Spin dynamics in the strong spin-orbit coupling regime

Xin Liu,[1] Xiong-Jun Liu,[1] and Jairo Sinova[1,2]
[1]*Department of Physics, Texas A&M University, College Station, Texas 77843-4242, USA*
[2]*Institute of Physics ASCR, Cukrovarnická 10, 162 53 Praha 6, Czech Republic*
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We study the spin dynamics in a high-mobility two-dimensional electron gas with generic spin-orbit interactions (SOI's). We derive a set of spin-dynamics equations that capture purely exponentially the damped oscillatory spin evolution modes observed in different regimes of SOI strength. Hence we provide a full treatment of the D'yakonov-Perel mechanism by using the microscopic linear-response theory from the weak to the strong SOI limit. We show that the damped oscillatory modes appear when the electron scattering time is larger than half of the spin precession time due to the SOI, in agreement with recent observations. We propose a way to measure the scattering time and the relative strength of Rashba and linear Dresselhaus SOI's based on these modes and optical grating experiments. We discuss the physical interpretation of each of these modes in the context of Rabi oscillation.

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I. INTRODUCTION

In recent years, research in semiconductor-based devices has incorporated the spin degree of freedom as a new state variable in novel electronic devices with potential for future applications. The spin-orbit interaction (SOI) is a key tool to electrically manipulate the spin and realize such devices. However, the SOI is a double-edged sword because it will also induce random spin precession through an angle $\Omega_{\rm so}\tau$ between collisions with impurities, where τ is the electron lifetime. This is known as the D'yakonov-Perel (DP) mechanism[1–3] and dominates the spin relaxation in the technologically important III-V semiconductors.[4] Therefore, it is very important to understand fully the DP mechanism for the possible application and further development of spintronics devices. Although study of the DP mechanism in semiconductors in the presence of SOI was initiated long ago, most of the theoretical research[5–10] was focused on the weak spin-orbit coupling (SOC) regime in which $\Omega_{\rm so}\tau \ll 1$. However, as high-mobility two-dimensional electron gas (2DEG) systems are created, it is now not difficult to reach the strong SOI regime experimentally where $\Omega_{\rm so}\tau > 1$ at low temperatures as long as the mobility is approximately larger than $1.2 \times 10^5 \mathrm{cm}^2/\mathrm{V\,s}$.[11] The spin evolution in this regime is observed to be damped oscillations in the uniform[11,12] and nonuniform spin-polarized system,[13–15] which cannot be described by spin-charge drift-diffusion equations derived for the weak SOC regime and lacks a clear theoretical explanation.

Here, we study the spin dynamics theoretically from the weak to strong SOC regime. The method we use is linear-response theory.[5,7,16] We derive a set of spin-dynamics equations in the uniform spin-polarized 2DEG with different SOI's in the presence of the short-ranged impurity scattering. For the experiments we consider, even in the strong SOC regime, it is dominated by neutral impurity or interface roughness scattering, which are short-ranged impurity scattering.[12] The weak localization effect on the spin relaxation[17] is neglected in our work because we consider the spin relaxation in the metallic regime, where the weak localization effect is small.

We show analytically that for $\Omega_{\rm so}\tau > \frac{1}{2}$, the damped oscillations appear. The decay rate in this case is proportional to $\frac{1}{\tau}$ instead of τ as in the weak SOC regime. The cubic Dresselhaus term is shown to reduce the oscillatory frequency and increase the decay rate in the strong SOC regime. The spin dynamics for nonuniform spin polarization with spatial frequency q in the strong SOC regime is obtained by solving the equations numerically. We discuss these dynamics by using the analogy with Rabi oscillations between two momentum states that are gapped by the SOI. Our results agree quantitatively with the experimental observations. We also show how to exploit our analysis to create an accurate measurement of the strength of Rashba and linear Dresselhaus SOI's in a 2DEG, hence allowing a full characterization of different device samples that will lead to a more accurate modeling and predictability of the optimal operating physical regimes.

II. MODEL HAMILTONIAN AND DENSITY-MATRIX RESPONSE FUNCTION

Normally in the 2D semiconductor heterostructures, we have three kinds of SOI's, namely the linear Rashba[18,19] term and the linear and cubic Dresselhaus[20] terms. The Hamiltonian takes the form

$$H = \frac{k^2}{2m} + \mathbf{h}(\mathbf{k}) \cdot \hat{\sigma}, \tag{1}$$

where $\mathbf{h}(\mathbf{k})$ is the effective magnetic and contains Rashba, linear, and cubic Dresselhaus terms, which are

$$\mathbf{h}^R(\mathbf{k}) = \alpha(-k_y,k_x), \tag{2}$$

$$\mathbf{h}^{D_1}(\mathbf{k}) = \beta_1(k_y,k_x), \tag{3}$$

$$\mathbf{h}^{D_3}(\mathbf{k}) = -2\beta_3 \cos 2\theta(-k_y,k_x), \tag{4}$$

where k_f is the Fermi wave vector. Here we take θ as the angle between the wave vector $\mathbf{k}$ and the [110] direction, which is the x axis in our coordinates. The above SOI's split the spin-degenerate bands and dominate the spin dynamics in the 2DEG. The corresponding SOC Hamiltonian and the spin precession frequency $\Omega_{\rm so}$ take the form:

$$H^{\rm so} = (\lambda_1 - 2\beta_3 \cos 2\theta)k_x\sigma_y + (\lambda_2 + 2\beta_3 \cos 2\theta)k_y\sigma_x, \tag{5}$$

where $\lambda_1 = \alpha + \beta_1$ and $\lambda_2 = \beta_1 - \alpha$.

