Coulomb drag in graphene single layers separated by a thin spacer

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(Received 12 May 2011; published 22 July 2011)

Motivated by very recent studies of Coulomb drag in a graphene-BN-graphene system we develop a theory of Coulomb drag for the Fermi-liquid regime, for the case when the ratio of spacer thickness *d* to the Fermi wavelength of electrons is arbitrary. The concentration (*n*) and thickness dependence of the drag resistivity is changed from $n^{-3}d^{-4}$ for the thick spacer to $n^{-1}|\ln (nd^2)|$ for the thin one.

DOI: 10.1103/PhysRevB.84.041407

PACS number(s): 72.80.Vp, 73.21.Ac, 73.63.Bd

Coulomb drag in bilayer semiconductor systems is a very interesting phenomenon, providing unique information about many-body effects.^{1–4} Since the role of electron-electron interactions in graphene, which is a two-dimensional material with extraordinary electronic and structural properties,^{5–9} is a controversial issue (for a review, see Ref. 10), it is important to study of Coulomb drag in graphene in order to clarify the situation.

Theoretical^{11,12} and experimental^{13,14} studies of Coulomb drag in graphene have been performed. The theory¹¹ deals with the case of a thick spacer ($k_F d \gg 1$, where k_F is the Fermi wave vector of graphene and *d* is the spacer thickness), and the results are in a good agreement with the corresponding experimental data¹³ (the effects of trigonal warping¹² seem to be negligible). Very recently, the group of Geim and Novoselov has performed experiments with graphene on a substrate of BN, also using BN as a spacer between two graphene single layers, and observed drag for much thinner spacers.¹⁴ Here we present a theory generalizing that of Ref. 11 for arbitrary values of $k_F d$.

We start with a general expression for drag conductivity^{1,2} based on the lowest-order perturbation theory in interlayer Coulomb interactions (we consider only the case of identical layers 1 and 2; qualitatively all the basic physics remains the same for the case of different doping of two layers):

$$\sigma_D = \frac{1}{16\pi k_B T} \sum_{\mathbf{q}} \int_{-\infty}^{\infty} \frac{d\omega}{\sinh^2\left(\frac{\hbar\omega}{2k_B T}\right)} \Gamma_x^2(\mathbf{q},\omega) |U_{12}(q,\omega)|^2,$$
(1)

where Γ_x is the nonlinear susceptibility of electrons in the layer, E_F is their Fermi energy, T is the temperature, and U_{12} is the Fourier component of the screened interlayer Coulomb interaction which reads, for the case of identical layers,

$$U_{12}(q,\omega) = \frac{u_c(q)}{[1 + v_c(q)\Pi(q,\omega)]^2 - [u_c(q)\Pi(q,\omega)]^2},$$
 (2)

where $v_c(q)$ and $u_c(q)$ are the Fourier components of bare Coulomb interactions within the layer and between the layers, respectively, and $\Pi(q,\omega)$ is the polarization operator of the electron gas in graphene. In vacuum, $v_c(q) = 2\pi e^2/q$ and $u_c(q) = v_c(q) \exp(-qd)$ and expression (2) coincides with Eq. (A2) from Ref. 1.

Since typical frequencies contributing to the integral in Eq. (1) is of the order of $k_B T/\hbar$, one can assume, for low enough temperatures, that the screening is static and replace $U_{12}(q,\omega)$ by $U_{12}(q,0)$.

In the ballistic regime (which means that the distance between the layers d is much smaller than the mean free path within the layer l) one can calculate for the case of graphene¹¹

$$\Gamma_{x}(\mathbf{q},\omega) \approx -\frac{4e\tau\omega}{\pi v} \frac{q_{x}}{q}$$
(3)

at $\omega \to 0$, where $\tau = l/v$ is the mean-free-path time and $v \approx 10^6$ m/s is the Fermi velocity.

Assuming that $\sigma_D \ll \sigma$ where σ is the inlayer conductivity, the drag resistivity is $\rho_D = -\sigma_D/\sigma^2$. The Drude formula for the case of graphene is

$$\sigma = \frac{e^2}{\pi \hbar^2} E_F \tau, \qquad (4)$$

thus, the drag resistivity does not depend on τ .

