Metamaterial-based model of the Alcubierre warp drive

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Electromagnetic metamaterials are capable of emulating many exotic space-time geometries, such as black holes, rotating cosmic strings, and the big bang singularity. This paper presents a metamaterial-based model of the Alcubierre warp drive and studies its limitations due to available range of material parameters. It appears that the material parameter range introduces strong limitations on the achievable "warp speed" so that ordinary magnetoelectric materials cannot be used. However, newly developed "perfect" bianisotropic nonreciprocal magnetoelectric metamaterials should be capable of emulating the physics of warp drive gradually accelerating up to $1/4c$.

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Metamaterial optics[1,2] greatly benefited from the field theoretical ideas developed to describe physics in curvilinear space-times.[3] An unprecedented degree of control of the local dielectric permittivity ϵ_{ik} and magnetic permeability μ_{ik} tensors in electromagnetic metamaterials has enabled numerous recent attempts to engineer highly unusual "optical spaces," such as electromagnetic black holes,[4–8] wormholes,[9] and rotating cosmic strings.[10] Phase transitions in metamaterials are also capable of emulating physical processes that took place during and immediately after the big bang.[11,12] These models can be informative for phenomena for which researchers have no direct experience and therefore limited intuition.

Since its original introduction by Alcubierre,[13] the warp drive space-time has become one of the most studied geometries in general relativity. In the simplest form, it can be described by the metric

$$ds^2 = c^2dt^2 - (dx - v(r)dt)^2 - dy^2 - dz^2, \qquad (1)$$

where $r = ((x - v_0t)^2 + y^2 + z^2)^{1/2}$ is the distance from the center of the "warp bubble," v_0 is the warp drive velocity, and $v = v_0 f(r)$. The function $f(r)$ is a smooth function satisfying $f(0) = 1$ and $f(r) \to 0$ for $r \to \infty$. This metric describes an almost flat spheroidal warp bubble, which is moving with respect to asymptotically flat external space-time with an arbitrary speed v_0. Such a metric bypasses the speed limitation due to special relativity: although nothing can move with a speed greater than the speed of light with respect to the flat background, space-time itself has no restriction on the speed with which it can be stretched. One example of fast stretching of space-time is given by the inflation theories, which demonstrate that immediately after the big bang our universe expanded exponentially during an extremely short period.

Unfortunately, when the space-time metric (Eq. (1)) is plugged into the Einstein's equations, it is apparent that exotic matter with negative energy density is required to build the warp drive. In addition, it was demonstrated that the eternal superluminal warp drive becomes unstable when quantum mechanical effects are introduced.[14] Another line of research deals with a situation in which a warp drive would be created at a very low velocity and gradually accelerated to large speeds. The physics of such a process are quite interesting.[15] The warp drive space-time cannot be reduced to a simple combination of white- and black-hole event horizons. Such a combination would be noncontroversial and "easy" to realize. The difference between the warp drive space-time and such a white-hole/black-hole combination is that the flat space-time region inside the warp bubble is moving as a whole with respect to the flat space-time outside the warp bubble [see the metric in Eq. (1)]. This nontrivial property of the warp drive space-time has led to conclusion that it cannot be realized even at subluminal speeds. Recently, it was demonstrated that even low-speed subluminal warp drives generically require energy condition–violating matter[16]: the T_{00} component of the energy-momentum tensor (the energy density distribution) appears to be negative even at subluminal speeds. Therefore, even subluminal warp drives appear to be prohibited by the laws of physics.

In this paper, I demonstrate that electromagnetic metamaterials are capable of emulating the warp drive metric [Eq. (1)]. Because energy conditions violations do not appear to be a problem in this case, metamaterial realization of the warp drive is possible. Our result is interesting because the body of evidence collected so far seems to indicate that the warp drives operating at any speed (even subluminal) were strictly prohibited by the laws of nature.