We derive the spin-dynamics equations from the density-matrix response function.[16] The spin diffusion is dominated by the pole of the spin-charge diffusion propagator or "diffuson:"[5]

$$\mathcal{D} = [1 - \hat{I}]^{-1} \tag{6}$$

and

$$\hat{I}_{\sigma_1\sigma_2,\sigma_3\sigma_4} = \frac{1}{2m\tau}\int \frac{d^2k}{(2\pi)^2} G^A_{\sigma_3\sigma_1}(k,0) G^R_{\sigma_2\sigma_4}(k+q,\Omega), \tag{7}$$

where σ_i is just a number that can be 1 or 2.[5] It is more convenient to write Eq. (7) in a classical charge-spin space,

$$I^{\alpha\beta} = \mathrm{Tr}(\sigma_\alpha \hat{I} \sigma_\beta), \tag{8}$$

where $\alpha,\beta = c,x,y,z$.[5]

If one calculates the response function by expanding in term of $\Omega_{so}\tau$ to the first order, the spin-relaxation behavior obtained by this approximate response function is only valid in the weak SOC regime, such as in Ref. 7. However, if one calculates the response function exactly without any expansion in the parameter $\Omega_{so}\tau$, this response function can give the spin relaxation in both the weak and strong SOC regime. In the appendix of Ref. 5, Burkov *et al.* give the expression of the spin-charge diffuson in the presence of the Rashba spin-orbit interaction. The authors in Ref. 5 are interested in finding a spin-charge drift diffusion equation, only applicable in the weak SOC regime, and therefore they expanded the expressions in terms of $\Omega_{so}\tau$ to first order. However, they claim that the expression should be useful in the strong SOC regime. Here, we will calculate the diffuson matrix exactly with the genetic SOI's and find the poles of this exact expression.

III. UNIFORM SPIN POLARIZATION

In the case of a uniform spin-polarized 2DEG system, i.e., $q = 0$, because the effective magnetic field due to the SOI has inversion symmetry in momentum space, only the diagonal elements of the diffuson matrix are nonzero, which means the spin x,y,z and charge are not coupled to each other. Therefore, when considering the uniform spin polarization along the z direction, only I^{zz} needs to be calculated. First, we neglect the cubic Dresselhaus term, which is normally much smaller than the linear Dresselhaus term. We find the pole of the diffusion matrix by solving the equation

$$1 - I^{zz} = 1 - \frac{1 - i\Omega\tau}{\sqrt{[(1-i\Omega\tau)^2 + (\Omega_{so}\tau)^2]^2 - \gamma^2(\Omega_{so}\tau)^4}} = 0, \tag{9}$$

where Ω is the frequency of the spin evolution, $\Omega_{so} = 2\sqrt{\alpha^2+\beta_1^2}k_f$, $\gamma = \frac{2\alpha\beta_1}{\alpha^2+\beta_1^2} = \frac{\lambda_1^2-\lambda_2^2}{\lambda_1^2+\lambda_2^2}$, k_f is the Fermi wave vector, and I^{zz} is obtained from the exact angular integration of Eq. (7). The details of calculating I^{zz} are shown in the Appendix. There are four solutions of Eq. (9), which take the form

$$\Omega\tau = -i\left(1 \pm \frac{\sqrt{2}}{2}\sqrt{1 - 2(\Omega_{so}\tau)^2 \pm \sqrt{1 - 4(\Omega_{so}\tau)^2 + 4(\Omega_{so}\tau)^4\gamma^2}}\right). \tag{10}$$

However, note that not all of these solutions give the spin evolution mode observed by the experiments.[11,12] To find the right one, we explore the values of the above four solutions in the limit of the weak spin-orbit coupling regime, say $\Omega_{so}\tau = 0$, and write them as

$$\begin{aligned}
\Omega_1\tau &= -i\left(1 - \frac{\sqrt{2}}{2}\sqrt{1 - 2(\Omega_{so}\tau)^2 + \sqrt{1 - 4(\Omega_{so}\tau)^2 + 4(\Omega_{so}\tau)^4\gamma^2}}\right) = 0,\\
\Omega_2\tau &= -i\left(1 - \frac{\sqrt{2}}{2}\sqrt{1 - 2(\Omega_{so}\tau)^2 - \sqrt{1 - 4(\Omega_{so}\tau)^2 + 4(\Omega_{so}\tau)^4\gamma^2}}\right) = -i,\\
\Omega_3\tau &= -i\left(1 + \frac{\sqrt{2}}{2}\sqrt{1 - 2(\Omega_{so}\tau)^2 + \sqrt{1 - 4(\Omega_{so}\tau)^2 + 4(\Omega_{so}\tau)^4\gamma^2}}\right) = -2i,\\
\Omega_4\tau &= -i\left(1 + \frac{\sqrt{2}}{2}\sqrt{1 - 2(\Omega_{so}\tau)^2 - \sqrt{1 - 4(\Omega_{so}\tau)^2 + 4(\Omega_{so}\tau)^4\gamma^2}}\right) = -i.
\end{aligned} \tag{11}$$

We know that for the spin relaxation dominated by the DP mechanism, $\Omega\tau \to 0$ when $\Omega_{so}\tau \to 0$, which indicates that only the first mode, Ω_1, in Eq. (11) gives the right behavior of the spin relaxation, say $\Omega \propto \tau$,[5,7] in the weak spin-orbit coupling regime. On the other hand, in the strong spin-orbit coupling regime, there is only one mode observed in the uniform spin-polarized case.[11,12] Therefore, we can conclude that only the first mode in Eq. (11) contributes to the spin relaxation. Therefore, the eigenmode of the spin dynamic evolution takes the form

$$i\Omega\tau = \tfrac{1}{2}(2 - \sqrt{2}\sqrt{1 - 2(\Omega_{so}\tau)^2 + \sqrt{1 - 4(\Omega_{so}\tau)^2 + 4(\Omega_{so}\tau)^4\gamma^2}}). \tag{12}$$

Note that $\gamma \leqslant 1$ and

$$[1 - 4(\Omega_{so}\tau)^2 + 4(\Omega_{so}\tau)^4\gamma^2] \leqslant \left(1 - 2\Omega_{so}^2\tau^2\right).$$

Therefore, as long as $1 - 4(\Omega_{so}\tau)^2 + 4(\Omega_{so}\tau)^4\gamma^2 < 0$, a nonequilibrium spin polarization will exhibit damped oscillation with respect to time (see Fig. 1).

