Fermionic Chern-Simons theory of SU(4) fractional quantum Hall effect

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We develop a Fermionic Chern-Simons (CS) theory for the fractional quantum Hall effect in monolayer graphene with SU(4) symmetry. We construct a general CS coupling matrix such that an even number of spin- and valley-dependent flux quanta is attached to all electrons and that any electron sees an integer number of flux attached to other electrons with different spin and valley quantum numbers. Using this matrix, we obtain a list of possible fractional quantum Hall states (FQHS) in graphene and propose wave functions for them. Our analysis unifies previously studied FQHS with different symmetries, predicts several states whose presence may be tested experimentally, and also applies to FQHS of bilayer spin polarized graphene and conventional bilayer quantum Hall systems. We thus provide a systematic way of charting FQHS in these systems and also reproduce earlier results as special cases.

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I. INTRODUCTION

The strong correlation arising out of a complete quench of the kinetic energy of the electrons in a two dimensional system in the presence of a strong perpendicular magnetic field B leads to the striking phenomenon called fractional quantum Hall effect (FQHE).[1] In the fractional quantum Hall regime, the applied magnetic field is strong enough to make the lowest Landau level have more states than the number of electrons in the system, leading to a huge degeneracy which is lifted only by the electron-electron interaction leading to the fractional quantum Hall states (FQHS).[2] A way to understand the nature of these states is provided by the composite Fermion(CF) theory[3] in which the state of the system is described in terms of CF quasiparticles which correspond to electrons bound to an even number ($2k$) of vortices of flux quantum $\phi_0 = hc/e$. Such a flux attachment can also be understood by carrying out Chern-Simon (CS) transformation[4] on the electron field operators, which leads to the introduction of a topological CS vector potential $\boldsymbol{a}$ resulting in a CS magnetic field $b(\boldsymbol{r}) \equiv (1/e)\boldsymbol{\nabla} \times \boldsymbol{a} = 2k\phi_0\rho(\boldsymbol{r})$ proportional to the electron density $\rho(\boldsymbol{r})$.[5,6] The factor $2k$ ensures that the statistics of the quasiparticles thus produced remain Fermionic. The FQHE of the electrons with filling fraction $\nu = \nu^*/(2k\nu^* + 1)$ correspond to integer quantum Hall effect (IQHE) of these quasiparticles with $\nu^* = p \in Z$. Such an analysis has also been applied to systems with SU(2) symmetry[7,8] and is found to correctly describe spin (layer) polarizations for FQHS for a single layer system with spin degree of freedom (bilayer system with frozen spin).

More recently, both IQHE[9,10] and FQHE[11–13] have been observed in single layer graphene whose effective low-energy theory is described in terms of Dirac-like quasiparticles which are low energy excitations around the Dirac cones centered around the edge of the graphene Brillouin zone.[14] There are two such inequivalent cones leading to two species of Dirac quasiparticles in graphene. These quasiparticles carry, apart from their physical spin, a valley quantum number which endows them with an additional internal symmetry. In the absence of any symmetry breaking interactions, their internal symmetry group is thus SU(4). FQHE in the lowest Landau level (LLL) for graphene has been studied in the past.[15–21] In Refs. 15 and 16, FQHS for spin-polarized electrons, i.e., with SU(2) symmetry due to the valley degrees of freedom, has been studied. However, given that the Zeeman energy in graphene is small compared to the Landau level splitting (the ratio of the two is approximately 10^{-4} for $B \sim 1T$), a full SU(4) symmetric FQHE seems to be more relevant in graphene. Such SU(4) symmetric FQHS has been studied using SU(4) generalized CF wave functions[17] and Halperin-like wave functions.[18] The former[17] described a restricted class of filling factor which arises from equal even integral flux $2k$ attached to each species of Dirac quasiparticles leading to $\nu = \nu^*/[2k\nu^* \pm 1]$, where $\nu^* = \nu_1 + \nu_2 + \nu_3 + \nu_4$ and $\nu_1, \nu_2, \nu_3, \nu_4$ are the effective integer filling factors of four different species of CFs. In this scheme, one obtains the spin and the valley polarizations of the FQHS for a given ν depending on the individual values of ν_i's (keeping their sum fixed). In contrast, Ref. 18 computes the spin and the valley polarization directly from the proposed Halperin-like wave functions and describes some FQHS which do not have definite spin, valley, or mixed polarizations. These states do not feature in Ref. 17. Other relevant studies involve computation of collective modes and skyrmion excitations in SU(4) quantum Hall ferromagnets,[19] computation of Hall conductivity for a general CS coupling matrix,[20] and construction of a general wave function for the SU(4) symmetric CS coupling matrix.[21] However, none of these studies obtains a list of possible FQHS. To the best of our knowledge, there is no unified formalism which reproduces all FQHS obtained by the above-mentioned schemes and provides a systematic method of generating the list of possible FQHS and their wave functions.

