Density of states anomalies in multichannel quantum wires

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We reformulate the Tomonaga–Luttinger liquid theory for quasi-one-dimensional Fermion systems with many subbands across the Fermi energy. Our theory enables us to obtain a rigorous expression of the local density of states (LDOS) for general multichannel quantum wires, describing how the power-law anomalies of the LDOS depend on inter- and intra-subbands couplings as well as the Fermi velocity of each band. The resulting formula for the exponents is valid in the cases of both bulk contact and edge contact, and thus plays a fundamental role in the physical properties of multicomponent Tomonaga–Luttinger liquid systems.

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I. INTRODUCTION

Bosonization is one of the most powerful techniques for describing the properties of one-dimensional (1D) interacting electron systems. In 1D systems, even a slight interaction between electrons strongly affects the quantum nature, resulting in the occurrence of Tomonaga–Luttinger liquid (TLL) states.[1–4] TLL states exhibit power-law anomalies in physical quantities, as predicted by the bosonization theory.[5] A prominent example is the power-law singularity of the single-particle density of states $D(E,T)$ near the Fermi energy $E_{\rm F}$, represented by $D(E,0) \propto |E - E_{\rm F}|^{\lambda}$ and $D(E_{\rm F},T) \propto T^{\lambda}$, with E and T being the energy and the temperature, respectively. The value of λ, called the TLL exponent, is dependent on the interaction strength[5] and other parameters characterizing the 1D system.[6–13] Recently, it has been suggested that a continuous variation in λ can be produced by an external field;[14–16] this implies artificial control of the transport properties of quasi-1D conductors, since λ governs the power-law behaviors of the differential tunneling conductance[9] $dI/dV \propto |V|^{\lambda}$ at high bias voltages ($eV \gg k_B T$) and the temperature-dependent conductance $G(T) \propto T^{\lambda}$ at low voltages ($eV \ll k_B T$).

Experimental realizations of TLL states encompass various systems showing highly anisotropic conductivity: metallic,[17,18] semiconducting,[19–25] and organic nanowires[26–30] and carbon nanotubes[9,10,31–34] are a few examples. These actual quasi-1D conductors possess a finite cross section, thus exhibiting a finite number of transmission channels in the transverse direction (except for a limited case in which $E_{\rm F}$ is small enough for only the lowest subband to be involved). The presence of multiple channels at $E_{\rm F}$ causes intersubband scatterings. Furthermore, different channels can have different Fermi velocities, i.e., the slope of the dispersion curve at $E_{\rm F}$ (see Fig. 1), and thus contributions from each channel to the TLL exponent differ from each other. Theories of a multichannel TLL have been developed for the Hubbard model in the presence of an external magnetic field,[35–37] where the discrepancies in the Fermi velocity between up and down spins are taken into account. A similar issue was also discussed in the study of quasi-1D Bose gases.[38] The effect of intersubband scattering on the TLL exponent has been investigated in connection with the TLL behavior of multiwall carbon nanotubes.[39] However, to the best of our knowledge, singular behavior in $D(E,T)$ remains unresolved for multichannel TLL systems with the coexistence of intersubband scatterings and Fermi-velocity variations. Hence, the rigorous expression of $D(E,T)$ in multichannel TLL systems is desirable for describing the transport properties and photoemission spectra that will be experimentally observed in actual quasi-1D conductors.

In this paper, we reformulate the multichannel TLL theory in order to derive the anomalous energy- and temperature-dependences of the local density of states of quasi-1D Fermion systems. Cases of locations both far from the boundary and close to it, which correspond to bulk contact and end contact of the transport properties, respectively, are discussed. We demonstrate clearly how the TLL exponents of multichannel systems depend on mutual interaction and Fermi velocities. The resulting formula for the exponents, as well as the theoretical framework we have established, will provide clues to exploiting the effects of subband couplings and Fermi velocity variations on the nature of TLLs in real 1D systems.

The paper is organized as follows. In Sec. II, the multichannel TLL theory is developed for N-channel quasi-1D Fermionic systems with different Fermi velocities. The local densities of states far from and close to the boundary are calculated in Sec. III. As a simple example, in Sec. IV, the theory is applied to two-channel spinless Fermion systems in the long-range interaction limit, by which effects of discrepancy in the Fermi velocity on the exponents are clarified. The paper closes with a summary in Sec. V. In the following, the unit $\hbar = k_B = 1$ is used, unless explicitly stated otherwise.

II. MULTICOMPONENT TOMONAGA-LUTTINGER LIQUIDS

A. Bosonization

We consider a quasi-1D Fermion system where N 1D energy bands cross $E_{\rm F}$. The band structure close to $E_{\rm F}$ is schematically shown in Fig. 1. Here, the Fermi velocity and the Fermi wave number of the νth band ($\nu = 1, \ldots, N$) are denoted by $v_{{\rm F}\nu}$ and $k_{{\rm F}\nu}$, respectively, and the one-particle state

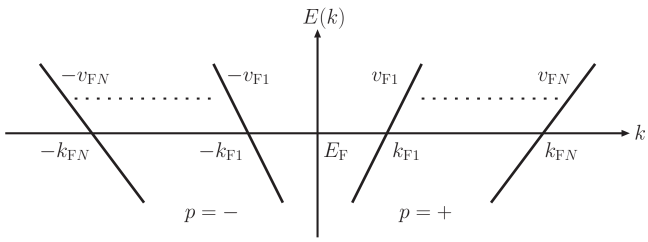

FIG. 1. Sketch of the energy dispersion of the present system, where N energy bands cross E_F. The Fermi velocity and the Fermi wave number of the νth band ($\nu = 1,\ldots,N$) are denoted by $v_{F\nu}$ and $k_{F\nu}$, respectively. The symbol $p = +(-)$ indicates a one-particle state moving toward the right (left).

moving to the right (left) is indicated by $p = +\ (-)$. The kinetic energy of the Hamiltonian, $\mathcal{H}_k$, is expressed by

$$\mathcal{H}_k = \sum_{\nu=1}^{N}\sum_{p=\pm}\sum_{k} p v_{F\nu} k c^\dagger_{k,p,\nu} c_{k,p,\nu}, \quad (1)$$

where the one-particle energy and the wave number k are measured from E_F and $pk_{F\nu}$, respectively. In Eq. (1), $c^\dagger_{k,p,\nu}$ denotes the creation operator of the Fermion with wave number k, branch p, and band index ν.

Let us introduce the density operator of the p branch of the ν th band, defined as

$$\rho_{p,\nu}(q) \equiv \begin{cases} \sum_k c^\dagger_{k+q,p,\nu}c_{k,p,\nu} & \cdots \quad q \neq 0 \\ N_{p,\nu} = \sum_k :c^\dagger_{k,p,\nu}c_{k,p,\nu}: & \cdots \quad q = 0 \end{cases}, \quad (2)$$

which satisfies the commutation relation $[\rho_{p,\nu}(-q),\rho_{p',\nu'}(q')] = \delta_{pp'}\delta_{\nu\nu'}\delta_{qq'}pqL/(2\pi)$. In terms of $\rho_{p,\nu}(q)$, $\mathcal{H}_k$ is expressed by[3,4]

$$\mathcal{H}_k = \sum_{\nu=1}^{N}\frac{\pi v_{F\nu}}{L}\sum_{p,q}\rho_{p,\nu}(q)\rho_{p,\nu}(-q), \quad (3)$$

where L is the length of the system. The most general form of the mutual interaction between Fermions leading to the N-component TLL is written as[35]

$$\mathcal{H}_{\rm int} = \frac{1}{2L}\sum_{\nu,\nu'=1}^{N}\sum_{p,q}\{\tilde{g}_2(\nu,\nu')\rho_{p,\nu}(q)\rho_{-p,\nu'}(-q) + \tilde{g}_4(\nu,\nu')\rho_{p,\nu}(q)\rho_{p,\nu'}(-q)\}. \quad (4)$$

The matrix elements $\tilde{g}_2(\nu,\nu')$ and $\tilde{g}_4(\nu,\nu')$ depend on the details of the model we consider. Specifically, the case with $\tilde{g}_2 \equiv \tilde{g}_4$ corresponds to the model for multiwall carbon nanotubes considered in Ref. 39. As an example, we will discuss the case of the spinless Fermion in Sec. IV.

