Casimir interaction between topological insulators with finite surface band gap

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The Casimir interaction between topological insulators with opposite topological magnetoelectric polarizabilities and finite surface band gaps has been investigated. For a large surface band-gap limit ($m \to \infty$), we can obtain results given by Grushin and Cortijo [Phys. Rev. Lett. **106**, 020403 (2011)]. For a small surface band-gap limit ($m \to 0$), the Casimir interaction between topological insulators is attractive and in analogy to the ideal mental model in a short separation limit. Generally, there is a critical value m_c, and when the surface band gap is greater than the critical value, the Casimir force is repulsive in an intermediate separation region. We estimate the critical surface band gap $m_c \sim 1/(2a)$, in which a is a critical separation where the Casimir force vanishes.

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I. INTRODUCTION

The time-reversal invariant topological insulator (TI)[1–3] is a new quantum state of matter which has a fully insulating gap in the bulk, but has gapless surface states that are protected topologically. This material has been extensively studied experimentally[4–9] and theoretically.[10–13] Two-dimensional TIs have been observed in the HgTe quantum well;[14,15] $Sb_{1-x}Te_x$ is the first material that has been reported to be a three-dimensional TI; and Bi_2Se_3, Bi_2Te_3, and Sb_2Te_3 have been predicted[16] to be TIs with a single Dirac cone on the surface. Novel properties of TIs have been predicted, for instance, the effective monopole[17] and topological magnetoelectric effect,[13] superconductor proximity-effect-induced Majorana fermion states,[18] etc.

Recently, an interesting property of TIs, that is, the tunable repulsive Casimir interaction between TIs with opposite topological magnetoelectric polarizability θ, has been proposed,[19] and the robustness of this repulsion in a small separation limit against the finite temperature and uniaxial anisotropy has also been analyzed.[20] The repulsive Casimir interaction has been discussed in a few proposals, with special geometry[21] or chiral metamaterials,[22] or filling high-refractive liquid between dielectrics.[23] The repulsion between TIs is the analogy to metamaterials, however, the time-reversal invariant TI is protected by gapless surface states. In order to observe the repulsive Casimir interaction, one must cover the TI surfaces with magnetic coating to open the band gap. The effect of finite surface band gap on this repulsive force is considerable.

In this paper, we analyze the influence of finite surface band gap on Casimir force between TIs with opposite topological magnetoelectric polarizability θ. We show that there is a minimal surface band gap m_c, and when surface band gap $m < m_c$, the repulsive Casimir force will disappear. We also estimate this critical surface band gap numerically.

We formulate the model as follows. When time-reversal symmetry is protected in the bulk, the topological nontrivial term $\alpha/(4\pi^2)\int d^3x dt \theta \boldsymbol{E}\cdot\boldsymbol{B}$ can be reexpressed as spin-momentum locked fermions on the interface of the TI and normal insulator; in this paper, we consider only one kind of fermion corresponding to $\theta = \pi$ or $-\pi$, where the generalization to multifermions is straightforward. The action of the Dirac fermion on the TI surface is

$$S_D = \int d^3x \bar{\psi}[i\gamma^a(\partial_a + ieA_a) - m]\psi, \tag{1}$$

where $a = 0,x,y$; $\gamma^0 = \sigma^z$; $\gamma^x = iv_F\sigma^y$; and $\gamma^y = -iv_F\sigma^x$. $\sigma^{x,y,z}$ are the Pauli matrices of the spin, and v_F is the Fermi velocity of the surface fermion, which has a magnitude of 10^{-3} speed of light (we set $\hbar = c = 1$ in this paper) and takes different values for different materials,[5,6] i.e., $v_F = 1.3\times10^{-3}$ for Bi_2Te_3, and 1.7×10^{-3} for Bi_2Se_3. Parameter m is the surface band gap opened by a magnetic coating on the TI, and we assume the chemical potential has been tuned into the surface band gap. A_a present the first three components of the vector potential, while the electromagnetic field is described by the Maxwell action,

$$S_{\rm EM} = -\frac{1}{8\pi}\int d^4x\left(\varepsilon\boldsymbol{E}^2 - \frac{1}{\mu}\boldsymbol{B}^2\right), \tag{2}$$

where $\boldsymbol{E}$ and $\boldsymbol{B}$ are electric and magnetic fields, and ε and μ are the permittivity and permeability of the TI in the bulk and equal to 1 in the vacuum.

