Spin-dependent inertial force and spin current in accelerating systems

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The spin-dependent inertial force in an accelerating system under the presence of electromagnetic fields is derived from the generally covariant Dirac equation. Spin currents are evaluated by the force up to the lowest order of the spin-orbit coupling in both ballistic and diffusive regimes. We give an interpretation of the inertial effect of linear acceleration on an electron as an effective electric field and show that mechanical vibration in a high-frequency resonator can create a spin current via the spin-orbit interaction augmented by the linear acceleration.

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I. INTRODUCTION

Studies of inertial effects on electrons have a long history dating back to the 1910s. Barnett investigated the magnetization induced by rotational motion.[1] Einstein and de Haas carried out the reverse experiment.[2] They measured the gyromagnetic ratio and the anomalous g factor of electrons before modern quantum physics was established. Stewart and Tolman estimated the electron mass by measuring the charge accumulation at the rim of a metal due to linearly accelerating motion.[3] Rapid progress in nanotechnology has allowed us to study the coupling of mechanical motion and electromagnetism in the quantum-mechanical regime. Effects of mechanical rotation on nanostructured magnetism are detected in microcantilevers[4,5] and torsional resonators.[6] The quantization of the rotational motion is observed in magnetic nanoparticles.[7] Single quantum excitations of vibration, namely, phonons, can be controlled in a piezoelectric acoustic wave resonator.[8] There has been theoretical work on effects of mechanical rotation on nanostructured magnetism,[9–15] the coupling of nanomechanical vibration and magnetism,[16–19] and the phonon-spin coupling related to spin relaxation.[20–26] However, the contribution of the spin-orbit interaction (SOI) in accelerating frames has not been thoroughly studied in previous papers.

Recent developments in spintronics,[27,28] which relies on not only an electron's charge but also its spin, have enabled us to utilize a "spin current," a flow of spins. In this context, the coupling of magnetization and spin current is of great interest in the field of spintronics.[29–32] To harness the spin current, understanding the spin-orbit interaction is indispensable. Recently, we proposed a fundamental theory describing the direct coupling of the mechanical rotation and spin current and predicted the spin-current generation arising from rotational motion.[33,34] Our finding is the first step to extending the theory of spin current in an inertial frame to that in a noninertial frame. In this paper, we provide a systematic approach to study spin-current generation from mechanical motion, including time-dependent rigid rotation and linear acceleration. First, we derive the spin-dependent inertial force in a rotating frame in the presence of an applied magnetic field. This force is responsible for the generation of a spin current due to mechanical rotation. Second, we investigate how linear acceleration generates a spin current. We show that nanomechanical vibration can create an ac spin current on the basis of the inertial spin-orbit coupling in accelerating systems.

The outline of the paper is the following. In Sec. II, we review the Dirac equation in a rotating frame. In Sec. III, the Pauli-Schrödinger equation in the rotating frame is derived. In Sec. IV, we derive the full expression for a spin-dependent force caused by mechanical rotation. Spin-current generation in the presence of impurity scattering is studied in Sec. V. Spin-current generation from mechanical vibration is investigated in Sec. VI. The paper ends with a few concluding remarks in Sec. VII and three Appendices. Appendix A contains a short summary of vierbein theory. Details of the Foldy-Wouthuysen-Tani transformation are given in Appendix B. Electromagnetic fields in a rotating frame are briefly summarized in Appendix C.

II. DIRAC EQUATION IN A ROTATING FRAME

In this section, we review the Dirac equation in a rotating frame. According to Einstein's principle of equivalence, gravitation can not be locally distinguished from inertial effects due to acceleration of the frame of reference. In general relativity, both gravitational and inertial effects are expressed by a metric and connection in a curved space-time. The fundamental equation of a spin-1/2 particle in curved space-time is the generally covariant Dirac equation[35]

$$\left[\gamma^\mu\left(\partial_\mu - \Gamma_\mu - \frac{iqA_\mu}{\hbar}\right) + \frac{mc}{\hbar}\right]\Psi = 0, \tag{1}$$

where $c,\hbar,q=-e$, and m are the velocity of light, the Planck constant, the charge and mass of an electron, $A_\mu = (A_0,\mathbf{A})$ is the U(1) gauge potential, and Γ_μ the spin connection.[35] The Clifford algebra in the curved space-time $\gamma^\mu = \gamma^\mu(x)$ satisfies

$$\{\gamma^\mu(x),\gamma^\nu(x)\} = 2g^{\mu\nu}(x), \tag{2}$$

where $g^{\mu\nu}(x)$ $(\mu,\nu = 0,1,2,3)$ is the inverse of the coordinate-dependent metric $g_{\mu\nu}(x)$. The coordinate transformation from a rigidly rotating frame to an inertial frame is given by

$$d\mathbf{r}' = d\mathbf{r} + (\mathbf{\Omega}\times\mathbf{r})dt, \tag{3}$$

where the rotation frequency with respect to an inertial frame is $\mathbf{\Omega}(t)$. Here, we assume that the rotation velocity $\mathbf{\Omega} \times \mathbf{r}$ is much less than the speed of light. The space-time line element in the rotating frame is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = [-c^2 + (\mathbf{\Omega} \times \mathbf{r})^2] dt^2 + 2(\mathbf{\Omega} \times \mathbf{r}) dt\, d\mathbf{r} + d\mathbf{r}^2. \quad (4)$$

Thus, the metric in the rotating frame becomes

$$g_{\mu\nu} = \begin{pmatrix} -1 + \mathbf{u}(x)^2 & u_x(x) & u_y(x) & u_z(x) \\ u_x(x) & 1 & 0 & 0 \\ u_y(x) & 0 & 1 & 0 \\ u_z(x) & 0 & 0 & 1 \end{pmatrix}, \quad (5)$$

with

$$\mathbf{u}(x) = \mathbf{\Omega}(t) \times \mathbf{r}/c. \quad (6)$$

This metric leads to the Clifford algebra and the spin connection in a rotating frame as

$$\gamma^0(x) = i\beta, \quad \gamma^i(x) = i\beta\alpha_i - u_i(x), \quad (7)$$

$$\Gamma_0 = \frac{\mathbf{\Omega} \cdot \mathbf{\Sigma}}{2c}, \quad \Gamma_i = 0, \quad (8)$$

where

$$\beta = \begin{pmatrix} I & O \\ O & -I \end{pmatrix}, \quad \boldsymbol{\alpha} = \begin{pmatrix} O & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & O \end{pmatrix} \quad (9)$$

are the Dirac matrices and $\mathbf{\Sigma}$ is the spin operator for a four-spinor defined by

$$\mathbf{\Sigma} = \frac{\hbar}{4i} \boldsymbol{\alpha} \times \boldsymbol{\alpha} = \frac{\hbar}{2} \begin{pmatrix} \boldsymbol{\sigma} & O \\ O & \boldsymbol{\sigma} \end{pmatrix} \quad (10)$$

with the Pauli matrix $\boldsymbol{\sigma}$ (details of the spin connection are given in Appendix A). From Eqs. (1)–(8), the Dirac equation in a rotating frame is written as

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi, \quad (11a)$$

$$H = \beta mc^2 + c\boldsymbol{\alpha} \cdot \boldsymbol{\pi} + qA_0 - \mathbf{\Omega} \cdot (\mathbf{r} \times \boldsymbol{\pi} + \mathbf{\Sigma}), \quad (11b)$$

where $\boldsymbol{\pi} = \mathbf{p} - q\mathbf{A}$ is the mechanical momentum and $\mathbf{r}$ is the position vector from the origin at the rotation axis.

