Nondeterministic ultrafast ground-state cooling of a mechanical resonator

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We present a feasible scheme for the cooling of a mechanical resonator via projective measurements on an auxiliary flux qubit which interacts with it. We find that ground-state cooling can be achieved with *several* random-time-interval measurements. The cooling efficiency hardly depends on the time intervals between any two consecutive measurements. The cooling scheme is also robust against environmental noise.

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I. INTRODUCTION

In the quantum regime, ground-state cooling of a small thermal object is an intriguing challenge and one of the most desirable quantum technologies. Physically, the ground-state cooling process can be formulated as a *transformation* from a thermal state of the small object into its ground state. The transformation is irreversible and cannot be performed when the object is isolated.

A mechanical resonator (MR) is a small macroscopic mechanical object, which can behave as a single-mode harmonic oscillator with a high frequency and a high quality factor. The physical realization of its quantum ground state, usually coupled to an auxiliary setup, has become more and more important in ultrahigh-precision measurements, classical to quantum transitions, preparations of nonclassical states, and quantum information processing.[1–3] Throughout the years, a considerable number of optomechanical[4–13] and electromechanical[14–19] schemes have been proposed for achieving ground-state cooling of MRs. Examples are bang-bang cooling through a Cooper pair box[15] or single-shot state-swapping cooling via a superconductor.[18] Recently, some of us[13] proposed a ground-state cooling scheme of an MR in an optomechanical system by controlling the optical drives. The best-studied ground-state cooling protocol for MRs is sideband cooling[4–6,8–12] designed originally for cooling atomic spatial motion. Theoretically, an MR could be cooled down to its ground state in the resolved sideband limit, i.e., reach a mean phonon number less than 1. Sideband cooling is now widely used for MR cooling experiments, and its recent record for the mean phonon number is about (or less than) 3.8.[19] Since ground-state cooling of MRs is difficult to achieve, new cooling approaches are required.

This work presents an alternate protocol for MR cooling using projective measurements on an auxiliary qubit. While it may be used in any stage of the cooling process of an MR, our protocol is aimed at ground-state cooling. Our study starts with cooling, and it uses repeated equal-time-interval measurements on the qubit, introduced for purification of quantum states[20] or measurement-based entanglement generation.[21,22]

While repeated equal-time-interval measurements work well for ground-state cooling, we find, unexpectedly, that cooling efficiency is significantly better when measurements are taken randomly. The ground state can be reached in several random-time-interval measurements in a very short time. We give an explanation for this unexpected phenomenon. These results suggest that our protocol is completely robust against measurement operational errors. In addition, the scheme is also robust against environmental noise.

II. MODEL

We employ a gradiometer-type flux qubit[23–25] as our auxiliary qubit, though our scheme may be applicable to any two-level system coupled to an MR. Figure 1 is a schematic diagram of our cooling setup. The doubly-clamped MR (the red bar in Fig. 1) is embedded in a flux-qubit circuit, which is composed of three superconducting loops with four Josephson junctions (JJs). An in-plane magnetic field B_0 induces qubit-MR coupling via a Lorentz force.[26] The top-right blue (bottom green) part is to measure (operate) the qubit, as described later.

The Hamiltonian of the flux qubit can be written as

$$H_q = \frac{\hbar\Delta}{2}\sigma_x + \frac{\hbar\epsilon}{2}\sigma_z, \quad (1)$$

where σ_z and σ_x are the Pauli matrices in the basis of two persistent current states $|\uparrow\rangle$ and $|\downarrow\rangle$.[27,28] Here, $\hbar\Delta$ is the tunneling amplitude between the two states. The bias energy $\hbar\epsilon$ depends linearly on the external flux bias and in our case is set to zero by pretrapping one flux quantum Φ_0 in the loop.[23,24]

The MR is modeled as a single-mode harmonic oscillator with a high-Q mode of frequency ω_m and an effective mass m. The entire system is characterized by the Hamiltonian[26]

$$H = H_q + \hbar\omega_m a^\dagger a - \hbar g(a + a^\dagger)\sigma_z, \quad (2)$$

where a and $a^\dagger$ are annihilation and creation operators, respectively, for the MR. The last term denotes the interaction between the MR and the flux qubit. The coupling constant is $g = B_0 I_p L_0 x_0/\hbar$, with B_0 the magnitude of the in-plane magnetic field, I_p the magnitude of the persistent current in

