Magnetic field modulated Josephson oscillations in a semiconductor microcavity

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We theoretically study an exciton-polariton Josephson junction in a planar semiconductor microcavity. When an external magnetic field is applied normal to the plane of the microcavity, remarkable competition is found between the Zeeman energy and the interactions of exciton polaritons. We can determine a critical magnetic field, below which there is only the dc Josephson effect, and above which the ac Josephson effect appears. The ac oscillations of extrinsic and intrinsic Josephson currents have the same frequency, which linearly increases with the magnetic field, analogous to the linear voltage dependence of the Josephson frequency in conventional superconducting junctions. The spontaneous polarization separation and the macroscopic quantum self-trapping can be realized by regulating the magnetic field. These results may be experimentally confirmed by investigating the magnetic-field modulated Josephson oscillations in semiconductor microcavities.

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I. INTRODUCTION

Exciton polaritons (EPs) are elementary excitations resulting from the strong coupling between quantum-well excitons and photons in semiconductor microcavities. They combine the properties of excitons and photons, for example, the effective mass of EPs is extremely small (about $10^{-5}m_e$, where m_e is the free-electron mass), and there exist interactions between EPs. In the recent years, many valuable physical phenomena have been discovered related to EPs such as the optical spin Hall effect,[1] superfluidity,[2] and vortices.[3,4] Meanwhile, a lot of potential applications have been put forward such as polariton lasers,[5] optical gates,[6] and optical switchers.[7]

It is worth noticing that the Bose-Einstein condensation of EPs has been reported in microcavities,[8–10] and the critical temperature is very high due to the small effective mass. The Josephson effect is then expected because of the spatial coherence of the EPs condensate,[11] and there have been some theoretical[12–15] and experimental[16] works associated with this effect. In experiments, there are several ways to realize the two-trap geometry in a microcavity, including applying stress,[10,17] using photolithographic techniques,[18,19] and allowing natural formation during sample growth.[16] Therefore one can get high-quality samples to study the Bose-Einstein condensation of EPs and the Josephson effect and explore more unique phenomena of the cavity quantum electrodynamics.

Spin is an essential degree of freedom of EPs, and many interesting phenomena are related to it.[20,21] The interaction between EPs in the triplet configuration is larger than that in the singlet configuration, and the interaction constant is positive in the former and negative in the latter. Taking advantage of these features, we utilize an external magnetic field to modulate the Josephson oscillations of EPs with spins, which have not been taken into account so far. This modulation will make the Josephson currents controllable, and pave the way for the future quantum interference devices in semiconductor microcavities.[22]

In this paper, we discuss the effects of an applied magnetic field on the Josephson oscillations in a semiconductor microcavity. There are two cases to be distinguished, corresponding to the Josephson oscillations below and above the critical magnetic field, respectively. In the former case, the interactions between EPs play a more important role in the Josephson effect. The paper is organized as follows: in Sec. II, using the single-mode mean-field approximation, we present the dynamics of the mode amplitudes with spins in both traps. In Sec. III, the numerical results are obtained and discussed. Finally, the conclusion of this work is given in Sec. IV.

II. THEORETICAL FORMULATION

We consider an EP Josephson junction in the presence of an external magnetic field normal to the plane of the semiconductor microcavity. In the single-mode mean-field approximation, we can describe a mode amplitude with a spin in any one of the two traps by the equations of motion ($\hbar = 1$),[12,23,24]

$$
\begin{aligned}
i\frac{\partial\psi_{j\sigma}}{\partial t} &= E_{j\sigma}\psi_{j\sigma} - \frac{i}{2}[\Gamma_{j\sigma} - R(n_{j\sigma})]\psi_{j\sigma} + J\psi_{\bar{j}\sigma} \\
&\quad + W\psi_{j\bar{\sigma}} + \alpha_1|\psi_{j\sigma}|^2\psi_{j\sigma} + \alpha_2|\psi_{j\bar{\sigma}}|^2\psi_{j\sigma} \\
&\quad + g_1 n_{j\sigma}\psi_{j\sigma} + g_2 n_{j\bar{\sigma}}\psi_{j\sigma}, \qquad (1)
\end{aligned}
$$

