Quantitative examination of the collapse of spin splitting in the quantum Hall regime

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We have quantitatively tested the theoretical model on the collapse of spin slitting in the quantum Hall effect regime proposed by Fogler and Shklovskii [Phys. Rev. B **52**, 17366 (1995)] in a high-mobility two-dimensional electron system (2DES) realized in a heterojunction insulated-gate field-effect transistor. In the 2DES density range between $n = 2 \times 10^{10}$ and 2×10^{11} cm^{-2}, the Landau level number N displays a power-law dependence on the critical electron density n_c where the spin splitting collapses and $N = 11.47 \times n_c^{0.64 \pm 0.01}$ (n_c is in units of 10^{11} cm^{-2}). This power-law dependence is in good agreement with the theoretical prediction in the low-density regime.

There is a great deal of current interest in understanding electron-spin physics in semiconductors for potential quantum computation applications. The quantum Hall effect in the two-dimensional electron system (2DES) has proved to be a unique system in this avenue due to a tunability in the difference of spin population and thus the strength of exchange interaction provided by the formation of Landau levels.[1]

At zero magnetic field B, the density of state (DOS) of an ideal 2DES is constant up to the Fermi energy. When placed in a high magnetic field, this continuous DOS breaks into discrete δ-function Landau levels. In a real specimen, disorder is inevitable. Consequently, the δ-function Landau levels become broadened, and Landau subbands are formed. Together, the two above scenarios have successfully explained the integer quantum Hall effect (IQHE) states[2] at the even-integer Landau level filling factors (ν).[3]

Zeeman splitting between the spin-up and spin-down electrons is responsible for the IQHE states at odd Landau level fillings (in this paper we focus on the IQHE states with $\nu > 1$), and their energy gap is expected to be roughly equal to $g\mu_B B$, where g is the Landé g factor and μ_B is the Bohr magneton. However, it has long been observed that at high B fields the odd IQHE states display much stronger transport features than expected from a bare Zeeman splitting.[4–8] The mechanism of an exchange-interaction-enhanced g factor[9–13] is believed to be responsible for their energy gaps being large. At low B fields, Fogler and Shklovskii[1] showed that, when the disorder broadening Γ is comparable to the strength of exchange interaction, the exchange enhancement is destroyed and a collapse of spin splitting occurs. Moreover, this collapse was shown to be a second-order phase transition in the presence of a finite g factor.[1] It was predicted *quantitatively* that in a typical high-mobility heterostructure the Landau level numbers N should display a power-law dependence on the critical electron density n_c where the spin splitting collapses and $N = 0.9dn_c^{2/3}/n_i^{1/6}$ when $n < n_i$. Here d is the so-called setback distance, and n_i is the areal density of random ionized impurity density in the modulation doping layer.

Soon after this theoretical work, experimental studies[14–19] were carried out to examine this collapse of spin splitting and

the possible phase transition. *Qualitative* consistence has been obtained. On the other hand, the theoretical prediction was never quantitatively examined in these previous studies. This lack of investigation is primarily due to a lack of high-quality samples where electron density can be varied *in situ* over a large range.

In this Rapid Communication, we present the results from our *quantitative* study of the collapse of spin splitting as a function of electron density at low magnetic fields B and low temperatures in a high-quality heterojunction insulated-gate field-effect transistor (HIGFET),[20] where the electron mobility μ of 2DES is larger than 2×10^6 cm^2/V s in the density range between $n = 2 \times 10^{10}$ and 2×10^{11} cm^{-2}, with a peak mobility over 10×10^6 cm^2/V s at $n \sim 1.5 \times 10^{11}$ cm^{-2}. The HIGFET device structure is very unique in that it allows us to carry out the measurements of R_{xx} (diagonal resistance) and R_{xy} (Hall resistance) at constant B field while sweeping n (or the gate voltage V_g). With this configuration, the Fermi energy E_F is changed with varying V_g, while the Landau level degeneracy and the screened fluctuating potential are kept fixed as B is fixed. Consequently, the issue of a B-field-dependent Γ (Refs. 21 and 22) incurred when n is fixed while B swept is alleviated, and the complication and uncertainty due to magnetic-field-induced scattering and screening are also minimized to their lowest level. In this HIGFET, the Landau level numbers were observed to display a power-law density dependence on the critical electron density n_c when the spin splitting collapses. In the density range between $n = 2 \times 10^{10}$ and 2×10^{11} cm^{-2}, N can be best fitted as $N = 11.47n_c^{0.64 \pm 0.01}$ (n_c is in units of 10^{11} cm^{-2}), in good agreement with the theoretical prediction.[1]