In this paper, we develop a CS theory for SU(4) FQHE which is relevant for monolayer graphene as well as for spin-polarized bilayer graphene and for conventional bilayer quantum Hall systems. The central point of our work is to introduce a general flux attachment scheme by using a CS coupling matrix. As shown schematically in Fig. 1, we choose the corresponding elements of this coupling matrix such that an even number of flux quanta, which may depend on the spin and valley quantum numbers, is attached to all electrons, and that any electron with a given spin and valley quantum number sees an integer number of flux attached to other

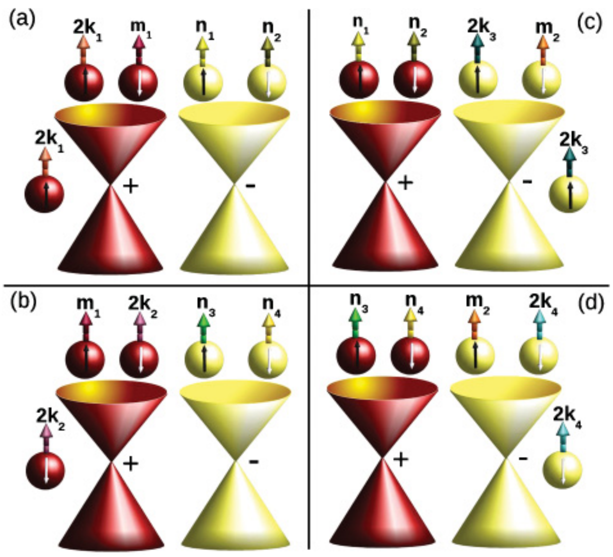

FIG. 1. (Color online) A pictorial representation of the flux attachment scheme. The spheres represent electrons and their colors denote the valleys $[K(+)$ and $K'(-)]$ in which the electrons belong. The arrows (thick arrows) represent the spin of (flux attached to) the electrons. The top left panel shows $2k_1$ flux attached to a $(\uparrow,+)$ electron, which sees m_1, n_1, and n_2 flux quanta attached to $(\downarrow,+)$, $(\uparrow,-)$, and $(\downarrow,-)$ electrons respectively. Other panels have similar representations.

electrons with different (spin and valley) quantum numbers. Using this CS matrix, we obtain a list of possible FQHS that might occur in graphene. We show that our formalism not only reproduces the FQHS obtained in Refs. 17 and 18 as special cases, but also provides a systematic method for listing all possible FQHS in such a system including those which do not have any SU(2) analogues. All of these FQHS emerge from Fermionic CS theory with IQHE of CS-CF quasiparticles. We also propose wave functions for these states and demonstrate that the applicability of the CF theory in the SU(4) case is much more robust than that studied in Ref. 17. Our analysis therefore constitutes a significant extension of the understanding of FQHS in these systems and is expected to provide a guideline to the forthcoming experiments on FQHE in monolayer and/or bilayer graphene.

The organization of the rest of the paper is as follows. In Sec. II, we develop the SU(4) CS theory, derive equations for the filling factor ν, the spin (S), the valley (V), and the mixed (M) polarizations using this theory, and propose wave functions which describe the obtained FQHS. This is followed by Sec. III, where we analyze these equations to provide an exhaustive list of possible FQHS for monolayer graphene. Finally, we summarize our results and conclude in Sec. IV.