We introduce the phase variables $\theta_\nu(x)$ and $\phi_\nu(x)$ ($\nu = 1,\cdots,N$), defined as

$$\theta_\nu(x) = -\frac{1}{\sqrt{2}}\sum_p p\left\{Q_{p,\nu} - \frac{2\pi p x}{L}N_{p,\nu} - \frac{2\pi i}{L}\sum_{q\neq 0} p\frac{e^{-iqx}}{q}\rho_{p,\nu}(q) - p\frac{\pi}{2}N_{-p,\nu}\right\}, \quad (5)$$

$$\phi_\nu(x) = -\frac{1}{\sqrt{2}}\sum_p \left\{Q_{p,\nu} - \frac{2\pi p x}{L}N_{p,\nu} - \frac{2\pi i}{L}\sum_{q\neq 0} p\frac{e^{-iqx}}{q}\rho_{p,\nu}(q) - p\frac{\pi}{2}N_{-p,\nu}\right\}, \quad (6)$$

where $[Q_{p,\nu},N_{p',\nu'}] = i\delta_{pp'}\delta_{\nu\nu'}$. In the summation in terms of q, the ultraviolet cutoff $\exp(-\alpha|q|/2)$ is implicitly included. The phase variables satisfy the commutation relation $[\theta_\nu(x),\phi_{\nu'}(x')] = i2\pi\delta_{\nu\nu'}\theta(x-x')$ for $L\to\infty$, with $\theta(x)$ being the conventional step function. In terms of the above phase variables, the Hamiltonian is written as

$$\mathcal{H} = \frac{1}{2}\sum_{\nu,\nu'=1}^{N}\int dx\{\Pi_\nu (K^{-1})_{\nu\nu'}\Pi_{\nu'} + \partial_x\theta_\nu V_{\nu\nu'}\partial_x\theta_{\nu'}\}, \quad (7)$$

where $\Pi_\nu = -\partial_x\phi_\nu/(2\pi)$. This is the general form of the phase Hamiltonian expressing the N-component TLL. The symmetric matrices K and V are defined as follows:

$$(K^{-1})_{\nu\nu'} = 2\pi v_{F\nu}\delta_{\nu\nu'} + \tilde{g}_4(\nu,\nu') - \tilde{g}_2(\nu,\nu'), \quad (8)$$

$$V_{\nu\nu'} = \frac{v_{F\nu}}{2\pi}\delta_{\nu\nu'} + \frac{\tilde{g}_4(\nu,\nu') + \tilde{g}_2(\nu,\nu')}{4\pi^2}. \quad (9)$$

The Fermion operator, defined by $\psi_{p,\nu}(x) = (1/\sqrt{L})\sum_k e^{ikx}c_{k,p,\nu}$, is related to the phase variables as

$$\psi_{p,\nu}(x) = \frac{\eta_\nu}{\sqrt{2\pi\alpha}}\exp\left\{i\frac{p}{\sqrt{2}}[\theta_\nu(x) + p\phi_\nu(x)]\right\}, \quad (10)$$

where η_ν expresses the Majorana Fermion operator satisfying $\eta_\nu = \eta_\nu^\dagger$ and $\{\eta_\nu,\eta_{\nu'}\} = 2\delta_{\nu\nu'}$.

B. Diagonalization

The Hamiltonian given by Eq. (7) has a bilinear form with respect to $\partial_x\phi_\nu$ and $\partial_x\theta_\nu$, and thus can be diagonalized by the standard unitary transformation, as shown below.

The equations of motion of the phase variables derived from Eq. (7) read as

$$\frac{\partial}{\partial t}\boldsymbol{\Pi} = V\frac{\partial^2}{\partial x^2}\boldsymbol{\theta}, \quad (11)$$

$$\frac{\partial}{\partial t}\boldsymbol{\theta} = K^{-1}\boldsymbol{\Pi}, \quad (12)$$

with $\boldsymbol{\theta} = (\theta_1,\theta_2,\cdots,\theta_N)^{\rm T}$ and $\boldsymbol{\Pi} = (\Pi_1,\Pi_2,\ldots,\Pi_N)^{\rm T}$. Here, the energy eigenvalue $\omega = v|k|$ and the eigenvector $\boldsymbol{X}$ corresponding to it are determined by

$$(v^2K - V)\boldsymbol{X} = 0, \quad (13)$$

whose solutions are denoted by v_j and $\boldsymbol{X}_j$ ($j = 1,2,\ldots,N$). The eigenvectors are normalized as $(\boldsymbol{X}_i,K\boldsymbol{X}_j) = \delta_{ij}$.

To obtain a concise representation of $\mathcal{H}$, we define the unitary transformation as

$$\boldsymbol{\theta} = X\boldsymbol{\Theta}, \quad (14)$$

$$\boldsymbol{\Pi} = KX\boldsymbol{\Xi}, \quad (15)$$

where $\boldsymbol{\Theta} = (\Theta_1,\Theta_2,\ldots,\Theta_N)^{\rm T}$ and $\boldsymbol{\Xi} = (\Xi_1,\Xi_2,\ldots,\Xi_N)^{\rm T}$. The $N\times N$ matrix X consists of the set of eigenvectors $\boldsymbol{X}_j$ as $X = (\boldsymbol{X}_1,\boldsymbol{X}_2,\ldots,\boldsymbol{X}_N)$, and satisfies $X^{\rm T}KX = 1$. Under the transformation, $[\Theta_j(x),\Xi_{j'}(x')] = i\delta_{jj'}\delta(x-x')$. By using the new variables, we obtain an alternative form of $\mathcal{H}$ given by

$$\mathcal{H} = \frac{1}{2}\sum_{j=1}^{N}\int dx\left\{\Xi_j^2 + v_j^2(\partial_x\Theta_j)^2\right\}, \tag{16}$$

and that of the field operator defined in Eq. (10)

$$\psi_{p,\nu}(x) = \frac{\eta_\nu}{\sqrt{2\pi\alpha}} \times \exp\left(i\frac{p}{\sqrt{2}}\sum_{j=1}^{N}\{X_{\nu j}\Theta_j(x) + p(KX)_{\nu j}\Phi_j(x)\}\right), \tag{17}$$

where $\phi_\nu = \sum_{j=1}^N (KX)_{\nu j}\Phi_j$.

III. DENSITY OF STATES

In this section, we discuss the local density of states $D(\omega,T,x)$ with $\omega \equiv E - E_F$ for $\omega \ll E_F$, where x denotes the position along the 1D direction. As noted earlier, the TLL exponent that characterizes the singularity of $D(\omega,T,x)$ near E_F is x dependent. From a practical view, it is specifically interesting to study the semi-infinite system with its end at the origin[6–8] and discuss the cases with $x \to 0$ and $x \to \infty$, which correspond to the end contact and the bulk contact, respectively. In the following argument, we therefore derive the TLL exponent for both cases, as well as the explicit forms of $D(\omega,T,x)$ as functions of ω and T.

