced splitting of atomic levels (the Lamb shift). We estimate the characteristic value of the band gap and find that it can sufficiently exceed the value of the Lamb shift.

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## I. INTRODUCTION

Graphene, a monolayer of carbon atoms, possesses unusual physical properties that make it attractive for various applications.[1–3] Usually treated as a platform for the novel high-speed electronics,[4,5] graphene is of great interest from the point of view of the fundamental physics as well. Indeed, the low-energy electron excitations in graphene are massless Dirac fermions with the linear energy spectrum $\varepsilon(\mathbf{k}) = \hbar v_F|\mathbf{k}|$.[6–9] That makes graphene a condensed-matter playground for the study of various relativistic quantum phenomena such as the Klein tunneling[10,11] and the Casimir effect.[12–14] Up to now, most of graphene-related studies were focused on its unusual transport properties, and quantum electrodynamics (QED) effects arising from interaction of electrons in graphene with a quantized electromagnetic field were neglected.[7] This paper is aimed to fill partially this gap in the theory. We show that, due to the giant Fermi velocity of electrons in graphene, $v_F \approx c/300$, QED effects are pronounced and can lead to qualitative modifications of the spectrum of elementary excitations.

The linear energy spectrum of electrons in graphene comes from its specific honeycomb lattice structure, which makes the band gap between the valence and conductivity bands to be exactly zero.[7] There is the long-standing problem of the opening of the band gap. The appearance of a controllable band gap is required for various electronical and optical applications of graphene.[1,15,16] Aside from this, it is interesting from the fundamental viewpoint to analyze how massless Dirac fermions can acquire a mass. This question is relevant, particularly, in the context of the observation of Majorana fermions in condensed-matter systems.[17] Several mechanisms of the band-gap opening in monolayer graphene have been proposed. Among them are breaking of the symmetry between two sublattices of the honeycomb lattice of graphene,[18,19] the spin-orbit coupling,[20] and the many-body interactions leading to the excitonic instability.[21,22]

Recently, one of us put forward the proposal of opening the band gap by illuminating graphene with a circularly polarized light.[23] In this case, the gap in the spectrum of elementary electron excitations appears due to the formation of composite electron-photon states, which are similar to polaritons in ionic crystals and quantum microcavities.[24–27] It should be noted that, within the framework of QED, the electron-photon interaction can be observed even if "real" photons are absent and electrons interact only with vacuum fluctuations of electromagnetic field due to emitting and reabsorbing virtual photons.[28] Therefore, one can expect that the photon-induced splitting of valence and conductivity bands in graphene[23] will take place due to the vacuum fluctuations even in the absence of an external field pumping. This QED effect is similar to the well-known Lamb shift in the atomic physics, i.e., the vacuum-induced splitting of the states $2s_{1/2}$ and $2p_{1/2}$ of a hydrogen atom with the characteristic splitting energy $\Delta \approx 4\,\mu$eV. The Lamb shift, discovered experimentally by Lamb and Retherford[29] and theoretically explained by Bethe[30] more than 60 years ago, is extremely important for understanding and verification of basic principles of QED. That is why it attracts the undivided attention of the physics community up to now.[31]

Since clockwise and counterclockwise polarized photons shift electron levels in graphene in mutually opposite directions,[23] the band-gap opening needs breaking of the symmetry between virtual photons with different circular polarizations. This can be achieved by placing a graphene monolayer inside a planar cavity filled with an optically active material (see Fig. 1). As it will be shown below, in this case, the vacuum fluctuations lead to the opening of the band gap in graphene even in the absence of an external circularly polarized optical pumping. It should be noted that the QED mass renormalization in an optically active media has been considered,[32] but, surprisingly, the most interesting case of massless Dirac fermions was not analyzed before.

## II. THE MODEL

Let us consider the problem of interaction between a single electron in graphene and a single photon mode of a planar microcavity. Generally, electron states in graphene near the Fermi energy are described by the eight-component wave function, which accounts for two elementary sublattices of graphene, two electron valleys, and two orientations of electron spin.[7] In what follows, intervalley scattering processes and spin-flip effects will be beyond consideration, which reduces the number of necessary components of wave function to two.

