Quantum Hall effect in narrow Coulomb channels

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Proposed is a scenario for the development of magnetic-field-induced electron states (MESs) in finite charged systems. These states arise due to incomplete screening of external electrostatic fields governing the electron density distribution and therefore exist within a certain static skin layer of width λ along the edge of a two-dimensional (2D) charged system (either classical or degenerate). In the magnetic field normal to the 2D system the electrons in the skin layer are dragged along the MES orbits by the Lorentz force in both classical and degenerate 2D systems. Details of the λ scenario for MESs in the narrow-channel quantum Hall effect problem are reported.

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Edge electron states arising in the presence of a magnetic field (magnetic edge state; MES) in finite two-dimensional (2D) charged systems frequently occur in finite conducting systems and have been studied in a variety of experimental setups. They can be traced back to Khaikin resonances[1–3] and, further, to the so-called transverse magnetic focusing in 3D metals and the 2D counterpart of this problem[4], effects of edge states on the magnetocapacity of 2D systems[5,6], and a Halperin-Büttiker formalism[7–10] treating the quantum Hall effect (QHE) as a manifestation of quantum properties in edge channel conduction. Edge states were also used in attempts aimed at detecting Luttinger liquid behavior in edge channels formed near sharp boundaries as well as within smooth domains of 2D systems[11–13]. In most cases, sufficient for qualitative analysis of experimental data is the very fact of the existence of MESs or the interesting property of the universality of their quantum conductivity. As to the details of the channel structure, they have almost never been discussed. Except for the isolated integer filling stripes[14] in 2D Hall samples, generally arising far from the 2D system boundary, the existence of even the simplest skipping orbit MES[1–4] requires a very natural but still hypothetical specular reflection of magnetized electrons from the ideal metal boundary.

The problem becomes even more interesting due to the current discussion in the literature on the physical essence of topological invariants heavily based on the existence of edge states (involving charge or spin) at the interface of two media with different topological indexes following from the most general analysis[15,16]. On the one hand, general theorems[16] do not require any knowledge of the boundary structure (in particular, the presence of a sharp infinite barrier is not required) and are sufficient to deduce the existence of MESs in semi-infinite magnetized 2D systems. On the other hand, the formalism considering MESs on the basis of electron (spin) behavior in homogeneous density domains far from the 2D system boundaries encounters difficulties when applied to strongly inhomogeneous problems (e.g., narrow Coulomb channels) where the electron density $n(x)$ is everywhere inhomogeneous, i.e., $n'(x) \neq 0$. At the same time, numerous experiments on narrow-channel transport[17,18] (the list of relevant references could easily be extended) certainly reveal the presence of the QHE, thus providing a serious indication of the existence of quasi-1D states. In other words, topological arguments are sufficient, but in no case necessary, for the existence of MESs.

Proposed in the present paper is an alternative (compared with Refs. 1–3 and 16) scenario of the MES development which is free, as far as possible, of any model simplifications (major virtue of the approach employed in Ref. 16) and does not fail under the conditions where $n'_{x,y}(x,y) \neq 0$ throughout the entire sample area. The scenario is based on the incomplete screening of external electrostatic fields shaping the electron density profile $n(x,y)$. Thus, the incompletely screened field is present in a ceratin static skin layer of width λ along the edge of a 2D charged system (either classical or degenerate). If a magnetic field normal to the 2D system is applied, the electrons in the skin layer are dragged along the MES orbits. Details of the λ scenario for development of MESs applied to available experimental data on the QHE in narrow channels are gathered at the end of this report (we consider the channel to be narrow if its typical transverse size w is less than the skin layer width λ).

(1) Let $V(x)$ be the external potential governing the channel electron density distribution $n(x)$. The so-called classical Coulomb equilibrium inside the 2D system with the density $n_0(x)$ and fixed number of electrons per unit channel length N is described by the equation

$$V(x) + e\hat{\varphi}[n_0(x)] = \text{const}, \qquad \int_{-b}^{+b} n_0(x)dx = N, \tag{1}$$

where $\hat{\varphi}[n_0(x)]$ is the electrochemical potential of electrons in the channel, which is actually a functional of the density $n_0(x)$ (the subscript 0 here stresses the Coulomb nature of the equilibrium) with the boundaries $\pm b$ satisfying the normalization condition, (1), still to be found; the axis OX is directed across the channel and its origin lies at the channel symmetry axis.