The local density of states is given by the summation of the contribution from each band: $D(\omega,T,x) = \sum_{\nu=1}^N D_\nu(\omega,T,x)$. The contribution from the νth band, $D_\nu(\omega,T,x)$, is given by

$$D_\nu(\omega,T,x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt e^{i\omega t}\langle\{\psi_\nu^\dagger(x,0),\psi_\nu(x,t)\}\rangle, \tag{18}$$

where $\psi_\nu(x,t) = e^{ik_{F\nu}x}\psi_{+,\nu}(x,t) + e^{-ik_{F\nu}x}\psi_{-,\nu}(x,t)$. Here, $\{A,B\} \equiv AB + BA$ and $\langle\cdots\rangle$ means the thermal average. The quantity in the integrand in Eq. (18) for $x \to \infty$ is proved to be

$$\langle\{\psi_\nu^\dagger(x\to\infty,0),\psi_\nu(x\to\infty,t)\}\rangle = \frac{1}{\pi\alpha}[F^{(b)}(t) + F^{(b)}(-t)], \tag{19}$$

$$F^{(b)}(t) = \left(\frac{\pi T t}{\sinh \pi T t}\right)^{\sum_{j=1}^N Y_{\nu,j}^{(b)}} \times \prod_{j=1}^{N}\frac{1}{(1 - iv_j t/\alpha)^{Y_{\nu,j}^{(b)}}}, \tag{20}$$

$$Y_{\nu,j}^{(b)} = \frac{1}{2}\left\{\frac{(X_{\nu,j})^2}{2\pi v_j} + 2\pi v_j[(KX)_{\nu,j}]^2\right\}, \tag{21}$$

where the superscript (b) means the case of a "bulk" contact. Similarly, the counterpart for the "edge" contact case $x \to 0$, labeled by (e), obeys Eqs. (19)–(21) with $Y_{\nu,j}^{(b)}$ replaced by

$$Y_{\nu,j}^{(e)} = 2\pi v_j[(KX)_{\nu,j}]^2. \tag{22}$$

The derivations of Eqs. (19)–(22) are shown in Appendix A.

We are ready to obtain the rigorous expression for $D_\nu^{(b/e)}(\omega,T) \equiv D_\nu(\omega,T,x\to\infty/0)$.[8] Since $D_\nu^{(b/e)}(0,0)$ vanishes as long as $\sum_{j=1}^N Y_{\nu,j}^{(b/e)} > 1$, we subtract it from the result and take the limit $\alpha \to 0$. Eventually, we attain the desired formulas:

$$\begin{aligned} D_\nu^{(b/e)}(\omega,T) &= \frac{1}{2\pi^2\alpha}\int_{-\infty}^{\infty} dt\left\{e^{i\omega t}\left(\frac{\pi T t}{\sinh\pi T t}\right)^{\sum_{j=1}^N Y_{\nu,j}^{(b/e)}} - 1\right\} \\ &\quad \times\left\{\prod_{j=1}^N \frac{1}{(1 - iv_j t/\alpha)^{Y_{\nu,j}^{(b/e)}}} + \prod_{j=1}^N \frac{1}{(1 + iv_j t/\alpha)^{Y_{\nu,j}^{(b/e)}}}\right\} \\ &= \frac{2}{\pi^2\alpha}\prod_{j=1}^N\left(\frac{\alpha}{v_j}\right)^{Y_{\nu,j}^{(b/e)}}\cos\left(\frac{\pi}{2}\sum_{j=1}^N Y_{\nu,j}^{(b/e)}\right) \\ &\quad \times\int_0^\infty dt\left\{\cos\omega t\left(\frac{\pi T t}{\sinh\pi T t}\right)^{\sum_{j=1}^N Y_{\nu,j}^{(b/e)}} - 1\right\}\frac{1}{t^{\sum_{j=1}^N Y_{\nu,j}^{(b/e)}}}. \end{aligned} \tag{23}$$

Equation (23) implies that the ω and T dependences of $D_\nu^{(b/e)}(\omega,T)$ are given by

$$D_\nu^{(b/e)}(\omega,0) = \frac{1}{\pi\alpha}\prod_{j=1}^N\left(\frac{\alpha}{v_j}\right)^{Y_{\nu,j}^{(b/e)}} \times \frac{1}{\Gamma\left[\sum_{j=1}^N Y_{\nu,j}^{(b/e)}\right]}\omega^{\sum_{j=1}^N Y_{\nu,j}^{(b/e)} - 1}, \tag{24}$$

$$D_\nu^{(b/e)}(0,T) = \frac{1}{\pi^2\alpha}\prod_{j=1}^N\left(\frac{\alpha}{v_j}\right)^{Y_{\nu,j}^{(b/e)}} \times \frac{\left\{\Gamma\left[\sum_{j=1}^N Y_{\nu,j}^{(b/e)}/2\right]\right\}^2}{\Gamma\left[\sum_{j=1}^N Y_{\nu,j}^{(b/e)}\right]}(2\pi T)^{\sum_{j=1}^N Y_{\nu,j}^{(b/e)} - 1}, \tag{25}$$

where $\Gamma[z]$ is the gamma function. It thus follows that the TLL exponent associated with the νth band reads as

$$\lambda^{(b)}(\nu) = \sum_{j=1}^N Y_{\nu,j}^{(b)} - 1 = \sum_{j=1}^N \frac{1}{2}\left\{\frac{(X_{\nu,j})^2}{2\pi v_j} + 2\pi v_j[(KX)_{\nu,j}]^2\right\} - 1 \tag{26}$$

for the bulk position, and

$$\lambda^{(e)}(\nu) = \sum_{j=1}^N Y_{\nu,j}^{(e)} - 1 = \sum_{j=1}^N 2\pi v_j[(KX)_{\nu,j}]^2 - 1 \tag{27}$$

for the edge. Equations (24) and (25) are the main findings of this article; they give explicit functional forms of the power-law density of states in N-channel TLL systems. We note that the exponents Eqs. (26) and (27) can be directly read off from the expression of the electron operators Eq. (17) together with Eqs. (A1)–(A3).[40]

Our results indicate that $D(\omega,T,x)$ is given by the summation of the contributions from each of the bands whose powers differ. Therefore, the smallest value would be observed in actual experiments, such as photoemissions and transport properties, because the band with the smallest value of power has the largest contribution to the local density of states (LDOS).

Specifically, if $N=2$ with $v_{F_1}=v_{F_2}=v_F$, the present model is reduced to the conventional 1D electron system, where the backward scattering between the different spins and the Umklapp scattering are ignored. In fact, by parameterizing as

$$\tilde{g}_2(1,1)=\tilde{g}_2(2,2)=g_{2\parallel}-g_{1\parallel}, \quad (28)$$

$$\tilde{g}_2(1,2)=\tilde{g}_2(2,1)=g_{2\perp}, \quad (29)$$

$$\tilde{g}_4(1,1)=\tilde{g}_4(2,2)=g_{4\parallel}, \quad (30)$$

$$\tilde{g}_4(1,2)=\tilde{g}_4(2,1)=g_{4\perp}, \quad (31)$$

Eqs. (26) and (27) lead to the familiar forms

$$\lambda^{(b)}(1)=\lambda^{(b)}(2)=\tfrac{1}{4}\left(K_\rho+K_\rho^{-1}+K_\sigma+K_\sigma^{-1}\right)-1, \quad (32)$$

$$\lambda^{(e)}(1)=\lambda^{(e)}(2)=\tfrac{1}{2}\left(K_\rho^{-1}+K_\sigma^{-1}\right)-1, \quad (33)$$

with

$$K_\rho=\sqrt{\frac{2\pi v_F+g_{4\parallel}+g_{4\perp}-g_{2\parallel}-g_{2\perp}+g_{1\parallel}}{2\pi v_F+g_{4\parallel}+g_{4\perp}+g_{2\parallel}+g_{2\perp}-g_{1\parallel}}}, \quad (34)$$

$$K_\sigma=\sqrt{\frac{2\pi v_F+g_{4\parallel}-g_{4\perp}-g_{2\parallel}+g_{2\perp}+g_{1\parallel}}{2\pi v_F+g_{4\parallel}-g_{4\perp}+g_{2\parallel}-g_{2\perp}-g_{1\parallel}}}. \quad (35)$$

IV. N-CHANNEL SPINLESS FERMIONS

In this section, we derive the matrix elements of the mutual interactions, which are included in the matrices K and V in Eqs. (8) and (9). As a simple example, we consider a quasi-1D spinless Fermion system where N 1D energy bands cross E_F. In addition, in order to clarify the effects of the Fermi velocity difference on the exponents, those for the $N=2$ case with a long-range mutual interaction are derived.