In classical mechanics, the Hamiltonian in the rotating frame has the additional term $\mathbf{\Omega} \cdot (\mathbf{r} \times \boldsymbol{\pi})$, which reproduces the inertial effects: Coriolis, centrifugal, and Euler forces.[36] The term $\mathbf{\Omega} \cdot \mathbf{\Sigma}$ is called the spin-rotation coupling.[37–39] The last term of Eq. (11b), $\mathbf{\Omega} \cdot (\mathbf{r} \times \boldsymbol{\pi} + \mathbf{\Sigma})$, can be regarded as a quantum-mechanical generalization of the inertial effects obtained by replacing the mechanical angular momentum $\mathbf{r} \times \boldsymbol{\pi}$ with the total angular momentum $\mathbf{r} \times \boldsymbol{\pi} + \mathbf{\Sigma}$.

III. PAULI-SCHRÖDINGER EQUATION IN A ROTATING FRAME

The Dirac equation is an equation of a four-spinor wave function, which contains the up- and down-spin electron and positron components. As the energy gap between the electron and positron state is much larger than the energy levels of condensed matter systems, we take the low-energy limit of the Dirac equation to obtain the Pauli-Schrödinger equation of up- and down-spin electrons. Following the low-energy expansion and block diagonalization of the Dirac Hamiltonian developed by Foldy, Wouthuysen,[40] and Tani,[41] we derive the Pauli-Schrödinger equation in a rotating frame (see Appendix B for the details of the derivation). The Hamiltonian (11b) is divided into the block-diagonal and off-diagonal parts denoted by $\mathcal{E}$ and $\mathcal{O}$, respectively:

$$H = \beta mc^2 + \mathcal{E} + \mathcal{O}, \quad (12)$$

with

$$\mathcal{E} = qA_0 - \mathbf{\Omega} \cdot (\mathbf{r} \times \boldsymbol{\pi} + \mathbf{\Sigma}), \quad (13a)$$

$$\mathcal{O} = c\boldsymbol{\alpha} \cdot \boldsymbol{\pi}. \quad (13b)$$

By successive Foldy-Wouthuysen-Tani transformations, the Hamiltonian up to the order of $1/m^2$ becomes

$$H = \beta \left[mc^2 + \frac{\mathcal{O}^2}{2mc^2} \right] + \mathcal{E} - \frac{1}{8m^2c^4} [\mathcal{O}, [\mathcal{O}, \mathcal{E}] + i\hbar \dot{\mathcal{O}}]. \quad (14)$$

Neglecting the rest-energy term in Eq. (14), the Pauli-Schrödinger equation for the upper component of a Dirac spinor, namely, the two-component electron wave function, is obtained in the rotating frame as[33]

$$i\hbar \frac{\partial \psi}{\partial t} = H_{\mathrm{PR}} \psi, \quad (15)$$

$$H_{\mathrm{PR}} = H_{\mathrm{K}} + H_{\mathrm{Z}} + H_{\mathrm{I}} + H_{\mathrm{S}} + H_{\mathrm{D}}, \quad (16)$$

where

$$H_{\mathrm{K}} = \frac{1}{2m} \boldsymbol{\pi}^2 + qA_0, \quad (17a)$$

$$H_{\mathrm{Z}} = \mu_B \boldsymbol{\sigma} \cdot \mathbf{B}, \quad (17b)$$

$$H_{\mathrm{I}} = -\mathbf{\Omega} \cdot (\mathbf{r} \times \boldsymbol{\pi} + \mathbf{S}), \quad (17c)$$

$$H_{\mathrm{S}} = \frac{q\lambda}{2\hbar} \boldsymbol{\sigma} \cdot (\boldsymbol{\pi} \times \mathbf{E}' - \mathbf{E}' \times \boldsymbol{\pi}), \quad (17d)$$

$$H_{\mathrm{D}} = -\frac{q\lambda}{2} \mathrm{div} \mathbf{E}', \quad (17e)$$

with

$$\mu_B = \frac{q\hbar}{2m}, \quad \lambda = \frac{\hbar^2}{4m^2c^2}, \quad \mathbf{S} = \frac{\hbar}{2} \boldsymbol{\sigma}, \quad (18)$$

and

$$\mathbf{E}' = \mathbf{E} + (\mathbf{\Omega} \times \mathbf{r}) \times \mathbf{B}. \quad (19)$$

The Hamiltonian in Eq. (15), H_{PR}, is a 2×2 matrix operator, and ψ is the two-spinor wave function of a single electron.

A. Lowest order of the expansion

In the lowest order of the expansion, the Hamiltonian to the order of $1/m$ is given by $H_{\mathrm{K}} + H_{\mathrm{Z}} + H_{\mathrm{I}}$. The spin-independent H_{K} contains the kinetic energy and the potential energy. The Zeeman energy H_{Z} contains the g factor of the electron equal to 2. Combining H_{K} with H_{Z}, the coupling with the magnetic field,

$$\frac{q}{2m} (\mathbf{r} \times \boldsymbol{\pi} + 2\mathbf{S}) \cdot \mathbf{B} \quad (20)$$

is obtained, which contrasts with Eq. (17c): the mechanical rotation couples to the total angular momentum of the electron

$$\mathbf{r} \times \boldsymbol{\pi} + \mathbf{S}. \tag{21}$$

The inertial effects, namely, the Coriolis, centrifugal, and Euler forces, are reproduced by the first term of $H_{\rm I}$ as mentioned above. The second term of $H_{\rm I}$ is the spin-rotation coupling term. Introducing the "Barnett field"

$$\mathbf{B}_{\boldsymbol{\Omega}} = (m/q)\boldsymbol{\Omega}, \tag{22}$$

we can combine the spin-rotation coupling with the Zeeman term, leading to a different form:

$$\mu_B \boldsymbol{\sigma} \cdot (\mathbf{B} + \mathbf{B}_{\boldsymbol{\Omega}}). \tag{23}$$

Equation (23) shows that the spin-rotation coupling can be interpreted as a correction to the Zeeman effect with an effective magnetic field $\mathbf{B}_{\boldsymbol{\Omega}}$.

Previous theoretical work[9–15] has been done on the basis of the Barnett field. In the following sections, we study inertial effects on a spin current using the spin-orbit interaction $H_{\rm S}$, which is obtained from the second order of the expansion.

B. Second order of the expansion

The expansion of the order of $1/m^2$ yields the SOI and Darwin terms with the mechanical rotation $H_{\rm S}$ and $H_{\rm D}$. In the absence of rotation $\boldsymbol{\Omega} = \mathbf{0}$, these terms reproduce the conventional SOI and Darwin terms in the rest frame. In the presence of rotation, we find that the electric field $\mathbf{E}$ in the two terms in the inertial frame is modified by an additional term $(\boldsymbol{\Omega} \times \mathbf{r}) \times \mathbf{B}$. This result is consistent with a general coordinate transformation of electromagnetic fields between the rest frame and the rotating frame[42] (the derivation of the transformation is given in Appendix C).