The single-particle Hamiltonian of electron in graphene coupled to the cavity mode reads (see the Appendix for details of the derivation) as

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_{\text{field}} + \hat{\mathcal{H}}_{\mathbf{k}} + \hat{\mathcal{H}}_{\text{int}}\,, \tag{1}$$

where

$$\hat{\mathcal{H}}_{\text{field}} = \sum_{\mathbf{q},\pm} \hbar\omega_{\mathbf{q},\pm} \hat{a}^{\dagger}_{\mathbf{q},\pm} \hat{a}_{\mathbf{q},\pm} \tag{2}$$

is the photonic part of the Hamiltonian written in the basis of circularly polarized states,

$$\hat{\mathcal{H}}_{\mathbf{k}} = \hbar v_F \hat{\boldsymbol{\sigma}} \cdot \mathbf{k} \tag{3}$$

is the electron Hamiltonian near the point where the valence and conductivity bands of graphene touch each other (the Dirac point), and

$$\hat{\mathcal{H}}_{\text{int}} = -e v_F \sqrt{\frac{2\hbar}{\epsilon_0 L S}} \sum_{\mathbf{q}} \left[ \frac{1}{\sqrt{\omega_{+,\mathbf{q}}}} \left(\hat{\sigma}^{-} \hat{a}_{+,\mathbf{q}} e^{i\mathbf{q}\mathbf{r}} + \hat{\sigma}^{+} \hat{a}^{\dagger}_{+,\mathbf{q}} e^{-i\mathbf{q}\mathbf{r}}\right) + \frac{1}{\sqrt{\omega_{-,\mathbf{q}}}} \left(\hat{\sigma}^{-} \hat{a}^{\dagger}_{-,\mathbf{q}} e^{-i\mathbf{q}\mathbf{r}} + \hat{\sigma}^{+} \hat{a}_{-,\mathbf{q}} e^{i\mathbf{q}\mathbf{r}}\right) \right] \tag{4}$$

is the Hamiltonian of electron-photon interaction in the cavity. For definiteness, we assume the graphene sheet to be placed in the center of the cavity. In Eqs. (2)–(4), the subscript indices ± correspond to the photon modes with clockwise and counterclockwise circular polarizations, $\mathbf{k} = \mathbf{e}_x k_x + \mathbf{e}_y k_y$ and $\mathbf{q} = \mathbf{e}_x q_x + \mathbf{e}_y q_y$ denote in-plane electron and photon wave vectors, respectively, $\mathbf{e}_{x,y}$ are unit vectors directed along the $x,y$ axes, $e$ is the electron charge, $\epsilon_0$ is the vacuum permittivity, $L$ is the distance between two mirrors of the planar cavity (the cavity length), $S$ is the area of graphene sample, $\omega_{\pm,\mathbf{q}}$ are the eigenfrequencies of clockwise and counterclockwise circularly polarized photons, and $\hat{a}_{\pm,\mathbf{q}}$ and $\hat{a}^{\dagger}_{\pm,\mathbf{q}}$ are photonic annihilation and creation operators. The Pauli vector operator $\hat{\boldsymbol{\sigma}}$ acts in the space of two orthogonal electron states $|\pm\rangle$, corresponding to the two sublattices of graphene in accordance with the following rules: $\hat{\sigma}_z|\pm\rangle = \pm|\pm\rangle$ and $\hat{\sigma}^{\pm}|\mp\rangle = |\pm\rangle$, where $\hat{\sigma}^{\pm} = (\hat{\sigma}_x \pm i\hat{\sigma}_y)/2$. Thus, it corresponds to the *pseudospin* of electron.