This paper explains what kind of metamaterial geometry is needed to emulate a laboratory model of the warp drive so that we can build a better understanding of the physics involved. It appears that the available range of material parameters introduces strong limitations on the possible "warp speed." Nevertheless, our results demonstrate that physics of a gradually accelerating warp drive can be modeled based on newly developed "perfect" magnetoelectric metamaterials.[17] Because even the low-velocity physics of warp drives is quite interesting,[15,16] such a lab model deserves further study.

To avoid unnecessary mathematical complications, consider a 1+1 dimensional warp drive metric of the form

$$ds^2 = (c/n_\infty)^2\, dt^2 - (dx - v_0 f(\tilde{x})dt)^2 - dy^2 - dz^2, \qquad (2)$$

where $\tilde{x} = (x - v_0t)$ and n_∞ is a scaling constant. In the rest frame of the warp bubble, it can be rewritten as

$$ds^2 = (c/n_\infty)^2dt^2 - (d\tilde{x} + v_0\tilde{f}(\tilde{x})dt)^2 - dy^2 - dz^2, \qquad (3)$$

where $\tilde{f}(0)=0$, and $\tilde{f}(\tilde{x}) \to 1$ for $\tilde{x} \to \pm\infty$. The resulting metric is

$$ds^2 = \left(\frac{1}{n_\infty^2} - \frac{v_0^2}{c^2}\tilde{f}^2(\tilde{x})\right)c^2dt^2 - d\tilde{x}^2 \\ -2v_0\tilde{f}(\tilde{x})d\tilde{x}dt - dy^2 - dz^2. \quad (4)$$

Following Ref. 18, Maxwell equations in this gravitational field can be written in the three-dimensional form as

$$\vec{D} = \frac{\vec{E}}{\sqrt{h}} + [\vec{H}\vec{g}],\ \vec{B} = \frac{\vec{H}}{\sqrt{h}} + [\vec{g}\vec{E}], \quad (5)$$

where $h = g_{00}$, and $g_\alpha = -g_{0\alpha}/g_{00}$. These equations coincide with the macroscopic Maxwell equations in a magnetoelectric material.[19] In the equivalent material,

$$\varepsilon = \mu = h^{-1/2} = \frac{1}{\sqrt{\frac{1}{n_\infty^2} - \frac{v_0^2}{c^2}\tilde{f}^2(\tilde{x})}}, \quad (6)$$

and the only nonzero component of the magnetoelectric coupling vector is

$$g_x = \frac{\frac{v_0}{c}\tilde{f}(\tilde{x})}{\frac{1}{n_\infty^2} - \frac{v_0^2}{c^2}\tilde{f}^2(\tilde{x})}. \quad (7)$$

In the subluminal $v_0 \ll c$ limit, Eqs. (6) and (7) become

$$\varepsilon = \mu \approx n_\infty\left(1 + \frac{v_0^2 n_\infty^2}{2c^2}\tilde{f}^2(\tilde{x})\right),\ g_x \approx n_\infty^2\frac{v_0}{c}\tilde{f}(\tilde{x}). \quad (8)$$

The magnetoelectric coupling coefficients in thermodynamically stable materials are limited by the inequality[20]:

$$g_x^2 \leqslant (\varepsilon - 1)(\mu - 1), \quad (9)$$

which means that a subluminal warp drive model based on the magnetoelectric effect must satisfy the inequality

$$\frac{v_0}{c}\tilde{f}(\tilde{x}) \leqslant \frac{n_\infty - 1}{n_\infty^2}. \quad (10)$$

This inequality demonstrates that although the "true" warp drive in vacuum ($n_\infty = 1$) is prohibited, $n_\infty > 1$ values in a material medium make a warp drive model thermodynamically stable—at least at subluminal speeds. This is an important result because the body of evidence collected so far seems to indicate that warp drives operating at any speed are strictly prohibited by the laws of nature. Equation (10) also provides an upper bound on the largest possible warp speed, which is achievable within the described metamaterial model. This upper bound is reached at $n_\infty = 2$ and equals $v_0 = 1/4c$. Therefore, at the least, we can build a toy model of a warp drive "operating" at $v_0 \sim 1/4c$. Coordinate dependence of the metamaterial parameters in such a model is shown in Fig. 1, assuming $\tilde{f}(\tilde{x}) = (1 + a^2/\tilde{x}^2)^{-1}$.