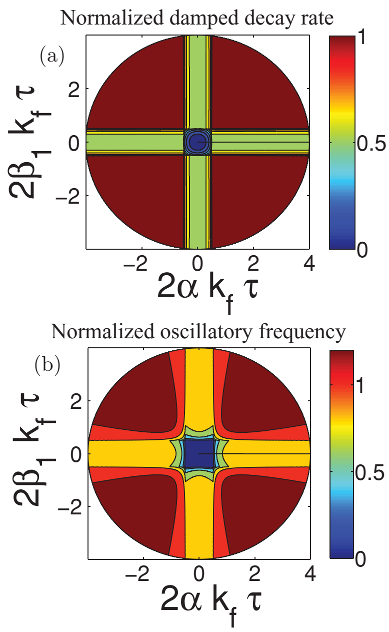

FIG. 1. (Color online) The uniform spin dynamics from the weak to the strong spin-orbit coupling regime in the presence of both Rashba and linear Dresselhaus terms. (a) The normalized exponential decay rate, $\mathrm{Im}(\Omega\tau)$, is shown as a function of normalized Rashba and linear Dresselhaus SOI. (b) The nonzero normalized oscillatory frequency, $\frac{\mathrm{Re}(\Omega)}{\Omega_{so}}$, is nonzero whenever $2\alpha k_f\tau \geqslant \frac{1}{2}$ or $2\beta_1 k_f\tau \geqslant \frac{1}{2}$.

In the case of $\alpha = 0$ or $\beta_1 = 0$, the eigenmode takes the form

$$i\Omega\tau = \tfrac{1}{2} - \tfrac{1}{2}\sqrt{1 - 4\Omega_{so}^2\tau^2}. \tag{13}$$

When $\Omega_{so}\tau > 1/2$, the decay rate changes from the exponential decay mode to the damped oscillation mode. The oscillatory frequency in the clean limit, $\tau \to \infty$, is Ω_{so}. Several experiments[11–14] observe the damped oscillation mode of spin evolution at low temperature. However, their analysis did not explain quantitatively when this kind of mode appears but just qualitatively argued that it appears in the regime where $\Omega_{so}\tau > 1$. Our theory agrees with a recent experiment[12] in which the authors observe that when the temperature is above 5 K, the oscillation will disappear. In their system, this corresponds to $\Omega_{so}\tau_p^* \approx 0.48$, which is close to our result $1/2$. Here τ_p^* is different from the transport scattering time τ_p obtained from the mobility; this difference is due to the Coulomb interaction effect on spin-currents and spin dephasing.[8,13] This *e-e* interaction treatment is beyond the scope of our paper and will not be discussed in this work. The τ here corresponds to τ_p^*. When the oscillatory mode appears, the damped decay rate is always equal to $\frac{1}{2\tau}$ when either $\alpha = 0$ or $\beta_1 = 0$. This result agrees with a recent experiment[11] in which the authors found that the decay rates for several different 2DEG's always equal $\frac{1}{1.9\tau}$ when the damped oscillatory mode appears, in agreement with our theoretical result.

As the linear and cubic Dresselhaus terms always coexist, we have to consider the effect of the cubic Dresselhaus term on Eq. (13). We do this in the simplest case, when the Rashba coefficient is zero. In this case, the diffuson matrix element I^{zz} takes the form

$$I^{zz} = \frac{1 - i\Omega\tau}{\sqrt{(1-i\Omega\tau)^2 + \Omega_{so}^2\tau^2\left[1 + 2\left(\frac{\beta_3}{\beta_1}\right)^2 - 2\frac{\beta_3}{\beta_1}\right]}} \times \frac{1}{\sqrt{(1-i\Omega\tau)^2 + \Omega_{so}^2\tau^2}}, \tag{14}$$

where $\Omega_{so} = 2\beta_1 k_f$ and $\delta = 2\frac{\beta_3}{\beta_1}(1 - \frac{\beta_3}{\beta_1})$. The corresponding spin decay rate is

$$i\Omega\tau = 1 - \frac{\sqrt{\left(1 + \sqrt{1 - 4\Omega_{so}^2\tau^2 + 2\Omega_{so}^2\tau^2\delta + \Omega_{so}^4\tau^4\delta^2}\right)^2 - \Omega_{so}^4\tau^4\delta^2}}{2}. \tag{15}$$

Equations (13) and (15) show that the cubic term will increase the exponential decay rate and decrease the oscillatory frequency. To show the effect of the cubic Dresselhaus term, the real (imaginary) value of the damped oscillatory frequency when $\beta_3 \neq 0$ is divided by the value when $\beta_3 = 0$. This ratio is plotted in Fig. 2 with respect to β_3/β_1 and $2\beta_1\tau$. When $\frac{\beta_3}{\beta_1} < 0.2$, the effect of the cubic term is very small and can be neglected. In this case, the damped decay rate is always equal to $\frac{1}{2\tau}$ as long as $\Omega_{so} > \frac{1}{2}$ and the oscillatory frequency Ω approaches Ω_{so} when $\Omega_{so}\tau \gg 1$. This provides a reliable way to measure the momentum scattering time τ. Further, the strength of the linear Dresselhaus SOI can be obtained from Eq. (13) once we know τ and the oscillatory frequency from the measurements. These will be discussed in a later section.