$$
\frac{dn_{j\sigma}}{dt} = F_{j\sigma} - \gamma_{j\sigma}n_{j\sigma} + Wn_{j\bar{\sigma}} - R(n_{j\sigma})|\psi_{j\sigma}|^2, \qquad (2)
$$

where $\psi_{j\sigma}$ is the mode amplitude with spin σ in the trap j ($\sigma = \uparrow,\downarrow$ and $j = L,R$), $n_{j\sigma}$ the density of EPs with the same spin in the corresponding reservoir, and $\bar{j}(\bar{\sigma})$ labels the trap (spin) opposite to $j(\sigma)$. $E_{j\sigma}$ is the energy without coupling and $\Gamma_{j\sigma}$ and $\gamma_{j\sigma}$ denote the loss rate of EPs in the condensate and reservoir, respectively. The term $R(n_{j\sigma})$ is the stimulated scattering rate from the reservoir into the condensate and $R(n_{j\sigma}) = \beta n_{j\sigma}$ with β being a constant.[3,4,23] J is the hopping energy between the two traps and W is the coupling strength between the opposite spins, corresponding to the spin-flip processes.[25,26] $\alpha_1(\alpha_2)$ and $g_1(g_2)$ are the interaction constants within the EPs condensate and between the condensate and the reservoir, respectively, for the triplet (singlet) configuration. In contrast with Eq. (1) describing the mode motion in the condensate, Eq. (2) is used to consider the balance of EPs in the reservoir with the pumping rate $F_{j\sigma}$, the loss and spin-flip rate, and the scattering rate into the condensate.

When an external magnetic field is applied on the microcavity, it is reasonable to fully take into account the Zeeman splitting and neglect the Landau quantization of electrons and holes.[27–30] The Zeeman energy $\Omega_z = g\mu_B B$ can be taken as the single parameter, where g is the polariton Landé factor, μ_B the Bohr magneton, and B the magnetic field. Then, $E_{j\uparrow} = E_0 - \frac{1}{2}\Omega_z$ and $E_{j\downarrow} = E_0 + \frac{1}{2}\Omega_z$, where E_0 is the energy in the absence of the magnetic field. It is estimated that $\Omega_z = 1$ meV when $B \approx 10$ T. According to Ref. 28, a magnetic field does not change the confining potential, because EPs are electrically neutral. Thus J is not affected by the magnetic field. It might be argued that the interaction constants α_1 and α_2 in Eq. (1) can be influenced by the magnetic field through the excitonic component of EPs. It is known that these constants originate from the exciton-exciton interaction and the saturation of the oscillator strength. In the presence of an external magnetic field, the electron-hole relative wave function of an exciton shrinks in the real space,[31] and this increases the exciton binding energy and the oscillator strength. These two terms are both influenced by the magnetic field. To judge the relative importance of the magnetic field effect in comparison to the Coulomb binding energy of excitons, it is convenient to consider the ratio λ_{2D}/l_B to determine the magnetic field effect on the interaction constants between excitons, where λ_{2D} is the two-dimensional exciton radius and l_B is the magnetic length. In our study here, this ratio is always smaller than one even at $B = 10$ T, so the magnetic field effect on the interaction constants is weak and can be neglected.