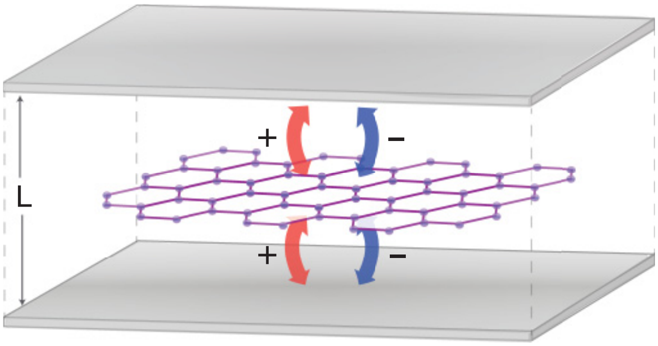


FIG. 1. (Color online) Sketch of the system. A graphene sample placed inside a planar cavity filled with an optically active media. The arrows with signs + and − correspond to clockwise and counterclockwise circularly polarized virtual photons, respectively, which are emitted and reabsorbed by electrons in graphene.

Eigenstates of the electron Hamiltonian (3) are given by the expression[7]

$$|\mathbf{k},\pm\rangle = \frac{e^{i\mathbf{k}\mathbf{r}}}{\sqrt{2S}} \left(e^{-i\theta_{\mathbf{k}}/2}|+\rangle \pm e^{i\theta_{\mathbf{k}}/2}|-\rangle\right), \tag{5}$$

where $\theta_{\mathbf{k}} = \arctan(k_y/k_x)$ and the signs ± correspond to electron states in the conductivity and valence bands of graphene (the upper and lower Dirac cones, respectively). The corresponding eigenenergies are $\varepsilon^{(0)}_{\pm,\mathbf{k}} = \pm\hbar v_F|\mathbf{k}|$. The eigenstates of the photon Hamiltonian (2) can be written as $|N_{\pm,\mathbf{q}}\rangle$, where $N_{\pm,\mathbf{q}}$ are the photon occupation numbers for photons with different circular polarizations (±) and wave vectors $\mathbf{q}$. Then, eigenstates of the full electron-photon Hamiltonian (1) can be decomposed in the basis of the orthogonal electron-photon states

$$|\mathbf{k},\pm,N_+,N_-\rangle = |\mathbf{k},\pm\rangle \otimes |N_{+,\mathbf{q}}\rangle \otimes |N_{-,\mathbf{q}'}\rangle \tag{6}$$

with the energies

$$\varepsilon^{(0)}_{\mathbf{k},\pm,N_+,N_-} = \pm\hbar v_F|\mathbf{k}| + \hbar\omega_{+,\mathbf{q}} N_{+,\mathbf{q}} + \hbar\omega_{-,\mathbf{q}'} N_{-,\mathbf{q}'}\,. \tag{7}$$

In order to find eigenstates and eigenenergies of the full Hamiltonian (1), we will use the perturbation theory, considering the interaction Hamiltonian (4) as a perturbation. To calculate the energy corrections in the lowest order of the perturbation, one needs to find the eigenvalues of the 2 × 2 matrix $\widetilde{\mathcal{H}}^{(1)}$ having matrix elements $\widetilde{\mathcal{H}}^{(1)}_{ss'}(\mathbf{k}) = \langle \mathbf{k},s,0,0|\hat{\mathcal{H}}_{\text{int}}|\mathbf{k},s',0,0\rangle$. However, it is easy to see that all matrix elements of this type are zero, and one needs to use the second order of the perturbation theory. Physically, we need to account for the following processes: the electron with a wave vector $\mathbf{k}$ emits a virtual photon with a momentum $\mathbf{q}$ and then reabsorbs it. Note that, in such a process, the momentum of the electron in the initial state $\mathbf{k}$ should be equal to its momentum in the final state, but the value of the index $s$ can be changed: the electron can remain in the same Dirac cone or move from one Dirac cone to another one. The last process becomes efficient around the $\mathbf{k} = 0$ point, where the energies of the Dirac cones are close to each other, which can lead to the lifting of the degeneracy as we show below. Therefore, to calculate the spectrum of the Hamiltonian (1), we need to use the perturbation theory for degenerate states.