C. Renormalization of SOI

The contribution of $H_{\rm S}$ to $H_{\rm I}$ in vacuum is negligible since the dimensionless spin-orbit coupling parameter

$$\eta_{\rm SO} = \frac{\lambda (mv)^2}{\hbar^2} = \left(\frac{v}{2c}\right)^2 \ll 1 \tag{24}$$

with the electron velocity v being much less than the speed of light. However, the spin-orbit coupling is enhanced in metals and semiconductors such as Pt.[45,46] The renormalization depends on detailed electronic structures and electron correlations.[47,48] In this paper, we do not go into detail about this procedure. Nevertheless, the results obtained in this paper are universal in nature. One can start with an effective Hamiltonian, e.g., a Luttinger[49] or Rashba[50] model, which treats the electromagnetic field (19) in a rotating frame. Replacing the momentum mv with the Fermi momentum $\hbar k_F$, the coupling $\eta_{\rm SO}$ becomes $\tilde{\lambda} k_F^2$, where $\tilde{\lambda}$ is an enhanced spin-orbit coupling parameter. The coupling $\eta_{\rm SO}$ of Pt is estimated as 0.59 by the nonlocal measurement of the spin Hall effect.[45,46] Electrons in a noninertial frame can not distinguish the inertial effect originating from the mechanical rotation $(\boldsymbol{\Omega} \times \mathbf{r}) \times \mathbf{B}$ in Eq. (17d) from conventional electric field $\mathbf{E}$. Thus, the new effect due to the SOI with the mechanical rotation can be significant in large SOI systems, as shown in the following sections.

IV. SPIN-DEPENDENT INERTIAL FORCE IN A ROTATING FRAME

Let us consider semiclassical equations of motion for an electron based on the Pauli-Schrödinger equation in a rotating frame. A quantum-mechanical analog of "force" $\mathcal{F}$ is defined by

$$\mathcal{F} = \frac{1}{i\hbar}[m\dot{\mathbf{r}}, H_{\rm PR}] + m\frac{\partial \dot{\mathbf{r}}}{\partial t}, \tag{25}$$

with $\dot{\mathbf{r}} = [\mathbf{r}, H_{\rm PR}]/i\hbar$. From Eq. (15), the spin-dependent velocity including the effect of mechanical rotation is obtained as

$$\dot{\mathbf{r}} = \mathbf{v} + \mathbf{v}_{\rm I} + \mathbf{v}_{\sigma}, \tag{26}$$

with

$$\mathbf{v} = \frac{1}{i\hbar}[\mathbf{r}, H_{\rm K}] = \frac{\boldsymbol{\pi}}{m}, \tag{27a}$$

$$\mathbf{v}_{\rm I} = \frac{1}{i\hbar}[\mathbf{r}, H_{\rm I}] = -\boldsymbol{\Omega} \times \mathbf{r}, \tag{27b}$$

$$\mathbf{v}_{\sigma} = \frac{1}{i\hbar}[\mathbf{r}, H_{\rm S}] = \frac{e\lambda}{\hbar}\boldsymbol{\sigma} \times \mathbf{E}'. \tag{27c}$$

The "force" $\mathcal{F} = \mathcal{F}_0 + \mathcal{F}_1 + \mathcal{F}_2 + \mathcal{F}_t$ is obtained as

$$\mathcal{F}_0 = q[\mathbf{E}' + \mathbf{v} \times (\mathbf{B} + 2\mathbf{B}_{\Omega})] + m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}), \tag{28a}$$

$$\begin{aligned}\mathcal{F}_1 = & -\frac{q^2\lambda}{\hbar}\{(\boldsymbol{\sigma} \times \mathbf{E}') \times (\mathbf{B} + \mathbf{B}_{\Omega}) - [(\mathbf{B} + \mathbf{B}_{\Omega}) \times \boldsymbol{\sigma}] \times \mathbf{E}'\} \\ & + \frac{qm\lambda}{\hbar}[\boldsymbol{\sigma} \cdot (\boldsymbol{\Omega} \times \mathbf{v})\mathbf{B} + 2(\mathbf{B} \cdot \boldsymbol{\Omega})\boldsymbol{\sigma} \times \mathbf{v} - (\mathbf{B} \cdot \mathbf{v})\boldsymbol{\sigma} \times \boldsymbol{\Omega} \\ & + \boldsymbol{\Omega} \cdot (\mathbf{r} \times \mathbf{B})\boldsymbol{\sigma} \times \boldsymbol{\Omega} - (\mathbf{B} \cdot \boldsymbol{\Omega})\boldsymbol{\sigma} \times (\boldsymbol{\Omega} \times \mathbf{r})],\end{aligned} \tag{28b}$$

$$\begin{aligned}\mathcal{F}_2 = & \frac{mq^2\lambda^2}{\hbar^2}\bigg(\frac{2}{\hbar}(\boldsymbol{\sigma} \cdot \mathbf{E}')m\mathbf{v} \times \mathbf{E}' \\ & + i[(\boldsymbol{\sigma} \cdot \mathbf{E}')\mathbf{B} \times \boldsymbol{\Omega} - (\boldsymbol{\sigma} \cdot \mathbf{B})\mathbf{E}' \times \boldsymbol{\Omega} + (\mathbf{B} \cdot \boldsymbol{\Omega})\mathbf{E}' \times \boldsymbol{\sigma}] \\ & + (\mathbf{E}' \times \mathbf{B}) \times \boldsymbol{\Omega} + 2(\mathbf{B} \cdot \boldsymbol{\Omega})\mathbf{E}'\bigg),\end{aligned} \tag{28c}$$

$$\mathcal{F}_t = m\mathbf{r} \times \frac{\partial \boldsymbol{\Omega}}{\partial t} + \frac{qm\lambda}{\hbar}\boldsymbol{\sigma} \times \left[\left(\mathbf{r} \times \frac{\partial \boldsymbol{\Omega}}{\partial t}\right) \times \mathbf{B}\right]. \tag{28d}$$

The spin-independent $\mathcal{F}_0$ consists of the electromagnetic force in a rotating frame $q(\mathbf{E}' + \mathbf{v} \times \mathbf{B})$, Coriolis force $q\mathbf{v} \times 2\mathbf{B}_{\Omega} = 2m\mathbf{v} \times \boldsymbol{\Omega}$, and centrifugal force $m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$. The first term in $\mathcal{F}_t$ is the Euler force. The other terms in $\mathcal{F}$ with the spin operator $\boldsymbol{\sigma}$ are responsible for spin-dependent transport of electrons. The full expression of the spin-dependent force in a rotating frame in the presence of electromagnetic fields is one of the principal results of this paper. In the absence of rotation $\boldsymbol{\Omega} = \mathbf{0}$, the above expression of $\mathcal{F}$ reproduces the previous results in an inertial frame.[51]

