Model for large arrays of Josephson junctions with unconventional superconductors

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We study large arrays of mesoscopic junctions made out of gapless unconventional superconductors where the tunneling processes of both particle-hole and Cooper pairs give rise to a strongly retarded effective action which, contrary to the standard case, cannot be readily characterized in terms of a local Josephson energy. This action can be relevant, for example, to grain boundary and c-axis junctions in layered high-T_c superconductors. By using a particular functional representation, we describe emergent collective phenomena in this system, ascertain its phase diagram, and compute electrical conductivity.

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Quantum dynamics of ultrasmall, normal, and superconducting (Josephson) junctions (JJ) has long been a field of active theoretical[1] and experimental[2] research. Recently, interest in this topic has been rekindled by a number of fresh ideas, such as the proposal of a "floating" phase in which context the effects of (spatially) long-range correlations were investigated at a greater length.[3]

Notably, though, most of the previous theoretical studies were limited to the JJs between conventional, fully gapped, s-wave superconductors. Although the case of the d-wave superconducting cuprates, such as biepitaxial grain boundary (in-plain) JJs in Yttrium Barium Copper Oxide (YBCO) or intrinsic c-axis (vertical) ones in Bi2212, has been rather extensively studied as well, all such analyses would routinely resort to a phenomenological description similar to that of the gapped (s-wave) superconductors.[4]

In contrast, the microscopic analysis of a single d-wave JJ carried out in Refs. 5 and 6 showed that the processes of both particle-hole and Cooper pair tunneling can give rise to equally nonlocal (in the time domain) terms in the effective action, thereby invalidating the very notion of a local Josephson energy.

In the present work we consider large arrays of such JJs and study possible effects of the previously ignored strong retardation intrinsic for this system. In particular, we analyze its phase diagram and transport properties, thereby predicting the possible existence of a new phase and demonstrating the need for using this microscopically justified as opposed to the phenomenological description of the unconventional JJs, including the experimentally relevant high-T_c ones.

The partition function of a generic JJ array reads (hereafter $\hbar = k_B = 1$)[1]

$$S = \int_0^{1/T} d\tau \frac{1}{2}\left[\sum_i C_{ii}\left(\frac{\partial\phi_i(\tau)}{\partial\tau}\right)^2 + \sum_{\langle ij\rangle} C_{ij}\left(\frac{\partial\phi_{ij}(\tau)}{\partial\tau}\right)^2\right] - \sum_{\langle ij\rangle}\int_0^{1/T} d\tau \int_0^{1/T} d\tau' \{\alpha(\tau-\tau')\cos t[\phi_{ij}(\tau)-\phi_{ij}(\tau')] + \beta(\tau-\tau')\cos[\phi_{ij}(\tau)+\phi_{ij}(\tau')]\}, \quad (1)$$

where $\phi_{ij}(\tau) = \phi_i(\tau) - \phi_j(\tau)$ is the phase difference across the link $\langle ij\rangle$, while the first two terms represent the effects of self- and mutual capacitances.

The double time integrals in (1) are governed by the kernels $\alpha(\tau)$ and $\beta(\tau)$, corresponding to the particle-hole and Cooper pair tunneling processes, respectively. To the leading order in the tunneling matrix element $T(k,k')$ they are given by the expressions[1]

$$\begin{bmatrix}\alpha(\tau)\\ \beta(\tau)\end{bmatrix} = -2\int \frac{d^Dk d^Dk'}{(2\pi)^{2D}} |T(k,k')|^2 \begin{bmatrix}\mathcal{G}_k(\tau)\mathcal{G}_{k'}(-\tau)\\ \mathcal{F}_k(\tau)\mathcal{F}_{k'}(-\tau)\end{bmatrix}, \quad (2)$$

where $\mathcal{G}$ and $\mathcal{F}$ are the normal and anomalous electron Green functions, correspondingly.

In a grainy superconductor the applicability of the effective "phase-only" model (1) can generally be justified once the superconducting phase coherence length becomes large as compared to the grain's size, which happens to be the case, e.g., in the vicinity of a phase transition.

