Bose-Einstein condensation of rotons

L. A. Melnikovsky*
P.L. Kapitza Institute for Physical Problems, Russian Academy of Sciences, 119334 Moscow, Russia
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Bose-Einstein condensation of rotons in helium was considered long ago. It was shown that the relative velocity of the normal motion in this state must be equal to the Landau critical velocity. We argue that the condensation can be attained at a smaller velocity if the temperature is low enough.

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I. INTRODUCTION

As a Bose gas is cooled down, the Bose-Einstein condensation (BEC) emerges to avoid a conflict between the statistics and the particle conservation requirement. Behavior of quasiparticles, unlike behavior of real particles, is not dominated by such conflict: their number is not an independent variable, it adjusts itself to maximize entropy in equilibrium. Conventional Bose-Einstein condensation of quasiparticles is therefore thought to be impossible; an ingenious mechanism[1] of the condensation for rotons via gap cancellation by a critical ($v_n - v_s \equiv v = v_L \equiv \Delta_0/P_0$, where Δ_0 is the roton energy gap and P_0 is its momentum) superfluid counterflow is required.[2]

Actually, even real particles hardly ever conserve exactly. Consider atoms of a cold gas in a trap: they can combine into molecules or even evaporate from the trap altogether. Nor is particle conservation present in exact relativistic theory. The BEC is still possible if it forms much faster than the particle population decays. It is therefore necessary to compare all relevant thermalization time scales. It turns out that any finite counterflow and low enough temperature $T \ll vP_0$ are the conditions favorable for the roton BEC creation.

Suppose these conditions are satisfied, and the roton number relaxation is much slower than that of their energy and momentum. The roton distribution is then characterized by the temperature, velocity, and finite chemical potential μ:

$$N_{\mathbf{P}} = \left(\exp\frac{\mathcal{E} - \mathbf{Pv} - \mu}{T} - 1\right)^{-1}, \tag{1}$$

where $\mathbf{P}$ is the roton momentum and $\mathcal{E} = \Delta_0 + (P - P_0)^2/(2\mu_0)$ is its energy. Let the z axis run along the $\mathbf{v}$ direction. The roton distribution argument can be expanded in powers of small deviation from its most probable value,

$$\begin{aligned}\mathcal{E} - \mathbf{vP} - \mu &\approx \Delta_0 - \mu - P_0 v - \mu_0 v^2 + \frac{f_z^2}{2\mu_0} + \frac{\left(f_x^2 + f_y^2\right) v}{2P_0} \\ &\equiv \Delta + \frac{f_z^2}{2\mu_0} + \frac{\left(f_x^2 + f_y^2\right) v}{2P_0} = \Delta + \frac{g^2}{2\mu_0},\end{aligned} \tag{2}$$

where $\mathbf{f} = \mathbf{P} - \mathbf{v}(\mu_0 + P_0/v)$,

$$\begin{aligned} g_{x,y} &= f_{x,y}\sqrt{\mu_0 v/P_0}, \\ g_z &= f_z. \end{aligned} \tag{3}$$

This expansion is applicable if

$$T \ll \mu_0 v^2. \tag{4}$$

Whether the roton BEC state is a Bogolyubov-like gas or a degenerate Bose liquid depends on the concentration. Critical concentration (particles per unit volume) required for Bose-Einstein condensation is

$$N_c = \zeta\left(\frac{3}{2}\right)\left(\frac{T}{2\pi}\right)^{3/2}\frac{\mu_0^{1/2}P_0}{v\hbar^3}. \tag{5}$$

The system is almost ideal (see Ref. 4) if

$$N\frac{V_0^3\mu_0 P_0^2}{2^6\pi^3\hbar^6 v^2} \ll 1, \tag{6}$$

where[5] $V_0 \sim 10^{-38}$ erg cm^3 is the interaction strength. Combining Eqs. (5) and (6) we get

$$T \ll \frac{2^5\pi^3\hbar^6}{\zeta(3/2)^{2/3}V_0^2\mu_0 P_0^2}v^2 \sim 10^3\mu_0 v^2.$$

This inequality is satisfied as a consequence of Eq. (4), implying that the condensation considered is a transition between gaseous phases.

II. RELAXATION

A. Roton number decay

Suppose initial roton distribution is characterized by some positive chemical potential μ. This means that the roton number is greater than that in complete equilibrium. The most important process at low temperature $T \ll vP_0$ for the chemical potential relaxation is the transformation of two rotons into one roton and one phonon. Little is known about the transformation probability in such collisions. It seems reasonable to assume that the transformation cross section (providing the process is allowed at all by the conservation laws) can be bounded from above by complete scattering cross section[5] known from the experimental viscosity data.

