# Interference with coupled microcavities: Optical analog of spin $2\pi$ rotations

O. Di Stefano, R. Stassi, A. Ridolfo, S. Patanè, and S. Savasta
*Dipartimento di Fisica della Materia e Ingegneria Elettronica, Università di Messina Viale S. D'Alcontres, I-98166 Messina, Italy*
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It is well known that spinor wave functions change their sign under $2\pi$ rotation. Several experiments have used magnetic precession of neutrons to implement rotations. Here we propose an all-optical analog of this effect based on time-resolved optical interference in coupled optical microcavities. We show that feeding the coupled-microcavity system with a pair of phase-locked probe pulses, separated by precise delay times, provides direct information on the sign change of the transmitted field after one complete Rabi-like oscillation period.

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## I. INTRODUCTION

All physicists at some time during their study of the quantum theory of angular momentum encounter the seemingly peculiar property of spinor, namely that it must be rotated by $4\pi$ radians to return to its initial state. A rotation by $2\pi$ radians, which intuitively ought to be equivalent to no rotation at all, multiplies the spinor wave function by $-1$. As observables in quantum theory are quadratic in a wave function, the change of sign cannot be detected by ordinary experiments. Theoretically the origin of such behavior follows from the form of unitary operators $U(\boldsymbol{\theta}) = \exp(-i\boldsymbol{\sigma}\cdot\boldsymbol{\theta}/2)$, ($\boldsymbol{\sigma}$ are the Pauli matrices) which rotates a state vector of angular momentum $\boldsymbol{\sigma}$ by an angle $\theta$ about the direction $\hat{\boldsymbol{\theta}}$. Hence, we have $U(2\pi) = -1$ independently of the rotation axis. As it is well known, for more general angular momenta the spin operator $\hbar\boldsymbol{\sigma}/2$ is replaced by the total angular momentum operator $\boldsymbol{J}$ with $2N$ eigenvalues $[N = J(J+1)]$. For $J = n/2$ (with $n$ integer) an analogous behavior under $2\pi$ rotation can be observed. The first Gedanken experiments aimed at the observation of the sign change of spinors under $2\pi$ rotations were published by Bernstein,[1] and independently by Aharonov and Susskind.[2] These two proposed experiments, the ffirst involving the interaction of a spin 1/2 particle with a magnetic field, and the second involving the tunneling of a current of free electrons, were conceptually similar. In both cases one system was split into two separate subsystems, one of them was affected by an additional $2\pi$ rotation relative to the other one, and then recombined. The first experimental verification of coherent spinor rotation was provided by Rauch *et al.*[3] and Werner *et al.*;[4] both groups employed unpolarized neutron interferometry as suggested in the Bernstein-Gedanken experiment. Klein and Opat reported the observation of $2\pi$ rotations by neutron Fresnel diffraction.[5] The similarity of the mathematical description (that is, the algebraic isomorphism) between spinor rotations and the transitions between two atomic or molecular states of any total angular momentum has been exploited to study analogies of $2\pi$ spin rotations with different experimental approaches that required no fermions.[6–9] One other system, where such an effect has been observed, consists of strongly interacting Rydberg atoms and microwave photons: After a full cycle of Rabi oscillation, the atom-cavity system experiences a global quantum phase shift $\pi$ (Ref. 10).

There exist interesting analogies between electron transport and the transport of optical waves in dielectric structures.[11] For example, photonic crystals are periodic dielectric systems that can exhibit a photonic band gap in analogy with the electronic band gap in semiconductors.[12] In disordered systems the optical counterpart of weak localization,[13,14] Anderson localization,[15] and universal conductance have been observed.[16] The optical analog of electronic Bloch oscillations in optical superlattices studied by means of time-resolved spectroscopy was also reported.[17,18] Often these processes are easier to study with light because the coherence time of optical wave packets is usually much longer than that of an electronic wave packet.

