Electrical current driven by a coherent spin wave in a bulk ferromagnetic semiconductor

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We theoretically investigate the effect of electrical current generation by a coherent spin wave propagated in a bulk ferromagnetic semiconductor. This is one of the effects in conductive magnetic materials that are based on spin-transfer torque concept first proposed by J. C. Slonszewski [J. Magn. Magn. Mater. **159**, L1 (1996)] and L. Berger [Phys. Rev. B **54**, 9353 (1996)]. Due to the relatively simple description of interaction between conduction electrons and a coherent spin wave (in the framework of s-d exchange), the spin-transfer torque effect is considered here *ab initio*. A systematic analysis of current generation effect is done by quantum kinetics methods; relaxation processes are considered within the τ approximation. We derive an analytical expression for the stationary current density and make estimations for a ferromagnetic semiconductor of the $CdCr_2Se_4$ type.

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Conductive magnetic materials with a strong s-d exchange interaction have the mutual influence of electronic and magnetic properties, resulting in rather "significant" effects. Thus the effects, which we call direct effects (DEs), have been a focus of attention from the midnineties until the present (Refs. 1–7 and references therein). DEs consist in manipulation of conductive magnetic material magnetization by electrical current. Also of great interest are inverse effects (IEs), consisting in current (voltage) generation in conductive magnetic materials as a result of temporal inhomogeneous magnetization variation.[8–16] The interpretation of both effects is based on the "spin-transfer torque" (STT) concept proposed by Slonszewski[3] and Berger.[1] The corresponding theoretical models have various approximations and constraints due to certain "scenarios"of experiments where DEs, as well as IEs, are observed. The main problem here is to find an exact solution of the time-dependent Schrodinger equation describing electron behavior in an s-d exchange field.

However, in some particular cases, the exact solution of the corresponding Schrodinger equation can be obtained and STT can be considered *ab initio*. A solution to the time-dependent Schrodinger equation describing an electron in an s-d exchange field of a coherent spin wave (CSW) propagated in a bulk magnetic material can be easily found. The present work, taking advantage of the possibility of finding such an exact solution, is dedicated to one of the IE variants corresponding to this case: this is the current generation effect in a bulk ferromagnetic material by a CSW. Let us note here that the work (Ref. 6; see also Ref. 2) studies DEs as an STT effect specifically for bulk systems. This effect consists in planar spin spiral (SS) rotation around its axis driven by electrical current. The analysis carried out in Ref. 6 also applies to conical SS. Rotation of conical SS around its axis is described mathematically similarly to the description of a CSW propagated in a spontaneous magnetization direction. Therefore, the effect studied in the present work is considered the inverse of the effect in question in Ref. 6.

In the present work, as a conductive magnetic material we consider a ferromagnetic semiconductor (FMS). We describe an exchange interaction of conduction electrons with lattice magnetization in the framework of the Vonsowsky s-d exchange model,[17,18] which is based on the Schubin-Vonsowsky polar model.[19–22] For definiteness, we assume that conduction electrons in the FMS are degenerate and completely spin-polarized. We consider time-dependent lattice magnetization due to CSW propagation as a classical quantity (given function of co-ordinate and time) and describe it in the continuous medium approximation by introducing a unit vector $\vec{S}(\vec{r},t)$ aligned with the local magnetization direction. Let us remark here that the classical picture of a CSW corresponds to large occupation numbers of magnon states, that is, a sufficiently high amplitude of CSW.[23] As we consider CSW propagation along the positive direction of the oz axis, the Cartesian components of $S_{x,y,z}(\vec{r},t)$ are

$$
\begin{aligned}
S_x(\vec{r},t) &= S_\perp \frac{1}{2}[\exp i(qz-\omega t)+\text{c.c.}],\\
S_y(\vec{r},t) &= S_\perp \frac{i}{2}[\exp i(qz-\omega t)-\text{c.c.}],\\
S_z(\vec{r},t) &= S_\parallel,
\end{aligned}
\tag{1}
$$

