

Quantum criticality in a dissipative (2+1)-dimensional XY model of circulating currents in high- T_c cuprates

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We present large-scale Monte Carlo results for the dynamical critical exponent z and the spatio-temporal two-point correlation function of a (2+1)-dimensional quantum XY model with bond dissipation, proposed to describe a quantum critical point in high- T_c cuprates near optimal doping. The phase variables of the model, originating with a parametrization of circulating currents within the CuO_2 unit cells in cuprates, are compact, $\{\theta_{\mathbf{r},\tau}\} \in [-\pi, \pi)$. The dynamical critical exponent is found to be $z \approx 1$, and the spatio-temporal correlation functions are explicitly demonstrated to be isotropic in space-imaginary time. The model thus has a fluctuation spectrum where momentum and frequency enter on equal footing, rather than having the essentially momentum-independent marginal Fermi-liquid-like fluctuation spectrum previously reported for the same model.

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Quantum critical points describe systems with diverging length scales at zero temperature, and have come into much focus in recent years as possible descriptions of anomalous phenomena in strongly correlated fermion systems and systems with competing orders.¹ One prime example of this is represented by the high- T_c superconducting cuprates, where various types of quantum critical phenomena have been proposed as essential for understanding the many unusual normal-state transport properties these systems exhibit. This has, over the past quarter of a century, represented one of the major challenges in condensed-matter physics.²

One successful phenomenological framework is to describe the the normal phase around optimal doping as a marginal Fermi liquid (MFL),³ the weakest possible violation of having a nonzero quasiparticle residue at the Fermi surface. Among the merits of the MFL phenomenology is that it describes transport properties in this strange metallic phase in good accordance with experiments. This follows naturally from the essentially momentum-independent, linear-in-frequency, fluctuation spectrum of the MFL hypothesis.³

More recent works have pursued a more microscopic foundation of MFL. The underlying picture is that there exists a quantum critical point (QCP) residing at $T = 0$ beneath the superconducting dome.⁴ The degrees of freedom associated with this QCP are circulating currents within the unit cells of the CuO_2 layers. The main idea is that the MFL phenomenology arises from the quantum critical fluctuations of these currents above the QCP at $T > T_c$. It has also been demonstrated how the same fluctuations may give rise to d -wave high- T_c superconductivity.⁵ The ordering of such circulating currents upon lowering the temperature from the strange metal region into the pseudogap region is a candidate for a possible competing order in this part of the phase diagram.⁶ Magnetic order conforming with such circulating currents has in fact been observed in several experiments.⁷⁻¹¹ It must be mentioned that others argue that such signatures may have a quite different origin,¹²⁻¹⁵ and also numerical results disagree on the presence of such circulating currents,¹⁶⁻¹⁸ but the model remains one of the central theories of the physics of high- T_c cuprates.^{2,19}

A remarkable implication of a q -independent fluctuation spectrum, such as that posited in MFL theory, is that the

associated QCP exhibits *local* quantum criticality (LQC). Defining the dynamical critical exponent z from the scaling of momentum and frequency at the quantum critical point, $\omega \sim q^z$, this means that, formally, $z = \infty$. It is a highly nontrivial question as to how such a remarkable property of a quantum critical point can arise in an extended system. Recently, it was argued^{20,21} that precisely such local criticality is found in a (2+1)-dimensional quantum XY model with bond dissipation of the Caldeira-Leggett²² form. The angle variables of this model were associated with circulating current degrees of freedom, as will be explained below.

The results of Ref. 20 would imply that the previously hypothesized MFL fluctuation spectrum has been derived from a microscopic theory applicable to cuprates. In a broader perspective, it is of considerable interest to investigate in detail if such unusual behavior can occur in model systems of condensed matter, as related variants of locality have also been considered in the context of gauge/gravity duality²³ and QCPs in disordered systems and heavy fermion compounds.¹

From naive scaling arguments^{24,25} applied to the dissipative model proposed in Ref. 20, one might expect that dissipation is irrelevant in the renormalization group sense. The result would then not be LQC, but instead conventional quantum criticality with isotropic scaling $z = 1$. Here, we report results from Monte Carlo simulations performed directly on the (2+1)-dimensional quantum XY model with bond dissipation and compact angle variables, considered in Ref. 20. Our results strongly indicate that in this model $z = 1$.