We show in the inset of Fig. 1 a schematic of the device structure for the HIGFET specimen we used in this study. After the growth of a GaAs overgrowth layer and a short-period superlattice of AlGaAs/GaAs quantum wells on a semi-insulating GaAs substrate, a 2-μm-thick high-quality GaAs layer is grown, followed by a 600-nm AlGaAs layer. The structure is finally capped by a heavily electron-doped GaAs layer (60 nm thick), which is also used as the front gate. Figure 1 shows the electron mobility versus the electron density. It increases from 2.8×10^6 cm^2/V s at $n = 2$
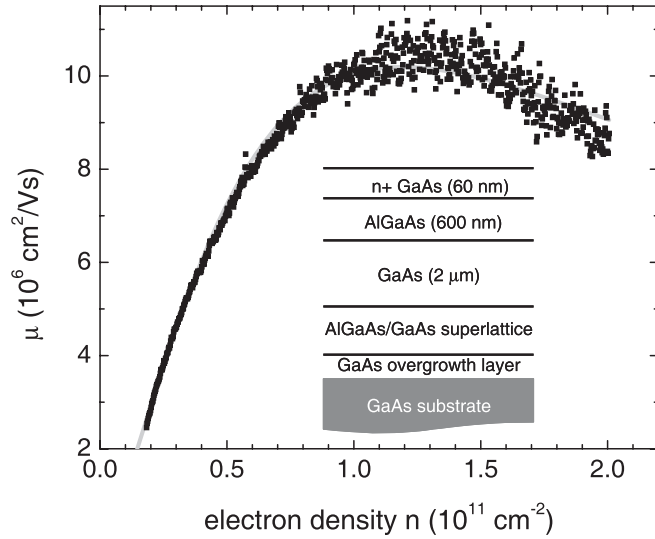
FIG. 1. Electron mobility vs density in our HIGFET device. The measurement temperature is 15 mK. The gray line is the fit discussed in the text. The inset shows a schematic of the device structure.

$\times\ 10^{10}\ \mathrm{cm}^{-2}$ to over $10 \times 10^6\ \mathrm{cm}^2/\mathrm{V}$ s at $n \sim 1.5 \times 10^{11}\ \mathrm{cm}^{-2}$.

R_{xx} and R_{xy} were measured using the standard low-frequency lock-in technique at the lowest fridge temperature of $T \sim 15$ mK. This ensures that the spin splitting in R_{xx} is best resolved. In Fig. 2, we show several selected R_{xx} traces at various B fields. The arrows mark the position of the odd-integer quantum Hall effect states at $\nu = 17$, 23, and 29. As B is reduced, the two peaks flanking the odd-integer quantum Hall states become closer and closer together, and eventually, a single peak develops at these fillings, indicating the collapse of spin splitting. Following Ref. 1, we identify the critical density n_c where the collapse occurs as the one when the difference of the Landau level fillings between the two flanking peaks is $\sim$0.5.
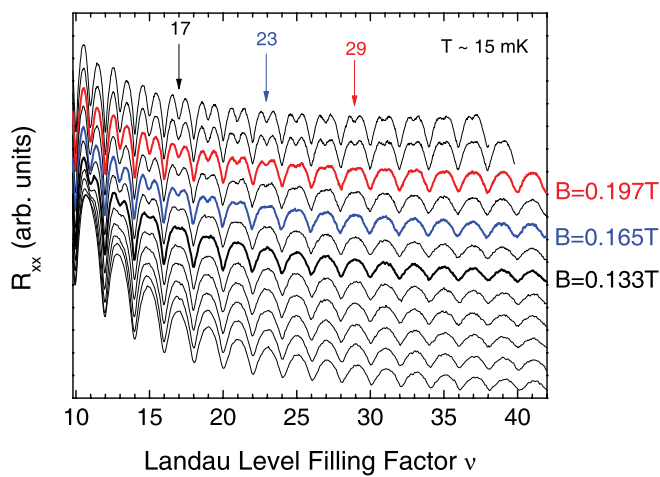