The purpose of the present paper is to provide a concrete and conceptually simple all-optical realization of the sign change under $2\pi$ rotations. We consider a system of two coupled planar microcavities (MCs). When one of the two is excited by an ultrafast resonant optical pulse, the energy oscillates between the two systems until losses through the external mirrors prevails. Periodic optical media and specifically stratified periodic structures play an important role in a number of applications. Modern crystal-growth techniques make it possible to grow multilayer media with well-controlled periodicities and with the thicknesses' precision below 1 nm. Over the past two decades semiconductor coupled planar MCs have been investigated as a way to further increase the flexibility in controlling both radiation and material degrees of freedom.[19–25] In such systems the coupling of the two cavity modes can be controlled by the transmission of the central mirror and the two resonant modes are the optical analogs of two atomic or molecular states, which, in turn, are isomorphic to a spin 1/2 system. Although we present detailed calculations for coupled planar semiconductor MCs, the results here presented can also be verified in three-dimensional coupled MCs such as coupled identical pillars etched from a semiconductor planar MC,[26] or coupled defects in photonic crystals.[27]

## II. COUPLED DOUBLE MICROCAVITY LIGHT MODES

A semiconductor planar MC is a structure formed by high reflecting dielectric mirrors [distributed Bragg reflectors (DBR)] on the two sides of a spacer (Sp) layer, of physical length $L_C$. Here, we consider a system composed by two planar MCs connected through a common DBR (see Fig. 1). The essential physical features of such a system may be understood through a simplified analytical model. An analytical quantum

    

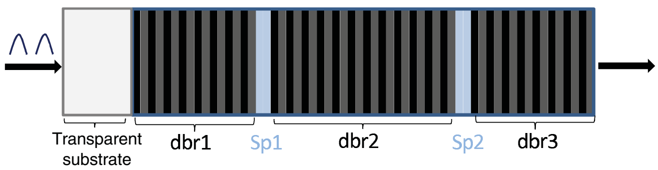


FIG. 1. (Color online) Scheme of the double microcavity.

statistical model for interacting quantum systems in the strong-coupling regime was adopted in Ref. 25. We assume that the two MCs have a high $Q$ factor and that the intracavity modes are coupled with the external field via two partially transmitting mirrors. More specifically we consider systems with coupling-induced splittings quite larger than the linewidth of the individual peaks.

To investigate the light propagation inside the heterostructure we exploit the transfer matrix approach.[28,29] Here we consider normal-incidence optical pulses propagating along the growth axis. The electric field distribution within each homogeneous layer can be expressed as the sum of an incident plane wave and a reflected plane one. The complex amplitudes of these two waves constitute the components of a column vector. The light propagation in each DBR of $N$ unit cells constituted by two alternating dielectrics with refraction indices $n_1$ and $n_2$, respectively, may be obtained by calculating initially how the electric field propagates inside the single cell. The electric field complex amplitudes emerging from the layer between a medium $l$ and a medium $r$ are calculated from the components of the incident light field through the relation[28,29]

$$\begin{pmatrix} a_{i-1} \\ b_{i-1} \end{pmatrix} = \mathbf{D}_{\mathrm{l}}^{-1}\mathbf{T}\mathbf{D}_{\mathrm{r}} \begin{pmatrix} a_i \\ b_i \end{pmatrix}, \tag{1}$$

with

$$\mathbf{T} = \mathbf{D}_2\mathbf{P}_2\mathbf{D}_2^{-1}\mathbf{D}_1\mathbf{P}_1\mathbf{D}_1^{-1}, \tag{2}$$

where 1 and 2 label the two different dielectrics

$$D_\alpha = \begin{pmatrix} 1 & 1 \\ n_\alpha & -n_\alpha \end{pmatrix}, \tag{3}$$

represents the effects of the dielectric interface, and

$$P_\alpha = \begin{pmatrix} e^{-i\phi_\alpha} & 0 \\ 0 & e^{i\phi_\alpha} \end{pmatrix}, \tag{4}$$

the propagation inside the single dielectric ($\phi_\alpha = 2\pi n_\alpha d_\alpha/\lambda$ where $d_\alpha$ is the layer thickness in the direction of the light propagation and $\lambda$ is the light wavelength). To obtain the amplitude of the electric field emerging from an $N$ double layer the calculation has to be iterated. We obtain

$$\begin{pmatrix} a_{i-1} \\ b_{i-1} \end{pmatrix} = \mathbf{D}_{\mathrm{l}}^{-1}\mathbf{T}^N\mathbf{D}_{\mathrm{r}} \begin{pmatrix} a_i \\ b_i \end{pmatrix}. \tag{5}$$