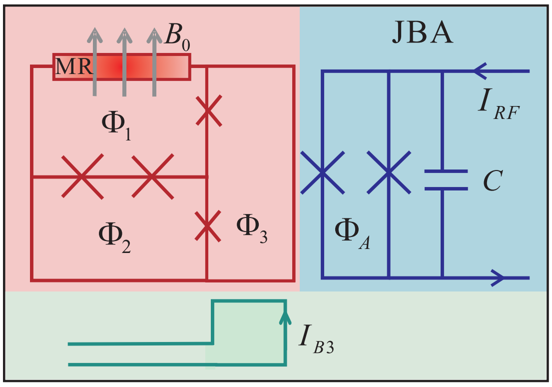

FIG. 1. (Color online) Schematic diagram of the circuit. The top-left part (red) is the coupled-flux-qubit-MR system, where each cross denotes a Josephson junction, and the bar denotes a doubly-clamped MR. The interaction between the MR and the flux qubit is modulated by an in-plane magnetic field B_0. The top-right part (blue) is a Josephson bifurcation amplifier (JBA) formed by a dc superconducting quantum interference device (SQUID) shunted by a capacitance. The bottom part (green) is a bias to control the qubit energy gap and the coupling strength between the JBA and the qubit.

the loop, L_0 the length of the MR, and $x_0 = \sqrt{\hbar/(2m\omega_m)}$ the zero-point displacement of the MR.

Here we consider an MR with fundamental mode frequency $\omega_m/2\pi \sim 100$ MHz. The qubit is tuned into resonance or near resonance with the MR by monitoring the tunneling Δ.[24,29] The coupling constant $g/2\pi$, e.g., of the order of MHz, is much smaller than the qubit frequency, such that the rotating wave approximation can be used to reduce the Hamiltonian (2) to the standard Jaynes-Cummings (JC) one ($\hbar = 1$),

$$H = \omega_m a^\dagger a + \frac{\Delta}{2}\tilde{\sigma}_z + g(a\tilde{\sigma}_+ + a^\dagger\tilde{\sigma}_-), \tag{3}$$

where $\tilde{\sigma}_{z,\pm}$ are the Pauli operators in the new basis of ground and excited states, $|g/e\rangle = (|\downarrow\rangle - / + |\uparrow\rangle)/\sqrt{2}$.

The operator $\hat{N}_c = a^\dagger a + |e\rangle\langle e|$ in the JC model is conserved, such that H in Eq. (3) can be represented by the direct sum of a one-dimensional block in the basis $|0,g\rangle$ and two-dimensional submatrices in pairs of bases $|n,g\rangle$ and $|n-1,e\rangle$, when $n \geqslant 1$. The Hamiltonian (3) can therefore be diagonalized,

$$H = -\frac{\Delta}{2}|0,g\rangle\langle 0,g| + \sum_{n\geqslant 1}\sum_{s=\pm}\varepsilon_n^s|ns\rangle\langle ns|, \tag{4}$$

where the dressed eigenstates are

$$\begin{aligned}|n+\rangle &= \cos\theta_n|n-1,e\rangle + \sin\theta_n|n,g\rangle,\\ |n-\rangle &= \sin\theta_n|n-1,e\rangle - \cos\theta_n|n,g\rangle\end{aligned} \tag{5}$$

and the corresponding eigenvalues are

$$\varepsilon_n^\pm = \left(n - \tfrac{1}{2}\right)\omega_m \pm \tfrac{1}{2}\sqrt{(\Delta-\omega_m)^2 + 4g^2 n}. \tag{6}$$

Here θ_n satisfies the equation

$$\tan 2\theta_n = \frac{2g\sqrt{n}}{\Delta - \omega_m}, \tag{7}$$

for $n \geqslant 1$.