Now, let us choose $\alpha = \beta_1$, which is a more unique case and gives us the persistent spin helix for special q values.[14,15,21]

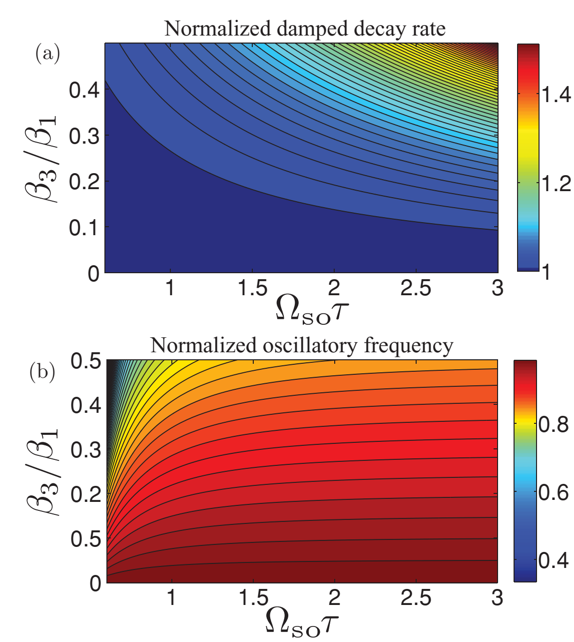

FIG. 2. (Color online) The uniform spin dynamics from the weak to the strong spin-orbit coupling regime in the presence of linear β_1 and cubic β_3 SOI. (a) The normalized exponential decay rate, $\mathrm{Re}(i\Omega\tau)$, is constant when β_3 is zero and slightly larger than $\frac{1}{2}$ when β_3 is nonzero. (b) The nonzero normalized oscillatory frequency, $\mathrm{Im}(i\Omega\tau)$, appears when $\Omega_{so}\tau > \frac{1}{2}$.

For the uniform spin polarization, the decay rate of the spin satisfies

$$i\Omega\tau = 1 - \sqrt{1 - 2(\Omega_{\rm so}\tau)^2}, \qquad (16)$$

where $\Omega_{\rm so} = 2\sqrt{\alpha^2 + \beta_1^2}k_f$. The damped oscillation mode will happen when $\Omega_{\rm so}\tau = 2\sqrt{2}\alpha k_f\tau > \sqrt{2}/2$, say $2\alpha k_f\tau > 1/2$, which is the same as the pure Rashba or Dresselhaus case. The oscillating frequency in the clean limit is $\sqrt{2}\Omega_{\rm so} = 4\alpha k_f$, which is the twofold frequency for the pure Rashba or Dresselhaus case. On the other hand, as the real part of $i\Omega\tau$ is equal to 1 when the damped oscillation mode appears, the damped decay rate is also the twofold case of the pure Rashba or Dresselhaus case.

IV. SPIN DYNAMICS AND RABI OSCILLATION

Before we discuss the spin dynamics for the nonuniform spin-polarization system, let us give a physical explanation of the result we have obtained. We can construct a simple physics picture to describe the spin-polarized wave theoretically. Taking the Rashba SOI, for example, we define the eigenstates $|\phi_k^a\rangle$ to denote the majority band and the $|\phi_k^b\rangle$ to denote the minority band. The spin of the eigenstate of the SOC 2DEG lies in the x-y plane. The majority electron has opposite spin to the minority electron when they have the same wave vector k.

As a result, the spin polarization along the z direction can be obtained by the superposition of the majority and minority bands as

$$\psi_{\uparrow,q} = A\left[\sum_k e^{(\epsilon-\epsilon_f)^2/4\sigma^2}\frac{1}{\sqrt{2}}\left(|\phi_k^a\rangle + |\phi_{k+q}^b\rangle\right)\right] + A\left[\sum_k e^{(\epsilon-\epsilon_f)^2/4\sigma^2}\frac{1}{\sqrt{2}}\left(|\phi_k^b\rangle + |\phi_{k+q}^a\rangle\right)\right], \qquad (17)$$

where A is the normalization coefficient, $\psi_{\uparrow,q}$ is the wave function of the system with positive spin polarization along the z direction with wave vector q, and the function $e^{(\epsilon-\epsilon_f)^2/4\sigma^2}$ restricts the spin-polarization electrons only in the narrow range $\frac{1}{2\sigma} \ll \epsilon_f$ around the Fermi energy ϵ_f. The expectation value $\langle\psi_{\uparrow,q}|\sigma_z\cos q'x|\psi_{\uparrow,q}\rangle$ is nonzero only when $q' = q$, which confirms that $\psi_{\uparrow,q}$ can describe the spin-polarized wave. The energy difference of these two electrons in the first (second) term on the right-hand side of Eq. (17) is $\Delta_{1(2)}$, as shown in Fig. 3. Therefore, $|\psi\rangle$ can be treated as a collective two-level system with two Rabi frequencies $\Omega_{1(2)} = \frac{\Delta_{1(2)}}{\hbar}$. The uniform spin polarization means $q = 0$ and there is only one Rabi frequency $\Omega_0 = \frac{\Delta_0}{\hbar}$, Fig. 3. When the system is very clean, our results, Eqs. (13) and (16), show that the spin evolution is damped oscillation and the oscillatory frequency is the Rabi frequency. It is a little surprising that when $\alpha = \beta$, although the SOC gap Δ_0 is not a constant, the oscillatory frequency corresponds to the maximum splitting energy $4\alpha k_f$ instead of the average splitting energy $2\sqrt{2}\alpha k_f$. In the weak SOC regime, the disorder is so strong that the splitting energy due to the SOI is completely submerged in the broadening of the band $\frac{\hbar}{\tau}$. Therefore, the spin polarization just decays exponentially. For the nonuniform spin-polarization case, since there are two Rabi oscillation frequencies Ω_1 and Ω_2, we expect to have two damped oscillatory modes in the clean system corresponding to energy differences Δ_1 and Δ_2, respectively, in Fig. 3.