where $S_\perp^2 + S_\parallel^2 = 1$, $\omega > 0$ is the CSW frequency, $\vec{q} = q\vec{\kappa}$ is the CSW wave vector, and $\vec{\kappa}$ is a unitary vector of the Cartesian coordinate system corresponding to the oz axis. In our consideration of the dispersion relation for CSW we keep to the approach stated in Ref. 23. Thus we are restricted to consideration of the exchange interactions between the lattice spins only and neglect the relativistic interactions as relatively weak. In this case and under the condition that the CSW length is sufficiently large, $aq \ll 1$ (a is a magnetic lattice constant), the quadratic dispersion relation is true:

$$
\omega \simeq \left(\frac{2\mu_B k_B T_c}{\hbar a M_0}\right) q^2, \tag{2}
$$

where T_c is the Curie temperature, M_0 is the saturation magnetization, and μ_B is the Bohr magneton.

The one-electron s-d Hamiltonian for conduction electrons is written as follows [with the use of Eq. (1)],

$$
\begin{aligned}
\hat{H}(t) = -\frac{\hbar^2}{2m_e}\triangle + \frac{S_\perp A}{2}\{(\hat{\sigma}_x - i\hat{\sigma}_y)\exp[-i(qz-wt)]\\
+(\hat{\sigma}_x + i\hat{\sigma}_y)\exp[i(qz-wt)]\} + S_\parallel A\hat{\sigma}_z,
\end{aligned}
\tag{3}
$$

where $2A > 0$ is the s-d exchange splitting value of the conduction band, $\hat{\sigma}_{x,y,z}$ are Pauli matrices, and m_e is the effective mass of the conduction electron. It is easily seen that

$$\left[\left(\hat{p}_z - \tfrac{1}{2}q\hbar\hat{\sigma}_z\right), \hat{H}(t)\right] = 0, \tag{4}$$

where $\hat{p}_z$ is the z component of the momentum operator, and square brackets denote the commutator. Therefore, the operator $\hat{\mathcal{P}} \equiv \hat{p}_z - \frac{1}{2}q\hbar\hat{\sigma}_z$ represents a conserved quantity.[24] It is obvious that the existence of a conserved quantity corresponding to operator $\hat{\mathcal{P}}$ means that there is an effective interaction between the orbital motion of the electrons and their spin.

Without a spin wave, that is, when $S_\perp = 0$, $S_\parallel = 1$, the Hamiltonian takes the form

$$\hat{H}(t) = -\frac{\hbar^2}{2m_e}\triangle + A\hat{\sigma}_z. \tag{5}$$

For its eigenfunctions and corresponding eigenvalues we get

$$\begin{aligned}\psi_{\vec{k}\downarrow} &= V^{-1/2}\begin{pmatrix}0\\-1\\a\end{pmatrix}\exp(i\vec{k}\vec{r}), \quad \varepsilon_{\vec{k}\downarrow}(t) = \varepsilon_{\vec{k}} - A,\\ \psi_{\vec{k}\uparrow} &= V^{-1/2}\begin{pmatrix}1\\0\end{pmatrix}\exp(i\vec{k}\vec{r}), \quad \varepsilon_{\vec{k}\uparrow}(t) = \varepsilon_{\vec{k}} + A,\end{aligned} \tag{6}$$

where $\vec{k} = \vec{i}k_x + \vec{j}k_y + \vec{\kappa}k_z$ and $\vec{r} = \vec{i}x + \vec{j}y + \vec{\kappa}z$; $\vec{i}$, $\vec{j}$, and $\vec{\kappa}$ are unitary vectors of the Cartesian coordinate system; the arrows ↑ and ↓ correspond to positive and negative projection of electron spin along the spontaneous magnetization direction of the FMS, $\varepsilon_{\vec{k}} = \hbar^2k^2/(2m_e)$; and V is the normalizing volume. We use $\psi_{\vec{k}\downarrow}$, $\psi_{\vec{k}\uparrow}$ as base functions. In this basis for matrix elements of Hamiltonian (3), we have