To have a better understanding of the spin-dependent force, we study the case of $\boldsymbol{\Omega} = (0,0,\Omega), \mathbf{B} = (0,0,B)$, $\mathbf{E} = \mathbf{0}, |\mathbf{B}|/|\mathbf{B}_{\Omega}| \gg 1$ and neglect the terms of the order of $\eta_{\rm SO}^2, |\Omega/\omega_c|^2$ with the cyclotron frequency

$$\omega_c = \frac{qB}{m}. \tag{29}$$

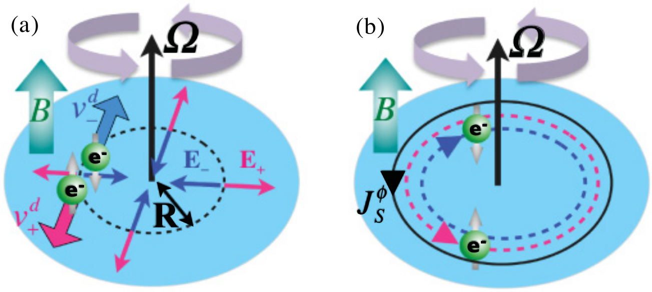

FIG. 1. (Color online) (a) Spin-dependent electric field $\mathbf{E}_\sigma$ and drift velocity $\mathbf{v}_\sigma^{\rm d}$ are illustrated. An external magnetic field $\mathbf{B}$ is applied along the rotation axis (z direction). For the z-polarized spins, the electric field $\mathbf{E}_+$ ($\mathbf{E}_-$) is induced in the radial outward (inward) direction. (b) The drift velocities $\mathbf{v}_\pm^{\rm d}$ in opposite directions result in the spin current J_s^ϕ in the azimuthal direction.

$\mathcal{F}$ is decomposed into the xy and z components $\mathcal{F}_\perp$ and $\mathcal{F}_\parallel$. Thus, we have

$$\mathcal{F}_\perp \approx q(\mathbf{E}_\mathbf{r} + \mathbf{E}_\sigma + \mathbf{v}\times\mathbf{B}), \tag{30}$$

with

$$\mathbf{E}_\mathbf{r} = (\mathbf{B}\cdot\boldsymbol{\Omega})\mathbf{r}, \tag{31a}$$

$$\mathbf{E}_\sigma = -\frac{q\lambda}{\hbar}(\mathbf{B}\cdot\boldsymbol{\sigma})(\mathbf{B}\cdot\boldsymbol{\Omega})\mathbf{r}. \tag{31b}$$

Here, $\mathbf{E}_\mathbf{r}$ is an electric field induced in the rotating frame with an applied magnetic field $\mathbf{B}$, and $\mathbf{E}_\sigma$ is an "effective spin-dependent electric field" induced by the SOI $H_{\rm S}$. Coexistence of electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ yields the $\mathbf{E}\times\mathbf{B}$ drift, which is the motion of the guiding center of a charged particle with the drift velocity[43]

$$\mathbf{v}^{\rm d} = \frac{\mathbf{E}\times\mathbf{B}}{B^2}. \tag{32}$$

From Eq. (30), we obtain two types of drift motion for an electron wave packet: one is the charge drift motion

$$\mathbf{v}_c^{\rm d} = \frac{\mathbf{E}_\mathbf{r}\times\mathbf{B}}{B^2}, \tag{33}$$

and the other is the spin-dependent drift motion

$$\mathbf{v}_\sigma^{\rm d} = \frac{\mathbf{E}_\sigma\times\mathbf{B}}{B^2}. \tag{34}$$

Figure 1(a) illustrates the relation of the rotation, magnetic field, induced spin-dependent field, and drift velocity. In a ballistic regime, the latter produces the spin current in the azimuthal direction

$$\mathbf{J}_s = en{\rm Tr}\sigma_z\mathbf{v}_\sigma^{\rm d} = 2ne\kappa\omega_c R\mathbf{e}_\phi, \tag{35}$$

where R is the distance from the rotation axis, $\mathbf{e}_\phi$ the unit azimuthal vector, n the electron density, and the dimensionless parameter

$$\kappa = \tilde{\lambda}k_F^2\cdot\frac{\hbar\Omega}{\epsilon_F} \tag{36}$$

with Fermi energy ϵ_F.[33] Setting $B = 1$ T, $\Omega = 1$ kH, $\tilde{\lambda}k_F^2 \approx 0.6$, $k_F \approx 10^{10}$ m, and $R = 10$ mm, $|\mathbf{J}_s|$ is estimated to be about 10^8 A/m^2.

Next, we consider fluctuation of the rotating axis, caused by a high-speed rotor in operation. Such fluctuation effects on spin current come from the $\boldsymbol{\Omega}$ dependence of $\mathbf{E}_\sigma$ in Eq. (31b) as well as the time-derivative of $\boldsymbol{\Omega}$ in the second term of $\mathcal{F}_t$. If the rotation frequency has a time-dependent written component as

$$\boldsymbol{\Omega}(t) = \boldsymbol{\Omega}_0 + \delta\boldsymbol{\Omega}(t), \tag{37}$$

where

$$\delta\boldsymbol{\Omega}(t) = \delta\boldsymbol{\Omega}_\perp(t) + \delta\boldsymbol{\Omega}_\parallel(t), \tag{38}$$

with $\delta\boldsymbol{\Omega}_\perp \perp \boldsymbol{\Omega}_0$, $\delta\boldsymbol{\Omega}_\parallel \parallel \boldsymbol{\Omega}_0$, and $|\delta\boldsymbol{\Omega}| \ll |\boldsymbol{\Omega}|$, a time-dependent drift motion is induced:

$$\delta\mathbf{v}_\sigma^{\rm d} = \frac{\delta\mathbf{E}_\sigma\times\mathbf{B}}{B^2} \tag{39}$$

with

$$\delta\mathbf{E}_\sigma = -\frac{q\lambda}{\hbar}(\boldsymbol{\sigma}\cdot\mathbf{B})(\mathbf{B}\cdot\delta\boldsymbol{\Omega})\mathbf{r}, \tag{40}$$

and the fluctuation of the spin current is obtained as

$$\delta\mathbf{J}_s(t) = \frac{2ne^2\lambda BR}{\hbar}\delta\boldsymbol{\Omega}_\parallel(t)\mathbf{e}_\phi. \tag{41}$$

The time derivative of Eq. (38) is divided into the xy and z components

$$\frac{\partial\delta\boldsymbol{\Omega}}{\partial t} = \left(\frac{\partial\delta\boldsymbol{\Omega}}{\partial t}\right)_\perp + \left(\frac{\partial\delta\boldsymbol{\Omega}}{\partial t}\right)_\parallel. \tag{42}$$

From the second term of $\mathcal{F}_t$, we have an additional effective spin-dependent electric field created in the xy plane $\delta\mathbf{E}'_\sigma$ given by

$$\delta\mathbf{E}'_\sigma(t) = \frac{qm\lambda}{\hbar}(\boldsymbol{\sigma}\cdot\mathbf{B})\mathbf{r}\times\left(\frac{\partial\delta\boldsymbol{\Omega}}{\partial t}\right)_\parallel. \tag{43}$$

This yields the spin current in the azimuthal direction

$$\delta\mathbf{J}'_s(t) = \frac{2nem\lambda R}{\hbar}\left(\frac{\partial\delta\boldsymbol{\Omega}}{\partial t}\right)_\parallel\mathbf{e}_\phi. \tag{44}$$

The ratio $|\delta J'_s/\delta J_s|$ is equal to $|\frac{\partial\delta\Omega}{\partial t}|/|\delta\Omega\omega_c|$. The time scale of the fluctuation of rotation is usually much smaller than that of the cyclotron frequency. Thus, the contribution from $\mathcal{F}_t$ to the spin-current fluctuation is negligible.