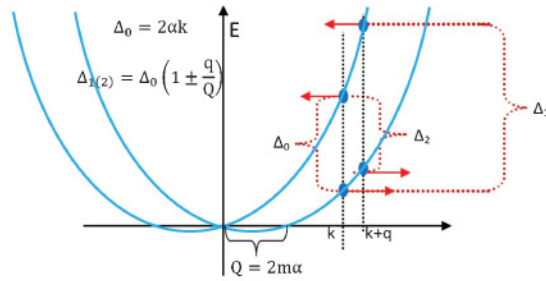

FIG. 3. (Color online) The dispersion relation due to the linear Dresselhaus SOI. The SOI induces the energy gap $\Delta_0 = 2\beta_1 k$, which is the spin precession frequency for the single-electron spin. However, when the system is excited to be a spin-polarization wave with wave vector q, the spin polarization along the z direction is constructed by the superposition of the two electrons with wave vectors k and $k+q$. In this case, the spin precession frequency will be $\Delta_{1(2)} \simeq \Delta_0(1 \pm \frac{q}{Q})$, where $Q = 2m\beta_1$.

V. NONUNIFORM SPIN POLARIZATION

In the case of the nonuniform spin-polarized 2DEG, the initial state is a spin wave with wave vector q, and the momentum $\mathbf{k}$ is coupled to $\mathbf{k}+\mathbf{q}$, which makes the center of the Fermi sea shift to near $\mathbf{q}$. The average magnetic field is nonzero and the of-diagonal elements of the diffusion matrix appear to couple the different spin components. When only considering the Rashba or linear Dresselhaus SOI, our numerical calculation does have two kinds of spin dynamical modes, which are shown in Figs. 4 and 5.

The two damped oscillatory modes and their oscillatory frequency satisfy our expectation based on the Rabi oscillation viewpoint. When q increases, the Rabi frequency of the faster mode always increases, which makes the damped oscillatory mode appear even when $\Omega_{\rm so}\tau < \frac{1}{2}$. This means we can expect to observe the oscillation for the nonzero spin polarization at higher temperature than for the uniform spin polarization. In Ref. 12, where the spin polarization is uniform, the damped oscillatory mode appears below 5 K. On the other hand, in Ref. 13, where the spin polarization is nonuniform, the damped oscillatory mode appears below 50 K. The material, Fermi energy, and mobility in these two papers are similar. This seems support our Rabi oscillation viewpoint. For the slow oscillatory mode, when q is around Q, the corresponding Rabi frequency Ω_2 is around 0, which means the spin precession is very slow. Because the Rabi frequencies are much smaller than $\frac{1}{\tau}$, the spin polarization just decays exponentially and the exponential decay rate has its minimum in this regime when q is around Q.

A particular case is when $\alpha = \beta_1$ and $\beta_3 = 0$. The analytical solutions of these two modes can be obtained by finding the poles of Eq. (20) of Ref. 21, and they have the form

$$i\Omega\tau = 1 - \sqrt{1 - (\Omega_{\rm so}\tau)^2\left(1 \pm \frac{q}{Q}\right)^2}, \qquad (18)$$

where $Q = 4m\alpha$. At $q = Q$, the Rabi frequency of the slower mode is zero for all of the electron momentum k. On the

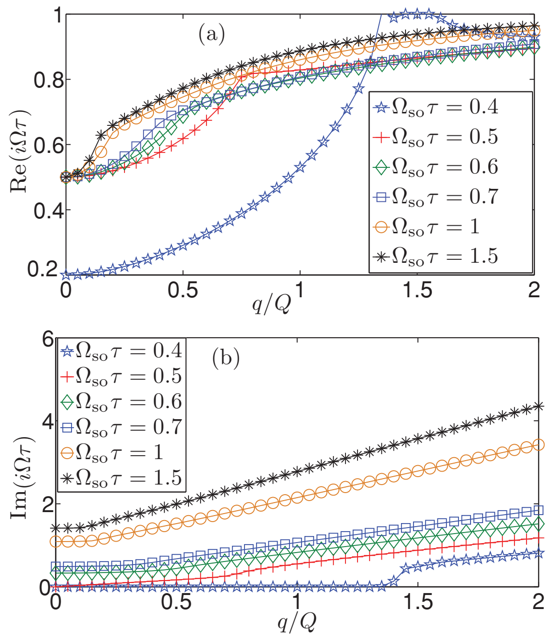

FIG. 4. (Color online) The fast oscillatory mode of the nonuniform spin dynamics in the strong SOC regime when the system only has bulk inversion asymmetry. (a) The normalized exponential decay rate, Re($i\Omega\tau$), increases with increasing q and approaches 1 at large q. (b) The nonzero normalized oscillatory frequency, Im($i\Omega\tau$), increases linearly at large q, the slope is close to $\Omega_{so}\tau$, and its value approaches $\Omega_{so}(1 + \frac{q}{Q})$, where $Q = 2m\beta_1$.