As a result, the drag resistivity for the case of identical graphene layers takes the form

$$\rho_D = -\frac{2h}{3e^2} \left(\frac{k_B T}{E_F}\right)^2 \sum_{\mathbf{q}} \left|\frac{U_{12}(q,0)}{\hbar v}\right|^2.$$
(5)

The quantity U_{12} in Eq. (5) is given by expression (2) with the static polarization operator for graphene (see, e.g., Refs. 15 and 16)

$$\Pi(q,0) = \begin{cases} \frac{2k_F}{\pi\hbar\nu}, & q < 2k_F, \\ \frac{2k_F}{\pi\hbar\nu} + \frac{q}{2\pi\hbar\nu} \left[\cos^{-1}\frac{2k_F}{q} - \frac{2k_F}{q} \sqrt{1 - \left(\frac{2k_F}{q}\right)^2} \right], & q > 2k_F. \end{cases}$$
(6)

To find $v_c(q)$ and $u_c(q)$, one needs to solve the electrostatic problem taking into account different screening by the substrate, spacer, and air. Let us assume that the dielectric medium is three layers, with the dielectric constant distribution

$$\varepsilon = \begin{cases} \varepsilon_1, & z > d, \\ \varepsilon_2, & d > z > 0, \\ \varepsilon_3, & z < 0. \end{cases}$$
(7)

The calculations are quite standard (see, e.g., Ref. 17). However, for the reader's convenience we present them here with some details.

We have to solve the Poisson equation

$$\frac{d}{dz}\left[\varepsilon(z)\frac{d\varphi(z)}{dz}\right] - q^2\varepsilon(z)\varphi(z) = -4\pi e\delta(z-\eta), \quad (8)$$

where $\varphi(z)$ is the electrostatic potential created by the point charge *e* situated at x = 0, y = 0, $z = \eta \rightarrow +0$. The only allowed solution at z < 0 is

$$\varphi(z) = A e^{qz} \tag{9}$$

and at z > d is

$$\varphi(z) = Be^{-qz}.$$
 (10)

For $\eta < z < d$ it should be tried in the most general form:

$$\varphi(z) = \alpha e^{qz} + \beta e^{-qz}.$$
 (11)

From continuity of the potential and the normal component of electric induction $D_n = -\varepsilon \frac{d\varphi}{dz}$ at the boundaries z = 0 and z = d we find, taking into account Eqs. (9) and (10),

$$\frac{\varphi'(-0)}{\varphi(-0)} = \frac{\varepsilon_3}{\varepsilon_2}q,\tag{12}$$

$$\frac{\varphi'(d)}{\varphi(d)} = -\frac{\varepsilon_1}{\varepsilon_2}q,\tag{13}$$

where the prime means the derivative with respect to z. The potential $\varphi(z)$ is continuous at $z = \eta [\varphi(-0) = \varphi(+0)]$ but its derivative has a jump. Due to Eqs. (8), (9), and (12),

$$\varphi'(+0) = \frac{\varepsilon_3}{\varepsilon_2} q A - \frac{4\pi e}{\varepsilon_2}.$$
 (14)

At last, we can find from Eqs. (14) and (13) the constants α and β . The final answer for $v_c(q) = e\varphi(z = 0)$ and $u_c(q) = e\varphi(z = d)$ reads

$$u_{c}(q) = \frac{8\pi e^{2}\varepsilon_{2} \exp(qd)}{q[(\varepsilon_{1} + \varepsilon_{2})(\varepsilon_{3} + \varepsilon_{2}) \exp(2qd) - (\varepsilon_{1} - \varepsilon_{2})(\varepsilon_{3} - \varepsilon_{2})]},$$

$$v_{c}(q) = \frac{8\pi e^{2}\varepsilon_{2} \exp(qd)[\varepsilon_{2} \cosh(qd) + \varepsilon_{1} \sinh(qd)]}{q[(\varepsilon_{1} + \varepsilon_{2})(\varepsilon_{3} + \varepsilon_{2}) \exp(2qd) - (\varepsilon_{1} - \varepsilon_{2})(\varepsilon_{3} - \varepsilon_{2})]}.$$
(15)