The dissipative (2+1)-dimensional [(2+1)D] XY action considered in Ref. 20 takes the form

$$\begin{aligned}
 S = & -K \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \sum_{\tau=1}^{L_\tau} \cos(\Delta\theta_{\mathbf{r},\mathbf{r}',\tau}) \\
 & -K_\tau \sum_{\mathbf{r}} \sum_{\tau=1}^{L_\tau} \cos(\theta_{\mathbf{r},\tau+1} - \theta_{\mathbf{r},\tau}) \\
 & + \frac{\alpha}{2} \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \sum_{\tau \neq \tau'}^{L_\tau} \left(\frac{\pi}{L_\tau} \right)^2 \frac{(\Delta\theta_{\mathbf{r},\mathbf{r}',\tau} - \Delta\theta_{\mathbf{r},\mathbf{r}',\tau'})^2}{\sin^2\left(\frac{\pi}{L_\tau} |\tau - \tau'|\right)} \quad (1)
 \end{aligned}$$

when put on a cubic $L \times L \times L_\tau$ lattice. The bond variables are given by $\Delta\theta_{\mathbf{r},\mathbf{r}',\tau} = \theta_{\mathbf{r},\tau} - \theta_{\mathbf{r}',\tau}$, where the sum over \mathbf{r} and \mathbf{r}'

goes over nearest neighbors in the x - y plane. Periodic boundary conditions are implicit in the imaginary time direction, and are also applied in the spatial directions.

Such a model has previously been employed as an effective description of a resistively shunted Josephson junction array,²⁶ and it may also be viewed as a generic quantum rotor model with dissipative currents. A third possible interpretation in the context of high- T_c cuprates is as follows. Suppose the angles *theta a priori* can take only four possible values. These four values then represent the directions of a pseudospin associated with the four possible ordered circulating current patterns within each CuO_2 unit cell (see, e.g., Fig. 1 of Ref. 27). The first two terms represent the standard interaction energies in space-imaginary time of these circulating currents in neighboring unit cells, and have been derived from microscopics.²⁷ The last term is the term responsible for dissipating the ordered circulating currents.²⁰

In Eq. (1), the angles are continuous variables. We will discuss a possible *a posteriori* justification for this later in this Rapid Communication by showing that an added fourfold anisotropy term is perturbatively irrelevant. Reference 20, moreover, appears to treat $\theta_{r,\tau}$ as compact variables, also in the presence of a dissipation term that apparently renders the action nonperiodic in the angle variables.²⁸ In order to investigate numerically the same model considered in Ref. 20, we therefore *compactify* the expression $\Delta\theta_{r,r',\tau} - \Delta\theta_{r,r',\tau'}$ so that it is defined modulo 2π . We will discuss alternative choices later.

The calculations of Ref. 20 were not restricted to any specific parameter regime, but predicted that every point on the $T = 0$ quantum critical surface in $\alpha - K - K_\tau$ (parameter) space (for $\alpha > 0$) should be a local QCP. Accordingly, we choose convenient coupling constants when searching for LQC in our simulations, and for the results presented here, the dissipation strength is fixed at $\alpha = 0.05$.

The phase diagram (not shown) is qualitatively very similar to those found for related compact (1+1)D models with bond dissipation.^{25,29} It features a single critical surface that separates a disordered from a fully ordered phase, and which is continuously connected to the 3D XY critical line at $\alpha = 0$. For similar models in (1+1) dimensions, only the region of relatively moderate dissipation was accessible to simulations, as increasing α increases finite-size effects, resulting in apparent values $z < 1$ for the dynamical critical exponent. As expected, this problem is no less severe in (2+1) dimensions. Available system sizes are restricted by the absence of cluster algorithms to treat models with bond dissipation appropriately,²⁵ and we are therefore confined to local Metropolis updates.

To locate the phase transition, we vary the spatial coupling K and use the crossing point for different system sizes L of the Binder cumulant $g = 1 - \langle |m|^4 \rangle / (2\langle |m|^2 \rangle^2)$. Here, $m = \sum_{r,\tau} \exp[i\theta_{r,\tau}]$ is the order parameter of the $U(1)$ -symmetric degrees of freedom. Due to the anisotropy of the interactions, we have to calculate g for multiple values of L_τ for each spatial system size L , as described in more detail, e.g., in Ref. 25. The value $L_\tau = L_\tau^*$ where the function $g(L_\tau)$ reaches its maximum corresponds to the *optimal* temporal extent for which the system appears as isotropic as it can be, the anisotropic interactions taken into account.

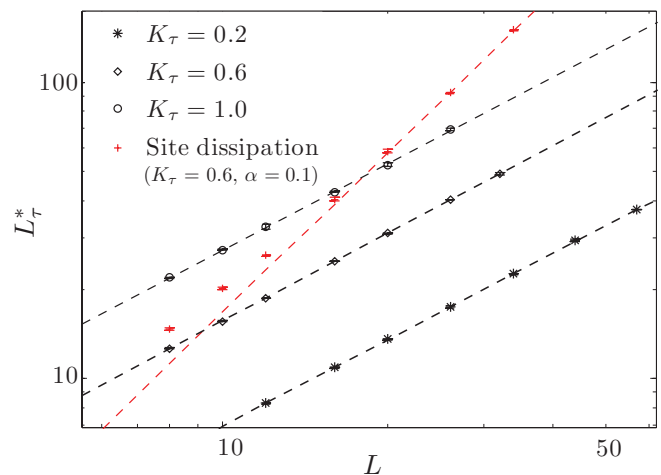


FIG. 1. (Color online) Finite-size analysis of the maximum L_τ^* of the Binder cumulant curves $g(L_\tau)$ as a function of spatial system size L . For the black data points, the dynamical critical exponents z as given in Table I are obtained from the slope of the fitting lines (dashed). The red (gray) points show similar results for site dissipation for comparison, where a fit of the three largest systems yields $z = 1.84(3)$.