This paper is organized as follows: In Sec. II, we evaluate an effective action for the electromagnetic field on the TI surface using the quantum field theory approach, and give the Maxwell equations of the electromagnetic field with proper boundary conditions. In Sec. III, we analyze the Casimir interaction between TIs via the Lifshitz theory. We discuss the results in Sec. IV, and give a conclusion in Sec. V.

II. EFFECTIVE LAGRANGIAN ON TI SURFACE AND MAXWELL EQUATIONS

In order to calculate the Casimir interaction caused by quantum fluctuation of the electromagnetic field between TIs, one needs to integrate the contribution from the surface fermion. An effective action for the external electromagnetic field in a (2 + 1) dimension can be found using the standard quantum field theory approach,[24–26] $S_{\rm eff}(A) = -i\ln\det[i\gamma^a(\partial_a + ieA_a) - m]$. We introduce a Feynman parameter, integrate out the fermion field up to a one-loop correction, and get the effective action in the following form:

$$S_{\text{eff}}(A) = \int d^3x \left[-\frac{\phi(\lambda)}{8\pi}\epsilon_{abc}A^a\partial^b A^c + \frac{\Phi(\lambda)}{4\pi|m|}\left(F_{0j}F^{0j} + v_F^2 F_{xy}F^{xy}\right)\right], \quad (3)$$

with dimensionless parameters ϕ and Φ which take the forms

$$\phi(\lambda) = \text{sign}(m)\alpha \int_0^1 dx \frac{1}{\sqrt{1-x(1-x)\lambda}}, \quad (4)$$

$$\Phi(\lambda) = \alpha \int_0^1 dx \frac{(1-x)x}{\sqrt{1-x(1-x)\lambda}}, \quad (5)$$

where sign(m) gives the sign of the surface band gap, which corresponds to different signs of topological magnetoelectric polarizability. $\alpha = 1/137$ is the fine-structure constant of electromagnetic interaction, $\lambda = [k_0^2 - v_F^2(k_x^2 + k_y^2)]/m^2$, and k_0, k_x, k_y are the frequency and momentum of the electromagnetic fields on the TI surface. A detailed derivation and a short discussion of this effective action (3) are given in the Appendix. We also note that in both limits $m^2 \to 0$ and $m^2 \to \infty$, ϕ and Φ are convergent. For the sake of Eq. (20), we derive the expressions of ϕ and Φ in imaginary-time formalism:

$$\tilde{\phi}(\gamma) = \text{sign}(m)\frac{2\alpha}{\sqrt{\gamma}} \arctan\left(\frac{\sqrt{\gamma}}{2}\right), \quad (6)$$

$$\tilde{\Phi}(\gamma) = \frac{\alpha}{2\gamma} + \left(\frac{\alpha}{4\sqrt{\gamma}} - \frac{\alpha}{\gamma^{3/2}}\right)\arctan\left(\frac{\sqrt{\gamma}}{2}\right), \quad (7)$$

where $\gamma = [k_0^2 + v_F^2(k_x^2 + k_y^2)]/m^2$. For the large surface band-gap limit ($|m| \to \infty$), $\tilde{\phi}(\gamma) \to \text{sign}(m)\alpha$, which shows the term proportional to $\phi(\lambda)$ in Eq. (3) is topological, and the term proportional to $\Phi(\lambda)$ in Eq. (3) is vanishing. For the small gap limit ($|m| \to 0$), $\tilde{\phi}(\gamma) \to 0$ and $\tilde{\Phi}(\gamma) \to 1/6$.

By adding the surface term given by Eq. (3) to the standard action of the electromagnetic fields given by Eq. (2), one can get the Maxwell equations with surface corrections:

$$\frac{1}{4\pi}\nabla\cdot\boldsymbol{D} = -\delta(z-z_i)\left(\frac{\phi_i}{4\pi}B_z - \frac{\Phi}{2\pi|m|}\nabla\cdot\boldsymbol{E}\right), \quad (8)$$

$$\frac{1}{4\pi}[\partial_t\boldsymbol{D} - (\nabla\times\boldsymbol{H})] = \delta(z-z_i)\left[\frac{\phi_i}{4\pi}\tilde{\boldsymbol{E}} + \frac{\Phi}{2\pi|m|}\left(\partial_t\boldsymbol{E} - v_F^2\nabla\times\boldsymbol{B}\right)\right], \quad (9)$$