V. EFFECTS OF IMPURITY SCATTERING

In this section, we discuss spin-current generation in the presence of impurity scattering. We consider a Pt thin film attached to a rotating disk with a uniform magnetic field parallel to the rotation axis, as shown in Fig. 2. Spin-dependent transport in a system with a strong SOI can be described by semiclassical equations[52]

$$\dot{\mathbf{r}} = \mathbf{v} + \mathbf{v}_{\sigma 0}, \tag{45a}$$

$$\hbar\dot{\mathbf{k}} = -e\,(\mathbf{E} + \dot{\mathbf{r}}\times\mathbf{B}). \tag{45b}$$

Here, $\mathbf{v} = \hbar\mathbf{k}/m$ represents the normal velocity, $\mathbf{k}$ is the wave vector, and $\mathbf{E}$ and $\mathbf{B}$ are applied electric and magnetic fields. The anomalous velocity $v_{\sigma 0}$, originating from the SOI in an inertial frame, is written as

$$\mathbf{v}_{\sigma 0} = \frac{e\lambda}{\hbar}\boldsymbol{\sigma}\times\mathbf{E}. \tag{46}$$

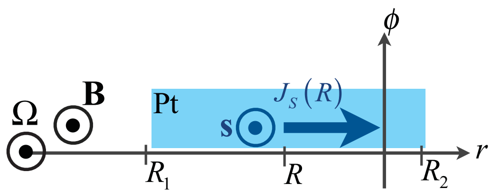

FIG. 2. (Color online) The z-polarized radial spin current is converted to the inverse spin Hall voltage in the azimuthal direction $\mathbf{e}_\phi$. The rotation frequency $\mathbf{\Omega}$ and the magnetic field $\mathbf{B}$ are applied along the z axis. The z-polarized spin current $J_s(R)$ is induced in the radial direction $\mathbf{e}_r$. Here, the spin polarization vector is denoted by $\mathbf{s}$.

It is straightforward to extend the equations in an inertial frame to a rotating frame using Eqs. (25) and (26):

$$\dot{\mathbf{r}} = \mathbf{v} + \mathbf{v}_{\rm I} + \mathbf{v}_\sigma, \tag{47a}$$

$$\hbar\dot{\mathbf{k}} = \mathcal{F}. \tag{47b}$$

We consider spin-current generation in a rotating normal metal with large spin-orbit coupling such as Pt in the presence of spin-independent impurity scattering. The electron distribution function depends on spins because of the spin dependence of the semiclassical equations (47). In this case, the transport equation of nonequilibrium steady states is written as

$$\dot{\mathbf{r}} \cdot \frac{\partial f_\sigma}{\partial \mathbf{r}} + \dot{\mathbf{k}} \cdot \frac{\partial f_\sigma}{\partial \mathbf{k}} = -\frac{f_\sigma - f_0}{\tau}, \tag{48}$$

where $f_\sigma = f_\sigma(\mathbf{r},\mathbf{k})$ is the spin-dependent distribution function, $f_0 = f_0(\varepsilon)$ is the Fermi-Dirac distribution function, and τ is the relaxation time.

Combining the semiclassical equations (47) with Eq. (48) and setting $\mathbf{E} = \mathbf{0}$, we obtain

$$\mathcal{F} = \mathcal{F}_\perp \approx -e[\mathbf{E}_\mathbf{r} + \mathbf{E}_\sigma + \mathbf{v} \times \mathbf{B}], \tag{49}$$

the solution of which is

$$f_\sigma = f_0 + e\mathbf{v} \cdot \tau \frac{\mathbf{E}'_\sigma + \tau\boldsymbol{\omega}_c \times \mathbf{E}'_\sigma}{1 + (\tau\omega_c)^2} \frac{\partial f_0}{\partial \varepsilon}, \tag{50}$$

with

$$\boldsymbol{\omega}_c = \frac{e\mathbf{B}}{m} \tag{51}$$

and

$$\mathbf{E}'_\sigma = \mathbf{E}_\mathbf{r} + \mathbf{E}_\sigma. \tag{52}$$

The solution (50) contains two "electric fields": the spin-independent part $\mathbf{E}_\mathbf{r}$ and the spin-dependent part $\mathbf{E}_\sigma$ as discussed in Sec. IV. The spin-independent part yields the conventional Hall effect in the rotating frame. If the ends in the radial (longitudinal) direction of a Pt film attached to the rotating disk (see Fig. 2) are electrically connected, the Hall voltage is obtained in the azimuthal (transverse) direction, while the spin-dependent part causes the spin-current generation.[53]

The z-polarized spin current generated by the mechanical rotation can be evaluated by

$$\mathbf{J}_s = -e \int d\mathbf{k}\, \mathrm{Tr}\,[\sigma_z f_\sigma(\mathbf{r},\mathbf{k})\dot{\mathbf{r}}]\,, \tag{53}$$

which leads to the explicit form of the spin current

$$\mathbf{J}_s(R) = J_s^r(R)\mathbf{e}_r + J_s^\phi(R)\mathbf{e}_\phi, \tag{54}$$

with

$$J_s^r = \frac{\tau\omega_c}{1 + (\tau\omega_c)^2} J_s^0, \tag{55a}$$

$$J_s^\phi = \frac{(\tau\omega_c)^2}{1 + (\tau\omega_c)^2} J_s^0. \tag{55b}$$

Here, $\mathbf{e}_r(\mathbf{e}_\phi)$ is the unit vector of the radial (azimuthal) direction as indicated in Fig. 2, and J_s^0 is given by

$$J_s^0(R) = 2ne\kappa\omega_c R. \tag{56}$$

In the large $\omega_c\tau$ limit, the radial spin current J_s^r vanishes, and we have

$$J_s^\phi \to J_s^0. \tag{57}$$

This reproduces the ballistic case, Eq. (35). Putting $\omega_c\tau \ll 1$, the radial spin current becomes much larger than the azimuthal one:

$$J_s^r \gg J_s^\phi. \tag{58}$$

VI. LINEAR ACCELERATION

In this section, we discuss spin-current generation by linear acceleration in the absence of electromagnetic fields. The Dirac Hamiltonian in a linearly and rotationally accelerating frame without electromagnetic fields was derived by Hehl and Ni (Ref. 39):