III. UNITARY EVOLUTION AND REPEATED MEASUREMENTS

Initially, we prepare the whole system in a separable state, $\rho_0 = |g\rangle\langle g| \otimes \rho_m$. Here ρ_m is the thermal state of the MR (or could also be an arbitrary state, where the MR ground state $|0\rangle$ is included). We then perform repeated but random unequal-time-interval measurements (UTIMs) on the flux qubit. The whole system evolves under the JC Hamiltonian (3) in between the measurements. The jth measurement is assumed to take place at the random time instant $t_j = j\tau + \delta t_j$, where τ is a given time interval, δt_js are random variations in time generated by uniformly distributed random numbers in the region $(-\tau/2,\tau/2)$, and $t_0 (= \delta t_0 \equiv 0)$ is the initial time. When all $\delta t_j \equiv 0$, the repeated process is reduced into equal-time-interval measurements (ETIMs) formulated in Ref. 20. After N such ETIMs on the qubits, and if all measurement outcomes are $|g\rangle$, the density matrix of the MR becomes

$$\rho_m^{(\tau)}(N) = V_g^N(\tau)\rho_m V_g^{\dagger N}(\tau)/P_g^{(\tau)}(N), \tag{8}$$

where

$$P_g^{(\tau)}(N) = \mathrm{Tr}\left[V_g^N(\tau)\rho_m V_g^{\dagger N}(\tau)\right] \tag{9}$$

is the survival probability.[20,21] Here

$$V_g(\tau) \equiv \langle g|e^{-iH\tau/\hbar}|g\rangle \tag{10}$$

is a non-Hermitian *effective evolution operator* only acting on the MR.

Here $V_g(\tau)$ for the model (3) is diagonal in the basis $\{|n\rangle\}$,

$$V_g(\tau) = \sum_{n\geqslant 0}\lambda_n(\tau)|n\rangle\langle n|, \tag{11}$$

where the eigenvalues are

$$\lambda_0 = e^{i\Delta\tau/2} \tag{12}$$

and

$$\lambda_n = e^{-i(n-1/2)\omega_m\tau}(\cos\Omega_n\tau + i\sin\Omega_n\tau\cos 2\theta_n) \tag{13}$$

with

$$\Omega_n = \sqrt{(\Delta-\omega_m)^2/4 + g^2 n}, \tag{14}$$

for $n \geqslant 1$. By carefully selecting the time interval τ, such that $\sin^2\Omega_n\tau \neq 0$ (for $n \geqslant 1$), all the values

$$|\lambda_n(\tau)| = \sqrt{1 - \sin^2\Omega_n\tau\sin^2 2\theta_n} \tag{15}$$

can be made less than 1, while $|\lambda_0(\tau)|$ is always equal to 1.

In the general case of the UTIMs, one has in the large-N limit

$$V_g(\tau_1)V_g(\tau_2)\cdots V_g(\tau_N) = \sum_{n\geqslant 0}\prod_{j=1}^{N}|\lambda_n(\tau_j)||n\rangle\langle n| \to |0\rangle\langle 0|, \tag{16}$$

where $\tau_j = \tau + \delta t_j - \delta t_{j-1}$ is the unequal time interval, and $|\lambda_0(t)| = 1 \geqslant |\lambda_n(t)|$ for $n \geqslant 1$ and arbitrary time t. All the density matrix elements of the MR will vanish, except that of ground state $|0\rangle$,

$$\rho_m^{(\tau)}(N) = \sum_{n\geqslant 0}\prod_{j=1}^{N}|\lambda_n^2(\tau_j)|\rho_m^{(n)}|n\rangle\langle n|/P_g^{(\tau)}(N) \rightarrow |0\rangle\langle 0|, \quad \text{for} \quad N\rightarrow\infty, \tag{17}$$

where $\rho_m^{(n)} \equiv \langle n|\rho_m|n\rangle$ and the survival probability

$$P_g^{(\tau)}(N) = \sum_{n\geqslant 0}\prod_{j=1}^{N}|\lambda_n^2(\tau_j)|\rho_m^{(n)} \rightarrow \rho_m^{(0)}. \tag{18}$$

It is shown above, for both ETIMs and UTIMs, that repeated measurements can drive the evolution from the initial state (e.g., the thermal state) to the ground state of an MR. The process has the same features as those of known ground-state cooling methods and is an alternate method for ground-state cooling of an MR. Furthermore, distinct from usual dynamical cooling schemes, this alternate cooling method is based on repeated nondynamical measurements.

