Multiphoton-induced nonlinear magnetoresistance oscillations in a dc-driven two-dimensional electron system irradiated by intense microwaves

X. L. Lei and S. Y. Liu

Department of Physics, Shanghai Jiaotong University, 1954 Huashan Road, Shanghai 200030, China

(Received 16 March 2011; revised manuscript received 15 June 2011; published 28 July 2011)

We examine the nonlinear magnetoresistance oscillations in a dc-biased high-mobility two-dimensional electron system irradiated by intense microwaves with a current-controlled balance-equation model for multi-photon-assisted magnetotransport. It is shown that the maxima/minima positions, particularly the oscillation period of the differential resistance as a function of the ratio of the microwave frequency ω to the cyclotron frequency ω_c, strongly depend on the radiation intensity under a large bias dc current density. Theoretical predictions well reproduce the recent experimental findings by Khodas *et al.* [Phys. Rev. Lett. **104**, 206801 (2010)], and support the multiphoton origin of these unusual magnetoresistance oscillations.

DOI: 10.1103/PhysRevB.84.035321 PACS number(s): 73.50.Jt, 73.40.−c, 73.43.Qt, 71.70.Di

I. INTRODUCTION

Microwave-induced magnetoresistance oscillation in high-mobility two-dimensional (2D) electron systems has been a subject of intensive experimental[1–16] and theoretical[17–35] studies over the past decade.

Under the irradiation of a frequency-ω microwave, the linear magnetoresistivity of a 2D system strongly oscillates as a function of the inverse magnetic field $1/B$, featuring the appearance of peak-valley pairs around the cyclotron resonance and its harmonics, $\epsilon_\omega \equiv \omega/\omega_c = 1,2,3,\ldots$ ($\omega_c = eB/m$ is the cyclotron frequency and m the electron effective mass), where the photoresistivity always vanishes. The basic period of the oscillation depends on the frequency of the microwave, irrespective of its intensity. Enhanced microwave radiation may increase the oscillation amplitude and produce secondary structures around fractional ϵ_ω positions, while the cyclotron resonance and its harmonics are always the node points of the primary peak-valley pairs of oscillation.[3,4,21,33]

A dc current flowing through the 2D system alone is also known to induce substantial oscillation of differential magnetoresistance as a function of the current density $J = N_s e v$ (N_s and v are the density and drift velocity of 2D electrons, respectively) or of the inverse magnetic field $1/B$. It is controlled by the parameter $\epsilon_j \equiv \omega_j/\omega_c$ ($\omega_j \equiv 2k_F v$, with k_F as the Fermi wave vector), exhibiting a periodicity $\Delta\epsilon_j \approx 1$.[36–42] Simultaneous application of a finite dc current and a microwave radiation leads to very interesting and complicated oscillatory behavior of resistance and differential resistance.[43–50] Experimental and theoretical studies have so far focused mainly on the oscillations of magnetoresistance with changing dc current or changing magnetic field in a system subjected to a given microwave radiation having modest strength. How the oscillation of nonlinear magnetoresistance, i.e., the differential magnetoresistance under a finite-bias dc current, would be affected when enhancing the radiation power of incident microwave has not yet been carefully explored.

Recently, Khodas *et al.*[51] investigated the effect of varying the radiation intensity on nonlinear magnetoresistance as a function of inverse magnetic field and reported a new class of magnetoresistance oscillations in high-mobility 2D electron systems exposed to high-power microwaves and subjected to a strong dc current. They are manifested by a series of multiple maxima and minima of the differential resistivity versus ϵ_ω, occurring in the proximity of the cyclotron resonance and its harmonics. The phases of these oscillations appear quite different from that of linear photoresistivity and change continuously with changing bias dc current. The maxima/minima positions, particularly the periods of oscillations, are strongly dependent on the radiation intensity for a given frequency microwave under a large bias dc current density. These unusual oscillation behaviors are referred to the effect of multiphoton processes.[51]

Exposed to an intense steady microwave the electrons are certainly heated even without a dc current passing through. When the radiation gets strong enough to induce an observable magnetoresistance oscillation, the electron temperature of the 2D system is generally in or above the range of a couple of degrees Kelvin in the regime of cyclotron resonance and its harmonics, $\omega_c/\omega < 1.5$, regardless of the lattice temperature.[21] Because of this, the thermalization time or the inelastic relaxation time is much shorter than the transport scattering time in the experimental high-mobility electron systems and the inelastic-mechanism contribution to radiation-induced magnetoresistance oscillations is negligible in comparison with that of the displacement mechanism.[15,34,52] This enables us to examine this unusual magnetoresistance oscillations using a microscopic balance-equation scheme[53] for photon-assisted magnetotransport directly controlled by the current.

It is demonstrated that for a given microwave frequency and polarization, the period of the oscillation is determined mainly by the intensity of the radiation, almost independent of the width of the Landau level and the range of the impurity potential, in spite of their remarkable influence on the amplitude of the resistance oscillation. Theoretical predictions well reproduce the experimental findings,[51] and confirm that these unusual magnetoresistance oscillations result from multiphoton processes.