$$\nabla\cdot\boldsymbol{B} = 0, \quad (10)$$

$$\partial_t\boldsymbol{B} + (\nabla\times\boldsymbol{E}) = 0, \quad (11)$$

where $\boldsymbol{D} = \varepsilon\boldsymbol{E}$ and $\boldsymbol{H} = \boldsymbol{B}/\mu$ are the electric displacement field and magnetizing field; $\tilde{E}_j = \epsilon_{jk}E_k$ $(j,k = x,y)$; $i = 1,2$; $z_1 = 0$ and $z_2 = a$ are positions of the TI surfaces (as shown in Fig. 1); and ϕ_1 and ϕ_2 are corresponding values of ϕ. Without loss of generality, we assume that the absolute values of the surface band gaps on two TIs are equal, and that different signs of surface band gaps stand for different signs of the topological term $\alpha\theta\boldsymbol{E}\cdot\boldsymbol{B}/(4\pi^2)$ in the Lagrangian of electromagnetic fields in TIs. We also note that in a large band-gap limit ($|m| \to \infty$), these Maxwell equations are equal to those given in Refs. 17 and 27 by redefining the electric displacement

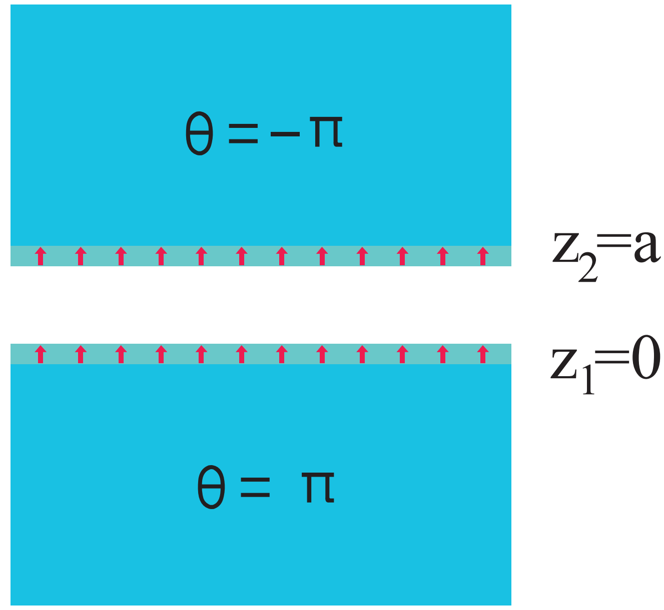

FIG. 1. (Color online) Schematic illustration of Casimir interaction between TIs with opposite topological magnetoelectric polarizability θ. We assume the thickness of the magnetic coating is much smaller than the separation between TIs.

and magnetizing field as $\boldsymbol{D} = \varepsilon\boldsymbol{E} + \alpha\frac{\theta}{\pi}\boldsymbol{B}$, $\boldsymbol{H} = \frac{1}{\mu}\boldsymbol{B} - \alpha\frac{\theta}{\pi}\boldsymbol{E}$. From the above Maxwell equations, we get the following discontinuous boundary conditions:

$$D_z(z_i^+) - D_z(z_i^-) = -\phi_i B_z + \frac{2\Phi}{|m|}(\partial_x E_x + \partial_y E_y), \quad (12)$$

$$H_x(z_i^+) - H_x(z_i^-) = \phi_i E_x - \frac{2\Phi}{|m|}(\partial_t E_y + v_F^2\partial_x B_z), \quad (13)$$

$$H_y(z_i^+) - H_y(z_i^-) = \phi_i E_y + \frac{2\Phi}{|m|}\left(\partial_t E_x - v_F^2\partial_y B_z\right), \quad (14)$$

where $z_i^{\pm}$ means $z_i \pm 0$. Also, E_x, E_y, and B_z are continuous on the interfaces.

III. CASIMIR INTERACTION

Now we analyze the Fresnel coefficients of reflection light on the TI-vacuum interface. The incident TE mode from the vacuum with wave vector (k_x,k_y,k_z) will induce a reflected TE and TM mode. If we assume the reflection coefficients are r_{ee} and r_{em}, respectively, then the electromagnetic waves in the vacuum read

$$\begin{aligned}\boldsymbol{E} &= (1+r_{ee})k_0(-k_y\boldsymbol{e}_x + k_x\boldsymbol{e}_y) + r_{em}(-k_z\boldsymbol{k} - k^2\boldsymbol{e}_z),\\ \boldsymbol{B} &= (-k_z\boldsymbol{k} + k^2\boldsymbol{e}_z) + r_{ee}(k_z\boldsymbol{k} + k^2\boldsymbol{e}_z)\\ &\quad + r_{em}k_0(-k_y\boldsymbol{e}_x + k_x\boldsymbol{e}_y),\end{aligned} \quad (15)$$