II. FORMALISM

The low-energy states in graphene can be described by an effective Dirac-like Hamiltonian

$$\mathcal{H} = \int d\boldsymbol{r}\psi_e^\dagger(\boldsymbol{r})H\psi_e(\boldsymbol{r}) + \frac{1}{2}\int d\boldsymbol{r}\int d\boldsymbol{r}'V(\boldsymbol{r}-\boldsymbol{r}') : \hat{\rho}_e(\boldsymbol{r})\hat{\rho}_e(\boldsymbol{r}'):, \tag{1}$$

where ψ_e is the eight component electronic annihilation operator whose components correspond to the sublattice, the valley, and the spin degrees of freedom,[14] $\hat{\rho}_e = \psi_e^\dagger\psi_e$ is the density operator, $:\ldots:$ denotes normal ordering, $V(\mathbf{r})$ represents electron-electron interaction whose precise form is unimportant for our purpose, and $H = v_F(\sigma_x\Pi_x + \tau_z\sigma_y\Pi_y)$ with $\boldsymbol{\Pi} = -i\nabla + e\boldsymbol{A}$, $\nabla \times \boldsymbol{A} = B\hat{z}$ and σ'(τ)'s are Pauli matrices which describe two sublattices (valleys) of graphene. Here and in the rest of this work, we shall set $\hbar = c = 1$ and use the shorthand notation $1 \equiv (\uparrow,+)$, $2 \equiv (\downarrow,+)$, $3 \equiv (\uparrow,-)$, and $4 \equiv (\downarrow,-)$, where $\uparrow,\downarrow$ $(+,-)$ represent the physical spin (valley) states of graphene electrons.

We now introduce the CS-CF quasiparticle creation operator $\psi^\dagger(\boldsymbol{r})$ in terms of $\psi_e^\dagger(\boldsymbol{r})$:

$$\psi_{e,\alpha}^\dagger(\boldsymbol{r}) \rightarrow \psi_\alpha^\dagger(\boldsymbol{r})e^{-i\mathcal{K}_{\alpha\beta}\int d\boldsymbol{r}'\arg(\boldsymbol{r}-\boldsymbol{r}')\rho_\beta(\boldsymbol{r}')}, \tag{2}$$

where the indices α and β take value from 1 to 4 as described above. Here $\arg(\boldsymbol{r}-\boldsymbol{r}')$ represents the angle made by the vector $(\boldsymbol{r}-\boldsymbol{r}')$ with the x axis, and the explicit form of the symmetric integer valued matrix $\mathcal{K}$ is chosen, as schematically shown in Fig. 1 and given by

$$\mathcal{K} = \begin{pmatrix} 2k_1 & m_1 & n_1 & n_2 \\ m_1 & 2k_2 & n_3 & n_4 \\ n_1 & n_2 & 2k_3 & m_2 \\ n_3 & n_4 & m_2 & 2k_4 \end{pmatrix}, \tag{3}$$

where k's, m's, and n's are positive integers including zero. The quasiparticles obtained by this transformation have the same interaction energy term as the electrons. However, the effective kinetic energy term for them is given by

$$H_{\rm eff} = \int d^2r\psi_\alpha^\dagger(\mathbf{r})v_F(\sigma_x\tilde{\Pi}_{\alpha,x} + \tau_z\sigma_y\tilde{\Pi}_{\alpha,y})\psi_\alpha(\mathbf{r}), \tag{4}$$

where $\tilde{\boldsymbol{\Pi}}_\alpha = -i\nabla + e\boldsymbol{A} - \boldsymbol{a}_\alpha$, with

$$\boldsymbol{a}_\alpha = \mathcal{K}_{\alpha\beta}\int d\boldsymbol{r}' g(\boldsymbol{r}-\boldsymbol{r}')\rho_\beta(\boldsymbol{r}') \tag{5}$$

and $g(\boldsymbol{r}) = (\hat{z}\times\boldsymbol{r})/r^2$. The corresponding singular CS magnetic field seen by these quasiparticles is

$$b_\alpha \equiv (1/e)\nabla \times \boldsymbol{a}_\alpha = \phi_0\mathcal{K}_{\alpha\beta}\ \rho_\beta(\boldsymbol{r}). \tag{6}$$

Note that the effective field depends on the quantum numbers (spin and valley) carried by the quasiparticles. Within a mean field approximation, the electron filling factor ν and the effective filling factor ν_α of CS-CF quasiparticles are thus related by

$$\rho_\alpha/\nu_\alpha = \rho/\nu - \mathcal{K}_{\alpha\beta}\rho_\beta, \tag{7}$$

with $\nu_\alpha > (<)0$ for $B > (<)b_\alpha$.