In terms of the condensate density $\rho_{j\sigma}$ and phase $\theta_{j\sigma}$ from the expression $\psi_{j\sigma} = \sqrt{\rho_{j\sigma}}e^{i\theta_{j\sigma}}$, the mode amplitude can be rewritten in Eq. (1) as well as Eq. (2). Then in the presence of a magnetic field, the following coupled equations are derived,

$$\frac{d\rho_{j\sigma}}{dt} = [\beta n_{j\sigma} - \Gamma_{j\sigma}]\rho_{j\sigma} + 2J\sqrt{\rho_{\bar{j}\sigma}\rho_{j\sigma}}\sin[(-1)^{\delta_{jL}}\Delta_\sigma] + 2W\sqrt{\rho_{j\bar{\sigma}}\rho_{j\sigma}}\sin[(-1)^{\delta_{\sigma\uparrow}}\Delta_j], \tag{3}$$

$$\frac{d\Delta_\sigma}{dt} = J\left(\sqrt{\frac{\rho_{L\sigma}}{\rho_{R\sigma}}} - \sqrt{\frac{\rho_{R\sigma}}{\rho_{L\sigma}}}\right)\cos\Delta_\sigma + W\sqrt{\frac{\rho_{R\bar{\sigma}}}{\rho_{R\sigma}}} \times \cos\Delta_R - W\sqrt{\frac{\rho_{L\bar{\sigma}}}{\rho_{L\sigma}}}\cos\Delta_L + \alpha_1(\rho_{R\sigma} - \rho_{L\sigma}) + \alpha_2(\rho_{R\bar{\sigma}} - \rho_{L\bar{\sigma}}) + g_1(n_{R\sigma} - n_{L\sigma}) + g_2(n_{R\bar{\sigma}} - n_{L\bar{\sigma}}), \tag{4}$$

$$\frac{d\Delta_j}{dt} = \Omega_z + J\sqrt{\frac{\rho_{\bar{j}\downarrow}}{\rho_{j\downarrow}}}\cos\Delta_\downarrow - J\sqrt{\frac{\rho_{\bar{j}\uparrow}}{\rho_{j\uparrow}}}\cos\Delta_\uparrow + W\left(\sqrt{\frac{\rho_{j\uparrow}}{\rho_{j\downarrow}}} - \sqrt{\frac{\rho_{j\downarrow}}{\rho_{j\uparrow}}}\right)\cos\Delta_j + (\alpha_1 - \alpha_2) \times (\rho_{j\downarrow} - \rho_{j\uparrow}) + (g_1 - g_2)(n_{j\downarrow} - n_{j\uparrow}), \tag{5}$$

$$\frac{dn_{j\sigma}}{dt} = F_{j\sigma} - \gamma_{j\sigma}n_{j\sigma} + Wn_{j\bar{\sigma}} - \beta n_{j\sigma}\rho_{j\sigma}, \tag{6}$$

where δ_{jL} and $\delta_{\sigma\uparrow}$ are two Kronecker-delta functions, $\Delta_\sigma = \theta_{L\sigma} - \theta_{R\sigma}$, and $\Delta_j = \theta_{j\uparrow} - \theta_{j\downarrow}$. In Eqs. (3)–(6), the condensate densities and phase differences are coupled, and we can find they are more complicated than the Josephson equations in conventional superconducting junctions. On the right side of Eq. (3), the first term represents the difference between EPs injected from the reservoir into the condensate and lost from the condensate, and this term does not exist in superconducting junctions. The second and third terms, varying with the corresponding phase differences, also reflect the motion of condensate densities, and are called the extrinsic and intrinsic Josephson currents, respectively, somewhat similar to that in superconducting junctions. In Eqs. (4) and (5), in addition to the tunneling between the two traps and the opposite spins in the same trap, the interactions between EPs affect the motion of the phase differences, which is different from that in superconducting junctions. Moreover, the condensate densities are dependent on the magnetic field.