Momentum conservation for this transformation imposes severe restriction on the angle ϕ between momenta of the incident rotons (see Fig. 1). Namely, this restriction is reduced to $\phi \gtrsim 2\pi/3$ if the inequality $\Delta \ll P_0 c$ is taken into account. Here c is the speed of sound. To simplify all assessments below we take $T \ll \Delta$, i.e., assume Boltzmann statistics for the rotons:

$$N_{\mathbf{P}} = \exp\frac{\mathbf{Pv} + \mu - \mathcal{E}}{T}. \tag{7}$$

This is certainly incorrect for the BEC state itself but must provide reasonable relative order of magnitude for different relaxation rates.

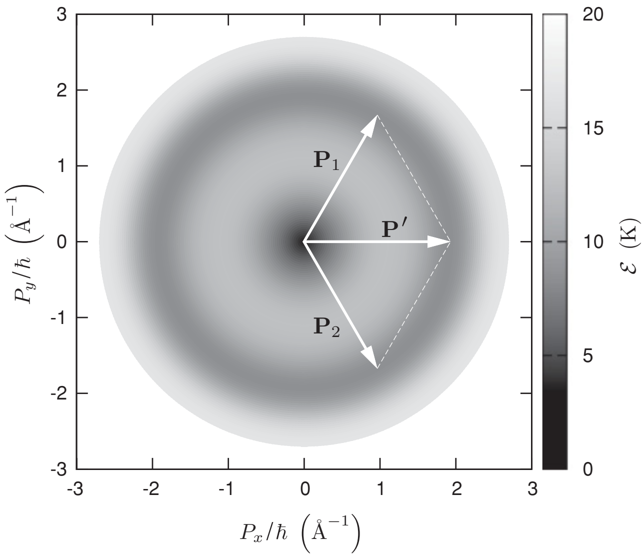

FIG. 1. Excitation spectrum of superfluid helium; dark-gray ring (spherical layer in real three dimensions) corresponds to the rotons. Transformation of two rotons with momenta $\mathbf{P}_1$ and $\mathbf{P}_2$ into one with momentum $\mathbf{P}'$ is impossible if the angle between incident rotons is less than $2\pi/3$. Extra energy can be carried away by a phonon (not shown) whose momentum is negligibly small.

As an estimate, not more than about $\exp[-2(1-\cos\pi/3)vP_0/T]$ fraction of all collisions end up with the transformation. For the chemical potential relaxation rate this gives (see Ref. 5)

$$\tau_\mu^{-1} \lesssim \frac{4N|V_0|^2 P_0\mu_0}{\hbar^4} \exp\frac{-vP_0}{T}, \tag{8}$$

where N is the total roton concentration. The relaxation rate here is defined according to

$$\dot{\mu} + \tau_\mu^{-1}\mu = 0.$$

B. Phonon-roton velocity relaxation

To find the upper boundary for the phonon-roton relaxation time it is sufficient to consider the two-particle scattering of rotons by phonons. Conservation laws for this process are (primes denote the final state)

$$\mathbf{p} + \mathbf{P} = \mathbf{p}' + \mathbf{P}', \quad \varepsilon + \mathcal{E} = \varepsilon' + \mathcal{E}', \tag{9}$$

where $\mathbf{p}$ and $\varepsilon = cp$ are the phonon momentum and energy. Landau and Khalatnikov have pointed out[5] that since $P \gg p$, it follows that the scattering of rotons by phonons is analogous to the scattering of heavy particles by light ones and the conservation laws (9) simply amount to the momenta parallelism of the incident and scattered rotons $\mathbf{P} \parallel \mathbf{P}'$. Similarly, the momenta magnitudes of the incident and scattered phonons are almost equal, $p \approx p'$. Really, substituting the first equation in Eq. (9) into the second one we get

$$p - p' = \frac{(\mathbf{m},\mathbf{p}-\mathbf{p}')}{\mu_0 c}(P - P_0) + \frac{(\mathbf{m},\mathbf{p}-\mathbf{p}')^2}{2\mu_0 c} \ll p, \tag{10}$$

where round brackets denote scalar product and $\mathbf{m}$ is the unit vector directed along $\mathbf{P}$ and $\mathbf{P}'$.