Equation (1) is an integral equation for local electron density $n_0(x)$. Its solution can be explicitly written as

$$\frac{e^2}{\kappa} n_0(x) = -\frac{1}{\pi^2}\sqrt{b^2 - x^2}\int_{-b}^{+b} \frac{ds\,V'(s)}{\sqrt{b^2 - s^2}(s - x)}, \tag{2}$$

where κ is the ambient medium dielectric constant. Making use of Eq. (2), one can easily express $2b$ in terms of the fixed total number of electrons N and potential energy $V(x)$.

Formally, Eq. (1) describes equilibrium in a system of Coulomb interacting particles with infinitely high masses $m \to \infty$ placed in the confining potential $V(x)$. The situation is different if $m \neq \infty$ and the general equilibrium condition at low temperatures $T \ll \epsilon_F$ should involve other terms, in addition to the usual Coulomb contribution. In particular, these additional terms can contain zero-oscillation energy in the Thomas-Fermi approximation (which, in the language employed in general schemes for electron density calculations[19], corresponds to the random phase approximation in zero magnetic field):

$$V(x) + e\hat{\varphi} + \frac{\hbar^2}{2m_*}n(x) = \mu, \quad \int_{-\infty}^{+\infty} n(x)dx = N. \quad (3)$$

Here μ is the electrochemical potential, and m_* is the effective electron mass.

Just as in the Coulomb case, (1), the equilibrium condition, (3), results in a certain density distribution $n(x)$ generalizing Eq. (2). It is easily seen that the channel edges (i.e., the neighborhoods of points $\pm b$) contain areas with incomplete [in the sense of Eq. (1)] screening of the external field, or static skin layers. We apply this term to the intervals of length λ filled with electrons where $U(x) = V(x) + e\varphi(x) \neq \text{const}$. In these areas the one-particle excitations feel the potential $U(x)$

$$U(x) = \text{const} - \hbar^2 n(x)/2m, \quad (4)$$

with $n(x)$ from (3), which, in the presence of a magnetic field H normal to the 2D electron plane, is capable of pushing electrons along the MES trajectories. Applications of this formalism to narrow channels in the absence of a magnetic field can be found in Refs. 19 and 20 [we use the term "narrow" for channels with $b \leqslant \lambda$, where $2b$ is the effective channel width, as defined, for example, by Eq. (2)].

The equation set (1)–(4) solves the problem of the existence and structure of the static skin layer where a nonzero field, (4), and an electron density $n(x)$ from (3) coexist, allowing the appearance of filled MESs.

(2) Now let us turn to the details of the λ scenario under the conditions of QHE. In the range of $\epsilon_F \leqslant \hbar\omega_c$, the basic requirement, just as in the problem, (3), in the absence of a magnetic field, is the general equilibrium condition $\mu = \text{const}$ across the channel. The extreme case of strong magnetic fields modifies the "chemical" part of the electrochemical potential, so that instead of Eq. (3), we have (first retaining, for simplicity, only the first two Landau levels and neglecting electron spin, similarly to Ref. 14)

$$\mu(H,\nu(x)) = \hbar\omega_c + U(x) + \zeta(H,\nu) = \text{const}, \quad (5)$$

$$\zeta = -T \ln S(H,\nu), \quad \epsilon = \exp\left(-\frac{\hbar\omega_c}{T}\right) \ll 1 \quad (6)$$

$$S(H,\nu) = 1/2(1/\nu - 1) + \sqrt{(1/4)\cdot(1/\nu - 1)^2 + \epsilon(2/\nu - 1)},$$

where $\nu(x) = \pi l_H^2 n(x)$ is the corresponding filling factor for the equilibrium electron density $n(x)$, l_H^2 is the squared magnetic length, and $U(x) = V(x) + e\varphi(x) \neq \text{const}$.