FIG. 2. (Color) R_{xx} vs ν at different magnetic fields. Traces are shifted vertically for clarity. From the top to bottom, the magnetic fields are 0.239, 0.229, 0.197, 0.176, 0.165, 0.144, 0.133, 0.122, 0.111, 0.101, 0.090, and 0.079 T. The three traces at $B = 0.197$, 0.165, and 0.133 T are highlighted, showing the collapse of spin splitting at $\nu = 29$, 23, and 17, respectively.
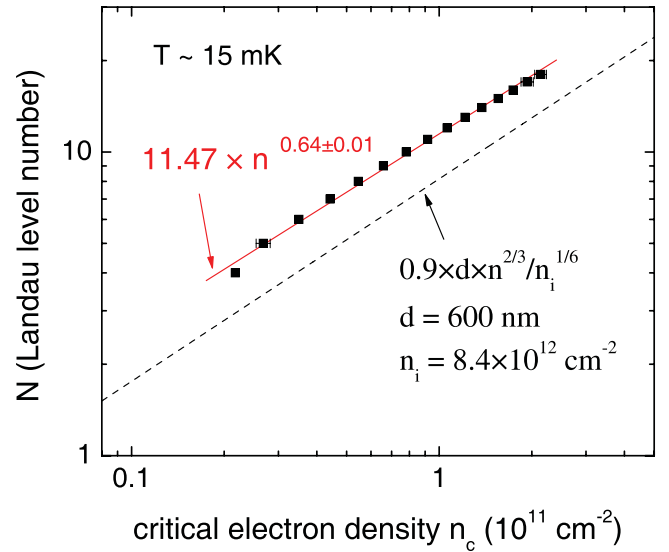

FIG. 3. (Color online) Landau level number as a function of the critical electron density where the collapse of spin splitting occurs. The solid line is a power-law dependence fit. The theoretical predicted is also shown as the dashed line.

In Fig. 3, we plot N, the Landau level number, defined by $\nu = 2N + 1$, as a function of the critical density n_c (in units of $10^{11}\ \mathrm{cm}^{-2}$). In the whole density range, N can be fitted as $N = 11.47 n_c^{0.64 \pm 0.01}$, in good agreement with the power-law dependence predicted by the theory.[1]

To carry out a quantitative comparison between our experimental result and the theoretical prediction, two sample parameters need to be determined, i.e., the random remote impurity area density in the modulation doping layer and the setback distance d. In our HIGFET device there is no modulation doping layer, and the 2D carriers are induced by the electrical field effect. On the other hand, the top GaAs layer is heavily doped, and the remote random impurities there contribute to the electron-scattering processes in the 2DES at the interface of AlGaAs and GaAs layers. In this regard, this top layer can be viewed as the modulation-doping layer in our HIGFET structure, and the setback distance is then the thickness of the AlGaAs layer, $d = 600$ nm. In Fig. 1, we show the fitting (gray trace) to the curve of the mobility versus density following the method used in Ref. 23. The obtained value for the random impurity density is $n_i = 8.4 \times 10^{12}\ \mathrm{cm}^{-2}$. This value is consistent with the growth parameter of a doping density of $\sim 2 \times 10^{18}\ \mathrm{cm}^{-3}$. Considering the thickness of the top heavily doped layer is 60 nm, we estimate an area doping density of $\sim 12 \times 10^{12}\ \mathrm{cm}^{-2}$. The value from the fitting is smaller than this number, which could be due to the fact that not all the donors are fully ionized or to the screening effect in the top layer. With the values of $n_i = 8.4 \times 10^{12}\ \mathrm{cm}^{-2}$ and $d = 600$ nm, we can then calculate the theoretically predicted density dependence of N, shown as the dashed line in Fig. 3. The theoretical formula we used, $n_c = 0.9 d n^{2/3}/n_i^{1/6} = 8.16 n^{2/3}$, is for the low-density regime where $n < n_i$ [Eq. (4c) in Ref. 1]. Overall, a quantitative agreement between the experimental data and the theoretical prediction is clearly seen and thus strongly supports the collapse of spin splitting as a quantum phase transition predicted in Ref. 1.

Though the good agreement is apparent, one slight discrepancy is also obvious. The experimental value is ~30% off the theoretical prediction, and the collapse of spin splitting is observed at lower electron densities. Before we discuss possible origins for this discrepancy, we want to point out that several assumptions on which the theoretical arguments were based were not met in the studied density range in our HIGFET device. In Ref. 1 it was assumed that $k_F a_B > 1$, where $k_F = (2\pi n)^{1/2}$ is the Fermi wave factor and $a_B \approx 10$ nm is the effective Bohr radius of the 2DES in GaAs. This assumption is not valid in our HIGFET, where $k_F a_B$ actually varies from 0.35 at 2×10^{10} cm^{-2} to 1.1 at 2×10^{11} cm^{-2}. This smaller $k_F a_B$ even leads to a negative $\alpha = \ln(2k_F a_B)/\pi k_F a_B$ at low electron densities, a scenario not considered in Ref. 1. Furthermore, in the density range between 2×10^{10} and 2×10^{11} cm^{-2}, the classical cyclotron radius R is of the same order of magnitude as d, and $R \sim d/2$. This does not satisfy the condition of $R < d$ considered in Ref. 1. As a result, the unaveraged potential used in Ref. 1 [Eq. (62)] needs to be modified, which can be expected to affect the numerical coefficient in the equation we used to calculate the theoretical curve in Fig. 3. Regarding the above discrepancies between the theoretical assumptions and the real experimental parameters, nevertheless, it is remarkable that the theoretical prediction and the experimentally measured value differ only by 30%.