The $N$th power of the unimodular matrix $\mathbf{T}$ can be simplified exploiting the Chebyshev identity.[28,29] The system under consideration is a double MC with one side on a substrate with refraction index $n_{\mathrm{sub}}$ and the other in contact with the air; formally, such a system may be modeled by three DBRs between two spacers with refraction index $n_{\mathrm{c}}$ representing the cavity in which the light propagates: the transfer matrix $\mathbf{T}^{\mathrm{d}}$ results

$$\mathbf{T}^{\mathrm{d}} = \mathbf{T}_{\mathrm{dbr}_1}\mathbf{P}_{\mathrm{c}}\mathbf{T}_{\mathrm{dbr}_2}\mathbf{P}_{\mathrm{c}}\mathbf{T}_{\mathrm{dbr}_3}, \tag{6}$$

where $\mathbf{T}_{\mathrm{dbr}_1} = \mathbf{D}_{\mathrm{air}}^{-1}\mathbf{T}^{N_1}\mathbf{D}_{\mathrm{c}}$, $\mathbf{T}_{\mathrm{dbr}_2} = \mathbf{D}_{\mathrm{c}}^{-1}\mathbf{T}^{N_2}\mathbf{D}_{\mathrm{c}}$, $\mathbf{T}_{\mathrm{dbr}_3} = \mathbf{D}_{\mathrm{c}}^{-1}\mathbf{T}^{N_3}\mathbf{D}_{\mathrm{sub}}$. Once the matrix $\mathbf{T}^{\mathrm{d}}$ is calculated, the transmitted $T$ and reflected $R$ intensity coefficients may be easily obtained from the relations

$$T = \frac{n_{\mathrm{sub}}}{n_{\mathrm{air}}}|\mathcal{T}|^2, \tag{7a}$$

$$R = |\mathcal{R}|^2, \tag{7b}$$

where we have defined the transmission $\mathcal{T} = [\mathbf{T}_{11}^{\mathrm{d}}]^{-1}$ and reflection $\mathcal{R} = \frac{\mathbf{T}_{21}^{\mathrm{d}}}{\mathbf{T}_{22}^{\mathrm{d}}}$ amplitudes. We consider a symmetric structure formed by two cavities of length $L_C = \lambda_0/(2n)$, with $\lambda_0 = 800$ nm embedded between two identical dielectric mirrors formed by eight double layers of material $A/B$. In addition, the mirror connecting the two cavities is formed by 12 double layers of the same $A/B$ dielectrics. For the calculations we have chosen LiF/ZnS Bragg mirrors[24] with refraction indexes $n_1 = 1.392$, $n_2 = 2.352$, the refraction index of the spacer is $n = 1.392$ hence $L_C = 287.3$ nm and the thickness of the two layers forming the single DBR is, respectively, $d_{\mathrm{A}} = \lambda_0/(4n_1) = 143.7$ nm and $d_{\mathrm{B}} = \lambda_0/(4n_2) = 85$ nm. The frequency dependence of the transmitted intensity is shown in Fig. 2: the Rabi peaks are the signature of the coupling between the two proper light modes; in the inset the transmission is shown in a longer energy range. We first consider excitation of the system by a Gaussian light pulse arriving from the left of the coupled system

$$E_{\mathrm{in1}}(t) = \frac{1}{\sigma\sqrt{2\pi}}e^{-i\omega_0(t-t_0)}e^{-\frac{(t-t_0)^2}{2\sigma^2}},$$