and the refracted light with the TE and TM mode in the TI take the forms

$$\begin{aligned}\boldsymbol{E} &= t_{ee}k_0(-k_y\boldsymbol{e}_x + k_x\boldsymbol{e}_y) + c\,t_{em}(p_z\boldsymbol{k} - k^2\boldsymbol{e}_z),\\ \boldsymbol{B} &= t_{ee}(-p_z\boldsymbol{k} + k^2\boldsymbol{e}_z) + \frac{t_{em}}{c}k_0(-k_y\boldsymbol{e}_x + k_x\boldsymbol{e}_y),\end{aligned} \quad (16)$$

where t_{ee} and t_{em} are refraction coefficients of the TE and TM mode, c is the relative velocity of light in TI bulk,

$\boldsymbol{k} = k_x \boldsymbol{e}_x + k_y \boldsymbol{e}_y$, $k^2 = k_x^2 + k_y^2$, and p_z is the z component of the wave vector in the TI. For the injected TM mode, one can write the analogy equations with reflection coefficients r_{me}, r_{mm} and refraction coefficients t_{me}, t_{mm}. After some tedious derivation, we obtain the Fresnel coefficients matrix $\mathcal{R}$ in imaginary-time formalism:

$$\mathcal{R} = \begin{pmatrix} r_{ee} & r_{em} \\ r_{me} & r_{mm} \end{pmatrix}, \tag{17}$$

with

$$\begin{aligned} r_{ee} &= -1 + \frac{2}{D}\left(1 + \varepsilon \frac{k_z}{p_z} + 2\tilde{\Phi}\frac{k_z}{m}\right), \\ r_{em} &= r_{me} = \frac{2}{D}\tilde{\phi}, \\ r_{mm} &= 1 - \frac{2}{D}\left(1 + \frac{1}{\mu}\frac{p_z}{k_z} + 2\lambda\tilde{\Phi}\frac{m}{k_z}\right), \end{aligned} \tag{18}$$

where the denominator

$$D = \left(1 + \varepsilon\frac{k_z}{p_z}\right)\left(1 + 2\gamma\tilde{\Phi}\frac{m}{k_z}\right) + \left(1 + \frac{1}{\mu}\frac{p_z}{k_z}\right) \times \left(1 + 2\tilde{\Phi}\frac{k_z}{m}\right) + \left(\frac{\epsilon}{\mu} + \tilde{\phi}^2\right) - (1 - 4\gamma\tilde{\Phi}^2). \tag{19}$$

For the large surface band-gap limit, we can obtain the same Kerr rotation and Faraday rotation angle as given in Refs. 13 and 27.

In imaginary-time formalism, the Casimir energy density between two parallel dielectric semispaces can be expressed in a closed form of dielectric permittivity,

$$\frac{E_C(a)}{A} = \int_0^\infty \frac{dk_0}{2\pi}\int \frac{d^2k_\parallel}{(2\pi)^2}\log\det[1 - \mathcal{R}^{(1)}\mathcal{R}^{(2)}\boldsymbol{e}^{-2k_3 a}], \tag{20}$$

where A is the surface area of TIs, $\mathcal{R}^{(1,2)}$ are Fresnel coefficients on the surfaces, and $k_3 = \sqrt{\boldsymbol{k}_\parallel^2 + k_0^2}$. In order to calculate the Casimir energy density numerically, we also need a form of frequency-dependent dielectric permittivity ε (we assume the permeability $\mu = 1$), which can be modeled by[28,29]

$$\varepsilon(ik_0) = 1 + \sum_{J=1}^{K}\frac{g_J}{k_0^2 + \omega_J^2 + \gamma_J k_0}. \tag{21}$$

Here we consider only one oscillator ($K = 1$) with oscillator strength g_J, oscillator frequency ω_J, and damping parameter γ_J. $\gamma_J \ll \omega_J$, and we omit the contribution from the damping parameter.