For a conventional QCP, at which the correlation length ξ_τ in imaginary time scales with the correlation length ξ in space as $\xi_\tau \sim \xi^z$ with a finite z , we expect to observe the scaling relation $L_\tau^* \sim L^z$. This scaling procedure then allows one to extract the dynamical critical exponent z from Binder cumulant data. For a local QCP formally having $z = \infty$, we expect this scaling to break down. Our strategy to search for possible LQC in the model (1) is therefore to perform the above procedure *assuming* conventional criticality, and then look for indications that this hypothesis should be rejected.

The results of this finite-size analysis is shown in Fig. 1, with the values of the dynamical critical exponent z given in Table I. Here, we have chosen three different values of the quantum coupling K_τ in order to investigate both the limit of relatively weak quantum coupling and the opposite limit leading to relatively strong system anisotropy.

The results show that the effective dynamical critical exponent is $z \lesssim 1$ for all the parameter sets considered, and we expect that we could obtain $z \approx 1$ if we were able to reach higher values of L . (For a smaller value $\alpha = 0.02$, we obtained $z = 1$ within statistical uncertainty.) It is conceivable that signatures of LQC would be visible only for systems larger than the admittedly moderate system sizes accessible to present algorithms. However, were that the case, the true

TABLE I. Critical coupling K_c and dynamical critical exponent z for different values of the quantum coupling K_τ , but for the same dissipation strength $\alpha = 0.05$. Uncertainty estimates for z have been calculated by a bootstrap procedure, including the uncertainty in K_c .

K_τ	K_c	z
0.2	0.48068(5)	0.968(8)
0.6	0.28244(4)	0.985(8)
1.0	0.18008(5)	0.970(11)

$z \rightarrow \infty$ nature of the model would likely reveal itself as strongly increasing effective values of z as a finite-size effect for increasing L . For comparison, we have also carried out simulations with equivalent parameters of a (2+1)D XY model with *site* dissipation, for which $z = 2$ is expected.^{25,30} The results are included in Fig. 1, and already for system sizes comparable to those for bond dissipation, we observe (finite-size) crossover behavior with $z \rightarrow 2$. For bond dissipation, we observe no tendency toward $z > 1$ for either of the parameter sets, and it is hard to imagine how crossover to $z \rightarrow \infty$ scaling should be much slower than crossover to $z = 2$ scaling.

For all results reported here, we have used parallel tempering³¹ to reduce autocorrelation times, and to ensure that the simulations are well equilibrated. To emulate the continuous $U(1)$ symmetry, the simulations are made for Z_q clock models, with $q = 128$ for $K_\tau = 0.2, 1.0$, and $q = 32$ for $K_\tau = 0.6$. The nature of the criticality remains unchanged also when increasing to $q = 1024$. The results are obtained using an implementation of the Mersenne Twister³² random number generator, but other random number generators produced consistent results.

Although we found no indication of LQC from the scaling of the Binder cumulant, we also considered the correlations of the order parameter field directly

$$C(\mathbf{r} - \mathbf{r}', \tau - \tau') = \langle e^{i\theta_{\mathbf{r},\tau}} e^{-i\theta_{\mathbf{r}',\tau'}} \rangle. \quad (2)$$

The correlation functions presented here are obtained for the parameter set $K_\tau = 0.6$, with $L_\tau = L_\tau^*$ and $K = K_c$ as obtained from the previous simulations, and therefore serve as a self-consistency check of the Binder scaling procedure. From Fig. 2, it is evident that the correlation function at the critical point decays isotropically in space-imaginary time. In other words, there are no signs of locality.