Equations (3)–(6) are our theoretical base for further study, and they include in fact twelve equations, from which we can perform the numerical calculations and give the asymptotic expressions. According to Refs. 12 and 13, there are two kinds of Josephson currents expressed by

$$I^e_\sigma = 2J\sqrt{\rho_{R\sigma}\rho_{L\sigma}}\sin\Delta_\sigma, \tag{7}$$

$$I^i_j = 2W\sqrt{\rho_{j\downarrow}\rho_{j\uparrow}}\sin\Delta_j, \tag{8}$$

in which I^e_σ is the extrinsic Josephson current, describing the tunneling of EPs between the two traps for the same spin σ, and this kind of current corresponds to the real-space motion of EPs. On the other hand, the intrinsic Josephson current I^i_j has nothing in common with the real spin current, and just reflects the spin beats with the opposite spins in the same trap j. It deserves to point out that we only consider the spin-conserving tunneling of EPs, because their coherence length is much larger than the distance between the two traps.[1]

III. RESULTS AND DISCUSSIONS

We can show that a magnetic field has important effects on the EP Josephson oscillations. We consider that a continuous-wave excitation is applied, and the initial densities of EPs can be adjusted by a Gaussian laser beam in both reservoirs. The parameters used in the calculations come from the typical GaAs semiconductor microcavity: $\beta = 0.012$ meVμm^2, $\alpha_1 = -10\alpha_2 = 6 \times 10^{-3}$ meVμm^2, and $g_1 = -10g_2 = 1.2 \times 10^{-2}$ meVμm^2.[4,26] For simplicity, let $\Gamma_{j\sigma} = \Gamma$, $\gamma_{j\sigma} = \gamma$, and $\gamma = 1.5\Gamma = 0.33$ meV.[4] The coupling constant between the opposite spins is chosen as $W = 0.05$ meV.[12] The hopping energy can be estimated as $J \approx 4He^{-\sqrt{2mH}d}$, where H is the effective depth of a trap, d is the distance between the traps, and m is the effective mass of EPs.[12] With the parameters $H = 5$ meV and $d = 1$ μm, we have $J \approx 0.14$ meV.

In our numerical investigation of the magnetic field effect on the Josephson oscillations, it is found that we should distinguish between two cases, i.e., the magnetic field below and above a critical value. When the magnetic field is small and below the critical value, the interactions between EPs play a more important role in the Josephson currents. Solving Eqs. (3)–(6) numerically, we obtain the extrinsic Josephson currents for both spins. The currents are time independent, as shown in Figs. 1(a) and 1(b), and this phenomenon is known as the dc Josephson effect. The phase differences Δ_σ and the condensate densities $\rho_{j\sigma}$ are all time independent but vary with the Zeeman energy. It is obvious that $|I^e_\uparrow| > |I^e_\downarrow|$ except $\Omega_z = 0$, and the difference between them becomes larger when

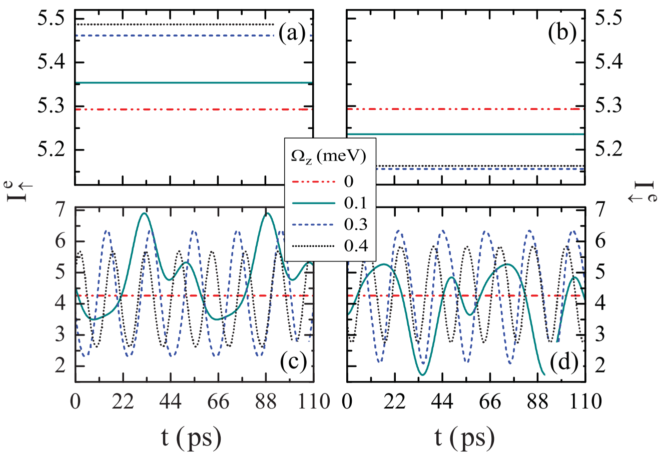

FIG. 1. (Color online) Time evolution of the extrinsic Josephson currents with the different Zeeman energies. In the upper row, the interactions between EPs are considered in the calculation, whereas there are no interactions in the lower row. The currents with spin $\uparrow$ and spin $\downarrow$ are shown in the left and right column, respectively. The parameters used are $F_{L\uparrow} = F_{L\downarrow} = 40$ meVμm^{-2} and $F_{R\uparrow} = F_{R\downarrow} = 30$ meVμm^{-2} in all the panels. The positive values mean that the Josephson currents are from the left to the right trap.