Let us briefly remind how the second-order corrections can be accounted for within the framework of perturbation theory for degenerate states (the details can be found, e.g., in Ref. 33). Imagine that we have a set of states $\{m\}$ that are close in energy to each other (this means that the energy distances $|\varepsilon_m^{(0)} - \varepsilon_{m'}^{(0)}|$ between them are comparable or smaller relative to a characteristic energy of the perturbation). The perturbation does not couple any states $m$ and $m'$ directly (otherwise, the standard first-order perturbation theory is applicable), but couples them to a set of the states $\{l\}$, the energies of which lie far from energies of the states $\{m\}$ (this means that the energy distances $|\varepsilon_m^{(0)} - \varepsilon_l^{(0)}|$ are large as compared with the characteristic energy of the perturbation). In our case, the set $\{m\}$ consists of the two states $\{|\mathbf{k},+,0,0\rangle, |\mathbf{k},-,0,0\rangle\}$, and the set $\{l\}$ corresponds to the states $|\mathbf{k}',\pm,N_{+,n,\mathbf{q}},N_{-,n,\mathbf{q}'}\rangle$ with $N_{+,n,\mathbf{q}} + N_{-,n,\mathbf{q}'} \neq 0$. Then, energies of the perturbed $\{m\}$ states can be obtained by diagonalization of the matrix Hamiltonian

$$\widetilde{\mathcal{H}} = \widetilde{\mathcal{H}}^{(0)} + \widetilde{\mathcal{H}}^{(2)}, \tag{8}$$

where $\widetilde{\mathcal{H}}^{(0)}$ is the matrix of unperturbed Hamiltonian (3) written in the subspace of states $\{m\}$, and the matrix elements of the Hamiltonian $\widetilde{\mathcal{H}}^{(2)}$ can be found as

$$\widetilde{\mathcal{H}}_{mm'}^{(2)} = \frac{1}{2}\sum_l \left(\frac{1}{\varepsilon_m^0 - \varepsilon_l^0} + \frac{1}{\varepsilon_{m'}^0 - \varepsilon_l^0}\right) \times \langle m|\hat{\mathcal{H}}_{\text{int}}|l\rangle\langle l|\hat{\mathcal{H}}_{\text{int}}|m'\rangle\,, \tag{9}$$

where the summation goes over all set of the states $\{l\}$. In the case we consider, the Hamiltonian $\widetilde{\mathcal{H}}_{\mathbf{k}}^{(2)}$ for a given electron wave vector $\mathbf{k}$ can be written as the $2 \times 2$ matrix

$$\widetilde{\mathcal{H}}_{\mathbf{k}}^{(2)} = \begin{pmatrix} \hbar v_F k + \widetilde{\mathcal{H}}_{\mathbf{k}}^{++} & \widetilde{\mathcal{H}}_{\mathbf{k}}^{+-} \\ \widetilde{\mathcal{H}}_{\mathbf{k}}^{-+} & -\hbar v_F k + \widetilde{\mathcal{H}}_{\mathbf{k}}^{--} \end{pmatrix}, \tag{10}$$

where the vacuum-fluctuation corrections $\widetilde{\mathcal{H}}_{\mathbf{k}}^{++}$, $\widetilde{\mathcal{H}}_{\mathbf{k}}^{--}$, and $\widetilde{\mathcal{H}}_{\mathbf{k}}^{+-} = (\widetilde{\mathcal{H}}_{\mathbf{k}}^{-+})^*$ are given by

$$\widetilde{\mathcal{H}}_{\mathbf{k}}^{++} = -\frac{e^2 v_F^2}{4\pi\epsilon_0 L}\sum_{\lambda=\pm}\int d^2\mathbf{q}\,\frac{\omega_{\lambda,\mathbf{q}} - v_F|\mathbf{k}|}{\omega_{\lambda,\mathbf{q}}[(\omega_{\lambda,\mathbf{q}} - v_F|\mathbf{k}|)^2 - v_F^2|\mathbf{k}-\mathbf{q}|^2]}\,, \tag{11}$$

$$\widetilde{\mathcal{H}}_{\mathbf{k}}^{--} = -\frac{e^2 v_F^2}{4\pi\epsilon_0 L}\sum_{\lambda=\pm}\int d^2\mathbf{q}\,\frac{\omega_{\lambda,\mathbf{q}} + v_F|\mathbf{k}|}{\omega_{\lambda,\mathbf{q}}[(\omega_{\lambda,\mathbf{q}} + v_F|\mathbf{k}|)^2 - v_F^2|\mathbf{k}-\mathbf{q}|^2]}\,, \tag{12}$$