IV. RESULTS AND DISCUSSION

In this paper, we analyze the Casimir interaction between TIs with finite surface band gap. First, for a large surface band-gap limit ($m \to \infty$), we can obtain the same results given by Grushin and Cortijo[19] from Eqs. (17)–(21). Second, for a small surface band-gap limit ($m \to 0$), the off-diagonal terms in Fresnel coefficient matrices will vanish and Casimir energy can be rewritten in imaginary-time formalism as

$$\frac{E_C(a)}{A} = \int_0^\infty \frac{dk_0}{2\pi}\int\frac{d^2k_\parallel}{(2\pi)^2}\big[\log\big(1 - e^{-2ka}r_{\mathrm{TE}}^{(1)}r_{\mathrm{TE}}^{(2)}\big). + \log\big(1 - e^{-2ka}r_{\mathrm{TM}}^{(1)}r_{\mathrm{TM}}^{(2)}\big)\big], \tag{22}$$

with

$$r_{\mathrm{TE}} = -1 + \frac{2}{1 + \frac{p_3}{k_3} + \frac{\pi\alpha}{4}\sqrt{\cos^2\theta + v_F^2\sin^2\theta}}, \tag{23}$$

$$r_{\mathrm{TM}} = 1 - \frac{2\frac{p_3}{k_3}\sqrt{\cos^2\theta + v_F^2\sin^2\theta}}{\frac{\pi\alpha}{4}\frac{p_3}{k_3} + \left(\frac{p_3}{k_3} + \varepsilon\right)\sqrt{\cos^2\theta + v_F^2\sin^2\theta}}, \tag{24}$$

where $k_3 = \sqrt{k_0^2 + \boldsymbol{k}_\parallel^2}$, $p_3 = \sqrt{\varepsilon k_0^2 + \boldsymbol{k}_\parallel^2}$, and $\theta = \cos^{-1}(k_0/k_3)$.

The Casimir energy between dielectric materials without special boundary conditions, that is, $\alpha \to 0$ in Eq. (23) and Eq. (24), has been studied.[28–30]

By considering the correction from the surface interaction for a large separation limit, we obtain the correction up to the first order of the fine-structure constant,

$$\frac{E_C^{(1)}(a)}{E_0} = -\frac{\pi\alpha}{4d^3}\left\{\frac{\varepsilon(0) - 1}{[\varepsilon(0) + 1]^3}\log\frac{1}{v_F} + \frac{\log\left[\frac{1}{2}(1 + \sqrt{\varepsilon(0)})\right]}{\varepsilon(0) - 1} - \frac{3 + 5\sqrt{\varepsilon(0)}}{4(1 + \sqrt{\varepsilon(0)})^3}\right\}, \tag{25}$$

where $E_0 = A\omega_J^3/(2\pi)^2$, which is set as the unit of Casimir energy, and $d = a\omega_J$ is the dimensionless separation.

For a small separation limit, in order to make the physics more clear, we also formally expand Eq. (22) in powers of α, up to the first-order correction, and the Casimir energy takes the following form (here we assume the relative oscillator strength $g_J/\omega_J^2 \ll 1$):

$$\frac{E_C^{(1)}(a)}{E_0} = -\frac{g_J}{\omega_J^2}\frac{\pi\alpha}{64d^3}\int_0^\infty dy y^2 e^{-y} \times \left[\frac{\theta(t)}{\sqrt{t}}\arctan\sqrt{t} + \frac{\theta(-t)}{\sqrt{-t}}\mathrm{arctanh}\sqrt{-t}\right], \tag{26}$$

where $t = -1 + v_F^2 y^2/4d^2$ and $\theta(t)$ is the Heaviside unit step function. The Casimir energy is dominated by surface Dirac fermion and turns into the ideal conductor case, which is proportional to $1/a^3$. This conclusion is also confirmed numerically in Fig. 2.

Finally, for a general surface band gap, we have two dimensionless parameters, m/ω_J and $d = \omega_J a$. (There are two other parameters in our model: the Fermi velocity of the surface fermion, v_F, and the optical oscillator strength in TIs, g_J/ω_J^2, both of which have quantitative influence on the Casimir energy.) For the large separation limit [$a \gg \max(1/\omega_J, 1/|m|)$], we expand the integral in Eq. (20) in the power of the fine-structure constant[31] α, and consider the correction up to α. In this case, the dielectric permittivity $\varepsilon(ik_0)$ can be approximated by the long wavelength limit value