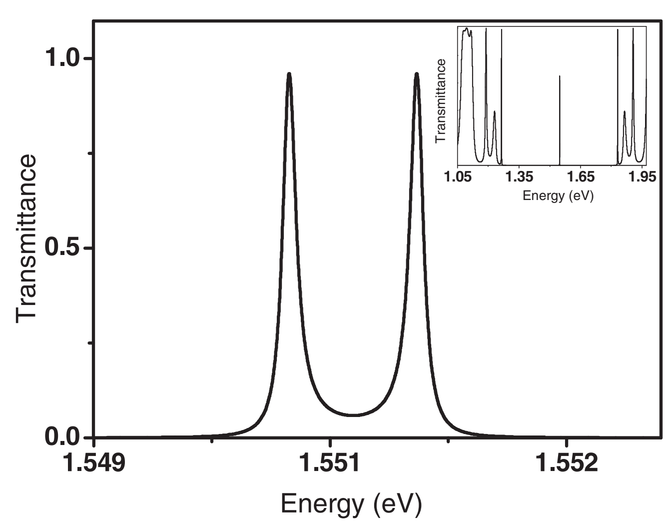


FIG. 2. Detail of the calculated transmitted intensity as a function of the energy (frequency) of incident light, showing the presence of two split resonances due to the coupling between the two MCs. In the inset is shown the calculated transmittance in a longer spectral range. Owing to the wide frequency range the double peak is not resolved in the inset.

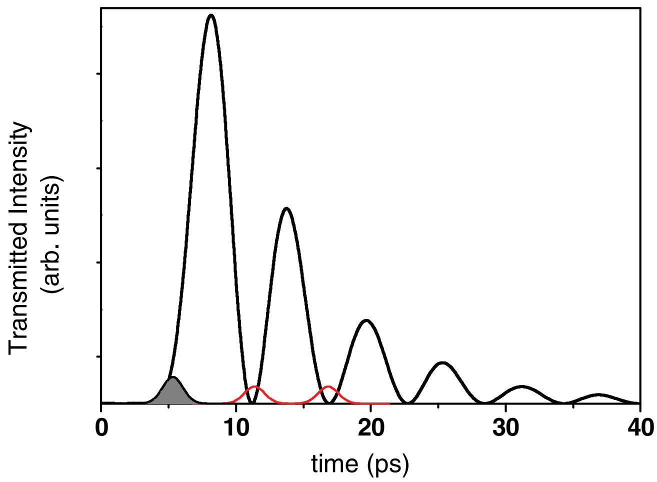


FIG. 3. (Color online) Light field transmitted intensity inside the cavity in function of time when a single excitation is sent at $t_0 = 5.3$ ps (in figure the Gaussian pulse with filled area). For reference are shown also the other two light pulses [in the text, respectively, $E_{\mathrm{in}1}(t)$ and $E_{\mathrm{in}2}(t)$] that may be sent such that the time delay between the two pulses corresponds to a complete Rabi-like oscillation and in correspondence of the first to minima (at 11.36 and 16.83 ps, respectively).

[with a full width at half maximum (FWHM) = 2.5 nm] modulated at the central frequency $\omega_0 = 2\pi c/\lambda_0 = 2.356 \cdot 10^{15}$ Hz $= 1.55$ eV. The pulse arrival time is at $t_0 = 5\sigma$ being $\sigma = 1.06$ ps. The time evolution of the transmitted electric field is obtained by exploiting the Fourier transform,

$$E_{\mathrm{t}}(t) = \int d\omega\, \mathcal{T}(\omega)\tilde{E}_{\mathrm{in}}(\omega)e^{-i\omega t}\,, \tag{8}$$

with $\tilde{E}_{\mathrm{in}}(\omega)$ being the Fourier transform of $E_{\mathrm{in}}(t)$. This approach may be used also in more complex situations when a nonclassical light beam is sent inside the cavity.[30–32] The calculated field intensity $|E_{\mathrm{t}}(t)|^2$, obtained for $E_{\mathrm{in}}(t) = E_{\mathrm{in}1}(t)$, is shown Fig. 3. The figure also displays (arb. units) the corresponding Gaussian input pulse (filled curve) as well as other two Gaussian pulses which will be employed in addition to the first one for studying time-resolved interference effects. The transmitted intensity displays a damped oscillatory time behavior (with Rabi frequency $\Omega_R$) originating from the combination of coherent energy exchange between the two MCs and losses through the external mirrors. To inspect the phase of the transmitted field after one or two Rabi-like oscillations, we now consider a second pulse in phase with the first one sent from the left into the double semiconductor planar MC (see Fig. 1). The total input field can be expressed as,

$$E_{\mathrm{in}2}(t) = E_{\mathrm{in}1}(t) + \frac{A}{\sigma\sqrt{2\pi}}e^{-i\omega_0(t-t_1)}e^{-\frac{(t-t_1)^2}{2\sigma^2}}\,,$$