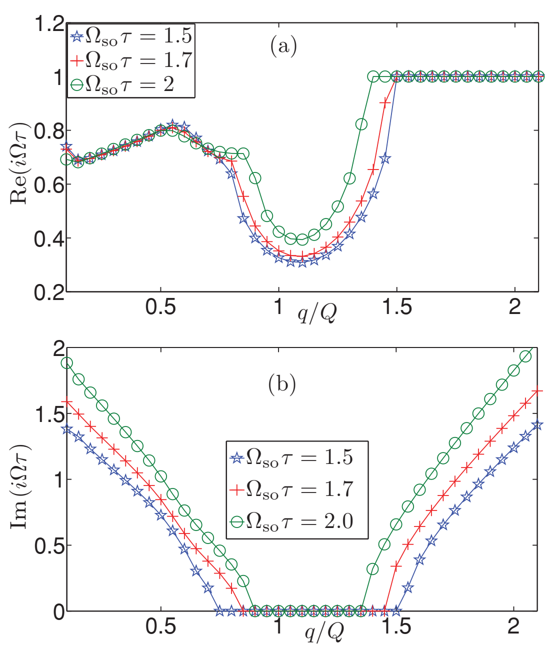

FIG. 5. (Color online) The slow oscillatory mode of the nonuniform spin dynamics in the strong SOC regime when the system only has bulk inversion asymmetry. (a) The normalized exponential decay rate, Re($i\Omega\tau$), has a minimum around $q = Q$ and approaches 1 at large q. (b) The nonzero normalized oscillatory frequency, Im($i\Omega\tau$), is always zero when q is around Q and increases linearly at large q. The slope is close to $\Omega_{so}\tau$ and the value approaches $\Omega_{so}(1 - \frac{q}{Q})$ at large q, where $Q = 2m\beta_1$.

other hand, the spin y is a good quantum number for all the electron states, which means the spin-independent disorder will never couple the two electrons in different bands with different spin directions. Therefore, the Rabi frequency of the slower mode is still exactly zero even in the presence of the spin-independent disorder no matter how strong it is. As a result, the spin along the z direction will never precess and has an infinitely long lifetime. This provides another way to understand the persistent spin helix.[14,15] However, the cubic Dresselhaus SOI induces a band transition in the presence of spin-independent impurities and makes the spin lifetime finite.[7] When $\alpha \neq \beta_1$, even at $q = Q$, the gap of the two electrons with momentum k and $k + q$ in different spin bands is dependent on k and fluctuates around the average value of the gap. The average value of the Rabi frequency of the slower mode is small but not zero. Therefore, the spin relaxation cannot be exactly suppressed. However, if the average value of the gap is much larger than the fluctuation, normally when $q \gg Q$, the spin relaxation can be well described by Eq. (18) for an arbitrary combination of α and β_1.

VI. PROPOSED EXPERIMENTS

The spin dynamics in the strong SOC regime have several special characters that can be used in experimental measurements.

Momentum scattering time τ_p^*. In the spin dynamics, the Coulomb interaction plays an important role in determining the momentum scattering time τ_p^*.[22,23] This is quite different from the charge-transport case, in which electron-electron (e-e) interaction will not change the ensemble momentum scattering τ_p, which determines the electron mobility. This difference is called spin Coulomb drag (SCD). In previous experimental work, SCD was observed through the spin diffusion coefficient $D_s = \frac{1}{2}v_f^2\tau_p^*$ by fitting the spin decay rate in the weak SOC regime. Here, we provide a way to observe SCD in the strong SOC regime by directly measuring the momentum scattering time τ_p^*. Based on Eqs. (13) and (15), when only Dresselhaus SOI is presented, the damped decay rate is always almost equal to $1/2$ as long as $\frac{\beta_3}{\beta_1} < 0.2$, which is easily realized in experiments.[11,15]

The strength of SOI's. Here, we would like to emphasize that $2\beta_1 k_f \tau = \frac{1}{2}$ is a very important case and corresponds to the transition point between the pure exponential decay mode and the damped oscillatory mode. The decay rate at this point is not only equal to $\frac{1}{2\tau}$ but also equal to $\frac{1}{2\beta_1 k_f}$ when $\alpha = 0$. This means that at this point we can obtain the strength of linear Dresselhaus SOI from the spin-polarization decay rate. When $2\beta_1 k_f \tau = \frac{1}{2}$, we can increase the Rashba SOI by adding a gate voltage. As long as $0 < \alpha < \beta_1$, according to Eq. (12), the spin evolution still decays exponentially and the decay rate is $[1 - \frac{\sqrt{2}}{2}\sqrt{1 - 2(\Omega_{so}\tau)^2}]/\tau$, where $\Omega_{so} = 2\sqrt{\alpha^2 + \beta_1^2}k_f$, which gives us the strength of Rashba SOI.

VII. CONCLUSION

We have discussed the spin dynamics in the strong spin-orbit coupling regime. We describe quantitatively the special characters of the damped oscillatory mode in this regime. We also compare our results to the previous experimental data and find they match very well. Based on our theoretical results, a reliable way is proposed to measure the Rashba and Dresselhaus coefficients and electron momentum scattering time, which does not correspond to the mobility due to the Coulomb interaction. Furthermore, we find that the spin dynamics in the 2DEG can be treated as a collective two-level system. This helps us to understand semiquantitatively the spin dynamics in the strong spin-orbit coupling regime. For the nonzero spin-polarization case, we predict that there exist double damped oscillatory modes at large q, and we explain the persistent spin helix mode from the Rabi oscillation point of view.

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APPENDIX: SPIN DYNAMIC MATRIX FOR THE UNIFORM SPIN POLARIZATION

In this section, we derive the spin evolution mode of the uniform spin polarization. According to Eq. (5), the strength of SOI is angle-dependent and can be written as

$$h_{\rm so} = \sqrt{\alpha^2+\beta_1^2}k_f\sqrt{1+\cos 2\psi\cos 2\theta+\left[2\left(\frac{\beta_3}{\lambda'}\right)^2-\frac{2\beta_3}{\lambda'}\sin(\psi+\pi/4)\right](1+\cos 4\theta)-\frac{4\beta_3}{\lambda'}\cos(\psi+\pi/4)\cos 2\theta}, \tag{A1}$$

where $\cos\psi = \lambda_1/\sqrt{\lambda_1^2+\lambda_2^2}$.