$$\widetilde{\mathcal{H}}_{\mathbf{k}}^{+-} = -\frac{e^2 v_F^2}{4\pi\epsilon_0 L}\sum_{\lambda=\pm}\lambda\int d^2\mathbf{q}\,\frac{\omega_{\lambda,\mathbf{q}}^2 - v_F^2(|\mathbf{k}|^2 + |\mathbf{k}-\mathbf{q}|^2)}{\left[(\omega_{\lambda,\mathbf{q}} + v_F|\mathbf{k}|)^2 - v_F^2|\mathbf{k}-\mathbf{q}|^2\right]\cdot\left[(\omega_{\lambda,\mathbf{q}} - v_F|\mathbf{k}|)^2 - v_F^2|\mathbf{k}-\mathbf{q}|^2\right]}\,, \tag{13}$$

and the symbol $\lambda = \pm$ corresponds to the two different circular polarizations of virtual photons. The physical meaning of the terms in the Hamiltonian (10) is the following. The matrix element $\widetilde{\mathcal{H}}_{\mathbf{k}}^{++}$ corresponds to the process: an electron in the upper Dirac cone emits a virtual photon and then reabsorbs this photon while returning to the same cone. The matrix element $\widetilde{\mathcal{H}}_{\mathbf{k}}^{--}$ corresponds to the same process for the electron in the lower Dirac cone. The off-diagonal matrix elements $\widetilde{\mathcal{H}}_{\mathbf{k}}^{+-} = \widetilde{\mathcal{H}}_{\mathbf{k}}^{-+}$ correspond to the processes in which the electron after reabsorption of the photon changes the Dirac cone. Diagrammatic representation of these terms is shown in Fig. 2. Diagonalization of the Hamiltonian (10) gives the renormalized energy spectrum of the elementary excitations in graphene,

$$\varepsilon^{\pm}(\mathbf{k}) = \widetilde{\mathcal{H}}_{\mathbf{k}}^{++}/2 + \widetilde{\mathcal{H}}_{\mathbf{k}}^{--}/2 \pm\sqrt{(\widetilde{\mathcal{H}}_{\mathbf{k}}^{++}/2 - \widetilde{\mathcal{H}}_{\mathbf{k}}^{--}/2 + \hbar v_F|\mathbf{k}|)^2 + (\widetilde{\mathcal{H}}_{\mathbf{k}}^{+-})^2}\,. \tag{14}$$

Taking into account that at $\mathbf{k} = 0$ we have $\widetilde{\mathcal{H}}_{\mathbf{k}}^{++} = \widetilde{\mathcal{H}}_{\mathbf{k}}^{--}$, the renormalized electron energy (14) at the Dirac point can be written as

$$\varepsilon^{\pm}(0) = \widetilde{\mathcal{H}}_0^{++} \pm |\widetilde{\mathcal{H}}_0^{+-}|\,. \tag{15}$$

It follows from Eq. (15) that the vacuum fluctuations of electromagnetic field in the cavity can open the band gap between the conductivity and valence bands of graphene at

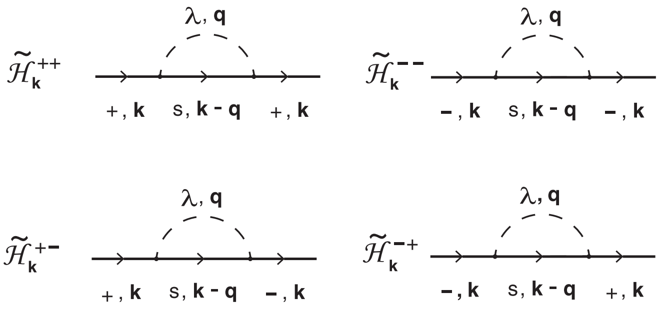


FIG. 2. Diagrammatic representation of the terms entering in the Hamiltonian (10). The solid lines correspond to the electrons and the dashed lines correspond to the virtual photons. Index $\lambda = \pm$ denotes the two different circular polarizations of the photons, and the index $s = \pm$ denotes the two different Dirac cones. Summation should be performed over both the indices $\lambda$ and $s$.