$$H = \beta mc^2 + c\boldsymbol{\alpha} \cdot \mathbf{p} + \frac{1}{2c}[(\mathbf{a} \cdot \mathbf{r})(\mathbf{p} \cdot \boldsymbol{\alpha}) + (\mathbf{p} \cdot \boldsymbol{\alpha})(\mathbf{a} \cdot \mathbf{r})] + \beta m(\mathbf{a} \cdot \mathbf{r}) - \mathbf{\Omega} \cdot (\mathbf{L} + \mathbf{S}), \tag{59}$$

with the linear acceleration $\mathbf{a}$ and the angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$. In the low-energy limit of this Hamiltonian up to the order of $1/m^2$, one has the Hamiltonian of the Pauli-Schrödinger equation of the electron's two-spinor wave function in the accelerating frame[39]

$$H = \frac{\mathbf{p}^2}{2m} + m\mathbf{a} \cdot \mathbf{r} - \mathbf{\Omega} \cdot (\mathbf{L} + \mathbf{S}) + \frac{\hbar}{4mc^2}\boldsymbol{\sigma} \cdot (\mathbf{a} \times \mathbf{p}), \tag{60}$$

where the rest-mass energy and the redshift effect of the kinetic energy[39] are neglected. We note that the last term, the so-called "inertial spin-orbit interaction," does not contain the mechanical rotation because of the absence of the magnetic field. It should be emphasized that the inertial effect of the linear acceleration on the electron can be interpreted as an "effective electric field." Introducing the "electric field"

$$\mathbf{E}_\mathbf{a} = (m/q)\mathbf{a}, \tag{61}$$

the inertial SOI can be rewritten as

$$H_{\mathrm{S},\mathbf{a}} = \frac{\lambda}{\hbar}\boldsymbol{\sigma} \cdot (\mathbf{p} \times q\mathbf{E}_\mathbf{a}). \tag{62}$$

Together with the second term of Eq. (60), $m\mathbf{a} \cdot \mathbf{r} = q\mathbf{E}_\mathbf{a} \cdot \mathbf{r}$, the inertial effects of the linear acceleration without rotation on the large spin-orbit interacting system can be analyzed by the same framework as the conventional Hamiltonian with SOI:

$$H = \frac{\mathbf{p}^2}{2m} + U + \frac{\lambda}{\hbar}\sigma \cdot \left[\frac{\partial U}{\partial \mathbf{r}} \times \mathbf{p}\right] \tag{63}$$

with the potential $U = q\mathbf{E_a} \cdot \mathbf{r}$.

The electron velocity in the linearly accelerating frame is given by

$$\dot{\mathbf{r}} = \frac{\mathbf{p}}{m} + \mathbf{v}_{\sigma,\mathbf{a}}, \tag{64}$$

where

$$\mathbf{v}_{\sigma,\mathbf{a}} = \frac{e\lambda}{\hbar}\boldsymbol{\sigma} \times \mathbf{E_a}. \tag{65}$$

We focus on the inertial effects of linear acceleration. The anomalous velocity $\mathbf{v}_{\sigma,\mathbf{a}}$ yields the mechanical analog of the spin Hall effect in a ballistic regime. The i-polarized spin current ($i = \{x,y,z\}$) generated by the linear acceleration is estimated as

$$\mathbf{J}_s^i = en\mathrm{Tr}[\sigma_i \mathbf{v}_{\sigma,\mathbf{a}}] = \frac{2nem\lambda}{\hbar}\mathbf{s}_i \times \mathbf{a}. \tag{66}$$

When the acceleration is induced by the harmonic oscillation with the frequency $\omega_\mathbf{a}$ and amplitude u in the x direction as

$$\mathbf{a} = u\omega_\mathbf{a}^2 e^{i\omega_\mathbf{a} t}\mathbf{e}_x, \tag{67}$$

the z-polarized spin current is created in the y direction

$$\mathbf{J}_s^z = \frac{2nem\lambda}{\hbar}u\omega_\mathbf{a}^2 e^{i\omega_\mathbf{a} t}\mathbf{e}_y. \tag{68}$$

Assuming the Pt film vibrates with $\omega_\mathbf{a} = 10$ GHz and $u = 10$ nm, the ac spin current is estimated to be $J_s^z \approx 10^7$ A/m^2 (see Fig. 3). It is a future challenge to probe the ac spin current in a high-frequency mechanical resonator.[54] In such a noninertial system, it is straightforward to extend the results in the ballistic regime to those in the diffusive regime using a well-established framework of the spin Hall effect by replacing the usual electric field $\mathbf{E}$ with the effective one $\mathbf{E_a}$.

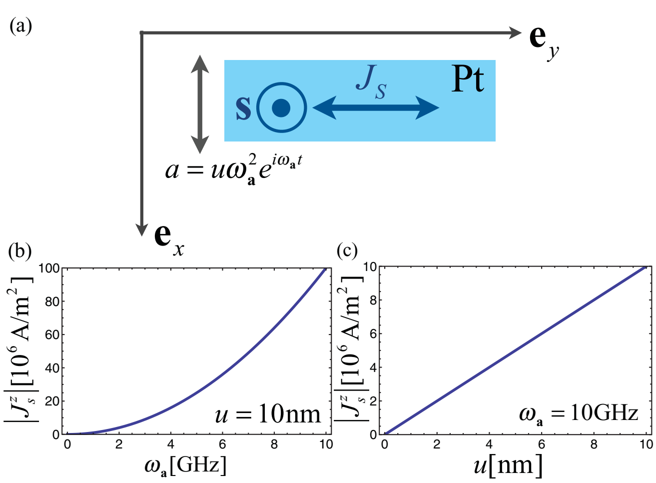

FIG. 3. (Color online) Spin-current generation in a linearly accelerating frame. (a) When a Pt film is attached to a mechanical resonator and vibrates along the x axis, the z-polarized ac spin current is created in the y direction. (b) The amplitude of J_s^z is plotted as a function of $\omega_\mathbf{a}$ at $u = 10$ nm. (c) The amplitude is plotted as a function of u at $\omega_\mathbf{a} = 10$ GHz.

VII. CONCLUSION

In this paper, we have investigated theoretically the generation of spin currents in both rotationally and linearly accelerating systems. The explicit form of the spin-dependent inertial force acting on electrons in a rotating frame in the presence of electromagnetic fields was derived from the generally covariant Dirac equation. We have shown that the force is responsible for the generation of spin currents by mechanical rotation in the first order of the spin-orbit coupling. For future experimental analysis, we discussed the effect of fluctuation of the rotation axis on the spin current using the time-dependent part of the force.

We have also studied the spin-current generation from the mechanical oscillation. The spin current can be created in a uniformly oscillating conductor with a large spin-orbit coupling because of the inertial spin-orbit interaction originating from the linear acceleration. We gave a concise interpretation of the inertial effect of linear acceleration on an electron as an effective electric field. This allows us to use the conventional theory of the spin Hall effect to describe the spin-current generation due to the linear acceleration by simply substituting the effective electric field for an ordinary one.

The framework proposed here offers a new route to study the inertial effects on electron transport phenomena, leading to an innovative combination of microelectromechanical systems (MEMS) and spintronics.