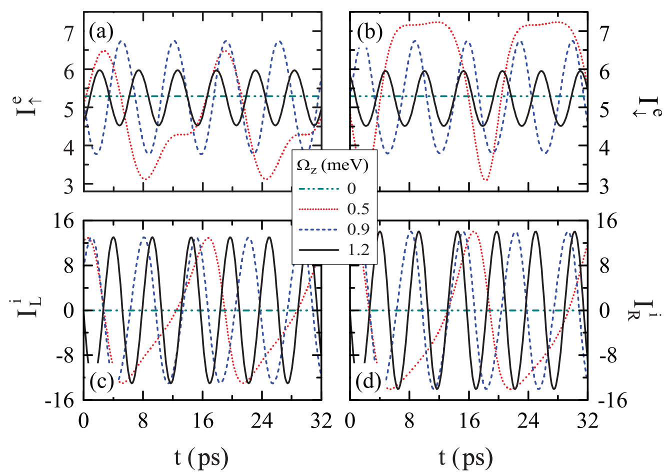

FIG. 2. (Color online) Extrinsic and intrinsic Josephson currents vs time with the different Zeeman energies in the upper and lower rows, respectively. The parameters used in the calculations are the same as in Fig. 1, and the interactions between EPs are considered in all the panels.

the magnetic field increases, because more EPs with the spin down will become the spin up in the condensate of each trap through the spin-flip process. It can be shown that the intrinsic Josephson currents are time-independent as well.

To find out the reason for the Josephson currents being time-independent with the above magnetic fields, we now neglect the interactions between EPs, and the results are shown in Figs. 1(c) and 1(d). The oscillations of the extrinsic Josephson currents appear for both spins except for $\Omega_z = 0$. For a fixed magnetic field, the frequencies of the oscillations are the same for EPs with the spin up and the spin down, but the frequency becomes higher if the magnetic field increases. When $\Omega_z = 0$, we have $I^e_\uparrow = I^e_\downarrow$ in the upper or lower row in Fig. 1, and the interactions increase the extrinsic Josephson currents by comparing Fig. 1(a) with Fig. 1(c). From Fig. 1, we conclude that the interactions can fully suppress the Zeeman splitting when the magnetic field is small, and there is no Josephson oscillations. This can be explained by the fact that the blue shifts, originating from the interactions, make the energy of EPs with the opposite spins equal. The Josephson oscillations reappear if the blue shifts are not considered. This suppression of the Zeeman splitting also embodies in the polarization of EPs in the condensate,[28,29] and a recent experimental work confirms this phenomenon.[27]

It is valuable to obtain the critical value of the Zeeman energy Ω_c, which has a connection with the interactions of EPs. The general criterion is derived from Eq. (5),

$$\zeta_j = \Omega_z - |(\alpha_1 - \alpha_2)(\rho_{j\downarrow} - \rho_{j\uparrow}) + (g_1 - g_2)(n_{j\downarrow} - n_{j\uparrow})|, \tag{9}$$

where only the dc Josephson effect appears if $\min(\zeta_L, \zeta_R) = 0$. Furthermore, we can find that $\zeta_L > \zeta_R$ if $\rho_{L\sigma} < \rho_{R\sigma}$, and then $\zeta_R = 0$ can give the critical Zeeman energy in the following expression:

$$\Omega_c = (\rho_{R\uparrow} - \rho_{R\downarrow}) \times \left[\alpha_1 - \alpha_2 - \frac{\beta F_{R\uparrow}(g_1 - g_2)}{(\gamma + \beta\rho_{R\uparrow})(\gamma + \beta\rho_{R\downarrow}) - W^2}\right]. \tag{10}$$

From it we obtain $\Omega_c = 0.45$ meV by using the parameters in Fig. 1.