the Dirac point, which is

$$\varepsilon_g = 2|\widetilde{\mathcal{H}}_0^{+-}|\,. \tag{16}$$

It should be stressed that, in the absence of an optically active media, the eigenfrequencies of clockwise and counterclockwise circularly polarized photons are equal, $\omega_{+,\mathbf{q}} = \omega_{-,\mathbf{q}}$. According to Eq. (13), in this case, the term $\widetilde{\mathcal{H}}_0^{+-}$ is zero and the band gap (16) vanishes. Therefore, for the band-gap opening, one needs to fill the cavity by an optically active media, which splits modes of virtual photons with different circular polarizations. In this case, the photonic dispersions read as

$$\omega_{\pm,\mathbf{q}} = c_\pm\sqrt{q^2 + q_z^2}\,, \tag{17}$$

where $q_z = \pi n/L$ is the quantized $z$ component of photon wave vector in the cavity, $n$ is the number of photon mode, $c_\pm = c/n_\pm$ are the speeds of light with clockwise and counterclockwise circular polarizations, and $n_\pm$ are the refractive indices for clockwise and counterclockwise polarized light, which are different in the optically active media $n_+ \neq n_-$.

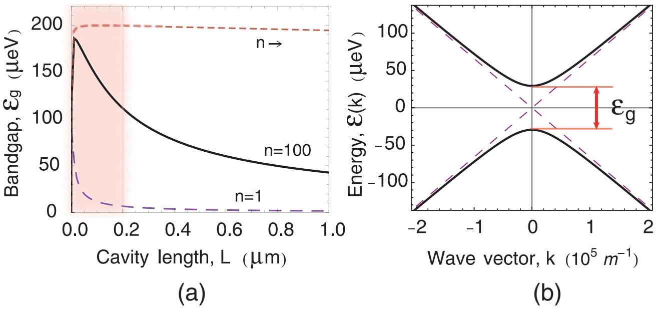
In the discussion above, we restricted our analysis to the single-mode approximation, accounting for the coupling of the electron in graphene with only one photon mode. Going beyond this approximation, one needs to perform the summation over all modes $n$ in Eqs. (11)–(13). Keeping in mind that photon modes with even numbers $n$ correspond to the zero-field intensity in the center of the cavity and, thus, do not interact with the graphene sheet, one gets the following expression for the band gap:

$$\varepsilon_g = \frac{e^2}{2\pi\epsilon_0 L}\left|\sum_{n=0}^{\infty}\sum_{\lambda=\pm}\frac{\lambda\beta_\lambda^2}{1-\beta_\lambda^2}\ln\left[1+\frac{q_0^2L^2(1-\beta_\lambda^2)}{\pi^2(2n+1)^2}\right]\right|, \tag{18}$$

where $\beta_\pm = v_F/c_\pm$, $q_0 \sim 1/a_0$ is the cutoff parameter of integration in Eq. (13) with $a_0$ being the lattice constant of graphene.

## III. RESULTS AND DISCUSSION

It is seen that Eq. (18) contains the summation over photon polarizations $\lambda = \pm$. As expected, the contributions of clockwise and counterclockwise polarized photons in the band gap (18) have opposite signs, and the band gap vanishes for $\beta_+ = \beta_-$ and appears only in the presence of an optically active media with $\beta_+ \neq \beta_-$. For instance, the cavity can be filled with a magnetogyrotropic media based on ferrite garnets, which possess the giant difference between the velocities of light with different circular polarizations.[34] The effect becomes even more pronounced if the cavity is filled with an active media with the circular dichroism.[35] In this case, one of the two circularly polarized photon modes in the cavity is suppressed and its contribution to the band gap (18) can be neglected. As a result, the summation over $\lambda$ in Eq. (18) can be omitted, which leads to the drastic increasing of the band gap.



(a) (b)

FIG. 3. (Color online) (a) The band gap in graphene induced by vacuum fluctuations, calculated by accounting different numbers of cavity modes $n$. (b) Energy spectrum of free electrons in graphene (dashed lines) and electrons dressed by virtual photons (solid lines). The calculation is performed for the cavity length $L = 300$ nm, and the number of accounted cavity modes is $n = 100$.