The mutual interaction $\mathcal{H}_{\rm int}$ of the spinless Fermion can be expressed generally as

$$\mathcal{H}_{\rm int}=\frac{1}{2}\iint dxdx'\psi^\dagger(x)\psi^\dagger(x')V(|x-x'|)\psi(x')\psi(x), \quad (36)$$

with $\psi(x)$ being the annihilation operator of the spinless Fermion. Since we are discussing low-energy physics, the interaction processes among the particles close to E_F are necessary. In order to obtain such interaction processes, the operator $\psi(x)$ is expanded, using the eigenfunctions of the states across E_F, $\phi_{\nu,K}(x)$, as

$$\psi(x)=\sum_{\nu=1}^{N}\sum_{K}a_{\nu,K}\phi_{\nu,K}(x), \quad (37)$$

where $a_{\nu,K}$ is the operator of the spinless Fermion with the eigenstate (ν,K). By inserting Eq. (37) into Eq. (36), $\mathcal{H}_{\rm int}$ is expressed as

$$\mathcal{H}_{\rm int}=\frac{1}{2}\sum_{\nu_1,\nu_2,\nu_3,\nu_4}\sum_{K_1,K_2,K_3,K_4}V_{\nu_1K_1,\nu_2K_2;\nu_3K_3,\nu_4K_4}\times a^\dagger_{\nu_1,K_1}a^\dagger_{\nu_2,K_2}a_{\nu_3,K_3}a_{\nu_4,K_4}, \quad (38)$$

where the matrix element of the mutual interaction is written as

$$V_{\nu_1K_1,\nu_2K_2;\nu_3K_3,\nu_4K_4}=\iint dxdyV(|x-y|)\times\phi^*_{\nu_1,K_1}(x)\phi^*_{\nu_2,K_2}(y)\phi_{\nu_3,K_3}(y)\phi_{\nu_4,K_4}(x). \quad (39)$$

We note that as a result of momentum conservation, the relation $K_1+K_2-K_3-K_4=nG$ holds, where G is the reciprocal lattice vector and n is an integer. In the following, we discuss the case where the filling of each band is incommensurate. Then, only the normal processes satisfying $n=0$ are taken into account as

$$V_{\nu_1K_1,\nu_2K_2;\nu_3K_3,\nu_4K_4}=\delta_{K_1+K_2,K_3+K_4}\times V_{\nu_1,\nu_2;\nu_3,\nu_4}(K_1,K_2;K_3,K_4). \quad (40)$$

In this case, $\mathcal{H}_{\rm int}$ is expressed by

$$\mathcal{H}_{\rm int}=\frac{1}{2}\sum_{\nu_1,\nu_2,\nu_3,\nu_4}\sum_{p_1,p_2,p_3,p_4}\sum_{k_1,k_2,k_3,k_4}\times\delta_{p_1k_{F\nu_1}+p_2k_{F\nu_2},p_3k_{F\nu_3}+p_4k_{F\nu_4}}\delta_{k_1+k_2,k_3+k_4}\times V_{\nu_1,\nu_2;\nu_3,\nu_4}(p_1k_{F\nu_1},p_2k_{F\nu_2};p_3k_{F\nu_3},p_4k_{F\nu_4})\times c^\dagger_{k_1,p_1,\nu_1}c^\dagger_{k_2,p_2,\nu_2}c_{k_3,p_3,\nu_3}c_{k_4,p_4,\nu_4}, \quad (41)$$

where $c_{p,k,\nu}=a_{\nu,pk_{F_\nu}+k}$ and we set $K_i=p_ik_{F\nu_i}+k_i$, with $|k_i|\ll k_{F\nu_j}$ $(i,j=1,\ldots,N)$. Assuming $k_{F\nu}\neq k_{F\nu'}$ for $\nu\neq\nu'$, Eq. (41) is written as $\mathcal{H}_{\rm int}=\mathcal{H}_{\rm int,1}+\mathcal{H}_{\rm int,2}+\mathcal{H}_{\rm int,4}$, where

$$\mathcal{H}_{\rm int,1}=\frac{1}{2}\sum_{k,k',q}\sum_{p=\pm}\sum_{\nu,\nu'=1}^{N}V_{\nu,\nu';\nu,\nu'}\times(pk_{F\nu},-pk_{F\nu'};pk_{F\nu},-pk_{F\nu'})\times c^\dagger_{k+q,p,\nu}c^\dagger_{k'-q,-p,\nu'}c_{k',p,\nu}c_{k,-p,\nu'}, \quad (42)$$

$$\mathcal{H}_{\rm int,2}=\frac{1}{2}\sum_{k,k',q}\sum_{p=\pm}\sum_{\nu,\nu'=1}^{N}V_{\nu,\nu';\nu',\nu}\times(pk_{F\nu},-pk_{F\nu'};-pk_{F\nu'},pk_{F\nu})\times c^\dagger_{k+q,p,\nu}c^\dagger_{k'-q,-p,\nu'}c_{k',-p,\nu'}c_{k,p,\nu}, \quad (43)$$

$$\begin{aligned}\mathcal{H}_{\rm int,4}=&\frac{1}{2}\sum_{k,k',q}\sum_{p=\pm}\sum_{\nu=1}^{N}V_{\nu,\nu;\nu,\nu}(pk_{F\nu},pk_{F\nu};pk_{F\nu},pk_{F\nu})\\&\times c^\dagger_{k+q,p,\nu}c^\dagger_{k'-q,p,\nu}c_{k',p,\nu}c_{k,p,\nu}+\frac{1}{2}\sum_{k,k',q}\sum_{p=\pm}\sum_{\nu\neq\nu'}^{N}\\&\times\{V_{\nu,\nu';\nu',\nu}(pk_{F\nu},pk_{F\nu'};pk_{F\nu'},pk_{F\nu})\\&\times c^\dagger_{k+q,p,\nu}c^\dagger_{k'-q,p,\nu'}c_{k',p,\nu'}c_{k,p,\nu}\\&+V_{\nu,\nu';\nu,\nu'}(pk_{F\nu},pk_{F\nu'};pk_{F\nu},pk_{F\nu'})\\&\times c^\dagger_{k+q,p,\nu}c^\dagger_{k'-q,p,\nu'}c_{k',p,\nu}c_{k,p,\nu'}\}.\end{aligned} \quad (44)$$