First we consider the case for $\beta_3 = 0$. The Hamiltonian is written as

$$H = \frac{k^2}{2m} + (\alpha+\beta)k_x\sigma_y - (\alpha-\beta)k_y\sigma_x = \frac{k^2}{2m} + \lambda_1 k_x\sigma_y + \lambda_2 k_y\sigma_x, \tag{A2}$$

where k_x is along the [110] direction, $\lambda_1 = \alpha+\beta$, and $\lambda_2 = -(\alpha-\beta)$. The Green's function for this Hamiltonian takes the form

$$G^{R(A)} = \frac{E-\frac{k^2}{2m}\pm\frac{i}{2\tau}+(\alpha+\beta)k_x\sigma_y-(\alpha-\beta)k_y\sigma_x}{\left(E-\frac{k^2}{2m}\pm\frac{i}{2\tau}\right)^2-(\alpha^2+\beta^2)k^2\left(1+\frac{2\alpha\beta}{\alpha^2+\beta^2}\cos 2\theta\right)} = \frac{E-\frac{k^2}{2m}\pm\frac{i}{2\tau}+\lambda_1 k_x\sigma_y+\lambda_2 k_y\sigma_x}{\left(E-\frac{k^2}{2m}\pm\frac{i}{2\tau}\right)^2-\frac{(\lambda_1^2+\lambda_2^2)}{2}k^2(1+\gamma\cos 2\theta)}, \tag{A3}$$

where τ is the momentum scattering time, and $\gamma = \frac{2\alpha\beta}{\alpha^2+\beta^2} = \frac{\lambda_1^2-\lambda_2^2}{\lambda_1^2+\lambda_2^2}$. It is more convenient to write down the element of the 2×2 Green's function, Eq. (A3), as

$$\begin{aligned}
G_{11} = G_{22} &= \frac{1}{2}\left(\frac{1}{E-\frac{k^2}{2m}-\lambda k\sqrt{1+\gamma\cos 2\theta}\pm\frac{i}{\tau}}+\frac{1}{E-\frac{k^2}{2m}+\lambda k\sqrt{1+\gamma\cos 2\theta}\pm\frac{i}{\tau}}\right),\\
G_{12} &= \frac{1}{2}\left(\frac{1}{E-\frac{k^2}{2m}-\lambda k\sqrt{1+\gamma\cos 2\theta}\pm\frac{i}{\tau}}-\frac{1}{E-\frac{k^2}{2m}+\lambda k\sqrt{1+\gamma\cos 2\theta}\pm\frac{i}{\tau}}\right)\frac{\sqrt{2}(-i\cos\psi\cos\theta+\sin\psi\sin\theta)}{\sqrt{1+\gamma\cos 2\theta}},\\
G_{21} &= \frac{1}{2}\left(\frac{1}{E-\frac{k^2}{2m}-\lambda k\sqrt{1+\gamma\cos 2\theta}\pm\frac{i}{\tau}}-\frac{1}{E-\frac{k^2}{2m}+\lambda k\sqrt{1+\gamma\cos 2\theta}\pm\frac{i}{\tau}}\right)\frac{\sqrt{2}(i\cos\psi\cos\theta+\sin\psi\sin\theta)}{\sqrt{1+\gamma\cos 2\theta}},
\end{aligned} \tag{A4}$$

where

$$\lambda=\sqrt{\left(\lambda_1^2+\lambda_2^2\right)/2} = \sqrt{\alpha^2+\beta^2}, \quad \cos\psi=\lambda_1/\sqrt{\lambda_1^2+\lambda_2^2}, \quad \text{and} \quad \gamma = \cos 2\psi.$$

According to Eq. (8), the diagonal element of the spin polarization along the z direction has the form

$$I^{zz} = I_{11,11} - I_{11,22} - I_{22,11} + I_{22,22} = \frac{1}{2m\tau}\int\frac{d^2k}{(2\pi)^2}\left(G_{11}^A G_{11}^R - G_{21}^A G_{12}^R - G_{12}^A G_{21}^R + G_{22}^A G_{22}^R\right). \tag{A5}$$

The first term and the fourth term in Eq. (A5) are equal to each other and have the form

$$\begin{aligned}
\frac{1}{2m\tau}\int\frac{d^2k}{(2\pi)^2}G_{11}^A G_{11}^R &= \frac{1}{2m}\int\frac{d^2k}{(2\pi)^2}\frac{1}{4}\left(\frac{1}{E-\epsilon_+(k)-\frac{i}{2\tau}}+\frac{1}{E-\epsilon_-(k)-\frac{i}{2\tau}}\right)\\
&\quad\times\left(\frac{1}{E+\Omega-\epsilon_-(k)+\frac{i}{2\tau}}+\frac{1}{E+\Omega-\epsilon_-(k)+\frac{i}{2\tau}}\right)\\
&= \frac{1}{16m\pi}\int_0^{2\pi}\frac{d\theta}{v_f}\left(\frac{k^+}{1-i\Omega\tau}+\frac{k^-}{1-i\Omega\tau+2i\lambda k\sqrt{1+\gamma\cos 2\theta}}+\frac{k^+}{1-i\Omega\tau-2i\lambda k\sqrt{1+\gamma\cos 2\theta}}+\frac{k^-}{1-i\Omega\tau}\right)\\
&\simeq \frac{1}{16\pi}\int_0^{2\pi}d\theta\left(\frac{1}{1-i\Omega\tau}+\frac{1}{1-i\Omega\tau+2i\lambda k\sqrt{1+\gamma\cos 2\theta}}+\frac{1}{1-i\Omega\tau-2i\lambda k\sqrt{1+\gamma\cos 2\theta}}+\frac{1}{1-i\Omega\tau}\right),
\end{aligned} \tag{A6}$$

where $v_f = \frac{\partial E_f}{\partial k}$. In the polar coordinate, $\int d^2k = \int d(k^2/2)d\theta$. As we assume that $\lambda k_f \ll E_f$, $d(k^2/2) \simeq m dE$, where m is the effective mass.