II. FORMULATION FOR PHOTOASSISTED NONLINEAR MAGNETOTRANSPOPRT

We deal with an isotropic 2D system of short thermalization time, consisting of N_s electrons in a unit area of the x-y

plane. These electrons, scattered by random impurities and by phonons in the lattice, are subjected to a uniform magnetic field $\boldsymbol{B} = (0,0,B)$ in the z direction. When an electromagnetic wave of angular frequency ω illuminates perpendicularly onto the 2D plane with the incident electric field

$$\boldsymbol{E}_{\rm i}(t) = \boldsymbol{E}_{\rm is}\sin(\omega t) + \boldsymbol{E}_{\rm ic}\cos(\omega t) \tag{1}$$

at $z = 0$ and a bias dc current flows within the plane, the electric field inside the 2D system involves a dc component $\boldsymbol{E}_0$ and an ac component $\boldsymbol{E}(t)$.

The steady-transport state under the radiation (1) of strength relevant to magnetoresistance oscillation can be described by the drift velocity of electron integrative (the center of mass) motion, consisting of a dc part $\boldsymbol{v}$ and a stationary time-dependent part of the form

$$\boldsymbol{V}(t) = \boldsymbol{v} + \boldsymbol{v}_s\sin(\omega t) + \boldsymbol{v}_c\cos(\omega t), \tag{2}$$

together with an average temperature T_e, characterizing the isotropic thermal distribution of electrons in the reference frame moving with the center of mass.[53] They satisfy the following force and energy-balance equations:[21]

$$N_s e\boldsymbol{E}_0 + N_s e(\boldsymbol{v}\times\boldsymbol{B}) + \boldsymbol{F} = 0, \tag{3}$$

$$N_s e\boldsymbol{E}_0\cdot\boldsymbol{v} + S_p - W = 0. \tag{4}$$

Here,

$$\boldsymbol{F} = \sum_{\boldsymbol{q}_\parallel}|U(q_\parallel)|^2\sum_{n=-\infty}^{\infty}\boldsymbol{q}_\parallel J_n^2(\xi)\Pi_2(q_\parallel,\omega_0 - n\omega) \tag{5}$$

is the time-averaged damping force against the electron drift motion, S_p is the time-averaged rate of the electron energy gain from the ac field, having an expression obtained from the right-hand side of the above equation by replacing the $\boldsymbol{q}_\parallel$ factor with $n\omega$. In Eq. (5), $U(q_\parallel)$ is the effective impurity potential, $\Pi_2(q_\parallel,\Omega)$ is the imaginary part of the electron density-correlation function at temperature T_e in the presence of the magnetic field without electric field, $\omega_0 \equiv \boldsymbol{q}_\parallel\cdot\boldsymbol{v}$, and $J_n(\xi)$ is the Bessel function of order n with the argument $\xi \equiv [(\boldsymbol{q}_\parallel\cdot\boldsymbol{v}_s)^2 + (\boldsymbol{q}_\parallel\cdot\boldsymbol{v}_c)^2]^{\frac{1}{2}}/\omega$. Note that, although contributions of phonon scattering to $\boldsymbol{F}$ and S_p are neglected in comparison with those of impurity scattering at the considered low lattice temperature, it provides the main channel for electron-energy dissipation to the lattice with a time-averaged energy-loss rate W, having an expression as given in Ref. 21.

The ac components $\boldsymbol{v}_s$ and $\boldsymbol{v}_c$ of electron drift velocity should be determined self-consistently from the incident ac field $\boldsymbol{E}_{\rm i}$ by the electrodynamic equations connecting both sides of the 2D system, taking into account the scattering-related damping forces $\boldsymbol{F}_s$ and $\boldsymbol{F}_c$.[21] However, for high-mobility systems at low temperatures, effects of these scattering-related damping forces are much weaker in comparison to that of radiative decay[32] and thus negligible, whence $\boldsymbol{v}_s$ and $\boldsymbol{v}_c$ are in fact directly determined by the incident fields $\boldsymbol{E}_{\rm is}$ and $\boldsymbol{E}_{\rm ic}$ based on the setup of the 2D system in the sample substrate.[21]

The effect of interparticle Coulomb interaction is included in the density-correlation function to the degree of energy-level broadening, in addition to the screening considered in the effective impurity and phonon potentials. The remaining $\Pi_2(q_\parallel,\Omega)$ function in Eq. (5) is that of a noninteracting 2D electron gas in the magnetic field, which can be written in the Landau representation as[54]

$$\Pi_2(q_\parallel,\Omega) = \frac{1}{2\pi l_B^2}\sum_{n,n'}C_{n,n'}\big(l_B^2 q_\parallel^2/2\big)\Pi_2(n,n',\Omega), \tag{6}$$

$$\Pi_2(n,n',\Omega) = -\frac{2}{\pi}\int d\varepsilon[f(\varepsilon) - f(\varepsilon+\Omega)] \times {\rm Im}G_n(\varepsilon+\Omega)\,{\rm Im}G_{n'}(\varepsilon), \tag{7}$$

where $l_B = \sqrt{1/|eB|}$ is the magnetic length, $C_{n,n+l}(Y) \equiv n![(n+l)!]^{-1}Y^l{\rm e}^{-Y}[L_n^l(Y)]^2$ with $L_n^l(Y)$ the associate Laguerre polynomial, $f(\varepsilon) = \{\exp[(\varepsilon-\mu)/T_e]+1\}^{-1}$ is the Fermi function at electron temperature T_e, and ${\rm Im}G_n(\varepsilon)$ is the density-of-states (DOS) function of the broadened Landau level n.