Here, $\mathcal{H}_{\rm int,1}$ represents the backward scattering, $\mathcal{H}_{\rm int,2}$ denotes the forward scattering among the different branches, and $\mathcal{H}_{\rm int,4}$ expresses the forward scattering between the same branches. It should be noted that we neglect accidental situations in the momentum conservation, for example, $k_{F\nu_1}-k_{F\nu_2}=-k_{F\nu_3}+k_{F\nu_4}$ with $k_{F\nu_1}\neq k_{F\nu_4}$ and $k_{F\nu_2}\neq k_{F\nu_3}$, in the forward scattering among different branches. Equations (42), (43), and (44) are reduced to

$$\mathcal{H}_{\rm int} = \frac{1}{2L}\sum_{k,k',q}\sum_{p=\pm}\sum_{\nu,\nu'=1}^{N} \times \{g_1(\nu,\nu')c^\dagger_{k+q,p,\nu}c^\dagger_{k'-q,-p,\nu'}c_{k',p,\nu'}c_{k,-p,\nu} + g_2(\nu,\nu')c^\dagger_{k+q,p,\nu}c^\dagger_{k'-q,-p,\nu'}c_{k',-p,\nu'}c_{k,p,\nu} + g_4(\nu,\nu')c^\dagger_{k+q,p,\nu}c^\dagger_{k'-q,p,\nu'}c_{k',p,\nu'}c_{k,p,\nu}\}, \tag{45}$$

where

$$g_1(\nu,\nu') = LV_{\nu,\nu';\nu,\nu'}(k_{{\rm F}\nu},-k_{{\rm F}\nu'};k_{{\rm F}\nu},-k_{{\rm F}\nu'}), \tag{46}$$

$$g_2(\nu,\nu') = LV_{\nu,\nu';\nu',\nu}(k_{{\rm F}\nu},-k_{{\rm F}\nu'};-k_{{\rm F}\nu'},k_{{\rm F}\nu}), \tag{47}$$

$$g_4(\nu,\nu') = L\{V_{\nu,\nu;\nu,\nu}(k_{{\rm F}\nu},k_{{\rm F}\nu};k_{{\rm F}\nu},k_{{\rm F}\nu})\delta_{\nu,\nu'} + [V_{\nu,\nu';\nu',\nu}(k_{{\rm F}\nu},k_{{\rm F}\nu'};k_{{\rm F}\nu'},k_{{\rm F}\nu}) - V_{\nu,\nu';\nu,\nu'}(k_{{\rm F}\nu},k_{{\rm F}\nu'};k_{{\rm F}\nu},k_{{\rm F}\nu'})](1-\delta_{\nu,\nu'})\}. \tag{48}$$

We have used the relation $\phi^*_{\nu,K}(x) = \phi_{\nu,-K}(x)$, which is a result of time-reversal symmetry. Note that $g_1(\nu,\nu')$, $g_2(\nu,\nu')$, and $g_4(\nu,\nu')$ are the real symmetric matrices. By comparing Eq. (45) with Eq. (4), we obtain $\tilde{g}_2(\nu,\nu') = g_2(\nu,\nu') - g_1(\nu,\nu')$ and $\tilde{g}_4(\nu,\nu') = g_4(\nu,\nu')$.

The following discussion makes clear how the discrepancy in the Fermi velocity components (i.e., $v_{\rm F_1} \neq v_{\rm F_2} \neq \cdots \neq v_{{\rm F}_N}$) causes a variety in the values of the exponents $\lambda^{(b/e)}(\nu)$. For simplicity, we consider the $N=2$ case, in which $g_2(\nu,\nu') = g_4(\nu,\nu') = g$ and $g_1(\nu,\nu') = 0$ that are effective approximations for long-range interactions.[39] The velocities of the two excitations are obtained as

$$v_\pm = \sqrt{\frac{1}{2}\{\xi_+ \pm \sqrt{\xi_-^2+\eta^2}\}}, \tag{49}$$

with

$$\xi_\pm = v_{\rm F_1}^2 \pm v_{\rm F_2}^2 + \frac{g}{\pi}(v_{\rm F_1} \pm v_{\rm F_2}), \tag{50}$$

$$\eta = \frac{2g}{\pi}\sqrt{v_{\rm F_1}v_{\rm F_2}}, \tag{51}$$

and the corresponding eigenvectors are given by

$$\boldsymbol{X}_\pm = \left(\sqrt{2\pi v_{\rm F_1}}\cos\theta_\pm, \pm\sqrt{2\pi v_{\rm F_2}}\sin\theta_\pm\right)^{\rm T}, \tag{52}$$

with

$$\tan\theta_\pm = \frac{\sqrt{\xi_-^2+\eta^2}\mp\xi_-}{\eta}. \tag{53}$$

It then follows from Eqs. (26) and (27) that the TLL exponents read as

$$\lambda^{(b)}(1) = \frac{1}{2}\left(\frac{v_{\rm F_1}}{v_+}+\frac{v_+}{v_{\rm F_1}}\right)\cos^2\theta_+ + \frac{1}{2}\left(\frac{v_{\rm F_1}}{v_-}+\frac{v_-}{v_{\rm F_1}}\right)\cos^2\theta_- - 1, \tag{54}$$

$$\lambda^{(b)}(2) = \frac{1}{2}\left(\frac{v_{\rm F_2}}{v_+}+\frac{v_+}{v_{\rm F_2}}\right)\sin^2\theta_+ + \frac{1}{2}\left(\frac{v_{\rm F_2}}{v_-}+\frac{v_-}{v_{\rm F_2}}\right)\sin^2\theta_- - 1, \tag{55}$$

$$\lambda^{(e)}(1) = \frac{v_+}{v_{\rm F_1}}\cos^2\theta_+ + \frac{v_-}{v_{\rm F_1}}\cos^2\theta_- - 1, \tag{56}$$

$$\lambda^{(e)}(2) = \frac{v_+}{v_{\rm F_2}}\sin^2\theta_+ + \frac{v_-}{v_{\rm F_2}}\sin^2\theta_- - 1. \tag{57}$$

Figure 2 shows the exponents $\lambda^{(b/e)}(1)$ and $\lambda^{(b/e)}(2)$ as a function of the Fermi velocity difference u; $v_{\rm F_1} = v_{\rm F}(1+u/2)$ and $v_{\rm F_2} = v_{\rm F}(1-u/2)$ in the plot (a), $v_{\rm F_1} = v_{\rm F}(1+u)$ and $v_{\rm F_2} = v_{\rm F}$ in (b), and $v_{\rm F_1} = v_{\rm F}$ and $v_{\rm F_2} = v_{\rm F}(1-u)$ in (c). For all the plots, the quantity $g/(\pi v_F)$ is fixed to be unity. Difference in the values of $\lambda^{(b/e)}(1)$ and $\lambda^{(b/e)}(2)$ becomes significant and grows monotonically with an increase in u. Besides, it is commonly observed in Figs. 2(a)–2(c) that the exponents are enhanced (reduced) with decreasing (increasing) the associated Fermi velocity. The latter feature is because decreasing Fermi velocity leads to effective enhancement of the electronic correlation and vice versa. As previously noted, the smallest $\lambda^{(b/e)}$ (i.e., the contribution from the largest-$v_{{\rm F}\nu}$ energy band) should take a primary role in determining experimental observations for actual 1D multiband systems.

Finally, it should be noted that the present result for the $4N$-channel systems with equal Fermi velocity and the matrix elements $g_2(\nu,\nu') = g_4(\nu,\nu') = g$ and $g_1(\nu,\nu') = 0$ corresponds to the multiwall carbon nanotubes composed of N metallic graphene sheets studied in Ref. 39. In this case, the velocities of the excitation are obtained as $v_1 = v_{\rm F}\sqrt{1+4Ng/(\pi v_{\rm F})}$ and $v_j = v_{\rm F}$ $(j=2,\ldots,4N)$, and our formulas lead to the same results, $\lambda^{(b)}(\nu) = \{(v_{\rm F}/v_1 + v_1/v_{\rm F})/2 - 1\}/(4N)$ and $\lambda^{(e)}(\nu) = (v_1/v_{\rm F}-1)/(4N)$, as those in Ref. 39.