$$\begin{aligned}H(t)_{\vec{k}\uparrow;\vec{k}\uparrow} &= \varepsilon_{\vec{k}} + (S_\parallel A); H(t)_{\vec{k}\downarrow;\vec{k}\downarrow} = \varepsilon_{\vec{k}} - (S_\parallel A),\\ H(t)_{\vec{k}_1\uparrow;\vec{k}_2\downarrow} &= -(S_\perp A)e^{-i\omega t}\delta_{\vec{k}_1;\vec{k}_2+\vec{q}},\\ H(t)_{\vec{k}_1\downarrow;\vec{k}_2\uparrow} &= -(S_\perp A)e^{i\omega t}\delta_{\vec{k}_1;\vec{k}_2-\vec{q}},\end{aligned} \tag{7}$$

and for matrix elements of the velocity operator $\hat{\upsilon}_z = -i(\hbar/m_e)(\partial/\partial z)$,

$$(\upsilon_z)_{\vec{k}\uparrow;\vec{k}\uparrow} = (\upsilon_z)_{\vec{k}\downarrow;\vec{k}\downarrow} = \frac{\hbar}{m_e}k_z. \tag{8}$$

For consideration of the many-electron problem we introduce Heisenberg operators $a^+_{\vec{k}\uparrow,\downarrow}(t)$ and $a_{\vec{k}\uparrow,\downarrow}(t)$, corresponding to the states defined by (6). Then the Hamiltonian of the many-electron system takes the form

$$\begin{aligned}\hat{\mathcal{H}}(t) = \sum_{\vec{k}_1,\vec{k}_2}\Big\{&\left[\varepsilon_{\vec{k}_1} + (S_\parallel A)\right]\hat{n}_{\vec{k}_1\uparrow}(t) + \left[\varepsilon_{\vec{k}_1} - (S_\parallel A)\right]\hat{n}_{\vec{k}_1\downarrow}(t)\\ &- (S_\perp A)e^{-i\omega t}\hat{S}^+_{\vec{k}_1\uparrow,\vec{k}_2\downarrow}(t) - (S_\perp A)e^{i\omega t}\hat{S}_{\vec{k}_2\downarrow,\vec{k}_1\uparrow}(t)\Big\}.\end{aligned} \tag{9}$$

The operators of current density and electron magnetization are determined by the formulas

$$\hat{J}_z(t) = -V^{-1}\sum_{\vec{k}}\frac{\hbar|e|}{m_e}k_z\{\hat{n}_{\vec{k}\uparrow}(t) + \hat{n}_{\vec{k}\downarrow}(t)\}, \tag{10}$$

$$\hat{M}_{z,el}(t) = -V^{-1}\sum_{\vec{k}}\mu_B\{\hat{n}_{\vec{k}\uparrow}(t) - \hat{n}_{\vec{k}\downarrow}(t)\}, \tag{11}$$

where

$$\begin{aligned}\hat{n}_{\vec{k}\uparrow,\downarrow}(t) &\equiv \hat{a}^+_{\vec{k}\uparrow,\downarrow}(t)\hat{a}_{\vec{k}\uparrow,\downarrow}(t),\\ \hat{S}_{\vec{k}_2\downarrow;\vec{k}_1\uparrow}(t) &\equiv \hat{a}^+_{\vec{k}_2\downarrow}(t)\hat{a}_{\vec{k}_1\uparrow}(t)\delta_{\vec{k}_1-\vec{q};\vec{k}_2},\\ \hat{S}^+_{\vec{k}_1\uparrow;\vec{k}_2\downarrow}(t) &\equiv \hat{a}^+_{\vec{k}_1\uparrow}(t)\hat{a}_{\vec{k}_2\downarrow}(t)\delta_{\vec{k}_1-\vec{q};\vec{k}_2}.\end{aligned} \tag{12}$$

and e is the electron charge.