Figure 3(a) shows the dependence of the band gap $\varepsilon_g$ on the cavity length $L$ for the cavity filled by such a media with the circular dichroism. In the physically relevant region of the cavity lengths [the white area in Fig. 3(a)], the band gap calculated in the single-mode approximation ($n = 1$) is of several $\mu$eV, which is comparable with the Lamb shift.[29] The summation over higher modes increases this value by one to two orders of magnitude: for 100 modes, the value of the band gap increases to 50–100 $\mu$eV, while the summation over all modes gives value of about 200 $\mu$eV. However, the summation over infinite number of modes overestimates the band gap. Indeed, if the characteristic photon wavelength $2\pi/q_z = 2L/n$ is comparable to the interatomic distance, the macroscopic description of an optically active media becomes irrelevant. Therefore, the band gap can be reasonably estimated to be about tens of $\mu$eV, which is one order of magnitude bigger than the Lamb shift.[29]

The energy spectrum of massive Dirac fermions in graphene is plotted in Fig. 3(b). The renormalized dispersion relation can be approximated by the analytical expression

$$\varepsilon^{\pm}(\mathbf{k}) = \pm\sqrt{(\hbar v_F|\mathbf{k}|)^2 + \left(m^* v_F^2\right)^2}\,, \tag{19}$$

where the effective mass of electron dressed by virtual photons is $m^* = \varepsilon_g/2v_F^2$.

It should be noted that the considered single-electron problem can be easily generalized for the realistic situation when the valence band is filled by the Fermi sea of electrons. In this case, the Pauli principle forbids virtual transitions into the lower Dirac cone filled with electrons, which reduces both the matrix elements (11)–(13) and the band gap (18) by the factor of 1/2.

## IV. CONCLUSIONS

We predicted the quantum electrodynamical effect in graphene placed inside a planar cavity filled by an optically active media. Due to the vacuum fluctuations of electromagnetic field in the cavity, the spectrum of elementary excitations in graphene undergoes qualitative changes. Namely, the valence and conductivity bands of graphene are split at the Dirac points. The value of the vacuum-induced band gap can be one order of magnitude bigger then the famous Lamb shift in hydrogen atom.

## ACKNOWLEDGMENT

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## APPENDIX: DERIVATION OF INTERACTION HAMILTONIAN

The introduction of the electron-photon interaction in graphene can be done by the conventional replacement $\hbar\mathbf{k} \to \hbar\mathbf{k} - e\hat{\mathbf{A}}$, where $\hat{\mathbf{A}}$ is the operator of the vector potential of electromagnetic field. Then, the full Hamiltonian of the electron-photon system reads as

$$\hat{\mathcal{H}} = v_F\hat{\boldsymbol{\sigma}}\cdot(\hbar\mathbf{k} - e\hat{\mathbf{A}}) + \frac{1}{2}\int dV(\epsilon_0\hat{\mathbf{E}}^{\dagger}\hat{\epsilon}\hat{\mathbf{E}} + \mu_0\hat{\mathbf{B}}^{\dagger}\hat{\mu}^{-1}\hat{\mathbf{B}})\,, \tag{A1}$$

where $\hat{\mathbf{E}}$, $\hat{\mathbf{B}}$ are the operators of electric and magnetic fields, and $\hat{\epsilon}$, $\hat{\mu}$ are the tensors of electric and magnetic permittivity of the media, respectively. The integration in the last term, giving the energy of free electromagnetic field, goes over all space where the field is present. In this paper, we consider a graphene sheet placed in a planar microcavity. In this case, it is convenient to represent the operators of the fields in terms of the eigenmodes of the cavity as

$$\hat{\mathbf{A}}(\mathbf{r}) = \sum_{\lambda,n,\mathbf{q}}\hat{\mathbf{A}}_{\lambda,n,\mathbf{q}}(\mathbf{r})\,, \tag{A2}$$

$$\hat{\mathbf{E}}(\mathbf{r}) = \sum_{\lambda,n,\mathbf{q}}\hat{\mathbf{E}}_{\lambda,n,\mathbf{q}}(\mathbf{r})\,, \tag{A3}$$