However, considering that the coherence length of the superconducting gap's amplitude (which might be quite different from that of the phase) can be quite small (as it is in, e.g., the high-T_c materials), Eq. (2), derived under the assumption of a spatially homogeneous amplitude, may be affected by disorder and/or boundary roughness. In this case a more detailed analysis of the quasiparticle dynamics near a tunneling interface in the framework of, e.g., the Usadel equation (see Ref. 1) might be required for properly computing the kernels $\alpha(\tau)$ and $\beta(\tau)$. Nonetheless, the long-time asymptotic behavior of these kernels is determined by the scaling properties of the Green functions at small energies and is expected to remain robust against the gap's inhomogeneity.

The α term describes (non-Gaussian) dissipation due to the Andreev quasiparticle tunneling whose effects have been extensively discussed in the previous works,[1] while the β term represents the processes of (in general, nonsynchronous) pair tunneling. In the "gapful" conventional (s-wave) superconductors, it decays as $\beta(\tau) \propto e^{-\Lambda|\tau|}$, thereby effectively reducing the last term in (1) to a single time integral $E_J \int_0^{1/T} d\tau \cos 2\phi_{ij}(\tau)$ of what can then be identified as the local Josephson energy $E_J = \int_0^{1/T} d\tau \beta(\tau)$.

By contrast, in the case of an unconventional (necessarily, "gapless") superconductor one generally obtains strongly retarded kernels,

$$\alpha(\tau)/\alpha = \beta(\tau)/\beta = 1/\tau^{2D-\eta}, \quad (3)$$

although the prefactor in the β kernel still vanishes for any factorizable matrix element, $|T(k,k')|^2 = f(k)f(k')$, if symmetry of the function $f(k)$ under the lattice group is different from that of the gap. Nonetheless, one can obtain a nontrivial result (3) for $\beta(\tau)$ in the presence of a nonfactorizable term $|T(k,k')|^2 = g(\vec{k} - \vec{k}')$, the exponent η being its scaling dimension $[g(\lambda\vec{q}) = g(\vec{q})/\lambda^\eta]$.

In the two-dimensional case ($D = 2$) and under the conditions of momentum conservation implying $\eta = D$, both the tunneling terms decay as $\propto 1/\tau^2$, as found previously.[5,6] A short-time divergence can be naturally regularized by substituting $\tau \to \sqrt{\tau^2 + \Lambda^{-2}}$, where the cutoff scale Λ is set by the maximal superconducting gap in the bulk.

Conceivably one can encounter even longer-ranged correlations ($2D - \eta < 2$) due to, e.g., resonant tunneling through zero energy states supported by certain tunneling configurations, such as that of the $d_0/d_{\pi/4}$ in-plane grain boundary.[7] However, for the sake of concreteness, in this work we focus on the above case of $2D - \eta = 2$, where the coupling constants α and β appear to be dimensionless numbers of order unity, the former being proportional to the normal state conductance.

Turning now to the effective action (1), we find that the strongly retarded nature of the tunneling terms renders a customary dual representation based on the Villain transformation of the local Josephson term inapplicable, thereby making this model unsuitable for the standard mapping onto an effective vortex plasma.[1] Therefore a well-known description of the different phases in terms of bound vortex-antivortex complexes (dipoles, quadrupoles, etc.) also cannot be readily generalized to the problem at hand, thus forcing one to take a different approach.

To that end we introduce a bosonic field $\psi_i(\tau)$ alongside an associated Lagrange multiplier field enforcing the local constraint $\psi_i(\tau) = e^{i\phi_i(\tau)}$. This approach should be contrasted with the previously developed treatments of the conventional (local) Josephson term (see, e.g., Ref. 8) where a constrained bosonic variable would be used to represent the *pair* field $e^{2i\phi_i(\tau)}$. Indeed, an attempt to implement this technique in the present (nonlocal) case would require one to work with a technically intractable bilocal composite operator $\psi_i(\tau)\psi_i(\tau')$.

By integrating out the phase variable ϕ_i, keeping the leading terms of the corresponding cluster expansion (cf. with Ref. 8), and then integrating out the Lagrange multiplier field, one arrives at the partition function