V. CONCLUSION

In the present paper, we reformulated the TLL theory for multichannel 1D Fermion systems. The theory obtained enables derivation of rigorous expressions for the local density of states and the corresponding TLL exponents $\lambda^{(b/e)}(\nu)$ with respect to the νth band. The strategy for evaluating $\lambda^{(b/e)}(\nu)$ is summarized as follows:

(1) Define the functional forms of the 1D eigenfunction $\phi_{\nu,K}(x)$ and the interaction $V(|x-y|)$ appropriate for the system being considered.

(2) Calculate $V_{\nu_1K_1,\nu_2K_2;\nu_3K_3,\nu_4K_4}$ using Eq. (39).

(3) Using the above result, set the mutual interaction terms $\tilde{g}_i(\nu,\nu')$ that are necessity to define the interaction $\mathcal{H}_{\rm int}$ given by Eq. (4). Particularly when considering spinless Fermions, we can obtain $g_i(\nu,\nu')$ for $i=1,2,4$ by substituting the results of step 2 into Eqs. (46)–(48).

(4) Set $(K^{-1})_{\nu,\nu'}$ and $V_{\nu,\nu'}$ according to Eqs. (8) and (9).

(5) Solve the eigenvalue problem (13) to obtain v_j and $\boldsymbol{X}_j$ for $j=1,\ldots,N$.

(6) Evaluate $Y^{(b)}_{\nu,j}$ and $Y^{(e)}_{\nu,j}$ from Eqs. (21) and (22).

(7) Finally, we obtain the exponents $\lambda^{(b)}(\nu)$ and $\lambda^{(e)}(\nu)$ from Eqs. (26) and (27).

By applying the strategy to 2-channel spinless Fermion systems with different Fermi velocities and long-range mutual interaction, we have revealed the role of difference in the Fermi velocities on the TLL exponent. The exponents become enhanced (suppressed) for the band with a small (large) Fermi velocity since the smaller the Fermi velocity is, the

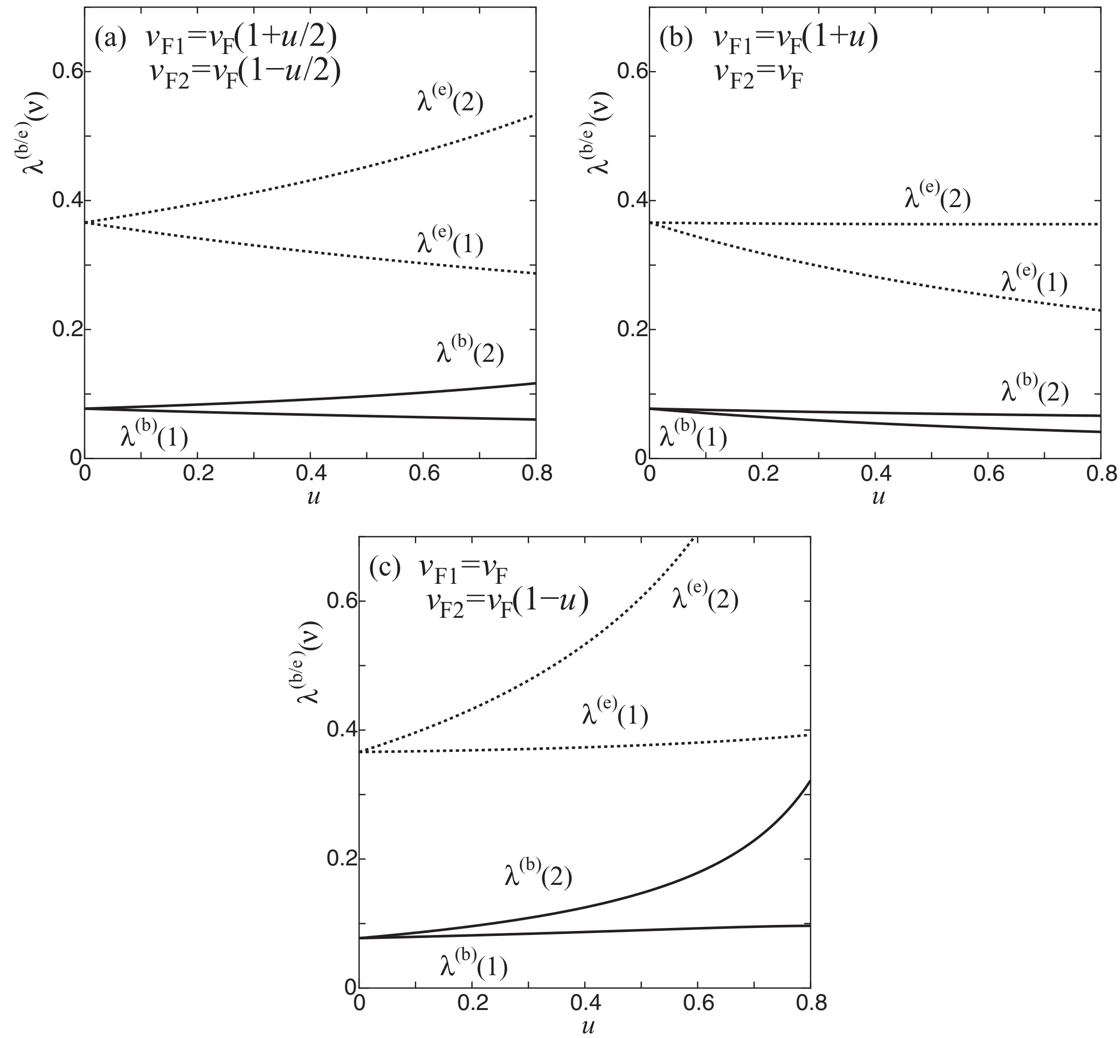

FIG. 2. TLL exponents $\lambda^{(b/e)}(\nu)$ for $g/(\pi v_F) = 1.0$ with $N = 2$ as a function of u for (a) $v_{F_1} = v_F(1 + u/2)$ and $v_{F_2} = v_F(1 - u/2)$, (b) $v_{F_1} = v_F(1 + u)$ and $v_{F_2} = v_F$, and (c) $v_{F_1} = v_F$ and $v_{F_2} = v_F(1 - u)$.

stronger the effective electronic correlation becomes. In the 1D multiband systems, the LDOS is given by the summation of the contribution from each band, and the largest component would be dominant in actual materials. Therefore, the smallest value of the exponent, which results from the band with largest Fermi velocity, would be observed in the actual experiments.