However, in classical magnetoelectric materials such as Cr_2O_3 and multiferroics, actual values of magnetoelectric susceptibilities are two orders of magnitude smaller than the limiting value described by Eq. (9),[21] so the warp drive model is impossible to make with ordinary materials. On the other hand, recently developed "perfect" magnetoelectric metamaterials,[17] which can be built based on such designs as split ring resonators and fishnet structures,[22] allow experimentalists to reach the limiting values described by Eq. (9) and make a lab model of the warp drive possible. Following Ref. 17, the effective susceptibilities of the split ring metamaterial can be written in the RLC-circuit model as

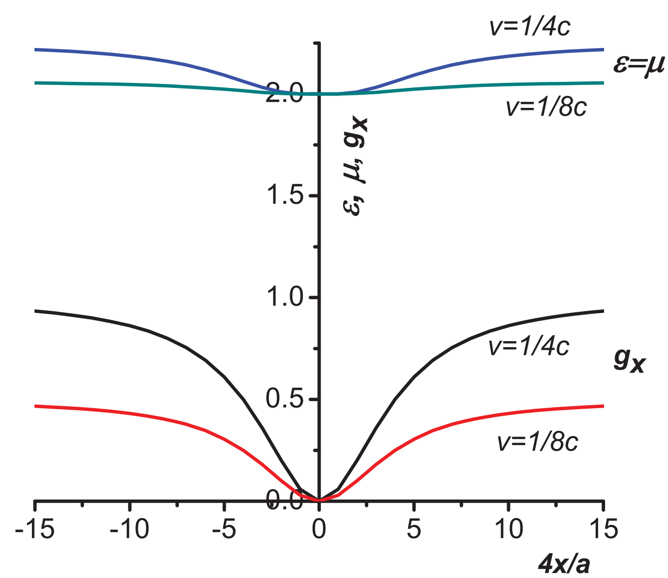

FIG. 1. (Color online) Spatial distributions of ε, μ, and g_x in the metamaterial model of a warp drive gradually accelerated up to $1/4c$.

$$\varepsilon = 1 + \frac{nCd^2\omega_0^2}{(\omega_0^2 - \omega^2 - i\omega\gamma)},\ \mu = 1 + \frac{nCS^2\omega^2\omega_0^2}{c^2(\omega_0^2 - \omega^2 - i\omega\gamma)}, \quad (11)$$

and

$$g = \frac{inCdS\omega\omega_0^2}{c(\omega_0^2 - \omega^2 - i\omega\gamma)}, \quad (12)$$

where n is the split ring density, d is the gap in the ring, S is the ring area, and C is the gap capacitance. These expressions explicitly demonstrate that the split ring metamaterial considered in Ref. 17 satisfies the upper bound given by Eq. (9) and therefore can be used as one of the building blocks in the metamaterial warp drive design. On the other hand, this particular split ring design cannot be used without modification, because this metamaterial is reciprocal.

An actual laboratory demonstration of a metamaterial warp drive space-time would require a nonreciprocal bianisotropic metamaterial in which both spatial and time-reversal symmetries are broken. In addition, the metamaterial loss issue has to be overcome. Because the issue of metamaterial loss compensation using gain media is well studied (e.g., see the recent experimental demonstration of loss compensation in a negative index metamaterial[23]), let us concentrate on the experimental ways of breaking spatial and time-reversal symmetries. Breaking the mirror $x \leftrightarrow -x$ symmetry is most easily achieved by deformation of the metamaterial, which can be easily done in one of the most popular split ring[17] or fishnet[22] metamaterial designs. As for breaking the time-reversal $t \leftrightarrow -t$ symmetry, there are two most natural ways to break this symmetry in solids: application of an external magnetic field[24,25] or spin-orbit interaction in a nonsymmorfic lattice.[26] In addition, such chiral superconductors as