Let the roton and phonon subsystems be separately in equilibrium. The roton and the phonon "bath" velocities will be $\mathbf{v}$ and $\mathbf{v} + \delta\mathbf{v}$, respectively. Velocity relaxation has two distinct time scales $\tau_{ph,v\parallel}$ and $\tau_{ph,v\perp}$; they correspond to $\delta\mathbf{v} \parallel \mathbf{v}$ and to $\delta\mathbf{v} \perp \mathbf{v}$. In linear approximation ($\delta v \ll v$) the relaxation is described by the equations

$$\dot{j}_{r\parallel} = \delta v_\parallel \rho_{r\parallel} \tau_{ph,v\parallel}^{-1},$$
$$\dot{j}_{r\perp} = \delta v_\perp \rho_{r\perp} \tau_{ph,v\perp}^{-1},$$

where $\mathbf{j}_r$ is the roton momentum density and ρ_r is the roton contribution to the differential normal density $\rho_n = \partial j/\partial v$. To calculate these contributions we note that in the low-temperature limit all rotons have equal momentum $P_0 + \mu_0 v$ directed along the velocity $\mathbf{v}$. This gives

$$\mathbf{j}_r = \left(\frac{P_0}{v} + \mu_0\right) N\mathbf{v}$$

and

$$\rho_{r\parallel} = \mu_0 N, \qquad \rho_{r\perp} = P_0 N/v. \tag{11}$$

Each elementary scattering changes the roton subsystem momentum by $\mathbf{P}' - \mathbf{P}$. The scattering rate is the Bose factor $(n'+1)nN_{\mathbf{P}}$, where

$$n = \left(\exp\frac{\varepsilon - \mathbf{p}(\mathbf{v}+\delta\mathbf{v})}{T} - 1\right)^{-1},$$

times the scattering probability $dw = c\,d\sigma$, where $d\sigma$ is differential cross section of this process (rotons are much slower than sound). Combining this together we obtain

$$\dot{\mathbf{j}}_r = \int (\mathbf{P}' - \mathbf{P})\,(n'+1)nN_{\mathbf{P}}\, c\,d\sigma\, \frac{d\mathbf{p}}{(2\pi\hbar)^3}\frac{d\mathbf{P}}{(2\pi\hbar)^3}. \tag{12}$$

According to the fundamental work,[5] the phonon-roton cross section is

$$d\sigma = \left(\frac{P_0 p^2}{4\pi\hbar^2\rho c}\right)^2 \left\{(\mathbf{n}+\mathbf{n}',\mathbf{m})(\mathbf{n},\mathbf{n}') + \frac{P_0}{\mu_0 c}(\mathbf{n},\mathbf{m})^2(\mathbf{n}',\mathbf{m})^2 + A\right\}^2 d\mathbf{n}', \tag{13}$$

where $\mathbf{n}$ and $\mathbf{n}'$ are the unit vectors directed along $\mathbf{p}$ and $\mathbf{p}'$, respectively. The parameter A is given[6] by

$$A = \frac{\rho^2}{P_0 c}\left[\frac{\partial^2\Delta_0}{\partial\rho^2} + \frac{1}{\mu_0}\left(\frac{\partial P_0}{\partial\rho}\right)^2\right].$$

Expression (13) remarkably involves only excitation spectrum parameters and does not rely on random assumptions about the phonon-roton interaction. It is derived by treating the roton as a small particle in a slowly varying hydrodynamic phonon field and is independent on the unknown internal structure of the roton.

At low temperature most rotons have momentum parallel to the velocity and the vector $\mathbf{m} \parallel \mathbf{v}$ can be regarded as a constant. To transform Eq. (12) we employ the usual relation between distribution functions:

$$(n'+1)nN_{\mathbf{P}} - (n+1)n'N_{\mathbf{P}'} \approx (n+1)n'N_{\mathbf{P}'}\frac{(\mathbf{P}'-\mathbf{P},\delta\mathbf{v})}{T},$$

and the principle of detailed balance:

$$\dot{\mathbf{j}}_r = \frac{1}{2T}\int(\mathbf{P}'-\mathbf{P})(\mathbf{P}'-\mathbf{P},\delta\mathbf{v})(n+1)n'N_{\mathbf{P}'}dw\,\frac{d\mathbf{p}}{(2\pi\hbar)^3}\frac{d\mathbf{P}}{(2\pi\hbar)^3} = \frac{c}{2T}\int p^2(\mathbf{n}-\mathbf{n}')(\mathbf{n}-\mathbf{n}',\delta\mathbf{v})(n+1)n'N_{\mathbf{P}'}d\sigma\,\frac{d\mathbf{p}}{(2\pi\hbar)^3}\frac{d\mathbf{P}}{(2\pi\hbar)^3}$$
$$= \frac{\pi^3P_0^2T^8}{60c^{10}\hbar^7\rho^2}\int N_{\mathbf{P}}\frac{d\mathbf{P}}{(2\pi\hbar)^3}\int(\mathbf{n}-\mathbf{n}')(\mathbf{n}-\mathbf{n}',\delta\mathbf{v})\left\{(\mathbf{n}+\mathbf{n}',\mathbf{m})(\mathbf{n},\mathbf{n}')+\frac{P_0}{\mu_0c}(\mathbf{n},\mathbf{m})^2(\mathbf{n}',\mathbf{m})^2+A\right\}^2 d\mathbf{n}'d\mathbf{n}. \tag{14}$$

For the relaxation rate this gives (lengthy but straightforward transformations are omitted)

$$\tau_{ph,v\parallel}^{-1} = \frac{8\pi^5P_0^2T^8}{15c^{10}\hbar^7\rho^2\mu_0}\left(\frac{4}{225}+\frac{2AP_0}{15\mu_0c}+\frac{P_0^2}{35\mu_0^2c^2}+\frac{A^2}{3}\right)\sim 500\frac{T^8P_0^2}{c^{10}\hbar^7\rho^2\mu_0}, \tag{15}$$

$$\tau_{ph,v\perp}^{-1} = \frac{8\pi^5vP_0T^8}{15c^{10}\hbar^7\rho^2}\left(\frac{8}{225}+\frac{2AP_0}{45\mu_0c}+\frac{P_0^2}{175\mu_0^2c^2}+\frac{A^2}{3}\right)\sim 300\frac{T^8P_0v}{c^{10}\hbar^7\rho^2}. \tag{16}$$

C. Phonon-roton temperature relaxation

Let us now find the temperature equilibration rate between the roton and the phonon subsystems. The phonon gas temperature is $T+\delta T$ and the phonon distribution is

$$n = \left(\exp\frac{\varepsilon-\mathbf{pv}}{T+\delta T}-1\right)^{-1}.$$

This function satisfies the equality

$$(n'+1)nN_{\mathbf{P}}-(n+1)n'N_{\mathbf{P}'}\approx(n+1)n'N_{\mathbf{P}'}\frac{\mathcal{E}'-\mathcal{E}}{T^2}\delta T.$$

From Eq. (4) it follows that the second term in Eq. (10) is much smaller than the first one,

$$\mathcal{E}'-\mathcal{E} = \frac{P-P_0}{\mu_0}(\mathbf{m},\mathbf{P}'-\mathbf{P}) = v(\mathbf{m},\mathbf{P}'-\mathbf{P}).$$

The energy inflow to the roton subsystem is

$$\dot{E}_r = \int(\mathcal{E}'-\mathcal{E})(n'+1)nN_{\mathbf{P}}\,dw\,\frac{d\mathbf{p}}{(2\pi\hbar)^3}\frac{d\mathbf{P}}{(2\pi\hbar)^3}$$
$$= \frac{v^2\delta T}{2T^2}\int(\mathbf{m},\mathbf{P}'-\mathbf{P})^2(n+1)n'N_{\mathbf{P}'}dw\,\frac{d\mathbf{p}}{(2\pi\hbar)^3}\frac{d\mathbf{P}}{(2\pi\hbar)^3}. \tag{17}$$

The phonon-roton temperature relaxation rate defined by

$$\dot{E}_r = C_r\tau_{ph,T}^{-1}\delta T,$$

where $C_r = 3N/2$ is the roton contribution to the specific heat per unit volume, can be immediately extracted using the obvious similarity between Eqs. (17) and (14):

$$\tau_{ph,T}^{-1} = \frac{v^2\rho_{r\parallel}}{C_rT}\tau_{ph,v\parallel}^{-1}$$
$$= \frac{16\pi^5P_0^2T^7v^2}{45c^{10}\hbar^7\rho^2}\left(\frac{4}{225}+\frac{2AP_0}{15\mu_0c}+\frac{P_0^2}{35\mu_0^2c^2}+\frac{A^2}{3}\right)$$
$$\sim 300\frac{T^7P_0^2v^2}{c^{10}\hbar^7\rho^2}. \tag{18}$$