$$\hat{\mathbf{B}}(\mathbf{r}) = \sum_{\lambda,n,\mathbf{q}}\hat{\mathbf{B}}_{\lambda,n,\mathbf{q}}(\mathbf{r})\,, \tag{A4}$$

where $n = 1,2,3,\ldots$ is the number of field modes in the cavity. Using the Coulomb gauge, we can write the field operators (A2)–(A4) as

$$\hat{\mathbf{A}}_{\lambda,n,\mathbf{q}}(\mathbf{r}) = \sqrt{\frac{\hbar}{2\epsilon_0\omega_{\lambda,n,\mathbf{q}}}}[\hat{a}_{\lambda,n,\mathbf{q}}\mathbf{u}_{\lambda,n,\mathbf{q}}(\mathbf{r}) + \hat{a}^{\dagger}_{\lambda,n,\mathbf{q}}\mathbf{u}^{*}_{\lambda,n,\mathbf{q}}(\mathbf{r})], \tag{A5}$$

$$\hat{\mathbf{E}}_{\lambda,n,\mathbf{q}}(\mathbf{r}) = i\sqrt{\frac{\hbar\omega_{\lambda,n,\mathbf{q}}}{2\epsilon_0}}[\hat{a}^{\dagger}_{\lambda,n,\mathbf{q}}\mathbf{u}^{*}_{\lambda,n,\mathbf{q}}(\mathbf{r}) - \hat{a}_{\lambda,n,\mathbf{q}}\mathbf{u}_{\lambda,n,\mathbf{q}}(\mathbf{r})], \tag{A6}$$

$$\hat{\mathbf{B}}_{\lambda,n,\mathbf{q}}(\mathbf{r}) = \sqrt{\frac{\hbar}{2\epsilon_0\omega_{\lambda,n,\mathbf{q}}}}[\hat{a}_{\lambda,n,\mathbf{q}}\nabla\times\mathbf{u}_{\lambda,n,\mathbf{q}}(\mathbf{r}) + \hat{a}^{\dagger}_{\lambda,n,\mathbf{q}}\nabla\times\mathbf{u}^{*}_{\lambda,n,\mathbf{q}}(\mathbf{r})]\,, \tag{A7}$$

where $\mathbf{u}_{\lambda,n,\mathbf{q}}$ are the cavity eigenmodes. If the cavity is filled with an optically active media, the eigenmodes are circularly polarized and can be found as

$$\mathbf{u}_{\pm,n,\mathbf{q}}(z,\mathbf{r}) = \mathbf{e}_{\pm}\sqrt{\frac{2}{LS}}\sin\left(\frac{\pi nz}{L}\right)e^{i\mathbf{q}\cdot\mathbf{r}}\,, \tag{A8}$$

where

$$\omega_{\pm,n,\mathbf{q}} = c_{\pm}\sqrt{q^2 + \left(\frac{\pi n}{L}\right)^2} \tag{A9}$$

are the photon eigenfrequencies. Therefore, the Hamiltonian of the interaction between the graphene sheet and the electromagnetic field in the cavity can be written as

$$\begin{aligned}\hat{\mathcal{H}}_{\text{int}} &= -ev_F\hat{\sigma}\cdot\hat{\mathbf{A}}(\mathbf{r})\\ &= ev_F\sqrt{2}\sum_{\lambda=\pm,n,\mathbf{q}}(\mathbf{e}_+\hat{\sigma}^- + \mathbf{e}_-\hat{\sigma}^+)\cdot\hat{\mathbf{A}}_{\lambda,n,\mathbf{q}}(\mathbf{r}).\end{aligned} \tag{A10}$$

Since the graphene sheet is placed in the center of the cavity (at $z = L/2$), it is coupled only with modes (A8) corresponding to odd numbers $n$. This means that the summation index $n$ in Eq. (A10) is odd: $n = 1,3,5,7,\ldots$. Then, using the expression (A5) for the vector potential operator $\hat{\mathbf{A}}_{\lambda,n,\mathbf{q}}(\mathbf{r})$, the interaction Hamiltonian (A10) can be rewritten in the form (4).

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