with $A = 0.8$ being a real amplitude. The transmitted intensity is calculated for two different physical situations as shown in Fig. 4. First we address the case when the arrival time of the second pulse is chosen so that the corresponding first maximum in the transmitted field is exactly in time with the second maximum originating from the first pulse [Fig. 4(a)]. In particular the time delay between the two pulses corresponds to a complete Rabi-like oscillation: $\Omega_R(t_1 - t_0) = 2\pi$. In this case we find that the total signal is strongly damped due to destructive interference. Hence, such an abrupt damping of the signal demonstrates that the transmitted field after a complete oscillation $[\Omega_R(t_1 - t_0) = 2\pi]$ acquires a $\pi$ phase (minus sign). If the arrival time of the second pulse is chosen so that $\Omega_R(t_1 - t_0) = 4\pi$ [see Fig. 4(a)] the total signal gets amplified due to constructive interference. This condition is verified when the corresponding first maximum in the transmitted field is exactly in time with the next (third) maximum originating from the first pulse. As a further analysis, we calculate the interference effects in the transmitted intensity when the phase shift of the second pulse with respect to the first one is continuously varied. Figure 5 shows, in the same physical situations described in Fig. 4, the transmitted field intensity as a function of the time and of the phase shift $\phi$ between the two input fields. The dots along the time axis indicate the starting time of the light pulses. As expected in Fig. 5(a) a destructive interference effect is observed at zero phase shift instead, when the phase between the two excitations is opposite, we observe constructive interference. A complementary behavior is observed when the second pulse is sent with a time delay $\Omega_R(t_1 - t_0) = 4\pi$.

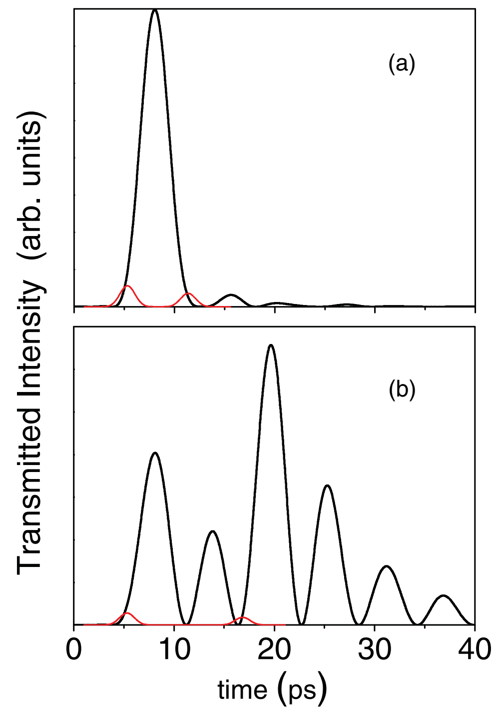


FIG. 4. (Color online) Transmitted intensity calculated when, after the initial stimulation, a second pulse (red curve) in phase with the first one is sent such that the time delay between the two pulses corresponds to a complete Rabi-like oscillation (a) in correspondence of the first minimum, (b) in correspondence of the second minimum. The abrupt damping of the signal in (a) (destructive interference) shows that the transmitted intensity after a complete oscillation ($2\pi$ phase shift) has a opposite phase with respect to the pulse, in (b) the constructive interference (zero phase shift between the two signals) is recovered after two complete oscillations ($4\pi$ phase shift).