By defining the spin (S), the valley (V), and the mixed (M) polarizations of these quasiparticles in a given FQHS as

$$\begin{aligned} S &= (\rho_1 + \rho_3 - \rho_2 - \rho_4)/\rho, \\ V &= (\rho_1 + \rho_2 - \rho_3 - \rho_4)/\rho, \\ M &= (\rho_1 + \rho_4 - \rho_2 - \rho_3)/\rho, \end{aligned} \tag{8}$$

the above-mentioned relation can be expressed as

$$\begin{aligned}
&\gamma_1\left(2k_1+\frac{1}{\nu_1}\right)+\gamma_2 m_1+\eta_1(n_1+n_2)+\delta_1(n_1-n_2)=\frac{4}{\nu},\\
&\gamma_2\left(2k_2+\frac{1}{\nu_2}\right)+\gamma_1 m_1+\eta_1(n_3+n_4)+\delta_1(n_3-n_4)=\frac{4}{\nu},\\
&\gamma_3\left(2k_3+\frac{1}{\nu_3}\right)+\gamma_4 m_2+\eta_2(n_1+n_2)+\delta_2(n_1-n_2)=\frac{4}{\nu},\\
&\gamma_4\left(2k_4+\frac{1}{\nu_4}\right)+\gamma_3 m_2+\eta_2(n_3+n_4)+\delta_2(n_3-n_4)=\frac{4}{\nu},
\end{aligned} \tag{9}$$

where $\eta_{1(2)}=1-(+)V$, $\delta_{1(2)}=S-(+)M$, $\gamma_{1(2)}=\eta_2+(-)\delta_2$, and $\gamma_{3(4)}=\eta_1+(-)\delta_1$.

Equation (9) represents the central result of this work and provides a relation between the total filling factor ν and the spin (S), the valley (V), and the mixed (M) polarizations of a FQHS in terms of the attached flux numbers k's, m's, and n's. Thus it provides a systematic way of charting out the possible FQHS for a given filling factor ν at the saddle point level and specifying the polarizations M, S, and V for these states. Moreover, in terms of these flux attachment numbers, one can write down, via a straightforward generalization of methods used in Refs. 7 and 8, a variational wave function (neglecting obsequious Gaussian factors) of these FQHS for the filling factor ν as

$$\Psi(\{u^\alpha\})=\mathcal{P}_L\left[\prod_{\alpha=1..4}\Phi_{\nu_\alpha}\left(u_1^\alpha,\ldots,u_{N_\alpha}^\alpha\right)\right]\prod_{i<j}^{N_\alpha}\left(u_i^\alpha-u_j^\alpha\right)^{2k_\alpha}\times\prod_{i,j;\alpha,\beta;\alpha\neq\beta}^{N_\alpha,N_\beta}\left(u_i^\alpha-u_j^\beta\right)^{\mathcal{K}_{\alpha\beta}}, \tag{10}$$

where $u^\alpha=x^\alpha-iy^\alpha$ denotes complex coordinates for the particles of species α,[2] N_α denotes the number of quasiparticles of species α, Φ_{ν_α} represents the IQHE wave function for ν_α-filled Landau levels of these quasiparticles, and $\mathcal{P}_L$ represents projection into the LLL. Note that the CS theory alone cannot lead to Eq. (10); the IQHE wave functions and the projection into the LLL receive input from the CF theory.[3] Below, we solve Eq. (9) to obtain the set of parameters for a specific ν, S, V, and M. The definite wave function (10) is then obtained for a FQHS in terms of these parameters. In this respect, Eq. (10) differs from the previously suggested Halperin-like wave functions[18] and is expected to provide an accurate starting point for numerical studies of FQHS in these systems.