The Landau-level broadening results from impurity, phonon, and electron-electron scatterings. In the experimental GaAs-based 2D systems having mobility higher than 10^3 m^2/V s, the dominant elastic scatterings, which come from residual impurities or defects in the background rather than from remote donors,[55] are short-ranged and phonon and electron-electron scatterings are generally also not long-ranged because of the screening. On the other hand, since the magnetoresistance oscillations occur at low temperatures and low magnetic fields in high-carrier-density samples, the cyclotron radius of electrons involved in transport is generally much larger than the correlation length or the range of the dominant scattering potentials. In this case, the level broadening is expected to have a Gaussian form [$\varepsilon_n = (n+\frac{1}{2})\omega_c$ is the center of the nth Landau level, $n = 0,1,2,\ldots$],[56]

$${\rm Im}G_n(\varepsilon) = -(2\pi)^{\frac{1}{2}}\Gamma^{-1}\exp[-2(\varepsilon-\varepsilon_n)^2/\Gamma^2] \tag{8}$$

with a $B^{\frac{1}{2}}$-dependent half-width expressed as

$$\Gamma = (2\omega_c/\pi\tau_s)^{\frac{1}{2}}, \tag{9}$$

where τ_s is the single-particle lifetime or quantum scattering time in the zero magnetic field that depends on impurity, phonon, and electron-electron scatterings. The total DOS (double spins) of a 2D system of unit area in the magnetic field is

$$g(\varepsilon) = -\sum_n {\rm Im}G_n(\varepsilon)/\pi^2 l_B^2. \tag{10}$$

III. NONLINEAR DIFFERENTIAL RESISTIVITY

For an isotropic system where the frictional force $\boldsymbol{F}$ is in the opposite direction of the drift velocity $\boldsymbol{v}$ and the magnitudes of both the frictional force and the energy-dissipation rate depend only on $v \equiv |\boldsymbol{v}|$, we can write $\boldsymbol{F}(\boldsymbol{v}) = F(v)\boldsymbol{v}/v$ and $W(\boldsymbol{v}) = W(v)$. In the Hall configuration with velocity $\boldsymbol{v}$ in the x direction $\boldsymbol{v} = (v,0,0)$ or the current density $J_x = J = N_s ev$, and $J_y = 0$, the longitudinal differential resistivity

$r_{xx} = -[\partial F(v)/\partial v]/(N_s^2 e^2)$ at given v derived from Eq. (3), can be written in the form

$$r_{xx} = -\frac{1}{4\pi^2}\int dq_\parallel q_\parallel^3 \frac{|U(q_\parallel)|^2}{N_s^2 e^2}\int d\theta \cos^2\theta \times \sum_{n=-\infty}^{\infty} J_n^2(\xi)\, \Pi_2'(q_\parallel, q_\parallel v\cos\theta - n\omega), \quad (11)$$

where $\Pi_2'(q_\parallel,\Omega) \equiv \partial \Pi_2(q_\parallel,\Omega)/\partial\Omega$.

This expression for the nonlinear differential magnetoresistivity of an irradiated 2D system in the presence of a finite drift velocity $\boldsymbol{v}$ results from impurity-induced electron transitions between Landau levels with the assistance (emission or absorption) of n ($n = 0,1,2,\ldots$) photons, together with an energy $\omega_0 = \boldsymbol{q}_\parallel \cdot \boldsymbol{v}$ supplied by the integrative motion of the system to an electron having momentum $\boldsymbol{q}_\parallel$ during its transition. In the case of low temperature (T_e much less than the Fermi energy ε_F) and high Landau-level filling, the density-correlation function $\Pi_2'(q_\parallel,\Omega)$ sharply peaks around $q_\parallel \simeq 2k_F$. As a result, in the system having drift velocity v the electron involving in a transition obtains an extra energy

$$\omega_j \equiv 2k_F v = \sqrt{8\pi/N_s}\, J/e \quad (12)$$

in addition to the energy $n\omega$ or $-n\omega$ by the absorption or emission of n photons of the radiation field having frequency ω. The oscillation of r_{xx} originates from the periodic function $\Pi_2'(q_\parallel,\Omega)$. A resonance appears when the energy change of the electron transition matches the integral Landau-level spacings: $\omega_j \pm n\omega \approx \pm l\omega_c$ ($n = 0,1,2,\ldots$ and $l = 0,1,2,\ldots$). Therefore, at a fixed current density, the most probable maxima (exhibiting the shortest oscillation period) of the n-photon-process-contributed r_{xx} component versus $\epsilon_\omega \equiv \omega/\omega_c$, are expected to emerge around

$$\epsilon_\omega \approx \frac{l}{\gamma_j + n} \quad (l = 0,1,2,\ldots), \quad (13)$$

where $\gamma_j \equiv \omega_j/\omega$.