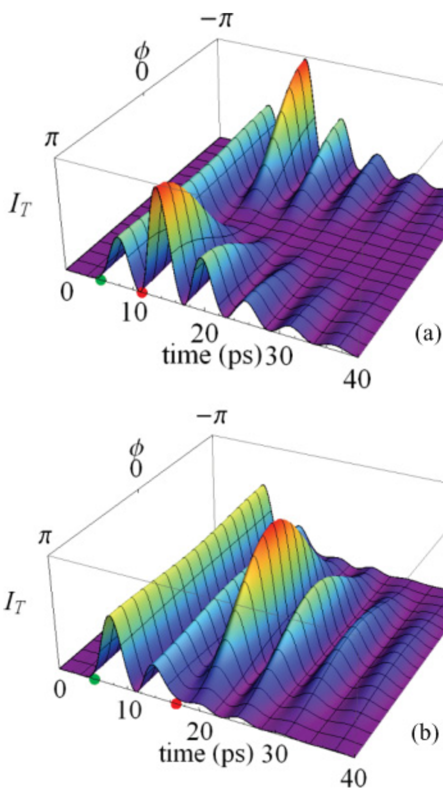


FIG. 5. (Color online) Transmitted intensity calculated analogously to what is shown in Fig. 4. The second pulse (see the dots in the panels showing when the two pulses are sent) are allowed to vary the phase shift with respect to the first excitation.)

## III. ANALYTICAL MODEL: TWO COUPLED OSCILLATORS WITH SOURCE TERM

The numerical results obtained in the previous section for coupled light modes in the double MC system may be better understood through a simplified analytical model. We adopt the quasimode approach. The discrete cavity modes (one for each MC) interact with an external multimode field. The quasimode approximation allows us to describe such systems analogously to a two interacting oscillators system. In particular, we consider a system of two coupled harmonic oscillators (the light modes of the two coupled cavities) with an external source $\epsilon(t)$. The Hamiltonian of such a system can be written as

$$\mathbf{H} = \hbar\omega_0 \mathbf{a}^\dagger \mathbf{a} + \hbar\omega_0 \mathbf{b}^\dagger \mathbf{b} - \hbar g(\mathbf{a}^\dagger \mathbf{b} + \text{H.c.}) + [\epsilon(t)\mathbf{a}^\dagger + \text{H.c.}], \tag{9}$$

where **a** and **b** are, respectively, the bosonic operators relative to the single mode in each cavity, the coupling $g$ depends on the reflectivity of the central mirror, and $\epsilon(t)$ describes the feeding of the cavity by a classical input beam. The resulting evolution equations for the photon operators inside the two cavities are

$$\begin{aligned} i\hbar\frac{d}{dt}\langle\mathbf{a}\rangle &= \hbar\omega_0\langle\mathbf{a}\rangle - \hbar g\langle\mathbf{b}\rangle - \frac{i\hbar\gamma}{2}\langle\mathbf{a}\rangle + \epsilon(t), \\ i\hbar\frac{d}{dt}\langle\mathbf{b}\rangle &= \hbar\omega_0\langle\mathbf{b}\rangle - \hbar\langle\mathbf{a}\rangle - \frac{i\hbar\gamma}{2}\langle\mathbf{b}\rangle, \end{aligned} \tag{10}$$

where $\langle\cdot\rangle$ indicates the mean value of the operator, and $\gamma$ takes into account the damping and losses of a field inside the structure and may be considered as a phenomenological parameter or as obtained from the master equation for two coupled oscillators interacting with a zero-temperature thermal reservoir.[34] In the rotating frame (putting $\omega_0 = 0$), if losses are neglected ($\gamma = 0$) and considering the input field in the cavity as a sharp pulse sent at $t = t_0$ whose functional expression may be modeled as $\epsilon(t) = Ae^{-i\omega_0 t}\delta(t - t_0)$, we obtain

$$\begin{aligned} \langle\mathbf{a}\rangle &= -i\frac{A}{\hbar}\cos\left[\frac{\Omega_{\rm R}(t-t_0)}{2}\right], \\ \langle\mathbf{b}\rangle &= \frac{A}{\hbar}\sin\left[\frac{\Omega_{\rm R}(t-t_0)}{2}\right], \end{aligned} \tag{11a}$$

$$\begin{aligned} \langle\mathbf{a}^\dagger\mathbf{a}\rangle &= A^2\frac{1+\cos[\Omega_{\rm R}(t-t_0)]}{2\hbar^2}, \\ \langle\mathbf{b}^\dagger\mathbf{b}\rangle &= A^2\frac{1-\cos[\Omega_{\rm R}(t-t_0)]}{2\hbar^2}, \end{aligned} \tag{11b}$$