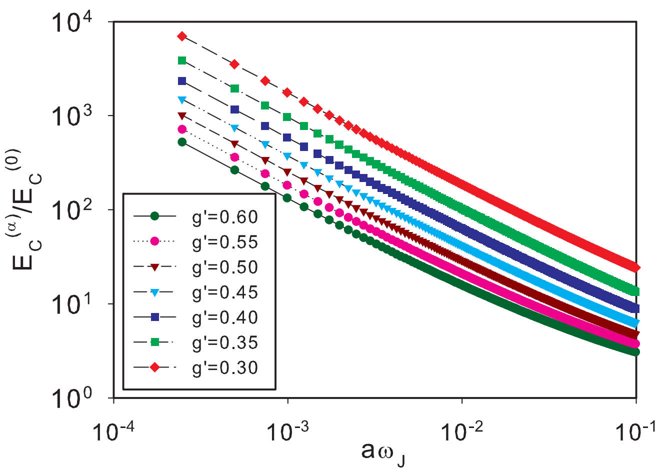

FIG. 2. (Color online) The ratio $E_C^{(\alpha)}/E_C^{(0)}$ as a function of dimensionless separation $a\omega_J$ for different oscillator strengths $g' = \sqrt{g_J}/\omega_J$ in the closed surface band-gap limit $m = 0$, where $E_C^{(\alpha)}$ ($E_C^{(0)}$) is the Casimir energy with (without) surface correction. Here fermion velocity $v_F = 1.0 \times 10^{-3}$.

$\varepsilon(0)$, and the Casimir energy correction from the interaction between the surface fermions and electromagnetic field reads

$$\frac{E_C^{(1)}(a)}{E_0} = -\frac{|m|\alpha}{\omega_J d^2}\int_0^1 \frac{dx}{v_F^2 + x^2}\left\{\frac{r^2[\varepsilon(0) - r]}{[\varepsilon(0)+r]^3} + \frac{x^2(1-r)}{(1+r)^3}\right\}, \quad (27)$$

where $r = \sqrt{1 + [\varepsilon(0) - 1]x^2}$, and $v_F \ll 1$.

For the small separation limit ($d \to 0$), in order to make the physics more clear, we also formally expand the Casimir energy in the power series of α. In this case, the Casimir energy is dominated by surface terms; the term which contains $\tilde{\Phi}^2$ and is proportional to $1/m^2a^5$ is important. However, this dominant term will be suppressed if $|m|a \to \infty$, and the topological term which contains $\mathrm{sign}(\theta_1\theta_2)\tilde{\phi}^2$ and is proportional to $1/a^3$ will provide a large repulsive potential between TIs when $\mathrm{sign}(\theta_1\theta_2) = -1$. So the surface terms in the Casimir energy will dominate, and $|m|a$ is a good parameter to estimate the Casimir force: when $|m|a \gg 1$, the Casimir force will be repulsive, and when $|m|a \ll 1$, the Casimir force will be attractive.

In Fig. 3, we give the boundary of the repulsive and attractive Casimir interaction as a function of dimensionless separation $d = a\omega_J$ and product $|m|a$. We find that there is a critical value $(|m|a)_c \sim 1/2$, and when $|m|a < (|m|a)_c$, the Casimir interaction is attractive for any separation length. The independence of $(|m|a)_c$ on oscillator strength g_J/ω_J^2 shows that the Casimir interaction, in a small separation limit, is dominated by surface terms. More intuitively, we calculated the Casimir energy as a function of dimensionless separation $d = a\omega_J$ for different surface band gaps, as shown in Fig. 4, for given parameters $g_J/\omega_J^2 = 0.45^2$ and $v_F = 1.0 \times 10^{-3}$. We find the critical surface band gap, where the repulsive peak disappears, $m_c \approx 300\omega_J$ (the blue-square dotted line in Fig. 4).

We note that our calculations can be generalized to a multivalue of topological magnetoelectric polarizability $\theta = (2n+1)\pi$ (n is an integer) straightforwardly by introducing

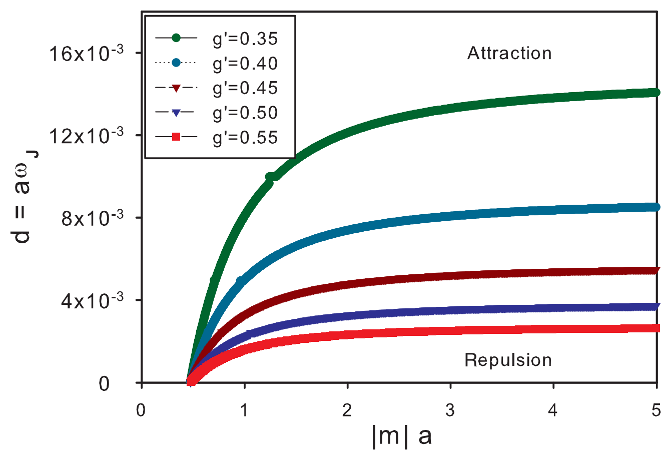

FIG. 3. (Color online) Boundary of repulsive and attractive Casimir interaction in the plane of dimensionless separation $d = a\omega_J$ and product $|m|a$ for different oscillator strengths $g' = \sqrt{g_J}/\omega_J$. When the parameters d and $|m|a$ have been taken in the upper left region over these lines, the Casimir interaction is attractive; when d and $|m|a$ have been taken in the lower right region of these lines, the Casimir interaction is repulsive. (The relative Fermi velocity v_F has been taken to be 1.0×10^{-3}.)