III. RESULTS

For the rest of this work, we make special choices of the parameters $n_1=n_2=n_3=n_4=n$ and $k_1=k_2, k_3=k_4$ for which Eq. (9) admits analytical solutions. First, we note that for $2k_1=2k_3=m_1=m_2=n=2k$, there is one dynamical CS gauge field $\mathcal{A}^\mu=a_1^\mu+a_2^\mu+a_3^\mu+a_4^\mu$. The rest of the CS gauge fields decouple.[7] Solving Eq. (9), we find

$$\nu=\frac{\nu^*}{2k\nu^*+1},\quad V=\frac{2(\nu_1+\nu_2)-\nu^*}{\nu^*},\quad S=\frac{2(\nu_1+\nu_3)-\nu^*}{\nu^*},\quad M=\frac{2(\nu_1+\nu_4)-\nu^*}{\nu^*}. \tag{11}$$

This is precisely the Toke-Jain sequence in graphene and the wave function [Eq. (10)] obtained for these sets of parameters exactly matches with the CF wave function.[17] The sequence of FQHS generated with this set of parameters is the same as the SU(2) sequence for $\nu^*\leqslant 2$. We find that in the limit $\nu^*\to\infty$, $\nu=1/2k$ and hence these even denominator states correspond to Fermi sea of CFs.[6] Second, we consider $2k_1=2k_3=m_1=m_2=2k$ and $n\neq 2k$: For this choice of parameters, we find

$$\begin{aligned}
\nu&=\frac{\nu^*+2(2k-n)(\nu_1+\nu_2)(\nu_3+\nu_4)}{2k\nu^*+1+(4k^2-n^2)(\nu_1+\nu_2)(\nu_3+\nu_4)},\\
V&=\frac{2(\nu_1+\nu_2)-\nu^*}{\nu^*+2(2k-n)(\nu_1+\nu_2)(\nu_3+\nu_4)},\\
S&=\frac{2(\nu_1+\nu_3)-\nu^*+2(2k-n)(\nu_1\nu_3-\nu_2\nu_4)}{\nu^*+2(2k-n)(\nu_1+\nu_2)(\nu_3+\nu_4)},\\
M&=\frac{2(\nu_1+\nu_4)-\nu^*+2(2k-n)(\nu_1\nu_4-\nu_2\nu_3)}{\nu^*+2(2k-n)(\nu_1+\nu_2)(\nu_3+\nu_4)}.
\end{aligned} \tag{12}$$

Note that for these solutions, the even denominator FQHS with $\nu=1/2k$ occurs when $(\nu_1+\nu_2)=1=(\nu_3+\nu_4)$ and $n=2k\pm 1$; they do not correspond to $\nu^*\to\infty$ and hence do not lead to the formation of a Fermi sea of the quasiparticles. These states are similar to those obtained for bilayer SU(2) quantum Hall systems. Finally, for $2k_1=2k_3=2k$, $m_1=m_2=m\neq 2k$, and $n\neq 2k$, we find solutions with zero spin and mixed polarization ($M=S=0$) and with finite nonzero valley polarizations given by

$$\begin{aligned}
\nu&=[4\nu^*+4(2k+m-2n)(\nu_1+\nu_2)(\nu_3+\nu_4)]/\mathcal{D},\\
\mathcal{D}&=4+2(2k+m)\nu^*+((2k+m)^2-4n^2)\\
&\quad\times(\nu_1+\nu_2)(\nu_3+\nu_4),\\
V&=\frac{2(\nu_1+\nu_2)-\nu^*}{2\nu^*+2(2k+m-2n)(\nu_1+\nu_2)(\nu_3+\nu_4)}.
\end{aligned} \tag{13}$$

We note that these states do not have any analog in U(1) and SU(2) FQHE and can only occur for SU(4) symmetric FQHE. Similarly, a set of FQHS which has $M=V=0$ and $S\neq 0$ and/or $S=V=0$ and $M\neq 0$ can also be found. For $S\neq 0$, we find

$$\begin{aligned}
\nu&=\frac{4(\nu_2\nu_3-\nu_1\nu_4)}{\nu^*-2(\nu_1+\nu_4)+(2k+m+2n)(\nu_2\nu_3-\nu_1\nu_4)},\\
S&=\frac{\nu^*-2(\nu_1+\nu_2)}{(2k-m)(\nu_2\nu_3-\nu_1\nu_4)},
\end{aligned} \tag{14}$$

while for the states with $M\neq 0$ we get

$$\begin{aligned}
\nu&=\frac{4(\nu_1\nu_3-\nu_2\nu_4)}{\nu^*-2(\nu_2+\nu_4)+(2k+m+2n)(\nu_1\nu_3-\nu_2\nu_4)},\\
M&=\frac{2(\nu_1+\nu_2)-\nu^*}{(2k-m)(\nu_1\nu_3-\nu_2\nu_4)}.
\end{aligned} \tag{15}$$