IV. NUMERICAL RESULTS AND ANALYSIS

Consider a $2\pi \times 100$ MHz nanomechanical resonator with a quality factor $Q_m = 10^5$ ($\gamma_m/2\pi = 200$ Hz), coupled to a flux qubit with a tunneling splitting $\Delta \simeq \omega_m$. The MR is initially at its thermal equilibrium state at the ambient temperature $T = 20$ mK, and the corresponding mean phonon number is $\bar{n}(0) = 1/[\exp(\hbar\omega_m/k_BT) - 1] = 3.69$. Figure 2 shows, for ETIMs, the mean phonon number $\bar{n}(N)$, the survival probability $P_g^{(\tau)}(N)$, and the fidelity $F_g^{(\tau)}(N) \equiv \langle 0|\rho_m^{(\tau)}(N)|0\rangle$ as a function of N, the number of measurements. The lines with red triangles in Fig. 2 are for the resonant case, $\Delta = \omega_m$, and the lines with gray squares correspond to the nonresonant case, $\Delta = 1.1\omega_m$. Ground-state cooling can be reached in both cases. Figure 2 shows that while ground-state cooling requires 60 measurements [$\bar{n}(N = 60) \approx 10^{-4}$], the mean phonon number decreases 90% with only five measurements.

The ideal ETIMs, with time interval τ, may be difficult technically since it requires accurate control of measurement time intervals. Figure 3 shows, for UTIMs, the same physical quantities as Fig. 2 for the $\Delta = \omega_m$ case, but with the higher bath temperature $T = 40$ mK and randomly selected time intervals τ_j. The mean phonon number is $\bar{n}(0) = 7.84$, initially. It is noticeable that ground-state cooling can be achieved much more efficiently for the UTIMs than for the ETIMs. It is a remarkable advantage for experimentalists to achieve ground-state cooling of MRs with random time intervals.

The physical reason is clear. In general, for a fixed n ($\geqslant 1$), the smaller the maximal value $\Lambda_n = \max\{|\lambda_n(\tau_j)|\}_{j=1,\ldots,N}$ the faster the term $\prod_{j=1}^{N}|\lambda_n^2(\tau_j)||n\rangle\langle n|$ decays with N. For ETIMs, it is unavoidable that Λ_n ($n \geqslant 1$) exists very close to one, since for a given finite time interval τ the periodical function $\cos(\Omega_n\tau)$ versus Ω_n runs across 0 several times. The corresponding component $|n\rangle\langle n|$ therefore decays very slowly. In particular, the component $|n\rangle\langle n|$ will not decay with N for ETIMs when $\Lambda_n = 1$. However, for random UTIMs, the corresponding $\prod_{j=1}^{N}|\lambda_n^2(\tau_j)||n\rangle\langle n|$ decays faster since $\prod_{j=1}^{N}|\lambda_n^2(\tau_j)| < \Lambda_n^{2N}$ for any $n \geqslant 1$.

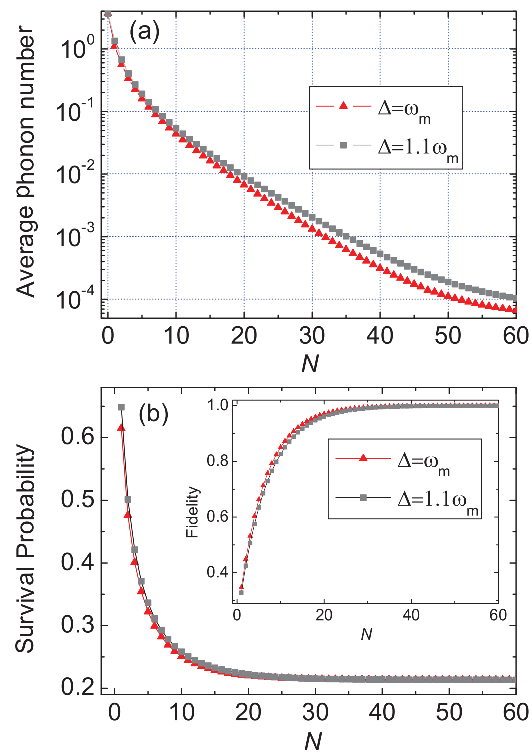

FIG. 2. (Color online) (a) The average phonon number $\bar{n}(N)$ after N ETIMs for the initial phonon number $\bar{n}(0) = 3.69$. (b) The corresponding survival probability $P_g^{(\tau)}(N)$ and fidelity $F_g^{(\tau)}(N)$ (inset). The lines with red triangles denote the resonant case $\Delta = \omega_m$. The lines with gray squares denote the nonresonant case $\Delta = 1.1\omega_m$. Here $g = 0.04\omega_m$ and $\tau = 10/\omega_m$.

Our cooling scheme is completely robust against measurement operational errors or randomness. It is also robust against relaxation effects of the MR and the qubit. Our UTIMs can cool an MR of frequency $\omega_m/2\pi \sim 100$ MHz, initially at a thermal state with mean phonon number $\lesssim 10$ and a quality factor $Q_m = 10^5$, in the time interval $10\tau \approx 100/\omega_m$, within 10 measurements—two orders of magnitude smaller than the MR's relaxation time, $\sim 1/[\bar{n}(0)\gamma_m] \approx 10000/\omega_m$. It can also be less than the relaxation time (in the 10 μs range) of the qubit we considered. However, since the final survival probability is proportional to the initial population probability at the ground state, our UTIMs are more suitable for further cooling based on a precooled MR at a thermal-like state with a smaller mean phonon number $\sim$10, e.g., by other cooling methods, such as sideband cooling.

We should comment that an equal-time-interval measurement-based cooling of an MR is proposed in Ref. 31 using a Cooper pair box as the auxiliary, where the measurement effect is averaged out, but there is no explicit analytical expression for the process. However, we find that method fails to achieve the MR cooling for our model after a considerable number of ETIMs. We should also remark that in the well-known sideband cooling the MR is coupled to a high-frequency auxiliary with the faster damping rate. The energy flows from the MR to the auxiliary and is then quickly lost to the bath. In contrast, the frequencies or the damping rates of the MR and the auxiliary qubit are of the same order

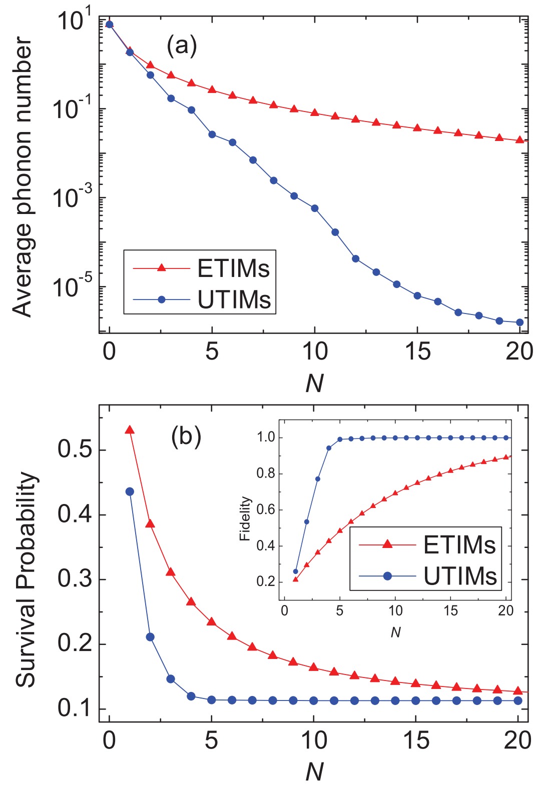

FIG. 3. (Color online) (a) The average phonon number $\bar{n}(N)$, (b) the survival probability, and [inset in (b)] the fidelity, after N measurements in the resonant case $\Delta = \omega_m$. Lines with red triangles denote ETIMs ($\tau = 8/\omega_m$) and lines with blue circles denote UTIMs.[30] Here the initial phonon number is $\bar{n}(0) = 7.84$, and the coupling strength is $g = 0.04\omega_m$.

in our model. Our scheme is nondeterministic and is based on repeated nondynamical measurements.