$$Z = \int D\psi_i^\dagger(\tau) D\psi_i(\tau) D\lambda_i(\tau) \exp\Bigg(- \sum_{\langle ij\rangle} \int_0^{1/T} d\tau_1 \int_0^{1/T} d\tau_2 \psi_i^\dagger(\tau_1)\big[W_{ij}^{-1}(\tau_1 - \tau_2) + \delta_{ij}\lambda_i(\tau_1)\delta(\tau_1 - \tau_2)\big]\psi_j(\tau_2)$$
$$+ \alpha(\tau_1 - \tau_2)\psi_i^\dagger(\tau_1)\psi_j^\dagger(\tau_2)\psi_i(\tau_2)\psi_j(\tau_1) + \beta(\tau_1 - \tau_2)\psi_i^\dagger(\tau_1)\psi_i^\dagger(\tau_2)\psi_j(\tau_2)\psi_j(\tau_1) + \text{H.c.}\Bigg), \quad (4)$$

where $\lambda_i(\tau)$ is an additional Lagrange multiplier enforcing the auxiliary constraint $\psi_i^\dagger(\tau)\psi_i(\tau) = 1$. (The latter is not automatically satisfied unless the integration over $\phi_i(\tau)$ is performed exactly.)

The correlation function appearing in Eq. (4),

$$\begin{aligned} W_{ij}(\tau) &= \langle e^{i\phi_i(\tau)} e^{-i\phi_j(0)}\rangle \\ &= \exp\left[- \int \frac{d\omega d^D k}{(2\pi)^{D+1}} \frac{1 - \cos(\omega\tau - \vec{k}\vec{R}_{ij})}{\omega^2 C(k)} \right] \\ &= \delta_{ij} e^{-E_c|\tau|}, \end{aligned} \quad (5)$$

is governed by the effective Coulomb energy $E_c = \int \frac{d^D k}{2(2\pi)^{D+1} C_k}$ proportional to the integral of the inverse capacitance $C_k = \sum_{<ij>} C_{ij} e^{i\vec{k}\vec{R}_{ij}}$ which converges, provided that the capacitance matrix progressively decreases with the separation between the sites.

The frequency integral in Eq. (5) diverges for any $\vec{R}_{ij} \neq 0$, which dictates that the correlation function $W_{ij}(\tau)$ remains strictly local in the real space. Also, Eq. (5) is written in the limit of vanishing temperature, while at finite T a proper account of large phase fluctuations with nontrivial winding numbers makes this (as well as any bosonic) function periodic with a period $1/T$ by virtue of the substitution $\tau \to \tau - T\tau^2$ (see Ref. 1).

At $\alpha = \beta = 0$ one then obtains a bare (normal) Green function

$$G_{ij}^{(0)}(\omega) = \frac{2\delta_{ij}}{\omega^2/E_c + E_c}, \quad (6)$$

while for finite α and β the quantum charge fluctuations give rise to the corrections which can be incorporated into the normal $G_{ij} = \langle\psi_i\psi_j^\dagger\rangle$ and anomalous $F_{ij} = \langle\psi_i\psi_j\rangle$ Green functions obeying the usual Dyson's equations

$$\begin{pmatrix} G_{ij} \\ F_{ij} \end{pmatrix} = \begin{pmatrix} G_{ij}^{(0)} \\ 0 \end{pmatrix} + G_{ik}^{(0)} \sum_{kl} \begin{pmatrix} \Sigma_{kl} & \Delta_{kl} \\ \Delta_{kl} & \Sigma_{kl} \end{pmatrix} \begin{pmatrix} G_{lj} \\ F_{lj} \end{pmatrix}, \quad (7)$$

where both the normal Σ_{ij} and anomalous Δ_{ij} self-energies can be computed as series expansions in powers of α and β.

The analysis of these expansions shows that they can be organized according to the powers of the inverse coordination number z (e.g., $z = 2D$ for a simple cubic lattice). In the leading approximation for $z \gg 1$, the self-energies are given by the equations

$$\begin{aligned} \Sigma_{ij}(\omega) &= \int \frac{d\omega'}{2\pi} \Bigg\{ \delta_{ij} \sum_l \alpha(\omega - \omega') G_{ll}(\omega') \\ &\quad + [\alpha(0) + \beta(0) + \beta(\omega - \omega')] G_{ij}(\omega') \Bigg\}, \\ \Delta_{ij}(\omega) &= \int \frac{d\omega'}{2\pi} \Bigg[\alpha(\omega - \omega') F_{ij}(\omega') \\ &\quad + \delta_{ij} \sum_l \beta(\omega - \omega') F_{ll}(\omega') \Bigg]. \end{aligned} \quad (8)$$

When ascertaining a general layout of the phase diagram of the JJ array, different components of the self-energy can serve as emergent order parameters. As such, one can distinguish between the local, $\Sigma_0 = \Sigma_{ii}$, and nonlocal, $\Sigma_1 = \frac{1}{z}\sum_\mu \Sigma_{i,i+\mu}$ (here the sum is taken over the z nearest neighbors), normal, as well the corresponding anomalous, $\Delta_0 = \Delta_{ii}$ and $\Delta_1 = \frac{1}{z}\sum_\mu \Delta_{i,i+\mu}$, self-energies.