where $\Omega_{\rm R} = 2g/\hbar$ represents the Rabi frequency. We now calculate the number of photons emerging from the cavity B, $\langle\mathbf{b}^\dagger\mathbf{b}\rangle$, that can be measured by a photodetector. Inspecting Eq. (11b), we observe that it oscillates with a Rabi of frequency $\Omega$. Instead, we observe, as is evident from Eq. (11a), that $\langle\mathbf{b}\rangle$ oscillates with a double period with respect to the light cavity population (i.e., at a frequency equal to $\Omega/2$). After a Rabi period $T = 2\pi/\Omega_{\rm R}$, we have $\langle\mathbf{b}\rangle_T = -\langle\mathbf{b}\rangle_0 = -A/\hbar$. Such behavior is the optical analog of the spin-1/2 system undergoing a $2\pi$ rotation in ordinary space.[3,4] In addition, if the time delay is $t = 2T = 4\pi/R$ (i.e., after a $4\pi$ Rabi oscillation) then $\langle\mathbf{b}\rangle_T = \langle\mathbf{b}\rangle_0 = A/\hbar$: the two signals are now in phase and we have the corresponding $4\pi$ rotation in a spin-1/2 system. We observe no phase change behavior in $\langle\mathbf{b}^\dagger\mathbf{b}\rangle$. The results in this section show that the simple analytical model here analyzed contains all the essential physics of the process discussed in the previous section, including the $\pi$ phase shift after a complete Rabi-like oscillation. It may be useful for gathering analytical results and behaviors in analogous systems.

## IV. DISCUSSION

The homodyne-like technique here described can be exploited to gather complete information about the phase of the electromagnetic field in general, coupled microcavity systems.[35] For example, the experiment here proposed can be generalized to more than two coupled MC systems. We expect that three coupled MCs are the optical analogs of $J = 1$ total angular momentum. In this case no phase shift after a complete oscillation should be observed in contrast with the case of two coupled MCs here described and in contrast to the case of 4 MCs ($J = 3/2$).

The homodyne-like method here employed can be also exploited to test the time-control of the wave-particle duality recently proposed.[33] Ridolfo *et al.* showed that the all-optical control of photonic coherence can be realized with a cavity embedded three-level emitter. In particular, the coherence of cavity photons can be suddenly switched on and off by exciting the emitter with suitable control pulses. The loss of coherence, after the arrival of a suitable control pulse, is inferred by calculating the coherent part of the cavity-photon population[33] $|\langle a\rangle|^2$, with $a$ being the photon annihilation operator of the MC mode. The experimental measurement of $\langle a\rangle$ requires a phase sensitive scheme. In the present case, a homodyne detection scheme could be implemented by superimposing the output MC field with a portion of the input resonant laser field feeding the MC (playing the role of local oscillator). However, owing to the light-matter strong-coupling regime, the amplitude and phase dynamics of the output MC field are drastically different from that of the input field, resulting in a complex and unclear dynamics hiding interference effects. In this case the method of two phase-locked probe pulses here adopted could be used to test the presence or absence of coherence. In this way the coherence of MC photons can be probed by simply measuring the cavity output photon-rate after excitation with pairs of phase-locked weak pulses. In the absence of coherence, no interference effects should be observed in the transmitted intensity as a function of time and phase in contrast to the results of Fig. 5.

Another intriguing application could be a double MC system with an active layer (or even a single quantum emitter for a three-dimensional MC) embedded in one of the two MCs. In this case, the optical control of the quantum state of the active layer by an additional beam could be exploited to introduce controllable phase shifts during light propagation.

## V. CONCLUSION

In this paper we proposed an all-optical analog of the well-known sign change of the spinor wave functions under $2\pi$ rotations. The system here investigated consists of two planar MCs coupled through a central mirror. We exploited the transfer-matrix approach and Fourier transform to study the time resolved response after the arrival of coherent optical pulses. Here the two modes (in the absence of coupling) play the role of the two spin states, whereas the coupling induces a quasiperiodic exchange of the optical excitation among the two modes after ultrafast optical excitation. A complete oscillation of the excitation from one mode to the other and back is the optical analog of a $2\pi$ spin rotation. We showed that by feeding the coupled-MC system with a pair of phase-locked probe pulses separated by precise delay times, we can gather direct information on the sign change of the transmitted field after one complete Rabi-like oscillation period. Such results were also explained qualitatively by a simplified physical model considering two coupled damped oscillators.


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