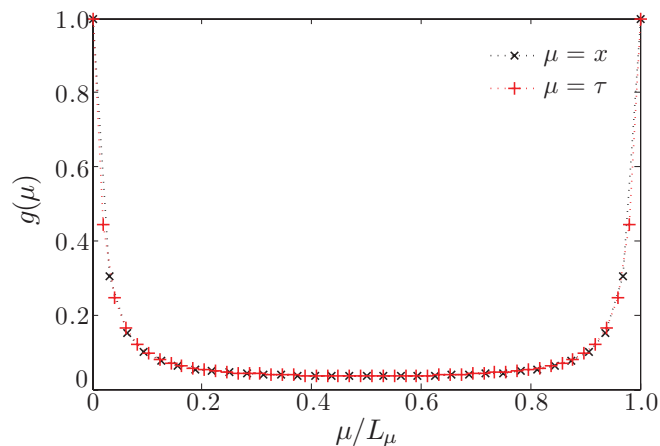


FIG. 2. (Color online) Correlation functions at the critical point $K_c = 0.28244$ for dissipation strength $\alpha = 0.05$ and quantum coupling $K_\tau = 0.6$. The system size $L = 32$, $L_\tau = 49 \approx L_\tau^*$ corresponds to the rightmost data point of the midmost data series in Fig. 1. The correlation function is defined in the spatial direction as $g(x) = g(|\mathbf{r} - \mathbf{r}'|) = C(\mathbf{r} - \mathbf{r}', 0)$ and in the temporal direction as $g(\tau) = C(0, \tau)$, with C defined in Eq. (2). Also, $L_x \equiv L$. Error bars are smaller than the linewidth, and the dotted lines are guides to the eye.

Furthermore, we have verified that the same conclusion may be drawn for the other values of K_τ considered, and also for larger system sizes with aspect ratios found from extrapolation based on the power law shown in Fig. 1. As an additional test, we compared the correlation functions shown here with those obtained by setting $\alpha = 0$ in Eq. (1). Letting $K_\tau > K$, values of L_τ and K_c were determined by the same procedure as for the dissipative model. There is no indication that adding dissipation changes the scaling of the temporal correlation length ξ_τ with respect to the spatial correlation length ξ .

Depending on how $\Delta\theta_{\mathbf{r},\mathbf{r}',\tau}$ is interpreted in the dissipation term, it may be argued either that the correct treatment is to compactify only the gradients $\Delta\theta_{\mathbf{r},\mathbf{r}',\tau}$, restricting them to the interval $[-\pi, \pi)$, or to do so to the difference $\Delta\theta_{\mathbf{r},\mathbf{r}',\tau} - \Delta\theta_{\mathbf{r},\mathbf{r}',\tau'}$ as well. Although we have chosen the latter, as in Ref. 29, we also performed simulations with the former compactification scheme. The results are qualitatively similar, with the difference merely amounting to a renormalization of the dissipative coupling α . In other words, the absence of LQC in this model is not contingent on the choice of compactification scheme.

As explained in connection with Eq. (1), the underlying circulating current degrees of freedom are most naturally described by discrete, Z_4 -symmetric variables. In Ref. 21, it was argued that a model with continuous $U(1)$ symmetry nonetheless would be a correct description. The result of LQC would then also apply to the four-state model of the original degrees of freedom since a fourfold anisotropy field, given by

$$S_4 = h_4 \sum_{\mathbf{r},\tau} \cos(4\theta_{\mathbf{r},\tau}), \quad (3)$$

would be irrelevant at the critical point of the action (1). We have investigated the effect of a fourfold anisotropy in our simulations by including the term (3) in the action. Using the approach of Ref. 33, we find the same result for the dissipative (2+1)D XY model as reported there for the classical 3D XY model, namely, that the h_4 term is perturbatively irrelevant.

The soft constraint represented by a (finite) anisotropy term is not obviously the same as the hard constraint constituted by the discrete Z_4 variables of the original model (the limit $h_4 = \infty$). We may only speculate whether a putative LQC fixed point for a $U(1)$ theory might survive in the limit $h_4 \rightarrow \infty$, but note that our simulations showed no signs of locality neither when enforcing a soft nor a strong Z_4 constraint on the variables.³⁴

Finally, we briefly consider variants of LQC other than that of Ref. 21, which predicts a strictly infinite z for $\xi_\tau \sim \xi^z$ so that ξ is strictly vanishing at criticality. Another conceivable sense in which $z \rightarrow \infty$ is by activated dynamical scaling,³⁵ i.e., scaling on the form $\ln \xi_\tau \sim \xi^\psi$. In this case, as we expect also in the first case, locality would manifest itself in our simulations as a strongly increasing value of $z > 1$ as the thermodynamical limit was approached. This is not observed in our results. We have also verified explicitly, by an appropriate modification of the scaling,³⁵ that our results are not consistent with activated dynamical scaling.

In conclusion, we find no signs of local quantum criticality in the compact (2+1)D XY model with bond dissipation, but instead conventional quantum criticality with indications of

isotropic scaling of imaginary time and space. This implies that the fluctuation spectrum of the model is a function of the combination $\sqrt{q^2 + \omega^2}$, rather than being dependent only on the frequency ω , but not on the momentum q (which would be a hallmark of local quantum criticality). Our results therefore differ in a fundamental way from those obtained from the same model in Ref. 20.

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