multifermion on the TI surface, and the critical value $(|m|a)_c$ is independent of the absolute value of θ (as shown in Fig. 5). This is because, in a short separation limit, the Casimir interaction is dominated by surface terms and each species fermion will contribute both a repulsive and attractive Casimir interaction if $\mathrm{sign}(\theta_1) = -\mathrm{sign}(\theta_2)$.

We can use this relationship to estimate the critical surface band gap for a repulsive Casimir interaction. For $TlBiSe_2$, as suggested in Ref. 19, the minimum of the Casimir energy appears at a separation of $a \sim 0.1$ μm, and the corresponding surface band gap needs to be greater than 1 eV, which reflects that the width of the surface band gap opened by magnetic coating is nonignorable and inaccessible experimentally.

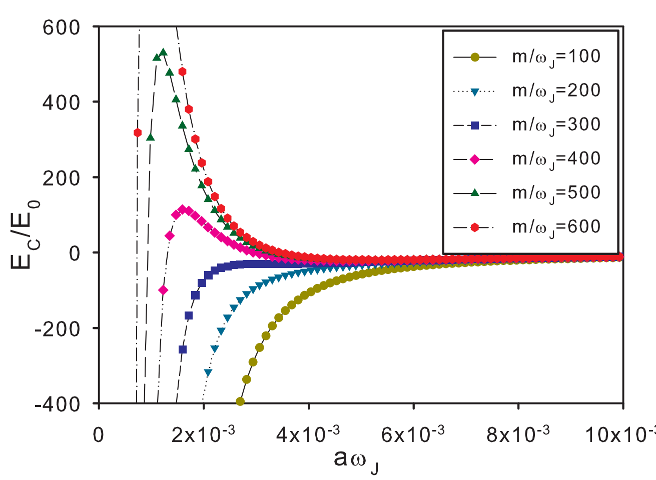

FIG. 4. (Color online) Casimir energy density E_C [in units of $E_0 = \omega_J^2/(2\pi)^2$] as a function of the dimensionless distance $d = a\omega_J$ with different surface band gaps m/ω_J. Here we take the dimensionless oscillator strength $g_J/\omega_J^2 = 0.45^2$ and Fermi velocity $v_F = 1.0 \times 10^{-3}$.

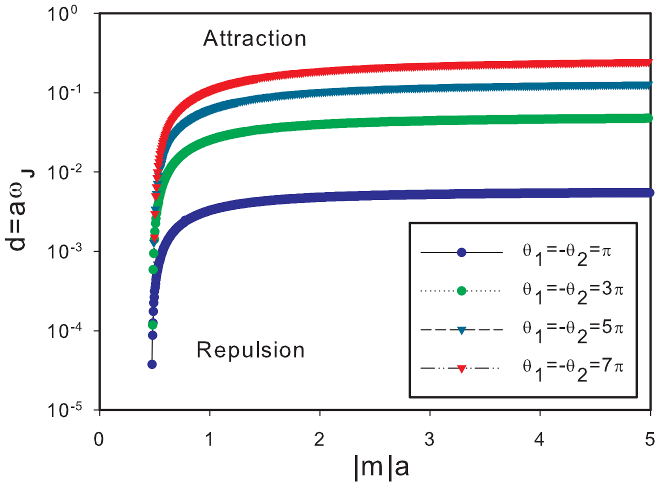

FIG. 5. (Color online) Boundary of repulsive and attractive Casimir interaction in the plane of dimensionless separation $d = a\omega_J$ and product $|m|a$ for different topological magnetoelectric polarizability. (The relative Fermi velocity v_F and oscillator strength g_J/ω_J^2 have been taken to be 1.0×10^{-3} and 0.45^2, respectively.)

V. CONCLUSION

We studied the Casimir energy between TIs with opposite topological magnetoelectric polarizability and finite surface band gap via the Lifshitz formula. We found that in a small separation limit, the Casimir force is dominated by the interaction between the surface fermion and electromagnetic field in the vacuum, and a great surface band gap $m > m_c \sim 1/(2a)$ is essential for a repulsive Casimir interaction.