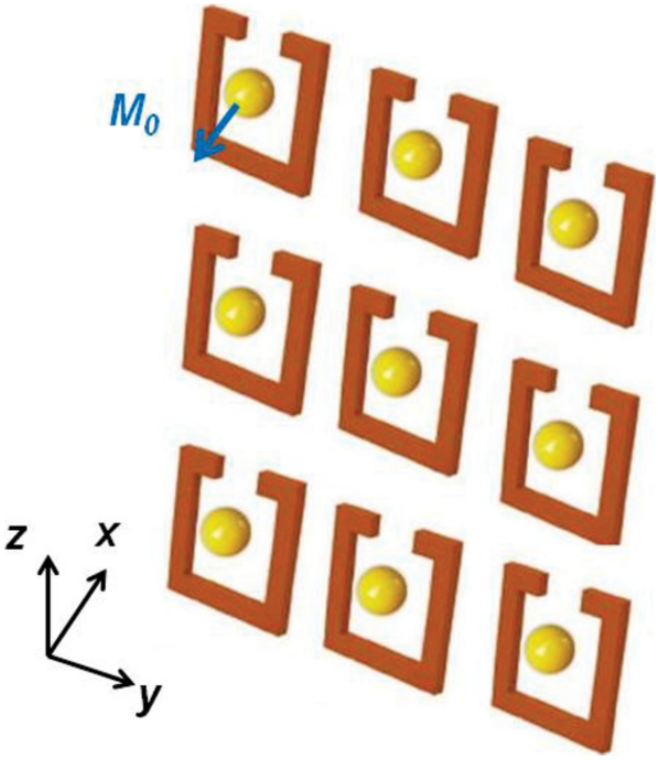

FIG. 2. (Color online) Example of a nonreciprocal bianisotropic metamaterial geometry, which explicitly violates spatial and time-reversal symmetries. An elementary unit of the split ring–based "perfect" magnetoelectric metamaterial design of Ref. 17 is supplemented with a magnetized ferrite particle. The particle is magnetized and shifted in the x direction with respect to the center of the split ring. The particle magnetization is proportional to the required g_x in a given location.

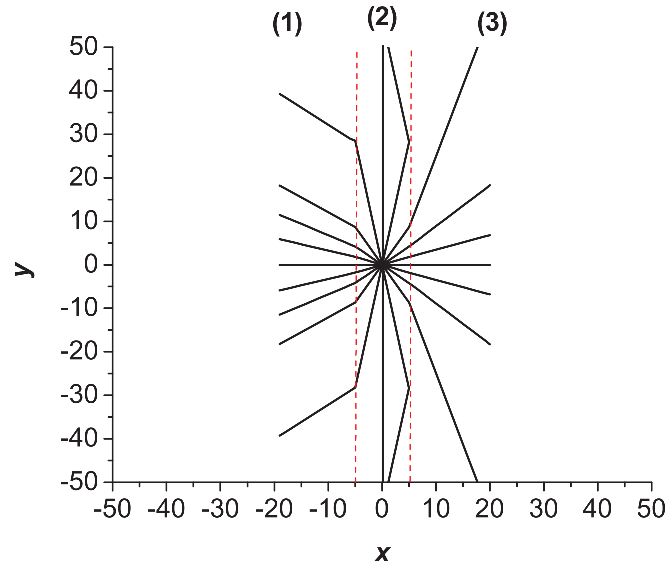

FIG. 3. (Color online) Light ray propagation inside the metamaterial model of the warp drive operating at $1/4c$. Rays are emitted by a point source located at the origin point (0,0,0) of the coordinate frame inside the warp bubble. Boundaries of the warp bubble are located at $x = \pm 5$. Metamaterial media 1 and 3 are identical.