Figure 1(a) shows the calculated differential resistivity r_{xx} and its component parts from 0-, 1-, 2-, 3-, 4-, 5-, and 6-photon processes, as functions of the normalized inverse magnetic field $\epsilon_\omega \equiv \omega/\omega_c$ for a GaAs-based quasi-2D system of $N_s = 3.7 \times 10^{15}$ m^{-2} and $\mu_0 = 1200$ m^2/V s from a mixture of background and short-range impurity scatterings at $T = 1.5$ K, irradiated by a 27 GHz x-polarized microwave [$\boldsymbol{E}_{\rm is} = (E_{{\rm i}\omega},0)$, $\boldsymbol{E}_{\rm ic} = (0,0)$] having incident electric field amplitude $E_{{\rm i}\omega} = 3.8$ V/cm and subjected to a dc current $J = 0.86$ A/m or $\gamma_j = 2.6$. The Landau level broadening is taken to be a Gaussian-type (8) having single-particle scattering time $\tau_s = 16.3$ ps. Although the real positions of maxima may be somewhat affected by the ϵ_ω variation of $J_n^2(\xi)$ factor in Eq. (11), the two closest maxima shown in Fig. 1 for each r_{xx} component on both sides of $\epsilon_\omega = 1$ are indeed around $2/\gamma_j \approx 0.77$ and $3/\gamma_j \approx 1.15$ for the 0-photon process, $3/(\gamma_j+1) \approx 0.83$ and $4/(\gamma_j+1) \approx 1.11$ for the 1-photon, $4/(\gamma_j+2) \approx 0.87$ and $5/(\gamma_j+2) \approx 1.09$ for the 2-photon, $5/(\gamma_j+3) \approx 0.89$ and $6/(\gamma_j+3) \approx 1.07$ for the 3-photon, $6/(\gamma_j+4) \approx 0.91$ and $7/(\gamma_j+4) \approx 1.06$ for the 4-photon, $7/(\gamma_j+5) \approx 1.92$ and $8/(\gamma_j+5) \approx 1.05$ for the 5-photon, and $8/(\gamma_j+6) \approx 0.93$ and $9/(\gamma_j+6) \approx 1.04$ for the 6-photon process,

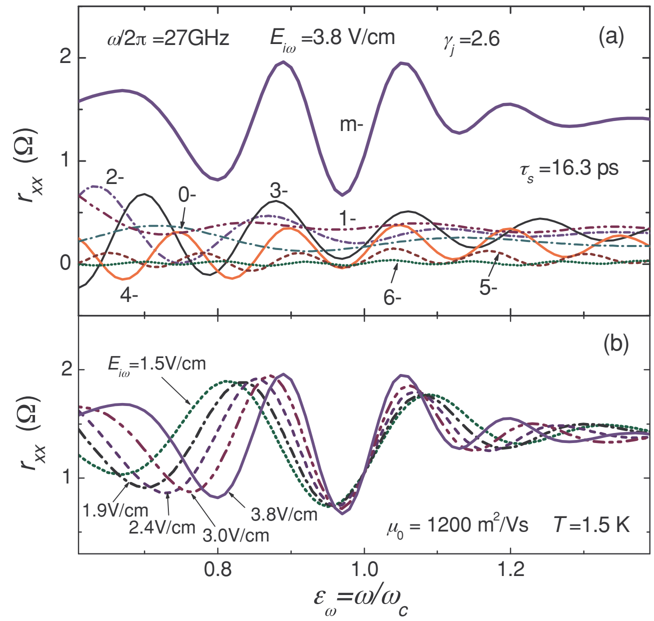

FIG. 1. (Color online) Magnetoresistivity r_{xx} vs $\epsilon_\omega = \omega/\omega_c$ at bias dc current $J = 0.86$ A/m or $\gamma_j = 2.6$ under the irradiation of 27 GHz microwaves of several incident strengths for a system of $N_s = 3.7 \times 10^{15}$ m^{-2} and $\mu_0 = 1200$ m^2/V s at $T = 1.5$ K having $\tau_s = 16.3$ ps. Curves with 0-, 1-, 2-, 3-, 4-, 5-, and 6- are separated contributions from zero-, single-, 2-, 3-, 4-, 5-, and 6-photon processes.

respectively. The oscillation period of the n-photon component, which is quite accurately given by $1/(\gamma_j + n)$, shrinks with increasing n. The total resistivity is the sum of all n-photon components. As a result, the oscillation period of the resistivity induced by higher-intensity microwaves would be smaller than that induced by lower-intensity microwaves, because higher-order photon processes play more important role in r_{xx} in the former. This can be seen clearly in Fig. 1(b), where we show the differential resistivity r_{xx} of the system under the same bias current density $J = 0.86$ A/m or $\gamma_j = 2.6$, irradiated by 27 GHz microwaves of different incident intensities: $E_{{\rm i}\omega} = 1.5, 1.9, 2.4, 3.0$, and 3.8 V/cm. Note that although $\epsilon_\omega = 1$ remains the node point at which all curves of different intensity cross, the phase of the resistivity oscillation at this bias current appears completely different from that of zero dc bias. The striking feature is that in this range of radiation intensity, the period of the oscillation decreases significantly with increasing microwave power, while its amplitude looks only a little different.

Figure 2 presents differential magnetoresistivity r_{xx} versus ϵ_ω for the same system described above but with $\tau_s = 9$ ps irradiated by 27 GHz microwaves: (a) under a given dc current of $\gamma_j = 2.6$ but different radiation intensities $E_{{\rm i}\omega} = 1.5, 1.9, 2.4, 3.0$, and 3.8 V/cm, and (b) with a given radiation strength $E_{{\rm i}\omega} = 2.4$ V/cm but at different bias dc current densities γ_j. All curves, other than the lowest ones, are offset upward for clarity. We see that the phase of the resistivity oscillation changes continuously with changing bias current density, while the oscillation amplitudes remain essentially the same.