Following Ref. 14 and assuming that the fields of neighboring stripes do not overlap, one can subtract condition (1) from Eq. (5) and consider this difference in the vicinity of one of the points $x = x_l$ satisfying the condition

$$\pi l_H^2 n(x_l) = \nu_l, \quad \nu_l = 1,2,3,4\ldots, \quad (7)$$

which defines the position x_l of integer filling factor points in the density profile. This procedure yields

$$\frac{2e^2}{\kappa}\int_{-a_l}^{+a_l} ds \frac{n(s) - n_0(s)}{x - s} \simeq -\left.\frac{\partial \zeta_l}{\partial n}\right|_{n=n_l} \frac{\partial n(x)}{\partial x}, \quad (8)$$

where $2a_l$ is the width of one of the integer shelves calculated according to the algorithm proposed in Ref. 14 [one such formula is given below; see Eq. (13)], and $\zeta_l(H,\nu)$, defined in Eq. (6), is the chemical part of the electrochemical potential in the vicinity of the points x_l, where $n_l = \nu_l/(\pi l_H^2)$.

In the areas where $x \neq x_l$ the derivative $\partial\zeta_l/\partial n$ [as well as the whole right part of Eq. (8)] is small. In these areas the external potential is almost completely screened, i.e., $[n(x) - n_0(x)] \to 0$. On the contrary, at points where $\pi l_H^2 n_0(x_l) = \nu_l$ the quantity $\partial\zeta_l/\partial n$ is sharply enhanced since the function ζ_l, (6), has steep jumps here. Thus, an expected failure of screening occurs in areas with nearly integer filling factors, resulting in nonzero difference $n(x) - n_0(x) \neq 0$. We omit the details and only report the following approximate formula for the derivative $n_l'(0)$ at the integer points which can be derived from Eq. (8):

$$n_l'(0) \simeq \frac{n_0'(0)}{1 + \left(\pi l_H^2 \kappa/a_l e^2\right)\partial\zeta_l/\partial\nu_{\max}}. \quad (9)$$

For $\partial\zeta_l/\partial\nu_{\max} \gg 1$, the derivative $n'(0) \ll n_0'(0)$ is small, thus symbolizing the development of required shelves [the well-known paper[14] employs the inequality $n'(0) \ll n_0'(0)$ as an assumption].

The information contained in Eqs. (5)–(9) allows us to examine the details of QHE in narrow channels by employing the data from Ref. 18 as a guide for our analysis. In this brief report we only comment on the initial stage of the ballistic conductivity development studied in Ref. 18. By employing the formulas from Ref. 21, which take into account the geometry of Ref. 18, one obtains, through Eqs. (10) and (11) the electron density profile $n(x)$ for external parameters used by the authors of Ref. 18 (see Fig. 1):

$$n(x) = n_s\sqrt{(b^2 - x^2)/(d^2 - x^2)}, \quad -b \leqslant x \leqslant +b, \quad (10)$$

$$\kappa V_g = 2\pi e n_s d[E(\sqrt{1 - t^2}) - t^2 K(\sqrt{1 - t^2})]. \quad (11)$$

Here $t = b/d$, n_s is the electron density in the gap between the electrodes for gate voltage $V_g \to 0$ [which is easily checked by employing the asymptotic behavior of elliptic integrals $K(s)$ and $E(s)$ of the first and second kind in the limit $b/d \to 1$], $2d$ is the nominal spacing between the electrodes, and $2b$ is again the Coulomb channel width, governed by V_g. The position of points x_l is defined by Eq. (7), and the reduced dimensionless effective voltages v_g^i are calculated as $v_g^i = V_g^i/(V_g^{\min} - V_g^{\max})$ by taking advantage of the fact that all the necessary values of V_g^i as well as the lowest gate voltage ($V_g^{\min}$ at which the point contact is formed together with the highest gate voltage $V_g^{\max}$) at which electrons are completely forced out of the contact area are also reported in Ref. 18.