For simplicity, we will consider further only the case $\varepsilon_1 = \varepsilon_2$ (which actually takes place in the experimental situation¹⁴ where BN is used both as a substrate and as a spacer). In this case, expression (15) is simplified dramatically:

$$u_c(q) = v_c(q) \exp(-qd),$$

$$v_c(q) = \frac{4\pi e^2}{q(\varepsilon_2 + \varepsilon_3)},$$
(16)

and

$$U_{12} = \frac{v_c}{2(v_c \Pi)^2 \sinh(qd) + (1 + 2v_c \Pi) \exp(qd)}.$$
 (17)

Substituting Eqs. (6), (15), and (17) into Eq. (5) we have

$$\rho_D = -\frac{h}{e^2} \frac{\pi}{48} \left(\frac{k_B T}{E_F}\right)^2 \frac{1}{\alpha^2} F(2k_F d),$$
(18)

where

$$\alpha = \frac{2e^2}{\hbar v(\varepsilon_2 + \varepsilon_3)} \tag{19}$$

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is the effective "fine structure" constant (for the combination of BN with $\varepsilon_2 \approx 4$ and air with $\varepsilon_3 = 1$ we have $\alpha \approx 0.87$) and the function *F* is represented as

$$F(y) = \int_0^\infty dx \frac{x^3}{\left[\varphi^2(x)\sinh(yx) + \frac{x(x+4\alpha\varphi(x))}{8\alpha^2}\exp(yx)\right]^2},$$
(20)

where

$$\varphi(x) = \begin{cases} 1, & x < 1, \\ 1 + \frac{x}{2} \left(\cos^{-1} \frac{1}{x} - \frac{\sqrt{x^2 - 1}}{x^2} \right), & x > 1. \end{cases}$$
(21)

In the limit $y \gg 1$,

$$F(y) \cong \frac{3\zeta(3)}{2y^4},\tag{22}$$

and Eqs. (18) and (20) give the known result¹¹

$$\rho_D = -\frac{h}{e^2} \frac{\pi \zeta(3)}{32} \left(\frac{k_B T}{E_F}\right)^2 \frac{1}{(k_F d)^2} \frac{1}{(\kappa d)^2},$$
 (23)

where $\kappa = 4\alpha k_F$ is the inverse screening radius.

In the opposite limit, $y \ll 1$, with typical values of $x \approx 1/y \gg 1$, and one can assume $\phi(x) \approx \pi x/4$, which gives

$$F(y) \cong \left(\frac{8\alpha^2}{1+\pi\alpha}\right)^2 \ln \frac{1}{y}.$$
 (24)

The behavior of drag resistivity as a function of charge carrier concentration for $k_F d$ of the order of one is shown



FIG. 1. Drag resistivity (18) as a function of charge carrier concentration, for $\varepsilon_3 = 1$, $\varepsilon_2 = 4$, T = 120 K: d = 3 nm (dashed line) and d = 4 nm (solid line).

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in Fig. 1. This result seems to be in a qualitative agreement with the experimental data,¹⁴ but at least it gives the correct order of magnitude for the drag resistivity. At the same time, for small enough $k_F d$ and $\alpha \approx 1$ the interlayer Coulomb interaction is in general not small, and it is not clear whether the lowest-order perturbation theory used here will be also quantitatively accurate or taking into account next-order contributions will be necessary. The issue requires further studies, both experimental and theoretical.

Recently two more works on the subject appeared.^{18,19} The results of this paper and the other two papers concerning concentration dependence of the drag resistivity are in an

agreement, namely, in Ref. 18 the same analytical concentration dependence as here, $\rho_D \propto n^{-1} |\ln (nd^2)|$, was obtained for the case of thin spacer, whereas in Ref. 19 the numerical data were fitted by $\rho_D \propto n^{-\alpha}$ with α of the order of one.

I am thankful to Andre Geim for stimulating discussions of unpublished experimental results¹⁴ and to Timur Tudorovskiy for helpful discussions. This work is part of the research program of the Stichting voor Fundamenteel Onderzoek der Materie (FOM), which is financially supported by the Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO).

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