D. Roton-roton velocity relaxation

Equilibrium within the roton subsystem is reached via the roton-roton collisions. Like a two-body problem in classical mechanics, these collisions are efficiently described [if Eq. (2) holds] in the center of inertia frame. Namely, suppose reduced momenta, defined according to Eq. (3), of the scattering rotons are $\mathbf{g}_1$ and $\mathbf{g}_2$. After a transformation

$$\mathbf{G} = \mathbf{g}_1+\mathbf{g}_2,$$
$$\mathbf{g} = (\mathbf{g}_1-\mathbf{g}_2)/2,$$

the net energy of two rotons is given by

$$\mathcal{E}_1+\mathcal{E}_2-\mathbf{vG}-2\mu = 2\Delta+\frac{G^2}{4\mu_0}+\frac{g^2}{\mu_0} \tag{19}$$

and the conservation laws are simplified to

$$\mathbf{G}' = \mathbf{G}$$
$$g' = g.$$

Accurate definition of the roton-roton equilibration time constants is hardly possible (cf. the discussion on the establishment of equilibrium of a phonon gas in Ref. 6). As an estimate we employ the relations similar to those for the phonon-roton relaxation:

$$\rho_{r\parallel}\tau_{r,v\parallel}^{-1} = \frac{1}{2T}\int(\mathbf{P}_1'-\mathbf{P}_1,\mathbf{m})^2N_{\mathbf{P}_1}N_{\mathbf{P}_2}dw\,\frac{d\mathbf{P}_1}{(2\pi\hbar)^3}\frac{d\mathbf{P}_2}{(2\pi\hbar)^3},$$
$$\rho_{r\perp}\tau_{r,v\perp}^{-1} = \frac{1}{2T}\int[\mathbf{P}_1'-\mathbf{P}_1,\mathbf{m}]^2N_{\mathbf{P}_1}N_{\mathbf{P}_2}dw\,\frac{d\mathbf{P}_1}{(2\pi\hbar)^3}\frac{d\mathbf{P}_2}{(2\pi\hbar)^3}.$$

Here the scalar $(\mathbf{P}_1'-\mathbf{P}_1,\mathbf{m})$ and vector $[\mathbf{P}_1'-\mathbf{P}_1,\mathbf{m}]$ products are used to decompose the momentum difference into parallel and perpendicular projections.

Unfortunately, no roton-roton counterpart of Eq. (13) is available: interaction between rotons does not yet allow for exact theoretical treatment. It eventually turns out (see below) that details of this interaction are not important and we adopt the simplest model[5] for the roton-roton scattering probability,

$$w = \frac{8\pi|V_0|^2}{\hbar}\delta(\mathcal{E}_1+\mathcal{E}_2-\mathcal{E}_{1'}-\mathcal{E}_{2'})\frac{d\mathbf{P}_{1'}}{(2\pi\hbar)^3}.$$

For the relaxation time this gives

$$\rho_{r\parallel}\tau_{r,v\parallel}^{-1} = \frac{4\pi|V_0|^2 e^{-2\Delta/T}}{\hbar T}\int (g_{1z'}-g_{1z})^2 \exp\left(-\frac{g_1^2+g_2^2}{2\mu_0 T}\right)\delta(\mathcal{E}_1+\mathcal{E}_2-\mathcal{E}_{1'}-\mathcal{E}_{2'})\frac{d\mathbf{P}_{1'}}{(2\pi\hbar)^3}\frac{d\mathbf{P}_1}{(2\pi\hbar)^3}\frac{d\mathbf{P}_2}{(2\pi\hbar)^3}$$
$$= \frac{4\pi|V_0|^2 P_0^3 e^{-2\Delta/T}}{\hbar T\mu_0^3 v^3}\int (g_{1z'}-g_{1z})^2 \exp\left(-\frac{G^2}{4\mu_0 T}-\frac{g^2}{\mu_0 T}\right)\delta\left(\frac{g^2-g'^2}{\mu_0}\right)\frac{d\mathbf{g}'}{(2\pi\hbar)^3}\frac{d\mathbf{g}}{(2\pi\hbar)^3}\frac{d\mathbf{G}}{(2\pi\hbar)^3}$$
$$= \frac{8N^2|V_0|^2 P_0\mu_0^{3/2}T^{1/2}}{3\pi^{3/2}\hbar^4 v}$$

and

$$\rho_{r\perp}\tau_{r,v\perp}^{-1} = \frac{8N^2|V_0|^2 P_0^2\mu_0^{1/2}T^{1/2}}{3\pi^{3/2}\hbar^4 v^2}.$$