Before closing, we remark that the present theory began with the electronic Hamiltonian of the mutual interaction in Eq. (4), which leads to the bosonic Hamiltonian Eq. (7) that contains no nonlinear terms. But there are possibilities that some interaction terms ignored in our setting of Eq. (4) induce nonlinear terms, and that those may be relevant to the nature of the spinless Fermions under consideration. Relevance of such terms may depend on the details of models, and thus it is quite challenging to examine without loss of generality. We can say that the present theory is useful even though the nonlinear terms arise whenever they are renormalized to zero. In such cases, it is necessary to take account of the renormalization of the parameters K and V by the diminishing nonlinear terms. We should also comment that the present method can be applied to the strictly 1D systems with multiple electron pockets, for example, 1D atomic nanowires formed by Au on Ge(001) that have two metallic electron pockets.[41]

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APPENDIX: DERIVATION OF EQS. (19)–(22)

We discuss a semi-infinite system with its end at the origin. For convenience, we scale the bosonic fields as

$$\tilde{\Theta}_j(x,t) = \sqrt{v_j}\,\Theta_j(x,t), \tag{A1}$$

$$\tilde{\Phi}_j(x,t) = \frac{1}{2\pi\sqrt{v_j}}\Phi_j(x,t), \tag{A2}$$

where $[\tilde{\Theta}_j(x,t),\tilde{\Phi}_{j'}(y,t)] = i\delta_{jj'}\theta(x-y)$. By using field operators, the Hamiltonian is written as

$$\mathcal{H} = \sum_{j=1}^{N} \frac{v_j}{2}\int dx\{\tilde{\Xi}_j^2 + (\partial_x\tilde{\Theta}_j)^2\}, \tag{A3}$$

where $\tilde{\Xi}_j = -\partial_x\tilde{\Phi}_j$. The boundary condition at the origin requires the Fermion field for the νth subband $\psi_\nu(0)=0$, i.e., $\psi_{-,\nu}(0) = -\psi_{+,\nu}(0)$. This condition leads to

$$\frac{1}{\sqrt{2}}\sum_{j=1}^{N}\frac{X_{\nu,j}}{\sqrt{v_j}}\tilde{\Theta}_j(0,t) = \left(n+\frac{1}{2}\right)\pi, \tag{A4}$$

with n being an arbitrary integer.

The mode expansion, together with the canonical quantization, leads to

$$\tilde{\Theta}_j(x,t) = C_j + \tilde{\Theta}'_j(x,t), \tag{A5}$$

$$\tilde{\Theta}'_j(x,t) = \frac{1}{\pi}\int_0^\infty dq\frac{\sin qx}{\sqrt{q}}\{-ie^{-iv_jqt}b_j(q) + ie^{iv_jqt}b_j^\dagger(q)\}, \tag{A6}$$

$$\tilde{\Phi}_j(x,t) = -\frac{1}{\pi}\int_0^\infty dq\frac{\cos qx}{\sqrt{q}}\{e^{-iv_jqt}b_j(q) + e^{iv_jqt}b_j^\dagger(q)\}, \tag{A7}$$

where C_j is the c number satisfying $(1/\sqrt{2})\sum_{j=1}^{N}(X_{\nu,j}/\sqrt{v_j})C_j = \{n+(1/2)\}\pi$, and $b_j(q)$ is the bosonic operator with $[b_j(q),b_{j'}^\dagger(q')] = \pi\delta_{jj'}\delta(q-q')$. The ultraviolet cutoff $\exp(-\alpha q/2)$ is inserted if necessary in the q integral in Eqs. (A6) and (A7). Note that $\partial_t\tilde{\Theta}_j(x,t) = -v_j\partial_x\tilde{\Phi}_j(x,t)$. As a result of Eqs. (A5)–(A7), the Fermion field of the νth band satisfies $\psi_{-,\nu}(x,t) = -\psi_{+,\nu}(-x,t)$. The Hamiltonian is written as

$$\mathcal{H} = \sum_{j=1}^{N}\frac{1}{\pi}\int_0^\infty dq\, v_j q b_j^\dagger(q)b_j(q). \tag{A8}$$

The quantity $\langle\{\psi_\nu^\dagger(x,0),\psi_\nu(x,t)\}\rangle$ is calculated as follows:

$$\begin{aligned}&\langle\{\psi_\nu^\dagger(x,0),\psi_\nu(x,t)\}\rangle\\ &\simeq \langle\{\psi_{+,\nu}^\dagger(x,0),\psi_{+,\nu}(x,t)\}\rangle + (x\to -x)\\ &= \frac{1}{2\pi\alpha}\left\{\prod_{j=1}^{N}G_{\nu,j}(x,t)H_{\nu,j}(x,t)\right.\\ &\quad\left.+\prod_{j=1}^{N}G_{\nu,j}(x,t)H_{\nu,j}^{-1}(x,t)\right\} + (x\to -x),\end{aligned} \tag{A9}$$

where

$$\begin{aligned}G_{\nu,j}(x,t) &= \left\langle \exp\left\{-i\frac{1}{\sqrt{2}}(f_{\nu,j}(x,0) - f_{\nu,j}(x,t))\right\}\right\rangle\\ &= \exp\left\{-\frac{1}{4}\langle(f_{\nu,j}(x,0)-f_{\nu,j}(x,t))^2\rangle\right\},\end{aligned} \tag{A10}$$

$$H_{\nu,j}(x,t) = \exp\left\{\frac{1}{4}[f_{\nu,j}(x,0),f_{\nu,j}(x,t)]\right\}, \tag{A11}$$

with $f_{\nu,j}(x,t) = X_{\nu,j}/\sqrt{v_j}\tilde{\Theta}'_j(x,t) + 2\pi\sqrt{v_j}(KX)_{\nu,j}\tilde{\Phi}_j(x,t)$. Here, we ignore the rapidly oscillating terms proportional to $\exp(\pm i2k_{F\nu}x)$ because these contributions can probably not be observed directly due to averaging over several lattice sites in the experiments. From Eqs. (A6) and (A7), together with (A8),

$$\begin{aligned}G_{\nu,j}(x,t) = \exp\Bigg\{&-\frac{A_{\nu,j}}{2}\int_0^\infty dq\frac{\sin^2 qx}{q}\\ &\times(1-e^{-iv_jqt})(1-e^{iv_jqt})[1+2g(v_jq)]\\ &-\frac{B_{\nu,j}}{2}\int_0^\infty dq\frac{\cos^2 qx}{q}(1-e^{-iv_jqt})\\ &\times(1-e^{iv_jqt})[1+2g(v_jq)]\Bigg\},\end{aligned} \tag{A12}$$

$$\begin{aligned}H_{\nu,j}(x,t) = \exp\Bigg\{&\frac{A_{\nu,j}}{2}\int_0^\infty dq\frac{\sin^2 qx}{q}(e^{iv_jqt}-e^{-iv_jqt})\\ &+\frac{B_{\nu,j}}{2}\int_0^\infty dq\frac{\cos^2 qx}{q}(e^{iv_jqt}-e^{-iv_jqt})\Bigg\},\end{aligned} \tag{A13}$$

where

$$A_{\nu,j} = \frac{(X_{\nu,j})^2}{2\pi v_j}, \quad B_{\nu,j} = 2\pi v_j[(KX)_{\nu,j}]^2, \tag{A14}$$

and $g(\epsilon) = (e^{\epsilon/T}-1)^{-1}$ is the Bose distribution function. As a result,

$$\begin{aligned}&\langle\{\psi_\nu^\dagger(x,0),\psi_\nu(x,t)\}\rangle\\ &= \frac{1}{\pi\alpha}\exp\left\{-\sum_{j=1}^{N}[C_+I_j(0,t)+C_-I_j(x,t)]\right\}\\ &\quad\times\left[\exp\left\{-\sum_{j=1}^{N}[C_+J_j(0,t)+C_-J_j(x,t)]\right\}\right.\\ &\quad\left.+(t\to -t)\right],\end{aligned} \tag{A15}$$