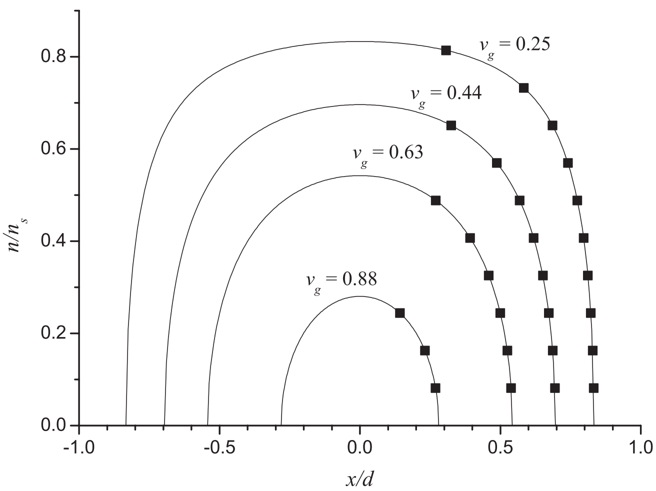

FIG. 1. Various density profiles $n_i(x)$ [(10), (11)] normalized to n_s for reduced values of $v_g^i = V_g^i/(V_g^{\min} - V_g^{\max})$ together with integer filling factor points x_l, (7), corresponding to the field $H = 0.6$ T.

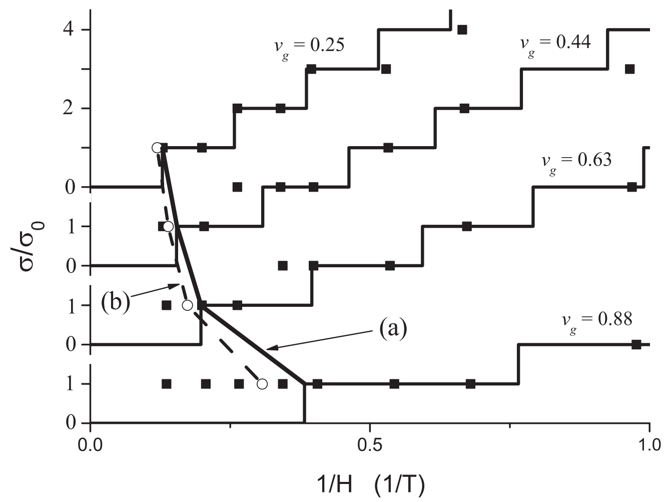

FIG. 2. Conductivity $\sigma_\parallel(H)$ in units of $\sigma_0 = e^2/h$ as a function of inverse magnetic field for appropriate values of reduced effective gate voltages. Solid lines: calculations according to the self-consistent model outlined in the text. Squares: experimental data from Ref. 18. Curve (a) connects the calculated threshold points where the nonzero conductance should arise for the 1D channel with $d =$ const. Dashed line: the same thresholds calculated with a weak dependence of d on y taken into account.

It is natural to assume that the appearance (disappearance) of a new ballistic channel as well as their total number correlate with the behavior of the points x_l in the electron density profiles $n_i(x)$ in Fig. 1 as the magnetic field is varied. Bearing in mind these heuristic considerations and employing the formalism outlined by Eqs. (7), (10), and (11), let us now turn to the general situation depicted in Fig. 2, which displays both the calculated and the measured[18] values of the channel conductivity $\sigma_i(H^{-1})$. The calculations obviously reproduce correctly the constant step length at each of the $\sigma_i(H^{-1})$ branches, the gradual increase in these lengths with growing v_g^i, and a rather large range of magnetic fields (especially for top, relatively small v_g^i) where the linear behavior $\sigma_i(H^{-1}) \propto H^{-1}$ typical of the strong field [(5) and (6)] limit holds. On the whole, our numerical results presented in Fig. 2 are consistent with the calculations of the contact conductivity reported in Ref. 18, which involved two adjustable parameters: (i) the actual width of a channel with infinitely high walls and (ii) the Fermi level for each value of v_g^i. On the contrary, our calculations contain a single adjustable parameter n_s, which was assumed to be 4.5×10^{11} cm^{-2}, which is close to the density of 3.56×10^{11} cm^{-2} reported in Ref. 18.