Sr_2RuO_4[27] may be used in superconducting metamaterial designs.[28] Application of external electric and magnetic fields is known to break both spatial and time symmetries of such materials as methyl-cyclopentadienyl-Mn-tricarbonil molecular liquids,[24] thus creating an illusion of a moving (nonreciprocal bianisotropic) medium.[25] Experimental results of Ref. 24 demonstrate this behavior (however, the emulated "medium velocity" is very low, on the order of 50 nm/s[25]). Because utilization of magnetized particles, such as ferrites, is easily applicable in the metamaterial design, all ingredients necessary for experimental realization of the Alcubierre metric have been demonstrated in the experiment. Moreover, it was recently asserted[29,30] that material parameters, which are necessary to achieve a warp drive imitation in a nanostructured metamaterial, are possible.

Coming back to the split ring–based, "perfect" magnetoelectric metamaterial design implemented in Ref. 17, this section demonstrates how the time-reversal symmetry may be broken in this metamaterial. One of the possible metamaterial geometries is shown schematically in Fig. 2. An elementary unit of the split ring–based "perfect" magnetoelectric metamaterial design of Ref. 17 is supplemented with a magnetized ferrite particle. The particle is magnetized and shifted in the x direction with respect to the center of the split ring. Thus, this geometry explicitly violates spatial and time-reversal symmetries, resulting in a nonreciprocal bianisotropic metamaterial. As demonstrated in Ref. 31, near the ferromagnetic resonance frequency in such a metamaterial,

$$g \sim \frac{\omega_m \omega_0}{\omega_0^2 - \omega^2}, \tag{13}$$

where ω_0 is the ferromagnetic resonance frequency and $\omega_m = \gamma M_0$. Thus, in the design presented in Fig. 2, g_x is proportional to the particle magnetization M_0 in a given location, which explicitly demonstrate the nonreciprocal nature of this metamaterial design. Time reversal $t \leftrightarrow -t$ leads to change of sign of g_x. Recently, a somewhat-related metamaterial design was proposed[32] that emulates medium motion at an arbitrary speed. While demonstrating the proof of principle, the designs presented in Fig. 2 and Ref. 32 may only be considered a first step. Such complicated metamaterial designs typically contain many unwanted terms in ε, μ, and g, which must be carefully eliminated by iteration so that the ideal form of Eq. (8) may be achieved.

Light ray propagation inside the metamaterial model of the warp drive operating at $1/4c$ is illustrated in Fig. 3. Metamaterial medium 1 outside the warp bubble is engineered to have properties of a medium moving toward the warp bubble with the designed warp speed, whereas medium 3 is "moving" away from the bubble. In the reference frame moving with the warp speed, these media look exactly the same as medium 2 at rest. Ray trajectories were calculated assuming a steplike $\tilde{f}(\tilde{x})$ profile. Rays are emitted by a point source located at the origin point (0,0,0) of the coordinate frame inside the warp bubble (marked as medium 2 in Fig. 3). Boundaries of the warp bubble are located at $x = \pm 5$ (marked by the dashed lines in Fig. 3). At large-enough incidence angles, light rays originating inside the warp bubble cannot penetrate into medium 3 (in the hypothetical superluminal warp drive, this would be true for any incidence angle: this boundary would look like a white-hole event horizon). On the other hand, all light rays propagating toward the other boundary of the warp bubble can propagate into medium 1. Metamaterial medium 1 is identical to medium 3.

In conclusion, this paper presents a metamaterial-based model of the Alcubierre warp drive metric. It appears that the material parameter range introduces strong limitations on the achievable warp speed so that ordinary magnetoelectric materials cannot emulate the warp drive. However, newly developed "perfect" magnetoelectric bianisotropic nonreciprocal metamaterials should be capable of emulating the physics of a gradually accelerating warp drive, which can reach warp speeds of up to $1/4c$.

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