Our final task is to calculate statistical average values of operators (10) and (11):

$$\begin{aligned}J_z(t) &= \langle\hat{J}_z(t)\rangle = -V^{-1}\sum_{\vec{k}}\frac{\hbar|e|}{m_e}k_z\{\langle\hat{n}_{\vec{k}\uparrow}(t)\rangle + \langle\hat{n}_{\vec{k}\downarrow}(t)\rangle\},\\ M_{z,el}(t) &= \langle\hat{M}_{z,el}(t)\rangle = -V^{-1}\sum_{\vec{k}}\mu_B\{\langle\hat{n}_{\vec{k}\uparrow}(t) - \hat{n}_{\vec{k}\downarrow}(t)\rangle\},\end{aligned} \tag{13}$$

where angle braces denote the statistical average values of the corresponding operators. In this connection, statistical averaging is carried out with a density matrix corresponding to thermodynamic equilibrium, which, as we assume, takes place in the absence of a CSW at $t = -\infty$. We also assume that interaction between conduction electrons and the CSW starts adiabatically slowly from $t = -\infty$ until the current moment of time. Here, we point out that we are interested in the stationary value of the current, reached within a time period much longer than the relaxation time which we consider later. Therefore, this stationary value of the current is "not sensitive" to the starting "scenario" of interaction between conduction electrons and the CSW.

Equations of motion for the statistical average values of operators (12) can be written as

$$\begin{aligned}\frac{\partial\langle\hat{n}_{\vec{k}\uparrow,\downarrow}(t)\rangle}{\partial t} &= i\hbar^{-1}\langle[\hat{\mathcal{H}}(t),\hat{n}_{\vec{k}\uparrow,\downarrow}(t)]\rangle + I^{\mathrm{rel}}\{\langle\hat{n}_{\vec{k}\uparrow,\downarrow}(t)\rangle\},\\ \frac{\partial\langle\hat{S}_{\vec{k}_2\downarrow;\vec{k}_1\uparrow}(t)\rangle}{\partial t} &= i\hbar^{-1}\langle[\hat{\mathcal{H}}(t),\hat{S}_{\vec{k}_2\downarrow;\vec{k}_1\uparrow}(t)]\rangle + I^{\mathrm{rel}}\{\langle\hat{S}_{\vec{k}_2\downarrow;\vec{k}_1\uparrow}(t)\rangle\},\\ \frac{\partial\langle\hat{S}^+_{\vec{k}_1\uparrow;\vec{k}_2\downarrow}(t)\rangle}{\partial t} &= i\hbar^{-1}\langle[\hat{\mathcal{H}}(t),\hat{S}^+_{\vec{k}_1\uparrow;\vec{k}_2\downarrow}(t)]\rangle + I^{\mathrm{rel}}\{\langle\hat{S}^+_{\vec{k}_1\uparrow;\vec{k}_2\downarrow}(t)\rangle\}.\end{aligned} \tag{14}$$

The last terms in (14) represent relaxation terms. The system of equations (14) can be easily solved if we do not consider relaxation terms. Taking into account the relaxation processes, we assume that in each moment of time t the system of electrons relaxes to the thermodynamic equilibrium state. This state corresponds to an instantaneous "frozen" distribution of magnetization in the CSW. A similar approach to relaxation processes is used for consideration of magnetic resonance problems.[25,26]

We denote thermodynamic equilibrium magnitudes in (14) corresponding to an instantaneous magnetization distribution in the CSW

$$\langle\overline{\hat{n}_{\vec{k}\uparrow,\downarrow}(t)}\rangle, \quad \langle\overline{\hat{S}_{\vec{k}_2\downarrow;\vec{k}_1\uparrow}(t)}\rangle, \quad \langle\overline{\hat{S}^+_{\vec{k}_1\uparrow;\vec{k}_2\downarrow}(t)}\rangle. \tag{15}$$

Relaxation includes both the relaxation profices of conductivity electron momentum and the relaxation process of its spin. We assume that relaxation processes are characterized by a common time τ. