

Nonlinear spin Hall effect in GaAs (110) quantum wellsV. I. Ivanov,¹ V. K. Dugaev,^{2,3} E. Ya. Sherman,^{4,5} and J. Barnas^{6,*}¹*Institute for Problems of Materials Science, Ukrainian Academy of Sciences, Vilde 5, 58001 Chernovtsy, Ukraine*²*Department of Physics, Rzeszów University of Technology, aleja Powstańców Warszawy 6, 35-959 Rzeszów, Poland*³*Department of Physics and CFIF, Instituto Superior Técnico, TU Lisbon, Avenida Rovisco Pais 1049-001 Lisbon, Portugal*⁴*Department of Physical Chemistry, Universidad del País Vasco, Bilbao, Spain*⁵*IKERBASQUE Basque Foundation for Science, 48011, Bilbao, Spain*⁶*Institute of Molecular Physics, Polish Academy of Sciences, ulica Smoluchowskiego 17, 60-179 Poznań, Poland*

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We consider a stationary spin current in a (110)-oriented GaAs-based symmetric quantum well due to a nonlinear response to an external periodic electric field. The model assumed includes the Dresselhaus spin-orbit interaction and the random Rashba spin-orbit coupling. The Dresselhaus term is uniform in the quantum well plane and gives rise to spin splitting of the electron band. The external electric field of frequency ω —in the presence of random Rashba coupling—leads to virtual spin-flip transitions between spin subbands, generating a stationary pure spin current proportional to the square of the field amplitude.

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I. INTRODUCTION

Spin-orbit (SO) interactions in semiconductors reveal a variety of fundamental spin-related phenomena.¹ Formation of stable spin helices with nontrivial temporal and spatial dynamics,^{2–4} spin optics,⁵ and spin-dependent sound radiation⁶ represent only some of these effects. Spin currents,⁷ solely attributed to the spin-orbit coupling, provide the possibility of inducing and controlling spin motion by electrical and optical fields, and therefore became one of the key elements of the modern spintronics oriented at new device applications of semiconductor-based structures. Thorough investigations of realistic systems enlarge the variety of both fundamental phenomena and possible applications. As an example we mention that disorder—always present in real systems—plays a crucial role in the spin Hall effect,⁸ as the spin Hall conductivity can be totally suppressed by any finite concentration of impurities.

Recently, symmetric GaAs (110) quantum wells (QWs) have become the subject of extensive experimental and theoretical investigations. This is related to the expectation of the longest spin relaxation times in these structures,^{9–16} which in turn can lead to interesting spin dynamics.^{17,18} A stationary pure spin current accompanying an electric current in (110) QWs was observed as reported in Ref. 19. SO interaction in these systems, described by the Dresselhaus term in the corresponding Hamiltonian,^{20–22} conserves the electron spin along the axis normal to the QW plane for any electron momentum \mathbf{k} . As a result, random motion of an electron does not lead to a random direction of the spin-orbit field and therefore does not lead to spin relaxation. In reality, however, this spin component relaxes very slowly, and its analysis provides a test for the rapidly developing low-frequency spin-noise spectroscopy²³ suitable for the measurements of long spin evolutions.

In the case of perfect $z \rightarrow -z$ symmetry (the axis z is perpendicular to the QW plane), the Rashba SO interaction is zero. In real structures, however, the Rashba coupling still exists in the form of a spatially fluctuating SO field (although it is zero on average).^{24–26} This interaction induces

spin-flip processes leading to spin relaxation,¹⁶ and can also be responsible for generation of a nonequilibrium spin density due to the absorption of an external electromagnetic field.²⁶ Recently, it was proposed that this random SO coupling can result in spin orientation by an external current²⁷ and also can play a role in the formation of the stripe structure of spin current distribution.²⁸

In this paper we propose the possibility of exciting a steady pure spin current by a periodic external field, extending thus the abilities of spin manipulation in real situations. In contrast to the conventional spin Hall effect, which is linear in the external electric field, the proposed spin current is quadratic in the external periodic field. The effect is a result of the interplay of constant Dresselhaus and spatially random Rashba terms, and is not related to the spin currents produced by gate manipulation of the Rashba coupling²⁹ or adiabatic pumping in graphene.³⁰ The exact mechanism of the effect does not necessarily involve real spin-flip transitions of electrons between the spin-split subbands in (110)-oriented GaAs QWs, but relies on virtual spin-flip processes which renormalize the wave functions of electrons in a nonequilibrium state. This makes such a nonlinear current a physically different phenomenon, which appears if one accounts for more realistic effects than those described by the conventional Rashba and Dresselhaus models.

In Sec. II we describe the model and Hamiltonian of the system. The spin current is calculated in Sec. III. The summary and final conclusions are presented in Sec. IV.

II. MODEL

The Hamiltonian of a two-dimensional electron gas with the constant Dresselhaus term H_D and spatially fluctuating Rashba spin-orbit interaction H_R , subjected to an external electromagnetic field described by the vector potential $\mathbf{A}(\mathbf{r}, t)$, takes the following form (we use units with $\hbar = 1$):

$$H = H_0 + H_D + H_R, \quad (1)$$

where the first two terms are

$$H_0 = -\frac{1}{2m} \left(\nabla - \frac{ie\mathbf{A}}{c} \right)^2, \quad (2)$$

$$H_D = -i\alpha\sigma_z \left(\nabla_x - \frac{ieA_x}{c} \right). \quad (3)$$

The Dresselhaus constant $\alpha = \gamma\pi^2/2w^2$, where γ is the corresponding bulk Dresselhaus coupling parameter, is inversely proportional to the square of the QW width w . The other components of the Dresselhaus interaction vanish due to the specific symmetry of the (110) orientation.^{16,21}

The last term in Eq. (1) stands for the effects of the spatially nonuniform Rashba SO interaction, which can be written as $H_R = H_R^0 + V$, where H_R^0 is the Rashba term for $\mathbf{A}(\mathbf{r}, t) = 0$,

$$H_R^0 = -\frac{i}{2}\sigma_x\{\nabla_y, \lambda(\mathbf{r})\} + \frac{i}{2}\sigma_y\{\nabla_x, \lambda(\mathbf{r})\}, \quad (4)$$

with $\{, \}$ denoting the anticommutator and $\lambda(\mathbf{r})$ being the random Rashba SO interaction. The term V , in turn, describes coupling of the electron spin to the external field $\mathbf{A}(\mathbf{r}, t)$ via the Rashba field,

$$V = -\frac{e}{c}\lambda(\mathbf{r})(\sigma_x A_y - \sigma_y A_x). \quad (5)$$

Due to the assumed symmetry with respect to z inversion, the spatially averaged Rashba interaction vanishes, $\langle \lambda(\mathbf{r}) \rangle = 0$. We assume that the random Rashba field can be described by the correlation function related to fluctuating density of impurities near the QW,^{24,26}

$$C_{\lambda\lambda}(\mathbf{r} - \mathbf{r}') \equiv \langle \lambda(\mathbf{r})\lambda(\mathbf{r}') \rangle = \langle \lambda^2 \rangle F(\mathbf{r} - \mathbf{r}'), \quad (6)$$

where the range function $F(\mathbf{r} - \mathbf{r}')$ depends on the type of disorder. We assume the correlator of the random Rashba interaction in the momentum space in the form^{26,27,31}

$$|\lambda_q|^2 = 2\pi \langle \lambda^2 \rangle R^2 e^{-qR}, \quad (7)$$

where R is the spatial scale of the fluctuations.

In the absence of external field and random Rashba SO interaction, the Hamiltonian $H_0 + H_D$ describes the spectrum of spin-polarized electrons, $\varepsilon_{\mathbf{k}\sigma} = (k_x^2 + k_y^2)/2m + \sigma\alpha k_x$. The energy bands of spin-up and spin-down electrons are thus shifted in opposite directions along the k_x axis. The corresponding Green function is then diagonal in the spin subspace,

$$\mathbf{G}_{\mathbf{k}\varepsilon}^{(0)} = \begin{pmatrix} G_{\mathbf{k}\varepsilon+} & 0 \\ 0 & G_{\mathbf{k}\varepsilon-} \end{pmatrix}, \quad (8)$$

$$G_{\mathbf{k}\varepsilon\sigma} = \frac{1}{\varepsilon - \varepsilon_{\mathbf{k}\sigma} + \mu + i\delta_{\mathbf{k}\sigma} \text{sgn}(\varepsilon)},$$

where $\sigma = +1$ for spin-up (\uparrow) electrons and $\sigma = -1$ for spin-down (\downarrow) electrons, whereas $\delta_{\mathbf{k}\sigma}$ is the momentum- and spin-dependent relaxation rate.

III. NONLINEAR SECOND-ORDER SPIN CURRENT

In the following we consider the z component of a pure spin current flowing along the x axis, which is the only component allowed by symmetry of the system under consideration. The

operators of the electron velocity \hat{v}_x and the corresponding spin current tensor component \hat{j}_x^z are

$$\hat{v}_x = i[H_0 + H_{D,x}] = \frac{k_x}{m} + \alpha\sigma_z - \lambda\sigma_y, \quad (9)$$

$$\hat{j}_x^z = \frac{1}{2}\{\hat{v}_x, \sigma_z\} = \frac{k_x}{m}\sigma_z + \alpha, \quad (10)$$

where the α -related terms correspond to the anomalous contribution to the velocity. The macroscopic spin current density is then given by

$$j_x^z = i \text{Tr} \sum_{\mathbf{k}} \int \frac{d\varepsilon}{2\pi} \hat{j}_x^z \mathbf{G}_{\mathbf{k}\varepsilon}, \quad (11)$$

where $\mathbf{G}_{\mathbf{k}\varepsilon}$ is the Green's function of the system interacting with the external electromagnetic field.

Upon substituting (10) into Eq. (11) one can note that the second term describes the current caused only by the electron density,

$$n = i \text{Tr} \sum_{\mathbf{k}} \int \frac{d\varepsilon}{2\pi} \mathbf{G}_{\mathbf{k}\varepsilon}, \quad (12)$$

conserved under any external perturbation. This conservation is achieved in calculations by an appropriate shift of the chemical potential μ . In equilibrium, however, there is no spin current in the system (expected, e.g., for QWs with other crystallographic orientations) since the integrated contributions from k_x/m and α terms in Eq. (10) exactly cancel each other. As a result, this type of structure does not demonstrate the Rashba paradox of the nonzero equilibrium pure spin current.³² This can be seen directly by calculating the spin current using Eq. (11) with the equilibrium Green's function in Eq. (8), or by taking into account the fact that Hamiltonian in Eq. (3) can be transformed by the SU(2) rotation to a form that does not have spin-dependent terms.³³

However, a nonzero pure spin current, which is the subject of interest here, can be generated by an external field in the presence of random Rashba coupling, as presented schematically by the Feynman graph in Fig. 1. In this graph we introduced the following notations:

$$V_{\mathbf{k}\mathbf{k}'} = \lambda_{\mathbf{k}\mathbf{k}'}[\sigma_x A_y(\omega) - \sigma_y A_x(\omega)], \quad (13)$$

$$V_{\mathbf{k}'\mathbf{k}} = \lambda_{\mathbf{k}'\mathbf{k}}[\sigma_x A_y(-\omega) - \sigma_y A_x(-\omega)], \quad (14)$$

for the transition matrix elements due to the external field and spin-orbit coupling.

With the Feynman graph shown in Fig. 1, we find the spin current density omitting the α term in the velocity operator

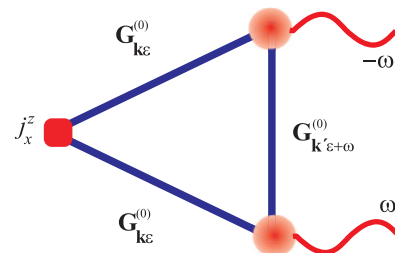


FIG. 1. (Color online) Feynman diagrams for the excited spin current in Eq. (11). The circles correspond to the matrix elements $V_{\mathbf{k}\mathbf{k}'}$ and $V_{\mathbf{k}'\mathbf{k}}$.

since its contribution is conserved if no additional electrons are injected into the system. The resulting expression for the contribution of a single spin component to the total spin current becomes

$$j_{x\sigma} = \frac{iA^2}{m} \sum_{\mathbf{k}\mathbf{k}'} \int \frac{d\varepsilon}{2\pi} k_x |\lambda_{\mathbf{k}\mathbf{k}'}|^2 G_{\mathbf{k}\varepsilon\sigma} G_{\mathbf{k}'\varepsilon+\omega\sigma'} G_{\mathbf{k}\varepsilon\sigma}, \quad (15)$$

where A is the vector potential amplitude, making the injected current independent of the external field orientation. The total spin current is given by $j_x^z = j_{x,\sigma=1} - j_{x,\sigma=-1}$ with $j_{x,\sigma=-1} = -j_{x,\sigma=1}$.

Using Eqs. (8) and (15), after rather tedious integration of the product of the three Green's functions in the complex ε plane, one obtains

$$\begin{aligned} j_{x\sigma} &= -\frac{A^2}{m} \sum_{\mathbf{k}\mathbf{k}'} k_x |\lambda_{\mathbf{k}\mathbf{k}'}|^2 \left[\frac{f(\varepsilon_{\mathbf{k}\sigma})}{[\varepsilon_{\mathbf{k}\sigma} - \varepsilon_{\mathbf{k}'\sigma'} + \omega + i\delta_{\mathbf{k}\sigma} + i\delta_{\mathbf{k}'\sigma'} \operatorname{sgn}(\varepsilon_{\mathbf{k}\sigma} + \omega - \mu)]^2} \right. \\ &\quad \left. + \frac{f(\varepsilon_{\mathbf{k}'\sigma'})}{[-\varepsilon_{\mathbf{k}\sigma} + \varepsilon_{\mathbf{k}'\sigma'} - \omega + i\delta_{\mathbf{k}'\sigma'} + i\delta_{\mathbf{k}\sigma} \operatorname{sgn}(\varepsilon_{\mathbf{k}'\sigma'} - \omega - \mu)]^2} \right] \\ &= \frac{A^2}{m} \sum_{\mathbf{k}\mathbf{q}} |\lambda_{\mathbf{q}}|^2 \left[\frac{k_x [f(\varepsilon_{\mathbf{k}\sigma}) - f(\varepsilon_{\mathbf{k}\sigma} + \omega)]}{(\varepsilon_{\mathbf{k}\sigma} - \varepsilon_{\mathbf{k}-\mathbf{q}\sigma'} + \omega + i\delta_+)^2} - \frac{(k_x + q_x) f(\varepsilon_{\mathbf{k}\sigma'})}{(\varepsilon_{\mathbf{k}+\mathbf{q}\sigma} - \varepsilon_{\mathbf{k}\sigma'} + \omega + i\delta_-)^2} + \frac{k_x f(\varepsilon_{\mathbf{k}\sigma} + \omega)}{(\varepsilon_{\mathbf{k}\sigma} - \varepsilon_{\mathbf{k}-\mathbf{q}\sigma'} + \omega + i\delta_-)^2} \right]. \quad (16) \end{aligned}$$

Here we introduced the notation $\delta_+ \equiv \delta_{\mathbf{k}\sigma} + \delta_{\mathbf{k}'\sigma'}$ and $\delta_- \equiv \delta_{\mathbf{k}\sigma} - \delta_{\mathbf{k}'\sigma'}$. Equation (16) shows that the injection of spin current is a coherent effect arising due to the change in electron wave function under the resonant electromagnetic radiation rather than the injection due to the two-photon absorption typical in nonlinear semiconductor optics.

Since the single-electron energy $\varepsilon_{\mathbf{k}\sigma}$ can be written as $[(k_x + \sigma\alpha m)^2 + k_y^2]/2m - m\alpha^2/2$, one can shift the chemical potential, $\mu \rightarrow \mu + m\alpha^2/2$. Then one can write $j_{x\sigma}$ in the form

$$\begin{aligned} j_{x\sigma} &= \frac{A^2}{m} \sum_{\mathbf{k}\mathbf{q}} |\lambda_{\mathbf{q}}|^2 \left[(k_x - \sigma\alpha m) \frac{f(\varepsilon_{\mathbf{k}}) - f(\varepsilon_{\mathbf{k}} + \omega)}{[(\mathbf{k} \cdot \mathbf{q}/m - q^2/2m + 2\sigma\alpha(k_x - q_x) - 2m\alpha^2 + \omega + i\delta_+)^2]} \right. \\ &\quad + (k_x - \sigma\alpha m) \frac{f(\varepsilon_{\mathbf{k}} + \omega)}{[\mathbf{k} \cdot \mathbf{q}/m - q^2/2m + 2\alpha\sigma(k_x - q_x) - 2m\alpha^2 + \omega - i\sigma\delta_-]^2} \\ &\quad \left. - (k_x + q_x + \sigma\alpha m) \frac{f(\varepsilon_{\mathbf{k}} + \omega)}{[(\mathbf{k} \cdot \mathbf{q}/m + q^2/2m + 2\alpha\sigma(k_x + q_x) + 2m\alpha^2 + \omega - i\sigma\delta_-)^2]} \right]. \quad (17) \end{aligned}$$

We emphasize here that the calculated spin current is a stationary coherent nonlinear effect proportional to the intensity of incident radiation,³⁴ in contrast to the spin current generated by pulse excitations, where the result is proportional to the total fluence in the pulse.²⁵ The transition processes produce real holes in the initially occupied subbands and electrons in those initially empty, changing the real occupation of the spin-up and spin-down states. The calculated current is also not related to the Drude-like linear response at frequency ω , suppressed by a factor of the order of $(\delta_+/\omega)^2$.

The results of numerical calculation of the spin current [taking real part of Eq. (17)] are presented in Fig. 2 for different values of the parameter $k_F R$. The parameters typical for the (110) quantum wells are $2\alpha m/k_F = 0.1$ and $\delta_+/\mu = 0.1$. For the momentum-dependent δ_- , we assume a typical value, $\delta_-/\mu = 0.05$. Furthermore, we used for GaAs $m = 0.067m_0$, where m_0 is the free-electron mass, Fermi momentum $k_F = 1.8 \times 10^6 \text{ cm}^{-1}$ (corresponding to electron concentration $5.2 \times 10^{11} \text{ cm}^{-2}$), and $\mu = 18.5 \text{ meV}$. The spin current in Fig. 2 is presented in units of j_0 , with j_0 defined as

$$j_0 = \frac{2m^2\alpha e^2}{c^2\pi^3} A^2 \frac{\langle \lambda^2 \rangle}{k_F^2}. \quad (18)$$

Taking into account the relation $A^2 = (c/\omega)^2 E^2$, where E is the electric field amplitude, we obtain $j_0 =$

$2m^2\alpha \langle \lambda^2 \rangle (eE/\omega)^2 / \pi^3 k_F^2$, with eE/ω being the amplitude of the momentum oscillation of a classical electron in a periodic electric field. It is interesting to mention that the maximum value of E , which can still be considered as a perturbation, is determined by $eE/\omega \sim k_F$, and, therefore, the maximum of j_0 is of the order of $m^2\alpha \langle \lambda^2 \rangle$, having the physical meaning of the equilibrium spin current (see first paper in Ref. 25) induced by the random Rashba spin-orbit coupling.

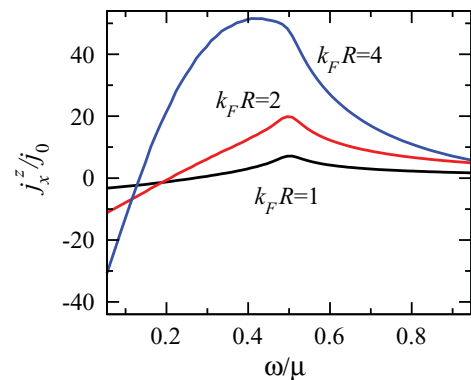


FIG. 2. (Color online) Total spin current, calculated by using Eq. (17) for indicated values of $k_F R$. The parameters used in numerical calculations are given in the main text.

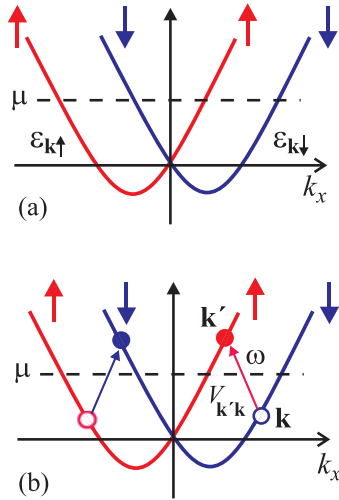


FIG. 3. (Color online) Schematic presentation of the light-induced resonant formation of spin holes in the energy bands occupied by electrons: (a) spin-split energy bands in GaAs (110) quantum well; (b) due to the coupling $V_{kk'}$ [cf. Eq. (13)] of the spin-up and spin-down states, the effective spin $\langle S_z \rangle$ in each subband decreases.

To understand better the physical mechanism of nonlinear spin current generation we consider a schematic picture presenting the electron energy bands as a function of k_x without Rashba random SO interaction and without external field; see Fig. 3. As we have already mentioned above, the Dresselhaus SO interaction leads to spin splitting of the electron states of a free-electron gas, which results in the energy bands $\varepsilon_{k\sigma}$ shown in Fig. 3(a) as a function of k_x (for $k_y = 0$). Even though the states $|\mathbf{k}\sigma\rangle$ of these bands are spin polarized, the spin current in equilibrium is exactly zero. This is related to the zero current associated with each of the subbands, $j_{\uparrow,\downarrow}$, calculated as the flux of electrons in each subband. Obviously, vanishing current j_{σ} in the subband σ means that the spin current j_{σ}^s is also zero. Distortions of the energy subbands either due to the random Rashba interaction in Eq. (4) or due to the external field in Eq. (2) do not break the condition $j = 0$.

Our calculations, however, showed that nonzero matrix elements of field-induced spin-flip intersubband transitions appear in the presence of random Rashba coupling. Accordingly, in the nonequilibrium situation the electron states in each

subband are a superposition of spin-up and -down states, so that the resulting state $|\mathbf{k}\pm\rangle$ [Fig. 3(b)] has a smaller effective spin. Such mixing of $|\mathbf{k}\sigma\rangle$ and $|\mathbf{k}'\sigma'\rangle$ states effectively depends on $|\mathbf{k} - \mathbf{k}'|$ and on $|\varepsilon_{k\sigma} - \varepsilon_{k'\sigma'} \pm \omega|$, so that the above-mentioned spin mixing is different at different parts of the dispersion curves $\varepsilon_{k\sigma}$. This is shown schematically in Fig. 3(b) for different spin subbands.

Thus, even though in nonequilibrium the current in each subband $\varepsilon_{k\sigma}$ is zero, the associated spin current is not zero anymore. For example, in the $\varepsilon_{k\uparrow}$ band more up spins flow in the $+x$ than in the $-x$ direction. Correspondingly, in the $\varepsilon_{k\downarrow}$ band more down spins flow in the $-x$ direction. This results in a net spin-up current in the $+x$ direction. Obviously, the direction of the spin current is related to the sign of the Dresselhaus coupling constant α . A remarkable change of the wave function by a strong electric field causes an injected pure spin current of the order or much stronger than the equilibrium spin currents arising as a result of the Rashba paradox.^{25,32}

IV. SUMMARY AND CONCLUSIONS

To conclude, we have proposed the effect of coherent nonlinear generation of steady pure spin currents in GaAs (110) quantum wells by electromagnetic waves. The injected spin current is proportional to the intensity of the external radiation, strongly depends on the frequency, and can be injected in the frequency range up to the Fermi energy of the two-dimensional electron gas. The physical mechanism of the effect is related to the virtual spin reorientation of electrons filling the spin subbands split by the Dresselhaus interaction in the presence of a randomly varying Rashba coupling. The latter may be introduced, e.g., by random doping of the quantum well. As a result, a virtual “spin hole” appears in the subband, leading to the light-induced spin current.

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