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APPENDIX A: METRIC, VIERBEIN, AND SPIN CONNECTION IN A ROTATING FRAME

In Sec. II, we use the vierbein representation of the Dirac equation. The relation of the matrices between curved and flat space-time is given by the vierbein (or tetrad) field $e^\mu_{(\alpha)}(x)$, the indices μ and (α) of which label the curved space-time coordinates and local flat space-time coordinates, respectively. The vierbein is a local orthonormal base $\{e^\mu_{(\alpha)}\}_{\alpha=0,1,2,3}$ and a kind of square root of the metric tensor $g^{\mu\nu}(x)$:

$$e^\mu_{(\alpha)}(x)\eta^{\alpha\beta}e^\mu_{(\beta)} = g^{\mu\nu}(x), \tag{A1}$$

with the Lorentz metric $\eta^{\mu\nu} = \mathrm{diag}(-1,1,1,1)$. Starting with the metric tensor

$$g_{00} = -1 + u^2, \quad g_{0i} = g_{i0} = u_i, \quad g_{ij} = \delta_{ij}, \tag{A2}$$

the inverse tensor is

$$g^{00} = -1, \quad g^{0i} = g^{i0} = u_i, \quad g^{ij} = \delta_{ij} - u_i u_j, \tag{A3}$$

where $u_i = (\mathbf{\Omega} \times \mathbf{r}/c)_i$ and $u^2 = u_1^2 + u_2^2 + u_3^2$. The vierbein in the frame is obtained from Eqs. (A1) and (A3):

$$e^0_{(0)} = 1, \quad e^0_{(j)} = 0,$$
$$e^i_{(0)} = -u_i, \quad e^i_{(j)} = \delta^i_j. \tag{A4}$$

We also have the inverse of the vierbein $e^{(\alpha)}_\mu(x) = g_{\mu\nu}(x)\eta^{\alpha\beta}e^\nu_{(\beta)}$:

$$e^{(\alpha)}_0 = \delta^\alpha_0 + \eta^{\alpha i}u_i, \quad e^{(\alpha)}_i = \delta^\alpha_i. \tag{A5}$$

The Clifford algebra in curved space-time can be written as $\gamma^\mu(x) = g^{\mu\nu}(x)e^{(\alpha)}_\nu(x)\bar{\gamma}_\alpha$ with $\bar{\gamma}_0 = i\beta, \bar{\gamma}_i = -i\beta\alpha_i$. Thus, we have the algebra in the rotating frame

$$\gamma^0(x) = -\bar{\gamma}_0 = -i\beta,$$
$$\gamma^i(x) = u_i\bar{\gamma}_0 + \bar{\gamma}_i = i\beta u_i - i\beta\alpha_i, \tag{A6}$$

where

$$\beta = \begin{pmatrix} I & O \\ O & -I \end{pmatrix}, \quad \boldsymbol{\alpha} = \begin{pmatrix} O & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & O \end{pmatrix}. \tag{A7}$$

In the vierbein representation, the spin connection $\Gamma_\mu(x)$ is expressed as

$$\Gamma_\mu(x) = -\tfrac{1}{4}\bar{\gamma}_\alpha\bar{\gamma}_\beta e^{(\alpha)}_\nu g^{\nu\lambda}\left(\partial_\mu e^{(\beta)}_\lambda - \Gamma^\sigma_{\mu\lambda}e^{(\beta)}_\sigma\right), \tag{A8}$$

where the affine connection $\Gamma^\lambda_{\mu\nu}$ is defined by

$$\Gamma^\lambda_{\mu\nu} = \tfrac{1}{2}g^{\lambda\sigma}(\partial_\nu g_{\sigma\mu} + \partial_\mu g_{\sigma\nu} - \partial_\sigma g_{\mu\nu}). \tag{A9}$$

Substituting (A2) into (A9),

$$\Gamma^0_{00} = \Gamma^0_{i0} = \Gamma^0_{ij} = \Gamma^i_{jk} = 0,$$
$$\Gamma^i_{00} = \frac{\epsilon_{ijk}\Omega_j u_k}{c} + \partial_0 u_i, \quad \Gamma^i_{j0} = -\frac{\epsilon_{ijk}\Omega_k}{c}. \tag{A10}$$

From the affine connection, Clifford algebra, and vierbein in the rotating frame, we arrive at the spin (spinor) connection

$$\Gamma_0(x) = \frac{\bar{\gamma}_i\bar{\gamma}_j\epsilon_{ijk}\Omega_l}{4c} = \frac{i\mathbf{\Sigma}\cdot\mathbf{\Omega}}{2c}, \tag{A11a}$$
$$\Gamma_i(x) = 0. \tag{A11b}$$

APPENDIX B: FOLDY-WOUTHUYSEN-TANI TRANSFORMATION

In Sec. II, we derive Pauli-Schrödinger equation in a rotating frame by Foldy-Wouthuysen-Tani transformations (FWTTs). The details of the derivation are shown below. FWTT is a unitary transformation and a block-diagonalization method of the Dirac Hamiltonian. First of all, we define the even and odd parts of the Hamiltonian as

$$H = \beta mc^2 + \mathcal{E} + \mathcal{O}, \tag{B1}$$

with

$$\mathcal{E} = qA_0 - \mathbf{\Omega}\cdot(\mathbf{r}\times\boldsymbol{\pi} + \mathbf{\Sigma}), \tag{B2a}$$
$$\mathcal{O} = c\boldsymbol{\alpha}\cdot\boldsymbol{\pi}. \tag{B2b}$$

The even part $\mathcal{E}$ is the block-diagonal part of the Hamiltonian, whereas the odd part $\mathcal{O}$ is the block–off-diagonal part. FWTT is defined by

$$H' = UHU^\dagger - U\partial_t U^\dagger \tag{B3}$$

with

$$U = \exp\left(-\frac{i\beta\mathcal{O}}{2mc^2}\right) = \exp\left(-\frac{i\beta c\boldsymbol{\alpha}\cdot\boldsymbol{\pi}}{2mc^2}\right). \tag{B4}$$

A low-energy expansion of the Hamiltonian H' into an exponential series of $1/m$ gives a systematic expansion in such a way that odd parts of H' vanish in any order. Up to the order of $1/m^2$, Eq. (B3) is reduced to

$$H' = \beta\left[mc^2 + \frac{\mathcal{O}^2}{2mc^2}\right] + \mathcal{E} - \frac{1}{8m^2c^4}[\mathcal{O},[\mathcal{O},\mathcal{E}] + i\hbar\dot{\mathcal{O}}]. \tag{B5}$$

Using the relation

$$\alpha_i\alpha_j = \delta_{ij} + i\epsilon_{ijk}\sigma_k, \tag{B6a}$$
$$[\pi_i,\pi_j] = i\hbar q\epsilon_{ijk}B_k, \tag{B6b}$$

the second term of Eq. (B5) becomes

$$\frac{\beta\mathcal{O}^2}{2mc^2} = \frac{\beta\alpha_i\alpha_j\pi_i\pi_j}{2m} = \beta\left(\frac{\boldsymbol{\pi}^2}{2m} - \frac{q\hbar}{2m}\boldsymbol{\sigma}\cdot\mathbf{B}\right). \tag{B7}$$