When $\Omega_z > \Omega_c$, that is, the Zeeman splitting can not be fully suppressed by the interactions between EPs, different physical effects appear. For the extrinsic Josephson currents, the periodic oscillations are displayed, except for $\Omega_z = 0$, with the spin up and spin down in Figs. 2(a) and 2(b), respectively. As the magnetic field increases, the oscillation frequency becomes higher, and each fixed magnetic field corresponds to a certain oscillation frequency, which is analogous to the ac Josephson effect in a superconducting Josephson junction. Here the effect of a magnetic field is analogous to that of an electric bias applied on a conventional superconducting Josephson junction. The magnetic-field dependent frequencies of I^e_σ are the same for both spins, which are similar to the results in Figs. 1(c) and 1(d) without interactions. It is shown that the amplitudes of I^e_σ become small when the Zeeman energy rises. The reason mainly lies in the periodic oscillations of $\rho_{j\sigma}$, but $\sin\Delta_\sigma$ changes very little with increasing the magnetic field. The oscillations of $\rho_{L\sigma}$ and $\rho_{R\sigma}$ are almost synchronized, and the difference between them is small, as shown in Fig. 3. When the magnetic field goes up, the amplitude of $\rho_{j\sigma}$ becomes smaller, so the amplitude of I^e_σ reduces.

Once a magnetic field is applied, the intrinsic Josephson oscillations can also appear in each of the two traps, as can be seen in Figs. 2(c) and 2(d), respectively. The amplitudes of the intrinsic oscillations in the left or right trap are almost equal with different magnetic fields, and it can be explained by the fact that $\sqrt{\rho_{j\downarrow}\rho_{j\uparrow}}$ changes little with increasing the magnetic field, but the phase difference Δ_j can change from 0 to 2π with time. When $\rho_{L\uparrow}$ becomes largest, $\rho_{L\downarrow}$

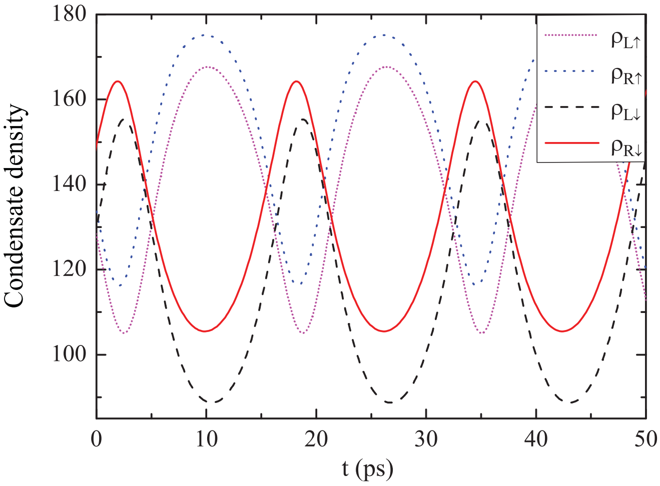

FIG. 3. (Color online) Time evolution of the condensate densities $\rho_{j\sigma}$ of EPs in the corresponding traps. The parameters used in the calculations are the same as in Fig. 1 and, particularly, $\Omega_z = 0.5$ meV.