V. IMPLEMENTATION OF MEASUREMENTS ON THE FLUX QUBIT

Our UTIMs on the flux qubit can be implemented by a Josephson bifurcation amplifier (JBA) in a fast and nondestructive way. As shown in Fig. 1, a JBA[32] (blue part) is coupled to the flux qubit inductively as the measurement device. The JBA consists of a dc superconducting quantum interference device (SQUID) shunted by a capacitance C, subject to a microwave drive $I_{RF}\cos(\omega_d t + \phi_A)$. The JBA-SQUID loop contains two JJs of identical critical current I_{A0} and different phase differences, φ_{A1} and φ_{A2}, respectively. The current in the loop is $I_A = \bar{I}_A(\Phi_A)\cos\varphi_A$, with Φ_A, the flux bias in the JBA SQUID, set by external coils, $\bar{I}_A(\Phi_A) = 2I_{A0}\sin(\pi\Phi_A/\Phi_0)$, and $\varphi_A = (\varphi_{A1} + \varphi_{A2})/2$. The JBA circuit forms a driven resonator with nonlinear Josephson inductance.

Since the JBA is positioned symmetrically with respect to qubit loops 1 and 2, the two loops are coupled to the JBA with equal mutual inductance, $M_1 = M_2$. Due to the gradiometer design, the total qubit flux is decoupled from the JBA.[23,24] However, the JBA still couples to qubit loop 3 through its influence on Φ_3. If $\pi M_3\langle I_A\rangle \ll \Phi_0$, this influence can be approximated as a linear coupling to the $\tilde{\sigma}_z$ operator of the flux qubit,[33,34] that is, an extra interaction term between the qubit and the measurement device $H_I = \lambda(\Phi_{3b})\tilde{\sigma}_z\cos\varphi_A$. Here the coupling coefficient is $\lambda(\Phi_{3b}) = -(\pi M_3\bar{I}_A/\Phi_0)\kappa(\Phi_{3b})$ with $\kappa(\Phi_{3b}) = 2\alpha_0\sin[\pi(\Phi_{3b}/\Phi_0)].(d\Delta/d\alpha)|_{\alpha=\bar{\alpha}}$ and $\bar{\alpha} = 2\alpha_0\cos(\pi\Phi_{3b}/\Phi_0)$, where α_0 is the ratio between the Josephson energy of the smaller junctions and that of the two bigger junctions in the flux qubit, and Φ_{3b} is the total flux bias of loop 3. Thus an external on-chip bias current I_{B3} (green part in Fig. 1) can be used to monitor the coupling strength $\lambda(\Phi_{3b})$.

Under a strong microwave drive, the Josephson energy of the junction, $-E_{JA}\cos\varphi_A$, is expanded beyond the harmonic approximation and the classical dynamics can be described by a Duffing oscillator.[35] For a certain range of drive conditions, the nonlinear oscillator exhibits bistable behavior with hysteresis.[32,35] The two possible stable states correspond to different oscillation amplitudes and phases, which can be distinguished by transmitted or reflected microwave signals.[36–38] Switching between the two stable states happens when the driving power reaches a threshold. The switching probability depends on the value of the nonlinear inductance, which in our case depends on the states of the qubit. This occurs because the effective Josephson energy of the junctions of the JBA is modified by the interaction H_I as $E_{JA}(\tilde{\sigma}_z) = \bar{I}_A\Phi_0/2\pi - \lambda(\Phi_{3b})\tilde{\sigma}_z$. Therefore, by measuring the phase of the transmitted microwave signal, the state of the qubit is collapsed to one of the eigenstates of its free Hamiltonian, $\tilde{\sigma}_z$ in our case.

In practical UTIMs, the measurement time, which is about 50 ns, should be considered. This is not a problem when measurements are performed as follows. First, the qubit-MR interaction is switched off through the in-plane magnetic field B_0. Then a measurement pulse with readout and latching plateau is sent to the JBA to read out the qubit state and induce the projection to either $|g\rangle$ or $|e\rangle$ state. After this projection, the qubit-MR interaction is switched on again. The process is repeated until the MR reaches its ground state. An alternative way to perform the measurement is to bias the qubit away from the degeneracy point and bias it back after the measurement. Although this will introduce an additional complication to the operations, the measurement fidelity could be higher since the coupling between the JBA and the qubit is stronger.