Specifically, Σ_1 signals the onset of a metallic behavior (hopping between neighboring sites), Δ_0 manifests an incipient local ψ-field pairing, Δ_1 serves as a precursor of the pairing coherence setting in across the entire JJ network, while a frequency-dependent part of the Σ_0 indicates the development of local time correlations.

With the on-site and nearest-neighbor terms taken into account, the spatial Fourier harmonics read

$$\begin{pmatrix}\Sigma(\omega,k)\\ \Delta(\omega,k)\end{pmatrix} = \begin{pmatrix}\Sigma_0(\omega)\\ \Delta_0(\omega)\end{pmatrix} + \begin{pmatrix}\Sigma_1(\omega)\\ \Delta_1(\omega)\end{pmatrix}\gamma(k) + \cdots, \quad (9)$$

where $\gamma(k) = \sum_\mu e^{ik\mu}$.

Equations (8) can be further improved by adding polarization corrections to the effective coupling terms

$$\begin{pmatrix}\tilde\alpha\\ \tilde\beta\end{pmatrix} = \begin{pmatrix}\alpha\\ \beta\end{pmatrix} + \begin{pmatrix}\Pi_E & \Pi_O\\ \Pi_O & \Pi_E\end{pmatrix}\begin{pmatrix}\alpha & \beta\\ \beta & \alpha\end{pmatrix}\begin{pmatrix}\tilde\alpha\\ \tilde\beta\end{pmatrix}, \quad (10)$$

where the polarization functions $\Pi_{E,O}(\omega) = \int \frac{d\omega'}{2\pi}\Gamma_{E,O}G(\omega')G(\omega-\omega')$ include the vertex corrections $\Gamma_{E,O}$ arising from the even and odd numbers of noncrossing β couplings,

$$\begin{pmatrix}\Gamma_E\\ \Gamma_O\end{pmatrix} = \begin{pmatrix}1\\ \beta\end{pmatrix} + \begin{pmatrix}\beta^2 & 0\\ 0 & \beta^2\end{pmatrix}\begin{pmatrix}\Gamma_E\\ \Gamma_O\end{pmatrix}. \quad (11)$$

With the vertex and polarization corrections included and in the absence of any emergent order parameters, the self-consistent equation for $\Sigma_0(\omega)$ reads

$$\Sigma_0(\omega) = z\int \frac{d\omega'}{2\pi}\bar\Gamma(\omega')\frac{\tilde\alpha(\omega-\omega')}{G_0^{-1}(\omega') - \Sigma_0(\omega')}. \quad (12)$$

The (static and spatially uniform) expectation value of the Lagrange multiplier $\lambda = \langle\lambda_i(\tau)\rangle$ can then be determined from the normalization condition $\int \frac{d\omega d^Dk}{(2\pi)^{D+1}}G(\omega,k) = 1$.

In order to find the phase boundaries we include a constant term $\Sigma_0(0) + \lambda$ into the definition of the renormalized Coulomb energy $\tilde E_c$ and expand Eqs. (8) to the first order in the emergent self-energies Σ_1, Δ_0, Δ_1, as well as the derivative of the (linear) frequency-dependent part of $\Sigma_0(\omega)$. Threshold values of the couplings, beyond which such self-energy components develop, are then given by the following eigenvalue equations:

$$\begin{aligned}
\Sigma_1(\omega) &= \int \frac{d\omega'}{2\pi}\Gamma[\tilde\alpha(0) + \tilde\beta(0) + \tilde\beta(\omega-\omega')]G_0^2(\omega')\Sigma_1(\omega'),\\
\Delta_0(\omega) &= z\int \frac{d\omega'}{2\pi}\Gamma\tilde\beta(\omega-\omega')G_0^2(\omega')\Delta_0(\omega'),\\
\Delta_1(\omega) &= \int \frac{d\omega'}{2\pi}\Gamma\tilde\alpha(\omega-\omega')G_0^2(\omega')\Delta_1(\omega'),\\
\frac{d\Sigma_0(\omega)}{d\omega} &= z\int \frac{d\omega'}{2\pi}\Gamma\tilde\alpha(\omega')G_0^2(\omega')\frac{d\Sigma_0(\omega')}{d\omega'}.
\end{aligned} \quad (13)$$