Next, we concentrate on the SU(4) singlet states which correspond to all $\nu_\alpha=1$. In this case, one has several possible solutions. First, there is a class of solutions which correspond to $M=S=V=0$ and are given by

$$\nu=\frac{8}{2+2k_1+2k_3+m_1+m_2+4n}, \tag{16}$$

provided that $2k_1+1\neq m_1$ and $2k_3+1\neq m_2$. Toke-Jain[17] SU(4) singlet states $\nu=4/(1+8k)$ such as 4/9 are special cases of this sequence. There are several other states in this category of which the wave function of 4/11 is the same as proposed in Ref. 18. We note that for the U(1) and SU(2) case where the state 4/11 occurs due to FQHE of CFs;[22] in contrast, here it arises due to IQHE of CS-CF quasiparticles. Second, there is a set of states that correspond to $M = S = 0$, but $V \neq 0$ and have

$$V = \frac{2(k_3 - k_1) + m_2 - m_1}{2(1 + k_1 + k_3) + (m_1 + m_2 - 4n)},$$
$$\nu = \frac{4(1 + k_1 + k_3) + 2(m_1 + m_2 - 4n)}{(2k_1 + 1 + m_1)(2k_3 + 1 + m_2) - 4n^2}. \tag{17}$$

A few examples of such even-denominator states are 1/2 and 3/8, and odd-denominator states are 3/5, 3/7, and 4/7. These states do not feature in previous studies. Third, within the SU(4) singlet FQHS with $k_1 = k_3 = k$ and $2k + 1 = m_1 = m_2 = m$, we find

$$\nu = \frac{2}{m + n}. \tag{18}$$

These correspond to a set of states with undetermined M and S. V can be determined only if $m \neq n$ (which corresponds to $V = 0$); for $m = n$, V is also undetermined. The filling factors for this sequence with $V = 0$ are 2/3, 1/2, and 2/5 for $k = 1$ and 2/5, 1/3, 2/7, 1/4, and 2/9 for $k = 2$. In contrast, the states for which V is also undetermined have filling factor 1/3 with $k = 1$ and 1/5 with $k = 2$. Among these, the wave functions [Eq. (10)] for the filling factors 2/3, 2/5, and 1/3 obtained from our formalism are precisely the Halperin-like wave functions proposed in Ref. 18. Note that this exact similarity of the wave functions are not generic but is a consequence of the condition $\nu_\alpha = 1$ for which Eq. (10) becomes identical to those in Ref. 18. Also, while the filling factor 2/3 occurs due to reverse flux attachment in CF theory,[17] it occurs here for parallel flux attachment as well. We note that the even denominator states in the above-mentioned sequence (such as 1/2 and 1/4) do not correspond to the Fermi sea of CFs. Fourth, if $2k + 1 = m_1 \neq m_2$, the solutions of Eq. (9) yield FQHS with $M = S$ but undetermined, and with the values of the filling fractions and the valley polarizations given by

$$\nu = \frac{3m_1 + m_2 - 4n}{m_1^2 + m_1 m_2 - 2n^2}, V = \frac{m_2 - m_1}{3m_1 + m_2 - 4n}. \tag{19}$$

These states do not appear in the work of Ref. 18. This demonstrates that the general formalism outlined here produces several FQHS which have not been charted out before. The filling factors for a few representative SU(4) singlet FQHS and their corresponding spin, valley, and mixed polarizations are tabulated in Tables I and II. We expect similar FQHS for spin-polarized bilayer graphene with S replacing the layer index and bilayer quantum Hall system with V denoting the layer index.