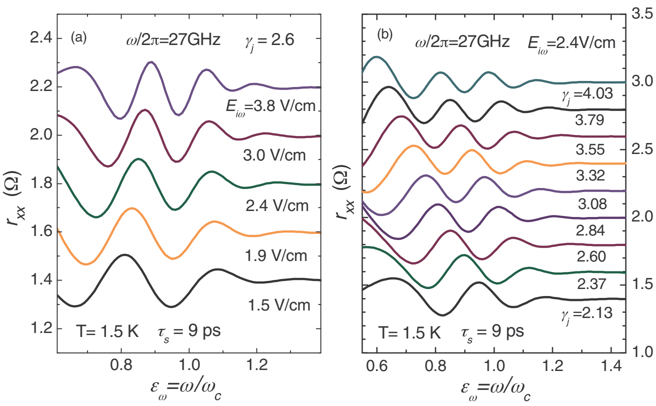

FIG. 2. (Color online) Magnetoresistivity r_{xx} vs ϵ_ω for the system as described in Fig. 1 but with $\tau_s = 9$ ps irradiated by 27 GHz microwaves: (a) subjected to a given dc current of $\gamma_j = 2.6$ but different radiation fields $E_{i\omega}$ and (b) exposed to a given radiation $E_{i\omega} = 2.4$ V/cm but at different bias dc current densities γ_j. All curves, other than the lowest ones, are offset upward.

IV. ANALYTICAL EXPRESSIONS IN THE OVERLAPPING LANDAU-LEVEL REGIME

Analytical expressions for r_{xx} can be derived at temperature T_e much lower than the Fermi level ($T_e \ll \varepsilon_F$) for short-range scattering in the overlapping Landau-level regime, where the Dingle factor

$$\delta = \exp\left(-\pi^2\Gamma^2/2\omega_c^2\right) = \exp(-\pi/\omega_c\tau_s) \tag{14}$$

is much smaller than 1. Retaining only terms of the lowest order in δ or of the fundamental harmonic oscillation, we have the approximate DOS expression for high Landau levels:

$$g(\varepsilon) \approx \frac{m}{\pi}[1 - 2\delta\cos(2\pi\varepsilon/\omega_c)]. \tag{15}$$

For circularly polarized incident radiation fields, the argument ξ of the Bessel function in Eq. (11) is not dependent on θ and the angular integration can be done exactly. Furthermore, at low temperature and high Landau-level filling the integral of $q_\parallel$ can be carried out in view of the function $C_{n,n'}(l_B^2 q_\parallel^2/2)$ sharply peaking around $q_\parallel = 2k_F$. After performing the summation over n in Eq. (11), we get the following expression for the nonlinear differential resistivity of the 2D system under an arbitrary microwave radiation and subject to an arbitrary bias dc current (excluding the Shubnikov-de Haas oscillation part):

$$\begin{aligned} r_{xx} = R_{i0}\{1 + 2\delta^2[&J_0(2\xi_b\sin(\pi\epsilon_\omega))G(2\pi\epsilon_j) \\ &-2\pi\epsilon_\omega\cos(\pi\epsilon_\omega)\xi_b J_1(2\xi_b\sin(\pi\epsilon_\omega))S(2\pi\epsilon_j)]\}, \end{aligned} \tag{16}$$

in which $R_{i0} = 1/(N_s e\mu_0)$ is the low-temperature linear resistivity of the 2D system in the absence of magnetic field without radiation and μ_0 is the linear mobility. In Eq. (16), $\epsilon_j \equiv \omega_j/\omega_c$, the $S(z)$ and $G(z)$ functions are

$$S(z) = J_0(z) - J_2(z), \tag{17}$$

$$G(z) = S(z) - \frac{z}{2}[3J_1(z) - J_3(z)] \tag{18}$$

[$J_k(z)$ stands for the Bessel function of order], and the parameter ξ_b is defined as

$$\xi_b^2 = e_\omega\eta, \tag{19}$$

where

$$e_\omega = \frac{e^2k_F^2E_\omega^2}{m^2\omega^4} \tag{20}$$

is an effective-radiation power index with E_ω as the effective amplitude of the incident radiation field and η is a polarization-related dimensionless coefficient including the effect of radiative damping.[21,32] We have

$$\eta = \eta_\pm = 2\frac{c_\pm^2 + d_\pm^2}{(a^2+b^2)^2} \tag{21}$$

for positive (+) or negative (−) circularly polarized incident radiation field [$\boldsymbol{E}_{is} = (E_{i\omega}/\sqrt{2},0)$, $\boldsymbol{E}_{ic} = (0,E_{i\omega}/\sqrt{2})$] or [$\boldsymbol{E}_{is} = (E_{i\omega}/\sqrt{2},0)$, $\boldsymbol{E}_{ic} = (0,-E_{i\omega}/\sqrt{2})$]. Here, $a = 1 - \lambda^2 + \gamma_\omega^2$, $b = 2\lambda\gamma_\omega$, $c_\pm = a(1\pm\lambda) - 2\lambda\gamma_\omega^2$, and $d_\pm = a\gamma_\omega \pm 2\lambda(1\pm\lambda)\gamma_\omega$, with $\lambda \equiv \omega_c/\omega$ and γ_ω being a radiative damping factor. The expressions for E_ω and γ_ω depend on the experimental setup of the 2D system. For 2D electrons contained in a thin sample suspended in vacuum, $E_\omega = E_{i\omega}$ and $\gamma_\omega = N_s e^2/(2m\epsilon_0 c\,\omega)$. If the 2D electrons are located under the surface plane of a semi-infinite semiconductor having a refractive index n_s, $E_\omega = 2E_{i\omega}/(1+n_s)$ and $\gamma_\omega = N_s e^2/[(1+n_s)m\epsilon_0 c\,\omega]$.[21]