where $C_\pm \equiv (B_{\nu,j}\pm A_{\nu,j})/2$ and

$$\begin{aligned}I_j(x,t) &= \int_0^\infty dq\frac{\cos 2qx}{q}(1-e^{-iv_jqt})(1-e^{iv_jqt})g(v_jq)\\ &= \frac{1}{2}\sum_{n=1}^{\infty}\left\{\log\left[1+\frac{(v_jt+2x)^2}{(nv_j/T)^2}\right]\right.\\ &\quad\left.+\log\left[1+\frac{(v_jt-2x)^2}{(nv_j/T)^2}\right] - 2\log\left[1+\frac{(2x)^2}{(nv_j/T)^2}\right]\right\}\\ &= \frac{1}{2}\left\{\log\frac{\sinh\pi T(t+2x/v_j)}{\pi T(t+2x/v_j)}\frac{2\pi Tx/v_j}{\sinh 2\pi Tx/v_j}\right.\\ &\quad\left.+\log\frac{\sinh\pi T(t-2x/v_j)}{\pi T(t-2x/v_j)}\frac{2\pi Tx/v_j}{\sinh 2\pi Tx/v_j}\right\},\end{aligned} \tag{A16}$$

$$\begin{aligned}J_j(x,t) &= \int_0^\infty dq\frac{\cos 2qx}{q}(1-e^{iv_jqt})\\ &= \frac{1}{2}\left\{\log\frac{\alpha-i(v_jt+2x)}{\alpha-i2x} + \log\frac{\alpha-i(v_jt-2x)}{\alpha+i2x}\right\}.\end{aligned} \tag{A17}$$

Since $I_j(\infty,t) = J_j(\infty,t) = 0$, these results lead to Eqs. (19)–(22).

[1]S. Tomonaga, Prog. Theor. Phys. **5**, 544 (1950).
[2]J. M. Luttinger, J. Math. Phys. **4**, 1154 (1963).
[3]D. C. Mattis and E. H. Lieb, J. Math. Phys. **6**, 304 (1965).
[4]F. D. M. Haldane, J. Phys. C **14**, 2585 (1981).
[5]For review, see, J. Voit, Rep. Prog. Phys. **58**, 977 (1995).
[6]M. Fabrizio and A. O. Gogolin, Phys. Rev. B **51**, 17827 (1995).
[7]S. Eggert, H. Johannesson, and A. Mattsson, Phys. Rev. Lett. **76**, 1505 (1996).
[8]A. E. Mattsson, S. Eggert, and H. Johannesson, Phys. Rev. B **56**, 15615 (1997).
[9]M. Bockrath, D. H. Cobden, J. Lu, A. G. Rinzler, R. E. Smalley, L. Balents, and P. L. McEuen, Nature (London) **397**, 598 (1999).
[10]Z. Yao, H. Postma, L. Balents, and C. Dekker, Nature (London) **402**, 273 (1999).
[11]H. W. C. Postma, M. de Jonge, Z. Yao, and C. Dekker, Phys. Rev. B **62**, R10653 (2000).
[12]V. Bubanja and S. Iwabuchi, Phys. Rev. B **79**, 035312 (2009).
[13]H. Shima, H. Yoshioka, and J. Onoe, Phys. Rev. B **79**, 201401(R) (2009); Physica E **42**, 1151 (2010).
[14]S. Bellucci, J. González, P. Onorato, and E. Perfetto, Phys. Rev. B **74**, 045427 (2006).
[15]M. Klanjšek, H. Mayaffre, C. Berthier, M. Horvatić, B. Chiari, O. Piovesana, P. Bouillot, C. Kollath, E. Orignac, R. Citro, and T. Giamarchi, Phys. Rev. Lett. **101**, 137207 (2008).
[16]H. Shima, S. Ono, and H. Yoshioka, Eur. Phys. J. B **78**, 481 (2010).
[17]E. Slot, M. A. Holst, H. S. J. van der Zant, and S. V. Zaitsev-Zotov, Phys. Rev. Lett. **93**, 176602 (2004).
[18]L. Venkataraman, Y. S. Hong, and P. Kim, Phys. Rev. Lett. **96**, 076601 (2006).
[19]O. M. Auslaender, A. Yacoby, R. de Picciotto, K. W. Baldwin, L. N. Pfeiffer, and K. W. West, Science **295**, 825 (2002).
[20]O. M. Auslaender, H. Steinberg, A. Yacoby, Y. Tserkovnyak, B. I. Halperin, K. W. Baldwin, L. N. Pfeiffer, and K. W. West, Science **308**, 88 (2005).
[21]Y. Tserkovnyak, B. I. Halperin, O. M. Auslaender, and A. Yacoby, Phys. Rev. Lett. **89**, 136805 (2002).
[22]Y. Tserkovnyak, B. I. Halperin, O. M. Auslaender, and A. Yacoby, Phys. Rev. B **68**, 125312 (2003).
[23]E. Levy, A. Tsukernik, M. Karpovski, A. Palevski, B. Dwir, E. Pelucchi, A. Rudra, E. Kapon, and Y. Oreg, Phys. Rev. Lett. **97**, 196802 (2006).
[24]H. Steinberg, G. Barak, A. Yacoby, L. N. Pheiffer, K. W. West, B. I. Halperin, and K. L. Hur, Nat. Phys. **4**, 116 (2008).
[25]Y. Jompol, C. J. B. Ford, J. P. Griffiths, I. Farrer, G. A. C. Jones, D. Anderson, D. A. Ritchie, T. W. Silk, and A. J. Schofield, Science **325**, 597 (2009).
[26]A. Schwartz, M. Dressel, G. Grüner, V. Vescoli, L. Degiorgi, and T. Giamarchi, Phys. Rev. B **58**, 1261 (1998).
[27]R. Claessen, M. Sing, U. Schwingenschlögl, P. Blaha, M. Dressel, and C. S. Jacobsen, Phys. Rev. Lett. **88**, 096402 (2002).
[28]M. Sing, U. Schwingenschlögl, R. Claessen, P. Blaha, J. M. P. Carmelo, L. M. Martelo, P. D. Sacramento, M. Dressel, and C. S. Jacobsen, Phys. Rev. B **68**, 125111 (2003).
[29]A. N. Aleshin, H. J. Lee, Y. W. Park, and K. Akagi, Phys. Rev. Lett. **93**, 196601 (2004).
[30]A. N. Aleshin, Adv. Mater. **18**, 17 (2006).
[31]A. Bachtold, M. de Jonge, K. Grove-Rasmussen, P. L. McEuen, M. Buitelaar, and C. Schonenberger, Phys. Rev. Lett. **87**, 166801 (2001).
[32]H. Ishii, H. Kataura, H. Shiozawa, H. Yoshioka, H. Otsubo, Y. Takayama, T. Miyahara, S. Suzuki, Y. Achiba, M. Nakatake, T. Narimura, M. Higashiguchi, K. Shimada, H. Namatame, and M. Taniguchi, Nature (London) **426**, 540 (2003).
[33]H. Yoshioka, Physica E **18**, 212 (2003).
[34]N. Tombros, S. J. van der Molen, and B. J. van Wees, Phys. Rev. B **73**, 233403 (2006).
[35]K. Penc and J. Sólyom, Phys. Rev. B **47**, 6273 (1993).
[36]A. Kitazawa, J. Phys. Condens. Matter. **15**, 2587 (2003).
[37]H. Frahm and T. Vekua, J. Stat. Mech. (2008) P01007.
[38]A. Tokuno and M. Sato, Phys. Rev. A **78**, 013623 (2008).
[39]R. Egger, Phys. Rev. Lett. **83**, 5547 (1999).
[40]I. Schneider and S. Eggert, Phys. Rev. Lett. **104**, 036402 (2010).
[41]S. Meyer, J. Schäfer, C. Blumenstein, P. Höpfner, A. Bostwick, J. L. McChesney, E. Rotenberg, and R. Claessen, Phys. Rev. B **83**, 121411(R) (2011).