Compared with a rather satisfactory general agreement between the calculations and the experimental data[18], there are considerable deviations of the predicted positions of the first steps in the $\sigma_i(H^{-1})$ staircases from the observed conductivity behavior. In principle, the first step position in every staircase should determine the width of all the shelves in that staircase, while experiment reveals that the width changes from shelf to shelf. In our opinion, the indicated deviations may be due to the fact that actually the split-gate channel[18] has a rather poorly controlled saddle shape. The electron density distribution $n(x,y)$ in the vicinity of the saddle point should reach higher values at the contact input and output areas compared with the density at the saddle point itself. Hence the same mechanism, (9), which results in the development of integer-filling shelves in a 1D problem, should force the electrons to flow from the hills toward the saddle points, shifting the observed conductivity thresholds to higher magnetic fields. This effect should be enhanced (and it is actually shown in Fig. 2) as the electron density at the saddle point is reduced.

To obtain an order-of-magnitude estimate of the saddle-point corrections to the ideal 1D picture, in the present paper we employ a quasi-1D approach. This means that we still use Eqs. (10) and (11) in the situation where the nominal gap $2d(y)$ appearing in these formulas has a weak dependence on y. Under these conditions we also use Eq. (8), assuming that the relevant integer filling points in the electron density profile $n(x,y)$ now reside on the curves

$$\alpha x^2 - \beta y^2 = c, \quad \alpha = n''_{x,x}(0,0), \quad \beta = n''_{y,y}(0,0). \tag{12}$$

In the limit $c \to 0$, Eq. (12) defines the separatrix of the saddle-like density distribution $n(x,y) \simeq n(0,0) - \alpha x^2 + \beta y^2$.

The calculations performed consisted in finding the widths $2a_0$ of the shelves hanging over the saddle point from the two sides of the contact,

$$e^2 a_0^2 \partial n(0,y_0)/\partial y \simeq \kappa \hbar \omega_c, \tag{13}$$

in the direction of the OY axis. The shelf wings extend toward the center of the contact, overlap there, and thus enhance the electron density at the saddle point. Therefore, the integer filling factor ν at the saddle point is reached at a higher magnetic field compared with the situation where the curvature of $d(y)$ is neglected. The solid line (a) in Fig. 2 depicts the conductivity threshold positions calculated for the latter case [curvature of $d(y)$ is neglected]. The overlapping condition $y_0 = a_0$ in Eq. (13) allows one to derive an estimate for the correction to the magnetic field strength at which a new channel is opened (i.e., a new integer filling factor ν is reached at the saddle point). The corresponding conductivity thresholds obtained by employing Eqs. (10)–(13) for the case where $\beta \sim 0.2\alpha$ are indicated in Fig. 2 by open circles connected by the dashed line

(b) in Fig. 2. It is clearly shown in the figure that accounting for the shelves' overlapping results in a qualitatively correct shift of thresholds toward higher magnetic fields.

In this report we have discussed a scenario for development (and, further, evolution with the magnetic field) of MESs arising near the boundary of finite 2D electron systems due to the existence of a static skin layer of width λ along the system boundary. The formalism is efficient over a wide range of parameters λ/w. For $\lambda \ll w$, where w is the typical system size, the results obtained within the proposed formalism practically coincide with other available approaches (Refs. 1,3–18) to a description of the MES behavior. However, in the interesting limit $\lambda \geqslant w$, our scenario currently has practically no alternatives. Within this approach we propose a systematic description of the QHE in ballistic conduction of narrow Coulomb channels based on the analysis of the so-called integer stripes naturally arising in the study of inhomogeneous 2D charged system. Calculations are worked out to the level allowing comparison with available data on QHE under appropriate conditions.

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