For linearly polarized radiation fields, the resistivity expressions (16)–(20) are also approximately usable with the η coefficient given by

$$\eta = \eta_x \approx \frac{3a^2 + 3c^2 + b^2 + d^2}{(a^2+b^2)^2} \tag{22}$$

for x-polarized field [$\boldsymbol{E}_{is} = (E_{i\omega},0)$, $\boldsymbol{E}_{ic} = (0,0)$], and

$$\eta = \eta_y \approx \frac{3b^2 + 3d^2 + a^2 + c^2}{(a^2+b^2)^2} \tag{23}$$

for y-polarized field [$\boldsymbol{E}_{is} = (0,E_{i\omega})$, $\boldsymbol{E}_{ic} = (0,0)$]. Here, $c = (1+\lambda^2+\gamma_\omega^2)\gamma_\omega$ and $d = (1-\lambda^2-\gamma_\omega^2)\lambda$.

The r_{xx} expression (16) applies for arbitrary radiation field and arbitrary bias dc current and is accurate enough to capture all the features of nonlinear differential magnetoresistivity, as long as the DOS of the system is described by Eq. (15) and the electron temperature is much lower than the Fermi level: $T_e \ll \varepsilon_F$. The temperature dependence of resistivity resides in the single-particle scattering rate $1/\tau_s$ involved in the δ^2 factor.

In the case of weak radiation field $\xi_b^2 \ll 1$, retaining terms of order of ξ_b^2 in Eq. (16), one gets

$$\begin{aligned} r_{xx} = R_{i0}\big(1 + 2\delta^2\big\{&G(2\pi\epsilon_j) - \xi_b^2[\sin^2(\pi\epsilon_\omega)G(2\pi\epsilon_j) \\ &+\pi\epsilon_\omega\sin(2\pi\epsilon_\omega)S(2\pi\epsilon_j)]\big\}\big), \end{aligned} \tag{24}$$

which is the expression of nonlinear differential resistivity resulting from zero- and single-photon processes under an arbitrary bias dc current.[52]

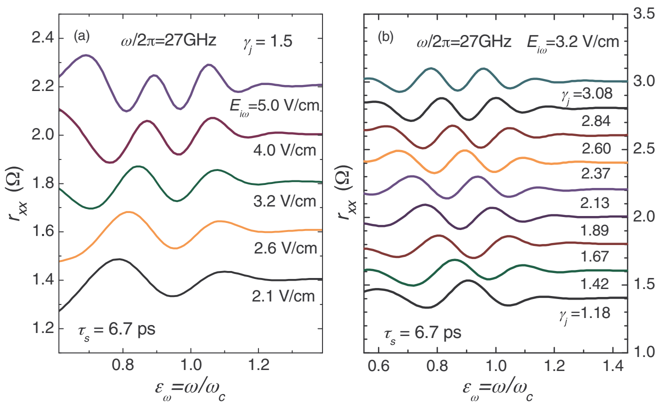

FIG. 3. (Color online) Magnetoresistivity r_{xx} vs ϵ_ω obtained from Eq. (16) for the system with $\tau_s = 6.7$ ps irradiated by 27-GHz microwaves: (a) under a given dc current $\gamma_j = 1.5$ but different radiation fields $E_{i\omega}$ and (b) exposed to a given radiation $E_{i\omega} = 3.2$ V/cm but at different bias dc currents γ_j. All curves, other than the lowest ones, are offset upward.

In the weak dc current limit ($2\pi\epsilon_j \to 0$), Eq. (16) reduces to an expression for linear magnetoresistivity under the irradiation of an arbitrary microwave:

$$\begin{aligned} r_{xx} = R_{i0}(1 + 2\delta^2\{&J_0[2\xi_b \sin(\pi\epsilon_\omega)] \\ &- 2\pi\epsilon_\omega \cos(\pi\epsilon_\omega)\xi_b J_1[2\xi_b \sin(\pi\epsilon_\omega)]\}). \end{aligned} \tag{25}$$

In the case of large bias current density $2\pi\epsilon_j \gg 1$, with the asymptotic expressions of $G(z)$ and $S(z)$ at $z \gg 1$, Eq. (16) can be written as

$$\begin{aligned} r_{xx} = R_{i0}\bigg(1 + 8\delta^2\epsilon_j^{\frac{1}{2}}\bigg\{&\cos\left(2\pi\epsilon_j + \frac{\pi}{4}\right)J_0[2\xi_b \sin(\pi\epsilon_\omega)] \\ &- \sin\left(2\pi\epsilon_j - \frac{\pi}{4}\right)\frac{\epsilon_\omega}{\epsilon_j}\cos(\pi\epsilon_\omega)\xi_b J_1[2\xi_b \sin(\pi\epsilon_\omega)]\bigg\}\bigg). \end{aligned} \tag{26}$$

Figure 3 shows the differential resistivity r_{xx} obtained from expression (16) for the above-described system ($N_s = 3.7 \times 10^{15}$ m^{-2} and $\mu_0 = 1200$ m^2/V s from short-range impurity scatterings) with $\tau_s = 6.7$ ps irradiated by 27-GHz x-polarized microwaves: (a) subjected to a fixed dc current of $\gamma_j = 1.5$ but different radiation-field strengths $E_{i\omega}$ and (b) exposed to a given radiation $E_{i\omega} = 3.2$ V/cm but at different bias dc currents γ_j. These curves closely follow those numerically calculated from Eq. (11) at $T = 1.5$ K, confirming the reliability of the approximation used in deriving Eq. (16). The theoretical predictions not only well reproduce the intensity variation of the oscillation period, but also capture the peak positions, amplitude, and phase of the r_{xx} oscillation experimentally observed by Khodas *et al.*[51] The r_{xx} expression (26) for $2\pi\epsilon_j \gg 1$, on the other hand, indicates that the present model gives rise to a r_{xx}-versus-ϵ_j behavior that is different from the result of the theoretical model in Ref. 51 at large bias current density.