Now, we turn to discuss several possibilities that might be responsible for the discrepancy. First, we note that the theoretical prediction was for $T = 0$. The real measurements, however, were carried out at an inevitably finite fridge temperature, here ~15 mK. The thermal broadening term due to this finite temperature (~1.3 μeV) is comparable to the disorder broadening (~1.7–8.6 μeV) and should be included in the broadening term. As a result, the collapse of spin splitting is expected to occur at higher densities compared to the theoretical prediction, opposite to what we observed. Indeed, the measurements at higher fridge temperatures confirm that the collapse occurs at higher B field, or higher n, for fixed N.

Second, the 2DES in our HIGFET has a finite width. It has been shown that the exchange enhancement is reduced when the finite thickness effect is taken into account.[10] Similar to the thermal broadening, a reduced exchange enhancement would move the collapse point to a higher electron density, instead of lower for a fixed N.

Third, it has been shown that the effective mass m^* of 2DES measured in a similar device structure varies with n, from ~1.1 m_0 at $n \sim 3 \times 10^{10}$ cm^{-2} to ~0.9 m_0 at $n \sim 2 \times 10^{11}$ cm^{-2}, where m_0 is the electron band mass in GaAs.[24] This nonconstant m^* might be responsible for the experimental data points in Fig. 3 not following a perfect power-law dependence. On the other hand, it is hard to explain a constant 30% difference between the experimental result and theoretical prediction over the whole density range.

Finally, the effect of Zeeman coupling needs to be considered. In GaAs, the value of the effective g factor is 0.44. In our experimental B field range, the Zeeman energy is ~2–6 μeV. This value is comparable to the sample disorder broadening and thus cannot be neglected. It was shown in Ref. 1 that a nonzero Zeeman coupling would smear the first-order quantum phase transition and increase the critical
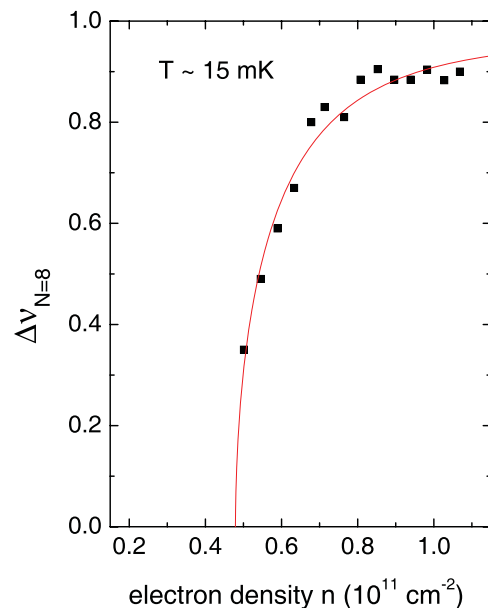

FIG. 4. (Color online) $\Delta \nu_{N=8}$ as a function of density for the two peaks flanking the odd-integer quantum Hall state at $\nu = 17$. The solid line is a fit to the equation discussed in the text.

Landau level value. This nonzero Zeeman coupling effect is consistent with our observation and may be responsible for the observed discrepancy.

To conclude this Rapid Communication, we plot in Fig. 4 $\Delta \nu_{N=8}$ as a function of density for the two peaks flanking the Landau level filling factor $\nu = 17$, following the pioneering work by Wong *et al.*[14] Similar to what was reported there, $\Delta \nu_{N=8}$ remains roughly constant at large densities and drops quickly around 5×10^{10} cm^{-2}. The red curve shows a fitting to the function adopted in Ref. 14, $\Delta \nu_N = a\coth[a(n-c)^{1/2}] - b\coth[b(n-c)^{1/2}]$, which resembles the behavior of the Brillouin function. Again, similar to what was obtained in Ref. 14, in our fitting the value of $|a-b|$ is close to 1. The data and fitting in Fig. 4 also support the conclusion that the termination of the spin-resolved IQHE is a quantum phase transition.

In summary, we tested quantitatively the model on the collapse of spin slitting in the quantum Hall effect by Fogler and Shklovskii[1] in a high-quality heterojunction insulated-gate field-effect transistor. In the density range between $n = 2 \times 10^{10}$ and 2×10^{11} cm^{-2}, the Landau level number N follows a power-law dependence on the critical electron density n_c, where the spin splitting collapses, and $N = 11.47 \times n_c^{0.64\pm0.01}$. This power-law dependence is in a good agreement with the theoretical prediction[1] in the low-density regime.

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