almost gets to the minimum, and this is similar to $\rho_{R\sigma}$, as shown in Fig. 3. Obviously, the frequencies of the intrinsic Josephson oscillations are the same as those of the extrinsic Josephson oscillations. In other words, the intrinsic and extrinsic Josephson oscillations have the same frequency when the Zeeman energy is fixed, which can be demonstrated by Eq. (3). Therefore the Josephson frequency ω_J varies with the Zeeman energy, as shown in Fig. 4. When the Zeeman energy is small ($\Omega_z \leqslant \Omega_c$), $\omega_J = 0$. As the magnetic field goes up, ω_J increases with Ω_z linearly. When $\Omega_z = 0$, the intrinsic Josephson currents are zero in both traps, as shown in Figs. 2(c) and 2(d), which are different from the extrinsic Josephson currents. Neglecting all the interactions between EPs, we can get a simple expression for ω_J, that is,

$$\omega_J \approx \Omega_z, \tag{11}$$

as shown by the solid line in Fig. 4. It is understood that the interactions make the Josephson frequency decrease slightly, which agrees with the result in Ref. 16. Therefore the interactions between EPs play an important part in the Josephson oscillations when the existing magnetic field is small. If the magnetic field is large, the condensate densities and phase differences are insensitive to the interactions, which can be explained by the fact that the variation of the phase differences is mainly dependent on the Zeeman splitting, whereas the difference of blue shifts with the opposite spins plays a secondary role in each trap. By noting $\Omega_z = g\mu_B B$, it deserves to point out from Eq. (11) that the linear magnetic-field dependence of the Josephson frequency in an EP junction is somewhat similar to the linear voltage dependence of the Josephson frequency in a conventional superconducting junction.

To study further the EP Josephson oscillations, we define the polarization in each trap,

$$\eta_j = \frac{\rho_{j\uparrow} - \rho_{j\downarrow}}{\rho_{j\uparrow} + \rho_{j\downarrow}}, \tag{12}$$

and the population imbalance between the traps,

$$Z_\sigma = \frac{\rho_{L\sigma} - \rho_{R\sigma}}{\rho_{L\sigma} + \rho_{R\sigma}}. \tag{13}$$

When $\Omega_z = 0$, the polarizations are zero in both traps, and the population imbalances of the opposite spins are time independent, as shown in Fig. 5. If a magnetic field is applied, and $\Omega_z > \Omega_c$, the periodic oscillations of η_j and Z_σ appear, which result from the periodic oscillations of $\rho_{j\sigma}$. When $\Omega_z = 0.4$ meV, η_L is smallest at $t = 5.4$ ps in Fig. 5(a), but η_R gets to the maximum in Fig. 5(b), in other words, there are more EPs with the spin up in the right trap and more EPs with the spin

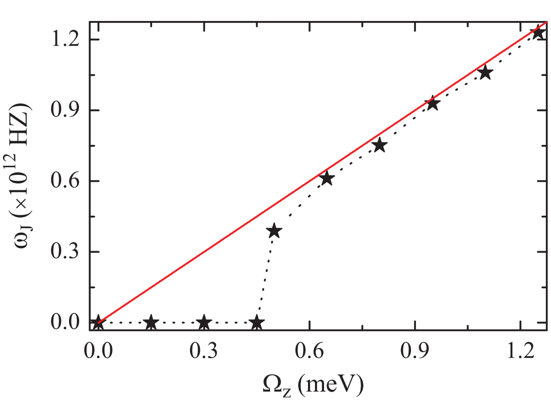

FIG. 4. (Color online) Josephson frequency ω_J vs the Zeeman energy Ω_z. The stars denote the Josephson frequencies calculated with the interactions between EPs. The solid line neglects all the interactions, and the dotted line connecting the stars is for comparison with the solid line. The parameters used in the calculations are the same as in Fig. 1.