V. SUMMARY

We have carried out numerical and analytical examinations on the nonlinear magnetoresistance oscillation induced by intense microwaves in a strongly dc-biased two-dimensional electron system using a photon-assisted magnetotransport scheme directly controlled by the current. The theoretical results, presented in Figs. 1 and 2 based on the Gaussian-type DOS (8) and Fig. 3 based on the cosine-type DOS (15), indicate that under the irradiation of intense microwaves of given frequency and polarization, the form of Landau-level broadening has only a slight influence on the behavior of the oscillation. The amplitude of differential resistance oscillation depends strongly on the single-particle lifetime τ_s, as well as on the range of the impurity-scattering potential in the system, and the phase of the oscillation is governed by the bias dc current density. The period of the oscillation, on the other hand, is determined almost solely by the intensity of the microwave radiation at a given bias current, irrespective of the single-particle lifetime τ_s and the range of the impurity-scattering potential. This radiation-intensity-dependent-only behavior of the resistance-oscillation period results from the crucial role of multiphoton processes under intense radiation and provides a unique way to determine the strength of the radiation field applied in the system.

ACKNOWLEDGMENTS

This work was supported by the projects of National Science Foundation of China (10734021 and 60876064) and National Basic Research Programs of China (2007CB310402 and 2011CB925603).

[1] M. A. Zudov, R. R. Du, J. A. Simmons, and J. L. Reno, Phys. Rev. B **64**, 201311(R) (2001).
[2] P. D. Ye, L. W. Engel, D. C. Tsui, J. A. Simmons, J. R. Wendt, G. A. Vawter, and J. L. Reno, Appl. Phys. Lett. **79**, 2193 (2001).
[3] R. G. Mani, J. H. Smet, K. von Klitzing, V. Narayanamurti, W. B. Johnson, and V. Umansky, Nature (London) **420**, 646 (2002); Phys. Rev. Lett. **92**, 146801 (2004).
[4] M. A. Zudov, R. R. Du, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **90**, 046807 (2003); **96**, 236804 (2006); Phys. Rev. B **73**, 041303(R) (2006).
[5] S. I. Dorozhkin, JETP Lett. **77**, 577 (2003).
[6] C. L. Yang, M. A. Zudov, T. A. Knuuttila, R. R. Du, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **91**, 096803 (2003).
[7] M. A. Zudov, Phys. Rev. B **69**, 041304(R) (2004).
[8] R. G. Mani, V. Narayanamurti, K. von Klitzing, J. H. Smet, W. B. Johnson, and V. Umansky, Phys. Rev. B **70**, 155310 (2004).
[9] R. L. Willett, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **93**, 026804 (2004).
[10] R. G. Mani, Phys. Rev. B **72**, 075327 (2005); Appl. Phys. Lett. **85**, 4962 (2004); **91**, 132103 (2007).