VI. CONCLUSION

We propose an ultrafast feasible scheme to cool an MR to its ground state via random-time-interval projective measurements on an auxiliary flux qubit. The measurement scheme is almost independent of the initial state of the MR. It works when the MR couples with the qubit either on-resonance or off-resonance, and it even works when the coupling g becomes time dependent. The cooling process is robust since it can be accomplished in a much shorter time than the relaxation times of the MR and the qubit.

We expect that our scheme will reduce experimental constraints since there are no requirements for control on the time intervals of measurements. In principle, the MR can be cooled to an arbitrarily low temperature with an arbitrarily small mean phonon number, within a very short time.

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[1]C. M. Caves, K. S. Thorne, R. W. P. Drever, V. D. Sandberg, and M. Zimmermann, Rev. Mod. Phys. **52**, 341 (1980); M. F. Bocko and R. Onofrio, *ibid.* **68**, 755 (1996); M. D. LaHaye, O. Buu, B. Camarota, and K. C. Schwab, Science **304**, 74 (2004).

[2]S. Mancini, V. Giovannetti, D. Vitali, and P. Tombesi, Phys. Rev. Lett. **88**, 120401 (2002); W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, *ibid.* **91**, 130401 (2003); L. F. Wei, Y.-X. Liu, C. P. Sun, and F. Nori, *ibid.* **97**, 237201 (2006); F. Xue, L. Zhong, Y. Li, and C. P. Sun, Phys. Rev. B **75**, 033407 (2007).

[3]K. Hammerer, M. Aspelmeyer, E. S. Polzik, and P. Zoller, Phys. Rev. Lett. **102**, 020501 (2009); L. Tian and P. Zoller, *ibid.* **93**, 266403 (2004).

[4]C. H. Metzger and K. Karrai, Nature (London) **432**, 1002 (2004).

[5]S. Gigan, H. R. Böhm, M. Paternostro, F. Blaser, G. Langer, J. B. Hertzberg, K. C. Schwab, D. Bäuerle, M. Aspelmeyer, and A. Zeilinger, Nature (London) **444**, 67 (2006).

[6]O. Arcizet, P.-F. Cohadon, T. Briant, M. Pinard, and A. Heidmann, Nature (London) **444**, 71 (2006).

[7]J. D. Thompson, B. M. Zwickl, A. M. Jayich, F. Marquardt, S. M. Girvin, and J. G. E. Harris, Nature (London) **452**, 72 (2008); J. C. Sankey, C. Yang, B. M. Zwickl, A. M. Jayich, and J. G. E. Harris, Nat. Phys. **6**, 707 (2010).

[8]I. Wilson-Rae, N. Nooshi, W. Zwerger, and T. J. Kippenberg, Phys. Rev. Lett. **99**, 093901 (2007).

[9]F. Marquardt, J. P. Chen, A. A. Clerk, and S. M. Girvin, Phys. Rev. Lett. **99**, 093902 (2007).

[10]Y. Li, Y.-D. Wang, F. Xue, and C. Bruder, Phys. Rev. B **78**, 134301 (2008); Y. Li, Z. D. Wang, and C. P. Sun, Eur. Phys. J. D **61**, 215 (2011).

[11]T. J. Kippenberg and K. J. Vahala, Opt. Express **15**, 17172 (2007).

[12]F. Marquardt, A. A. Clerk, and S. M. Girvin, J. Mod. Opt. **55**, 3329 (2008).

[13]Y. Li, L.-A. Wu, and Z. D. Wang, Phys. Rev. A **83**, 043804 (2011).

[14]L. Tian and P. Zoller, Phys. Rev. Lett. **93**, 266403 (2004).

[15]P. Zhang, Y. D. Wang, and C. P. Sun, Phys. Rev. Lett. **95**, 097204 (2005).

[16]A. Naik, O. Buu, M. D. LaHaye, A. D. Armour, A. A. Clerk, M. P. Blencowe, and K. C. Schwab, Nature (London) **443**, 193 (2006).