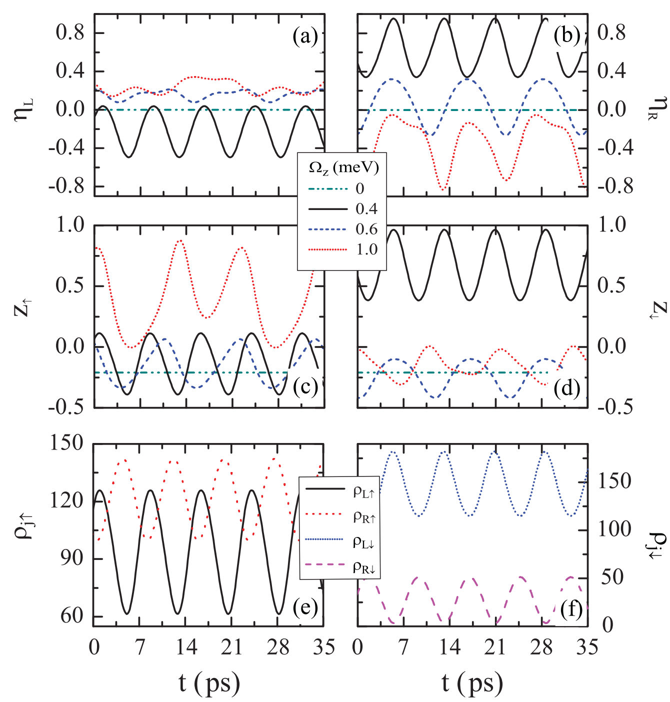

FIG. 5. (Color online) Time evolution of the polarization and population imbalance with the different Zeeman energies in the first and second rows, respectively; and the condensate densities $\rho_{j\downarrow}$ varying with time when $\Omega_z = 0.4$ meV in the third row. The parameters used are $F_{L\uparrow} = F_{L\downarrow} = 40$ meVμm^{-2} and $F_{R\uparrow} = F_{R\downarrow} = 14.2$ meVμm^{-2} in all the panels.

down in the left trap. Thus the spontaneous separation of EPs with the opposite spins is realized in the real space periodically. As the magnetic field increases, η_R becomes small because $\rho_{R\downarrow}$ becomes large through the tunneling from the left trap. From Fig. 5(c), we can find that the time evolution of $Z_\uparrow$ is not always positive with the different magnetic fields, and $\rho_{j\uparrow}$ is shown in Fig. 5(e) when $\Omega_z = 0.4$ meV, so EPs with the spin up are not self-trapped. However, the macroscopic quantum self-trapping occurs for EPs with the spin down, as shown by the solid line in Fig. 5(d), which originates from the interactions between EPs, inhibiting the large amplitude of oscillations.[15,32] Additionally, $\rho_{j\downarrow}$ is also shown in Fig. 5(f), and the tunneling of EPs is very small between the traps. Accordingly, by adjusting the magnetic field, we could obtain the Josephson oscillations with different frequencies and amplitudes. Moreover, the macroscopic quantum self-trapping for the spin down, as well as the spontaneous polarization separation in the traps periodically, can be realized with an appropriate magnetic field.

IV. CONCLUSION

We have studied an EP Josephson junction in a planar semiconductor microcavity with an external magnetic field applied normal to its plane. It is found that there is a competition between the Zeeman energy and the interactions of EPs, and a critical magnetic field can be determined. When the magnetic field is smaller than the critical value, there are time-independent extrinsic and intrinsic Josephson currents, i.e., the dc Josephson effect. This can be explained by the fact that the Zeeman splitting is fully suppressed by the interactions. On the other hand, when the magnetic field exceeds the critical value, the oscillations of the extrinsic and intrinsic Josephson currents occur with the same frequency, and this phenomenon is analogous to the voltage-modulated ac Josephson effect in a superconducting Josephson junction. The oscillation frequency becomes higher when the magnetic field rises. However, although the amplitudes of the intrinsic Josephson currents keep almost unchanged, the amplitudes of the extrinsic Josephson currents reduce when the magnetic field becomes larger. These facts can be explained by associating the variations of the amplitudes of condensate densities and their phase differences with the magnetic field. In addition, the oscillations of the polarization and population imbalance are periodic and the macroscopic quantum self-trapping as well as the spontaneous polarization separation, can be realized under appropriate conditions. It is believed that the magnetic field is a good modulating parameter to facilitate the experimental investigations of the EP Josephson effects in semiconductor microcavities.

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