[11] S. I. Dorozhkin, J. H. Smet, V. Umansky, and K. von Klitzing, Phys. Rev. B **71**, 201306(R) (2005).
[12] S. A. Studenikin, M. Potemski, A. Sachrajda, M. Hilke, L. N. Pfeiffer, and K. W. West, IEEE Transaction on Nanotechnology **4**, 124 (2005); Phys. Rev. B **71**, 245313 (2005).
[13] J. H. Smet, B. Gorshunov, C. Jiang, L. N. Pfeiffer, K. W. West, V. Umansky, M. Dressel, R. Meisels, F. Kuchar, and K. von Klitzing, Phys. Rev. Lett. **95**, 116804 (2005).
[14] C. L. Yang, R. R. Du, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **74**, 045315 (2006).
[15] A. T. Hatke, M. A. Zudov, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **102**, 066804 (2009); Phys. Rev. B **79**, 161308(R) (2009).
[16] R. G. Mani, C. Gerl, S. Schmult, W. Wegscheider, and V. Umansky, Phys. Rev. B **81**, 125320 (2010).
[17] V. I. Ryzhii, Sov. Phys. Solid State **11**, 2078 (1970).
[18] V. I. Ryzhii and R. Suris, J. Phys. Condens. Matter **15**, 6855 (2003); Phys. Rev. B **68**, 165406 (2003); V. I. Ryzhii, Jpn. J. Appl. Phys. Part 1 **44**, 6600 (2005).
[19] J. Shi and X. C. Xie, Phys. Rev. Lett. **91**, 086801 (2003).
[20] A. C. Durst, S. Sachdev, N. Read, and S. M. Girvin, Phys. Rev. Lett. **91**, 086803 (2003).
[21] X. L. Lei and S. Y. Liu, Phys. Rev. Lett. **91**, 226805 (2003); Phys. Rev. B **72**, 075345 (2005); Appl. Phys. Lett. **86**, 262101 (2005); X. L. Lei, J. Phys. Condens. Matter **16**, 4045 (2004).
[22] M. G. Vavilov and I. L. Aleiner, Phys. Rev. B **69**, 035303 (2004).
[23] M. Torres and A. Kunold, Phys. Rev. B **71**, 115313 (2005); J. Phys. Condens. Matter **18**, 4029 (2006).
[24] T. K. Ng and Lixin Dai, Phys. Rev. B **72**, 235333 (2005).
[25] J. Iñarrea and G. Platero, Phys. Rev. Lett. **94**, 016806 (2005); Phys. Rev. B **72**, 193414 (2005).
[26] A. Kashuba, Phys. Rev. B **73**, 125340 (2006).
[27] I. A. Dmitriev, A. D. Mirlin, and D. G. Polyakov, Phys. Rev. Lett. **91**, 226802 (2003); **99**, 206805 (2007); Phys. Rev. B **70**, 165305 (2004); **75**, 245320 (2007); I. A. Dmitriev, M. G. Vavilov, I. L. Aleiner, A. D. Mirlin, and D. G. Polyakov, *ibid.* **71**, 115316 (2005).
[28] J. Dietel, L. I. Glazman, F. W. J. Hekking, and F. von Oppen, Phys. Rev. B **71**, 045329 (2005); C. Joas, J. Dietel, and F. von Oppen, *ibid.* **72**, 165323 (2005); J. Dietel, *ibid.* **73**, 125350 (2006).
[29] A. V. Andreev, I. L. Aleiner, and A. J. Millis, Phys. Rev. Lett. **91**, 056803 (2003).
[30] J. Alicea, L. Balents, M. P. A. Fisher, A. Paramekanti, and L. Radzihovsky, Phys. Rev. B **71**, 235322 (2005).
[31] A. Auerbach, I. Finkler, B. I. Halperin, and A. Yacoby, Phys. Rev. Lett. **94**, 196801 (2005).
[32] S. A. Mikhailov, Phys. Rev. B **70**, 165311 (2004).
[33] X. L. Lei and S. Y. Liu, Appl. Phys. Lett. **88**, 212109 (2006); **89**, 182117 (2006); **94**, 232107 (2009).
[34] I. A. Dmitriev, M. Khodas, A. D. Mirlin, D. G. Polyakov, and M. G. Vavilov, Phys. Rev. B **80**, 165327 (2009).
[35] J. Iñarrea, R. G. Mani, and W. Wegscheider, Phys. Rev. B **82**, 205321 (2010).
[36] C. L. Yang, J. Zhang, R. R. Du, J. A. Simmons, and J. L. Reno, Phys. Rev. Lett. **89**, 076801 (2002).
[37] A. A. Bykov, J. Q. Zhang, S. Vitkalov, A. K. Kalagin, and A. K. Bakarov, Phys. Rev. B **72**, 245307 (2005).
[38] W. Zhang, H.-S. Chiang, M. A. Zudov, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **75**, 041304(R) (2007).
[39] J. Q. Zhang, S. Vitkalov, A. A. Bykov, A. K. Kalagin, and A. K. Bakarov, Phys. Rev. B **75**, 081305(R) (2007).
[40] A. A. Bykov, J. Q. Zhang, S. Vitkalov, A. K. Kalagin, and A. K. Bakarov, Phys. Rev. Lett. **99**, 116801 (2007).
[41] X. L. Lei, Appl. Phys. Lett. **90**, 132119 (2007); Physica E **42**, 63 (2009).
[42] M. G. Vavilov, I. L. Aleiner, and L. I. Glazman, Phys. Rev. B **76**, 115331 (2007).
[43] W. Zhang, M. A. Zudov, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **98**, 106804 (2007); Physica E **40**, 982 (2008).
[44] X. L. Lei, Appl. Phys. Lett. **91**, 112104 (2007).
[45] A. Auerbach, and G. V. Pai, Phys. Rev. B **76**, 205318 (2007).
[46] A. T. Hatke, H.-S. Chiang, M. A. Zudov, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **77**, 201304(R) (2008).
[47] X. L. Lei and S. Y. Liu, Appl. Phys. Lett. **93**, 082101 (2008).
[48] M. Khodas and M. G. Vavilov, Phys. Rev. B **78**, 245319 (2008).
[49] A. T. Hatke, H.-S. Chiang, M. A. Zudov, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **101**, 246811 (2008).
[50] X. L. Lei, Phys. Rev. B **79**, 115308 (2009).
[51] M. Khodas, H.-S. Chiang, A. T. Hatke, M. A. Zudov, M. G. Vavilov, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **104**, 206801 (2010).
[52] X. L. Lei, Mater. Sci. Eng., R **70**, 126 (2010).
[53] X. L. Lei and C. S. Ting, Phys. Rev. B **30**, 4809 (1984); **32**, 1112 (1985); X. L. Lei, J. L. Birman, and C. S. Ting, J. Appl. Phys. **58**, 2270 (1985).
[54] C. S. Ting, S. C. Ying, and J. J. Quinn, Phys. Rev. B **16**, 5394 (1977).
[55] V. Umansky, R. de Picciotto, and M. Heiblum, Appl. Phys. Lett. **71**, 683 (1997).
[56] M. E. Raikh and T. V. Shahbazyan, Phys. Rev. B **47**, 1522 (1993).
[57] P. T. Coleridge, Semicond. Sci. Technol. **12**, 22 (1997).