ACKNOWLEDGMENT

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APPENDIX: EFFECTIVE ACTION

We give a detailed derivation of the effective action (3) in this Appendix. The effective action from quantum field theory is

$$S_{\rm eff}(A) = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3} A_a(k)\Pi^{ab}(k)A_b(k), \tag{A1}$$

where $\Pi(k)$ is the polarization tensor, which takes the form

$$i\Pi^{ab}(k) = -e^2\int \frac{d^3p}{(2\pi)^3}\mathrm{tr}[(-i\gamma^a)G(k+p)(-i\gamma^b)G(k)], \tag{A2}$$

and $G(k) = i/(\gamma^a k_a + m)$ is the propagator of the fermion on the TI surface. From the standard calculation in quantum field theory, one can get the exact form of polarization tensor:

$$\Pi(k) = \Pi_1(k) + \Pi_2(k), \tag{A3}$$

$$\Pi_1^{ab}(k) = \frac{\phi(\lambda)}{4\pi}\epsilon^{abc} i k_c, \tag{A4}$$

$$\Pi_2(k) = \frac{\Phi(\lambda)}{2\pi|m|}\begin{pmatrix} k_x^2+k_y^2 & -k_0k_x & -k_0k_y \\ -k_0k_x & k_0^2 - v_F^2k_y^2 & v_F^2k_xk_y \\ -k_0k_y & v_F^2k_xk_y & k_0^2 - v_F^2k_x^2\end{pmatrix}, \tag{A5}$$

where $\phi(\lambda)$ and $\Phi(\lambda)$ has been given in Eqs. (4) and (5), and $k_{1,2}$ (k_0) are the momentum (frequency) of the electromagnetic field. One can check that the polarization tensor satisfies the Ward identity, $\sum_a k_a\Pi^{ab}(k) = \sum_b \Pi^{ab}(k)k_b = 0$. The Fourier transformation of Eq. (A1) gives Eq. (3).

We take $\Pi^{xy}(k)$ as an example to show more detailed calculations of polarization tensor. Taking the trace in Eq. (A2), one can get

$$i\Pi^{xy}(k) = -e^2\int\frac{d^3p}{(2\pi)^3} \times \frac{2v_F^2[-imk_0 + v_F^2(2k_xk_y + k_xp_y + k_yp_x)]}{[(p_0+k_0)^2 + m^2 - v_F^2(\boldsymbol{p}+\boldsymbol{k})^2][k_0^2+m^2-v_F^2\boldsymbol{k}^2]}. \tag{A6}$$

One can get the following form of $i\Pi^{xy}(k)$ by introducing a Feynman parameter x and redefining the integration variables $l'_a = p_a + xk_a$, $l_0 = l'_0$, and $\boldsymbol{l} = v_F\boldsymbol{l}'$:

$$i\Pi^{xy}(k) = -2e^2\int_0^1 dx\int\frac{d^3l}{(2\pi)^3}\frac{imk_0 + 2x(1-x)v_F^2k_xk_y}{(l_0^2 - \boldsymbol{l}^2 - \Delta)^2}, \tag{A7}$$

where $\Delta = m^2 - x(1-x)(k_0^2 - v_F^2\boldsymbol{k}^2) = m^2[1-\lambda x(1-x)]$. By making the Wick rotation $l_0 \to il_0^E$ and integration over l, we find

$$\begin{aligned} i\Pi^{xy}(k) &= ie^2\left[imk_0\int\frac{dx}{4\pi\sqrt{\Delta}} + v_F^2k_xk_y\int dx\frac{x(1-x)}{2\pi\sqrt{\Delta}}\right] \\ &= i\left[\frac{ik_0}{4\pi}\phi(\lambda) + \frac{v_F^2k_xk_y}{2\pi|m|}\Phi(\lambda)\right]. \end{aligned} \tag{A8}$$

In comparison with the effective action of the electromagnetic field in a monolayer graphene system, as shown in Ref. 32, we find that there is an additional topological term given by Eq. (A4) together with the normal vacuum polarization given by Eq. (A5). The first term is essential for the TI because this parity-odd term reflects the fact that there are always odd species of surface fermions which are spin-momentum locked. The contribution from the second term is in analogy to the Dirac fermion in a monolayer graphene system and reflects the dynamical response of the TI surface state to extra electromagnetic field.

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