In the case of marginal (ohmic) dissipation corresponding to $2D - \eta = 2$, the Fourier transforms of the (regularized) coupling functions behave as $\alpha(\omega)/\alpha = \beta(\omega)/\beta = \pi\Lambda e^{-|\omega|/\Lambda}$, thus resulting in only a weak frequency dependence of the self-energy at $\omega \ll \Lambda$.

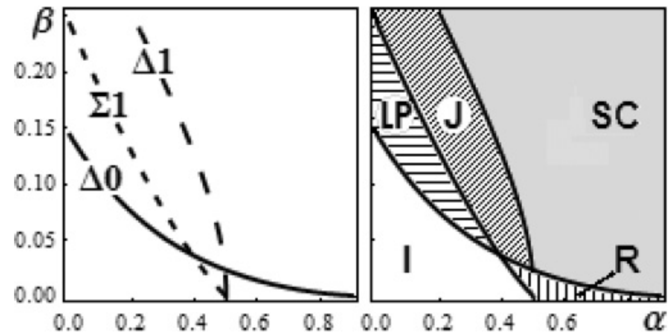

FIG. 1. Left panel: Onset of the intersite self-energy Σ_1 and both on-site and intersite anomalous self-energies Δ_0 and Δ_1. Right panel: phase diagram (see text).

The first three of the eigenvalue equations (13) then reduce to the algebraic ones,

$$\begin{aligned}
1 &= \left(\Gamma_E^2 + \Gamma_O^2\right)(2\tilde\beta + \tilde\alpha) + 2\Gamma_E\Gamma_O(2\tilde\alpha + \tilde\beta),\\
1 &= z\left(\Gamma_E^2\tilde\beta + 2\Gamma_E\Gamma_O\tilde\alpha + \Gamma_O^2\tilde\beta\right),\\
1 &= \Gamma_E^2\tilde\alpha + \Gamma_O^2\tilde\alpha + 2\Gamma_E\Gamma_O\tilde\beta,
\end{aligned} \quad (14)$$

from which one determines putative locations of the critical lines in the $\alpha - \beta$ plane (see Fig. 1).

Interestingly enough, Eqs. (14) suggest that for small α and large z the onset of local (on-site) pairing upon increasing β signaled by the emergent order parameter $\Delta_0 \neq 0$ may precede that of the metallic behavior signified by Σ_1, while for small β the intersite (bond) pairing Δ_1 can only emerge at sufficiently large α.

These observations suggest a general layout of the phase diagram presented in Fig. 1. The region of small α and β with $\Sigma_1 = \Delta_0 = \Delta_1 = 0$ is interpreted as uniformly insulating (I), while at $\beta \sim 1/z$ one expects the onset of local pairing (LP). The latter is a potential "pseudogap" phase where the classical Josephson effect remains suppressed by the Coulomb blockade. At still higher values of $\beta \sim 1$ one expects to enter a Josephson-like phase (J) with $\Delta_0, \Sigma_1 \neq 0$ but without global coherence. On the other hand, at $\alpha \sim 1$ the insulator gives way to the resistive phase (R) with $\Sigma_1, \Delta_1 \neq 0$, which supports both pair and single quasiparticle transport. Lastly, the uniformly superconducting phase (SC) with $\Sigma_1, \Delta_{0,1} \neq 0$ would be attained at $\alpha,\beta \gtrsim 1$. It should be noted, though, that these predictions are based on the approximate analysis and therefore not all the putative phase boundaries may actually be present in the real system. In particular, there may or may not be a physical distinction, other than a crossover, between the J and LP phases, or the latter regime might be absent altogether (as it is for $z = 2$).

Such caveats notwithstanding, the overall behavior appears to be somewhat reminiscent of that in the standard (s-wave) case: the system can be nudged closer to the superconducting state by increasing either the Cooper pair or particle-hole tunneling, the latter providing a mechanism for intrinsic dissipation which quenches phase fluctuations and promotes the classical Josephson effect.