Here we substituted the roton concentration according to

$$N = e^{-\Delta/T}\int e^{-g_1^2/(2T\mu_0)}\frac{d\mathbf{P}_1}{(2\pi\hbar)^3} = \frac{2^{3/2}\pi^{3/2}T^{3/2}\mu_0^{1/2}P_0}{v(2\pi\hbar)^3}e^{-\Delta/T}.$$

Eventually, using Eq. (11), we see that the two rates are equal,

$$\tau_{r,v\parallel}^{-1} = \tau_{r,v\perp}^{-1} = \frac{8N|V_0|^2 P_0\mu_0^{1/2}T^{1/2}}{3\pi^{3/2}\hbar^4 v}. \quad (20)$$

E. Roton-roton temperature relaxation

Employing the same approach as in Eq. (17) we express the roton-roton temperature relaxation rate through the parallel velocity relaxation rate (19),

$$\tau_{r,T}^{-1} = \frac{v^2\rho_{r\parallel}}{C_r T}\tau_{r,v\parallel}^{-1} = \frac{16N|V_0|^2 P_0\mu_0^{3/2}v}{9\pi^{3/2}\hbar^4 T^{1/2}}. \quad (21)$$

III. DISCUSSION

As we mentioned above, possibility of the Bose-Einstein condensation of rotons depends on the relative magnitude of the different relaxation rates. We begin with the notion that within the roton gas the temperature relaxation (21) is much faster than the velocity relaxation (20),

$$\frac{\tau_{r,T}^{-1}}{\tau_{r,v}^{-1}} = \frac{2\mu_0 v^2}{3T} \gg 1.$$

The latter in turn is always faster than the chemical potential relaxation (8),

$$\frac{\tau_{r,v}^{-1}}{\tau_{\mu}^{-1}} = \frac{2}{3\pi^{3/2}}\sqrt{\frac{T}{\mu_0 v^2}}\exp\frac{2vP_0}{T} \gg 1.$$

This completes the proof that a roton subsystem with an initially narrow momentum distribution around some under-critical momentum P such that $0 < P - P_0 < \mu_0\Delta_0/P_0$ must pass through a BEC state.

Another process is equilibration between the roton and phonon subsystems. Consider a roton cloud propagating with the velocity $\mathbf{v}$ and interacting with the phonon environment. The rotons cool down if the phonon temperature is lower than the roton one. Cooling rate is determined by the expressions obtained in Secs. II B and II C. Whether this "phonon cooling" can be experimentally used to condense a cloud of rotons with wide initial distribution depends not only on the relation between the rates of the phonon-roton relaxation (15), (16), (18), and of the roton number decay (8). The latter has exponential dependence on the temperature and can in principle be made arbitrary slow relatively, but at low temperature the phonon-roton relaxation is very slow itself and may take too long, therefore demanding a very large experimental cell. To overcome this difficulty one could try to condense rotons at rest in the laboratory frame of reference while the superfluid passes through a capillary. Andreev reflection of rotons[7] at low temperature will protect the distribution width in the roton-wall collisions.

Note that exponentially slow roton number decay (in contrast with specific power-law temperature dependency of other equilibration rates) is a general result and does not depend on exact roton-roton interaction.

Experimental observation of the roton BEC should be possible by a number of techniques:

(i) The coherent roton quantum state has finite momentum. Upon BEC formation, bulk helium acquires a spatial inhomogeneity,[8] a one-dimensional density wave. The wavelength is the roton wavelength and the modulation direction is the velocity $\mathbf{v}$ direction. The periodicity should manifest itself as a Bragg peak in the x-ray-scattering experiments. Actually this roton BEC state is a supersolid as it simultaneously has superfluid and crystalline order. Note that one-dimensional crystalline order is not destroyed by Landau-Peierls fluctuations[9,10] thanks to the true three-dimensional superfluid order.

(ii) Another option to probe BEC is to explore excitations of the condensate. The roton second sound is well studied in normal roton systems[11] and may be used for the condensate detection.

(iii) The roton distribution can be measured directly by the quantum evaporation.[12] The δ-shaped peak in the distribution function would become an explicit confirmation of the BEC formation.

Finally, let us remark that the stability analysis performed in Ref. 13 is irrelevant for the metastable BEC considered in present paper, because the roton number is fixed.

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*leva@kapitza.ras.ru

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