TABLE I. A chart of the possible filling fractions ν with numerator $\leqslant 4$ for the SU(4) singlet states and the corresponding polarizations S, V and M for different sets of parameters $\{k_1 = k_2, k_3 = k_4, m_1, m_2, n_i = n\}$. The symbol "—" for the polarizations denotes undetermined value.

ν	k_1	k_3	m_1	m_2	n	S	V	M
1/2	1	1	3	3	1	–	0	–
1/2	1	2	2	3	1	0	1/3	0
1/2	2	2	1	1	1	0	0	0
1/3	1	1	3	3	3	–	–	–
1/3	2	2	5	5	1	–	0	–
1/4	2	2	5	5	3	–	0	–
1/5	2	2	5	5	5	–	–	–
2/3	1	1	1	1	1	0	0	0
2/5	1	1	3	3	2	–	0	–
2/5	2	2	3	3	1	0	0	0
2/7	2	2	3	3	3	0	0	0
2/7	2	2	5	5	2	–	0	–
2/9	2	2	5	5	4	–	0	–
3/5	1	2	1	1	1	0	1/3	0
3/7	1	2	2	1	2	0	1/3	0
3/7	1	2	3	5	1	–	1/3	–
3/8	1	2	3	3	2	–	1/3	–
3/8	2	2	1	3	2	0	1/3	0
4/7	1	1	2	2	1	0	0	0
4/7	1	2	1	3	1	0	1/2	0
4/9	1	1	2	2	2	0	0	0
4/9	2	2	2	2	1	0	0	0
4/11	1	2	3	5	2	–	1/2	–
4/11	2	2	2	2	2	0	0	0
4/11	2	2	4	4	1	0	0	0
4/13	2	2	4	4	2	0	0	0
4/15	2	2	4	4	3	0	0	0
4/17	2	2	4	4	4	0	0	0

TABLE II. Same as in Table I but with numerators $\geqslant 5$.

ν	k_1	k_3	m_1	m_2	n	S	V	M
5/6	1	2	1	1	0	0	1/5	0
5/7	1	2	0	1	1	0	3/5	0
5/8	1	1	1	2	1	0	1/5	0
5/8	1	2	1	0	1	0	1/2	0
5/9	2	2	1	4	0	0	1/5	0
5/12	1	2	2	3	2	0	3/5	0
5/12	2	2	0	3	2	0	3/5	0
5/13	2	2	1	2	2	0	1/5	0
5/18	2	2	3	4	3	0	1/5	0
7/10	1	2	0	3	1	0	5/7	0
7/12	1	2	1	2	1	0	3/7	0
7/12	1	2	3	3	0	–	1/7	–
7/12	2	2	1	3	0	0	1/7	0
7/13	1	2	2	1	1	0	1/7	0
7/13	2	2	0	1	1	0	1/7	0
7/19	2	2	1	4	2	0	3/7	0
8/9	1	2	0	4	0	0	1/2	0
8/11	1	1	0	2	1	0	1/2	0
8/19	1	2	2	2	2	0	1/2	0
8/19	2	2	0	2	2	0	1/2	0

IV. CONCLUSION

In summary, we have developed a Fermionic CS theory for SU(4) FQHE and analyzed the possible FQHS obtained from such a theory. We have reproduced SU(4) FQHS arising from CF theory,[17] as well as Halperin-like[18] states within a single unified formalism. We have also proposed several other states which are not obtained in the previous studies. Although the filling factors and their polarizations presented here are for monolayer graphene, the analysis is valid for any SU(4) system. Two other examples of such systems where this theory could be applicable are bilayer quantum Hall systems and bilayer graphene[23] with complete spin or valley polarizations. Taking cue from the CS theory,[3] we have proposed wave functions for all of these FQHS. We note that for FQHS with a particular filling factor, the precise ground state wave function will depend on the exact nature of the interaction between electrons. It will be interesting to obtain the overlap of the ground state with our proposed wave function. Finally, the ground state for FQHS in graphene may be tuned by tuning either the Zeeman coupling or the intervalley coupling. It will certainly be interesting to use our proposed wave function to study the resulting transitions between the FQHS for all of these states by changing Zeeman coupling and obtain the corresponding phase diagram. We leave these issues for future studies.

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