Next, we focus on $[\mathcal{O},[\mathcal{O},\mathcal{E}] + i\hbar\dot{\mathcal{O}}]$:

$$\begin{aligned}[\mathcal{O},\mathcal{E}] + i\hbar\dot{\mathcal{O}} &= [c\boldsymbol{\alpha}\cdot\boldsymbol{\pi}, -qA_0] + c\boldsymbol{\alpha}\cdot(-iq\hbar)\partial_t\mathbf{A} \\ &\quad + [c\boldsymbol{\alpha}\cdot\boldsymbol{\pi}, -\mathbf{\Omega}\cdot(\mathbf{\Sigma} + \mathbf{r}\times\boldsymbol{\pi})] \\ &= -i\hbar cq\boldsymbol{\alpha}\cdot(-\nabla A_0 - \partial_t\mathbf{A}) \\ &\quad - i\hbar c\boldsymbol{\alpha}\cdot(\mathbf{\Omega}\times\boldsymbol{\pi}) + i\hbar c\boldsymbol{\alpha}\cdot(\mathbf{\Omega}\times\boldsymbol{\pi}) \\ &\quad - i\hbar cq\boldsymbol{\alpha}\cdot(\mathbf{\Omega}\times\mathbf{r})\times\mathbf{B} = i\hbar c\boldsymbol{\alpha}\cdot q\mathbf{E}'\end{aligned} \tag{B8}$$

with

$$\mathbf{E}' = -\nabla A_0 - \partial_t\mathbf{A} + (\mathbf{\Omega}\times\mathbf{r})\times\mathbf{B}. \tag{B9}$$

Thus, we have

$$\begin{aligned}&-\frac{1}{8m^2c^4}[\mathcal{O},[\mathcal{O},\mathcal{E}] + i\hbar\dot{\mathcal{O}}] \\ &\quad = -\frac{1}{8m^2c^4}[c\boldsymbol{\alpha}\cdot\boldsymbol{\pi}, -i\hbar c\boldsymbol{\alpha}\cdot q\mathbf{E}'] = \frac{i\hbar}{8m^2c^2}\alpha_i\alpha_j[\pi_i, qE'_j] \\ &\quad = \frac{q\hbar^2}{8m^2c^2}\mathrm{div}\mathbf{E}' + \frac{q\hbar}{8m^2c^2}\boldsymbol{\sigma}\cdot(\boldsymbol{\pi}\times\mathbf{E}' - \mathbf{E}'\times\boldsymbol{\pi}).\end{aligned} \tag{B10}$$

From Eqs. (B7) and (B10), we obtain the Hamiltonian H_{PR} in Sec. III by neglecting the rest-energy term.

APPENDIX C: ELECTROMAGNETIC FIELDS IN A ROTATING FRAME

In this appendix, we give a brief review of the general relativistic transformation of electromagnetic fields.

Electromagnetic fields in the rotating frame are related to those in the rest frame by

$$\mathbf{E}' = \mathbf{E} + (\mathbf{\Omega}\times\mathbf{r})\times\mathbf{B}, \tag{C1a}$$
$$\mathbf{B}' = \mathbf{B}, \tag{C1b}$$

when the rotation velocity is much less than the speed of light $|\mathbf{\Omega}\times\mathbf{r}| \ll c$, where $\mathbf{E}'$ and $\mathbf{B}'$ are electromagnetic fields in the rotating frame.

Equations (C1) are not Lorentz transformations in special relativity but a general coordinate transformation in general relativity. The Lorentz transformations of the electromagnetic fields are written as[43]

$$\mathbf{E}''/c = \gamma(\mathbf{E}/c + \boldsymbol{\beta}\times\mathbf{B}) - \frac{\gamma^2}{\gamma+1}(\boldsymbol{\beta}\cdot\mathbf{E}/c)\boldsymbol{\beta}, \quad \text{(C2a)}$$

$$\mathbf{B}'' = \gamma(\mathbf{B} - \boldsymbol{\beta}\times\mathbf{E}/c) - \frac{\gamma^2}{\gamma+1}(\boldsymbol{\beta}\cdot\mathbf{B})\boldsymbol{\beta}, \quad \text{(C2b)}$$

with

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \qquad \boldsymbol{\beta} = \frac{\mathbf{v}_0}{c}. \quad \text{(C3)}$$

Here, $\mathbf{E}''$ and $\mathbf{B}''$ are the electromagnetic fields in the inertial frame, which has a uniform velocity $\mathbf{v}_0$ relative to the rest frame. Replacing $\mathbf{v}_0$ with $\mathbf{v}(x) = \boldsymbol{\Omega}\times\mathbf{r}$ in Eqs. (C2) can never reproduce the correct transformation between the rotating frame and the rest frame (C1). The special relativistic transformations (C2) have an apparent symmetry for $\mathbf{E}/c$ and $\mathbf{B}$, whereas the relations (C1) do not. Such an asymmetry in (C1) originates from the space-time asymmetry of a general coordinate transformation that relates physical quantities in a rotating frame to those in a rest frame.

When the rotation axis is parallel to the z axis, the transformation between the rest frame and the rotating frame is expressed as

$$x'^{\mu} = L^{\mu}_{\nu} x^{\nu} \quad \text{(C4)}$$

with

$$L^{\mu}_{\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Omega t & \sin\Omega t & 0 \\ 0 & -\sin\Omega t & \cos\Omega t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \text{(C5)}$$

where rotating coordinates carry the prime. Equation (C4) leads to a general coordinate transformation[42,43]

$$R = \frac{\partial x^{\alpha}}{\partial x'^{\beta}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \Omega y/c & \cos\Omega t & \sin\Omega t & 0 \\ -\Omega x/c & \sin\Omega t & \cos\Omega t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad \text{(C6)}$$

According to the principle of general covariance, electromagnetic fields are components of a second-rank tensor in general coordinate transformations. Electromagnetic tensors in the rest frame F and those in the rotating frame F' are related by

$$F' = LR^T F R L^T, \quad \text{(C7)}$$

in which T denotes the transpose matrix, and

$$F = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & B_z & -B_y \\ E_y/c & -B_z & 0 & B_x \\ E_z/c & B_y & -B_x & 0 \end{pmatrix}. \quad \text{(C8)}$$

Equations (C7) and (C8) lead to

$$E'_x = E_x + x\Omega B_z, \quad \text{(C9a)}$$
$$E'_y = E_y + y\Omega B_z, \quad \text{(C9b)}$$
$$E'_z = E_z - \Omega(xB_x + yB_y), \quad \text{(C9c)}$$
$$B'_x = B_x, \quad \text{(C9d)}$$
$$B'_y = B_y, \quad \text{(C9e)}$$
$$B'_z = B_z. \quad \text{(C9f)}$$

Thus, we obtain the relations (C1) in Sec. III B. The relations shown in this section hold for $|\boldsymbol{\Omega}\times\mathbf{r}| \ll c$. Generalized relations for electromagnetic fields in a more rapidly rotating frame should include a Lorentz factor $\gamma = 1/\sqrt{1-(\boldsymbol{\Omega}\times\mathbf{r}/c)^2}$.[42] We omit discussion on Maxwell's equations in a rotating frame. The subject is originally studied by means of generally covariant Maxwell's equations.[44]

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