The other two terms are also equal to each other and can be written as

$$\frac{1}{2m\tau}\int \frac{d^2k}{(2\pi)^2}G^A_{21}G^R_{12} = \int \frac{d^2k}{(2\pi)^2}\frac{1}{4}\left(\frac{1}{E-\epsilon_+(k)-\frac{i}{2\tau_e}} - \frac{1}{E-\epsilon_-(k)-\frac{i}{2\tau_e}}\right)\left(\frac{1}{E+\Omega-\epsilon_+(k)+\frac{i}{2\tau_e}} - \frac{1}{E+\Omega-\epsilon_-(k)+\frac{i}{2\tau_e}}\right)$$
$$= \frac{1}{16m\pi}\int_0^{2\pi}\frac{d\theta}{v_f}\left(\frac{k^+}{1-i\Omega\tau} - \frac{k^-}{1-i\Omega\tau+2i\lambda k\sqrt{1+\gamma\cos 2\theta}} - \frac{k^+}{1-i\Omega\tau-2i\lambda k\sqrt{1+\gamma\cos 2\theta}} + \frac{k^-}{1-i\Omega\tau}\right).$$
$$\simeq \frac{1}{16\pi}\int_0^{2\pi}d\theta\left(\frac{1}{1-i\Omega\tau} - \frac{1}{1-i\Omega\tau+2i\lambda k\sqrt{1+\gamma\cos 2\theta}} - \frac{1}{1-i\Omega\tau-2i\lambda k\sqrt{1+\gamma\cos 2\theta}} + \frac{1}{1-i\Omega\tau}\right). \tag{A7}$$

Substituting Eqs. (A6) and (A7) into Eq. (A5), we have

$$I^{zz} = \frac{1}{4\pi}\int_0^{2\pi}d\theta\left(\frac{1}{1-i\Omega\tau+2i\lambda k\sqrt{1+\gamma\cos 2\theta}} + \frac{1}{1-i\Omega\tau-2i\lambda k\sqrt{1+\gamma\cos 2\theta}}\right)$$
$$= \frac{1}{2\pi}\int_0^{2\pi}d\theta\frac{1-i\Omega\tau}{(1-i\Omega\tau)^2+(\Omega_{so}\tau)^2(1+\gamma\cos 2\theta)} = \frac{1-i\Omega\tau}{2\pi[(1-i\Omega\tau)^2+(\Omega_{so}\tau)^2]}\int_0^{\pi}dx\frac{2}{[1+a\cos(x)]}, \tag{A8}$$

where $x = 2\theta$ and $a = \gamma(\Omega_{so}\tau)^2/[(1-i\Omega\tau)^2+(\Omega_{so}\tau)^2]$. The indefinite integral $\int dx \frac{1}{1+a\cos(x)} = \frac{2\,\text{arc tanh}\left[\frac{(-1+a)\tan[\frac{x}{2}]}{\sqrt{-1+a^2}}\right]}{\sqrt{-1+a^2}}$. Therefore, we have

$$I^{zz} = \frac{1-i\Omega\tau}{2\pi[(1-i\Omega\tau)^2+(\Omega_{so}\tau)^2]}\int_0^{\pi}dx\frac{2}{[1+a\cos(x)]}$$
$$= \frac{1-i\Omega\tau}{2\pi[(1-i\Omega\tau)^2+(\Omega_{so}\tau)^2]}2\left(\frac{2\,\text{arc tanh}\left[\frac{(-1+a)\tan[\frac{\pi}{2}]}{\sqrt{-1+a^2}}\right]}{\sqrt{-1+a^2}} - \frac{2\,\text{arc tanh}\left[\frac{(-1+a)\tan[\frac{0}{2}]}{\sqrt{-1+a^2}}\right]}{\sqrt{-1+a^2}}\right)$$
$$= \frac{1-i\Omega\tau}{2\pi[(1-i\Omega\tau)^2+(\Omega_{so}\tau)^2]}\frac{2\pi i}{\sqrt{-1+a^2}} = \frac{1-i\Omega\tau}{\sqrt{[(1-i\Omega\tau)^2+(\Omega_{so}\tau)^2]^2-\gamma^2(\Omega_{so}\tau)^4}}. \tag{A9}$$

When the Rashba SOI is zero, the strength of the SOI's takes the form

$$h_{so} = \beta_1 k_f\sqrt{1+\left(2\frac{\beta_3^2}{\beta_1^2}-2\frac{\beta_3}{\beta_1}\right)(1+\cos 4\theta)}. \tag{A10}$$

To obtain the spin diffusive matrix element I^{zz}, it is easy to prove that we only need to replace the term $\lambda k\sqrt{1+\gamma\cos 2\theta}$ in Eq. (A9) with h_{so} in Eq. (A10). Therefore, we have

$$I^{zz} = \frac{1}{4\pi}\int\left(\frac{2(1-i\Omega\tau)}{(1-i\Omega\tau)^2+(2h_{so}\tau)^2}\right)d\theta = \frac{1-i\Omega\tau}{\sqrt{(1-i\Omega\tau)^2+\Omega_{so}^2\tau^2}\sqrt{(1-i\Omega\tau)^2+\Omega_{so}^2\tau^2\left[1+2\left(\frac{\beta_3}{\beta_1}\right)^2-2\frac{\beta_3}{\beta_1}\right]}}. \tag{A11}$$

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