[17]Y.-D. Wang, Y. Li, F. Xue, C. Bruder, and K. Semba, Phys. Rev. B **80**, 144508 (2009).

[18]K. Jacobs, H. I. Nurdin, F. W. Strauch, and M. James, e-print arXiv:1003.2653; X. Wang, S. Vinjanampathy, F. W. Strauch, and K. Jacobs, e-print arXiv:1103.5750.

[19]T. Rocheleau, T. Ndukum, C. Macklin, and J. B. Hertzberg, Nature (London) **463**, 72 (2010); J. D. Teufel, T. Donner, D. Li, J. H. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, and R. W. Simmonds, *ibid.* **475**, 359 (2011).

[20]H. Nakazato, T. Takazawa, and K. Yuasa, Phys. Rev. Lett. **90**, 060401 (2003).

[21]L.-A. Wu, D. A. Lidar, and S. Schneider, Phys. Rev. A **70**, 032322 (2004).

[22]M. B. Plenio, S. F. Huelga, A. Beige, and P. L. Knight, Phys. Rev. A **59**, 2468 (1999).

[23]F. G. Paauw, A. Fedorov, C. J. P. M. Harmans, and J. E. Mooij, Phys. Rev. Lett. **102**, 090501 (2009).

[24]A. Fedorov, A. K. Feofanov, P. Macha, P. Forn-Diaz, C. J. P. M. Harmans, and J. E. Mooij, Phys. Rev. Lett. **105**, 060503 (2010).

[25]J. E. Mooij, T. P. Orlando, L. Levitov, L. Tian, C. H. van der Wal, and S. Lloyd, Science **285**, 1036 (1999).

[26]F. Xue, Y. D. Wang, C. P. Sun, H. Okamoto, H. Yamaguchi, and K. Semba, New J. Phys. **9**, 35 (2007).

[27]T. P. Orlando, J. E. Mooij, L. Tian, C. H. van der Wal, L. S. Levitov, S. Lloyd, and J. J. Mazo, Phys. Rev. B **60**, 15398 (1999).

[28]S. Saito, M. Thorwart, H. Tanaka, M. Ueda, H. Nakano, K. Semba, and H. Takayanagi, Phys. Rev. Lett. **93**, 037001 (2004).

[29]X. Zhu, A. Kemp, S. Saito, and K. Semba, Appl. Phys. Lett. **97**, 102503 (2010).

[30]Here the time variances δt_N $(N = 1,\ldots,20)$ are taken randomly, e.g., as $\{0.26, 0.14, -0.40, -0.07, 0.06, 0.24, -0.20, -0.44, -0.34, 0.01, -0.01, -0.12, 0.39, -0.28, -0.43 0.06, -0.16, 0.17, 0.47, 0.01\}$.

[31]C. Bergenfeldt and K. Mølmer, Phys. Rev. A **80**, 043838 (2009).

[32]I. Siddiqi, R. Vijay, F. Pierre, C. M. Wilson, M. Metcalfe, C. Rigetti, L. Frunzio, and M. H. Devoret, Phys. Rev. Lett. **93**, 207002 (2004).

[33]Y.-D. Wang, S. Chesi, D. Loss, and C. Bruder, Phys. Rev. B **81**, 104524 (2010).

[34]Y.-D. Wang, X. B. Zhu, and C. Bruder, e-print arXiv:1011.6529.

[35]L. D. Landau and E. M. Lifshitz, *Mechanics* (Butterworth-Heinemann, Oxford, England, 1976).

[36]N. Boulant, G. Ithier, P. Meeson, F. Nguyen, D. Vion, D. Esteve, I. Siddiqi, R. Vijay, C. Rigetti, F. Pierre, and M. Devoret, Phys. Rev. B **76**, 014525 (2007).

[37]A. Lupascu, S. Saito, T. Picot, P. C. D. Groot, C. J. P. M. Harmans, and J. E. Mooij, Nat. Phys. **3**, 119 (2007).

[38]F. Mallet, F. R. Ong, A. Palacios-Laloy, F. Nguyen, P. Bertet, D. Vion, and D. Esteve, Nat. Phys. **5**, 791 (2009).