However, should the tunneling β term happen to decay even more slowly ($2D - \eta < 1$), the analog of the effective Josephson energy would then diverge at large τ, thus making the infrared behavior essentially singular and possibly allowing for some drastic changes in the phase structure.

Conducting properties of the JJ array allow one to discriminate between the different phases. In particular, electrical conductivity can be computed as $\sigma_{\mu\nu}(\omega) = \frac{1}{i\omega}\frac{\delta^2 S[A]}{\delta A_\mu \delta A_\nu}$ with the use of the action of Eq. (1) in the presence of an external vector potential A_μ, resulting in

$$\sigma_{\mu\nu}(\omega) = \int_0^{1/T} d\tau \left\{ \alpha(\tau)\frac{1-e^{i\omega\tau}}{\omega}\langle \cos[\nabla_\mu\phi(\tau) - \nabla_\nu\phi(0)]\rangle + \beta(\tau)\frac{1+e^{i\omega\tau}}{\omega}\langle \cos[\nabla_\mu\phi(\tau) + \nabla_\nu\phi(0)]\rangle \right\} + \dots , \tag{15}$$

where the dots stand for paramagnetic terms containing higher powers of α and β, which are small compared to the above (diamagnetic) contributions for $\alpha,\beta \lesssim 1$ (cf. with the discussion of a normal granular metal, $\beta = 0$, in Ref. 9).

The thus-obtained longitudinal conductivity reads

$$\sigma_{\mu\mu}(\omega) \approx \int_0^{1/T} d\tau \left\{ \alpha(\omega)\frac{1-e^{i\omega\tau}}{\omega}\left[G_1^2(0) + G_0^2(\tau) + F_1^2(\tau)\right] + \beta(\omega)\frac{1+e^{i\omega\tau}}{\omega}\left[G_1^2(0) + G_1^2(\tau) + F_0^2(\tau)\right]\right\}, \tag{16}$$

and upon performing the frequency integrations, one obtains

$$\sigma_{\mu\mu}(\omega) \approx \alpha\left[\frac{E_c}{T}e^{-2E_c/T}\left(1 + \frac{\Delta_1^2}{E_c^2}\right) + \frac{\Sigma_1^2}{E_c^2}\right] + \beta\delta(\omega)\frac{\Sigma_1^2 + \Delta_0^2}{E_c}, \tag{17}$$

where, for the sake of simplicity, we chose $T \ll E_c = \Lambda$.

At $\Sigma_1 = \Delta_1 = 0$ the first term in Eq. (17) reproduces the result obtained for a granular metal.[9] The emergent metallicity order parameter Σ_1 promotes a metal-like (temperature-independent at $T \to 0$) conductivity, thereby distinguishing it from the activation-type behavior characteristic of the insulating regime. Interestingly enough, it also contributes to the superfluid density, alongside the local pairing Δ_0, while the nonlocal one Δ_1 does not (at least, to the lowest order in β).

It is conceivable, though, that there might be a (partial) cancelation between the diamagnetic and paramagnetic terms at $\alpha,\beta \sim 1$, as a result of which the conductivity could remain universal along the critical lines, akin to the situation in the conventional, s-wave, JJ networks.[10] (It is worth reiterating that in the present case that one cannot readily invoke the charge-vortex duality on which the universality argument is based[1] due to the inapplicability of the underlying Villain transformation.)

To summarize, in the present work we study large arrays of unconventional JJs with effective long-range (in the time domain) interactions resulting from the presence of gapless quasiparticle excitations. On the technical side, the problem presents a challenge by not being amenable to any standard approach which exploits the intrinsic locality of the standard Josephson effective action (see Ref. 1).

By using an alternative representation we ascertain this system's phase diagram which can feature all or some of the following: insulating, uniformly superconducting, Josephson, local pairing (pseudogap), and metallic phases, all being associated with the corresponding emergent order parameters. We also predict that this picture might be further altered in the presence of resonant tunneling between zero energy states where the phase fluctuations appear to be even longer-range correlated in time.

Given the relevance of our work to such practically important examples as large assemblies of high-T_c JJs,[2] also envisioned as a suitable platform for quantum computations,[4] we conclude with a hope that this exploratory analysis will prompt a further investigation into (and provide an alternative means for interpreting the experimental data on) such systems beyond the scope of the customary phenomenological approach adapted from earlier studies of s-wave superconductors.

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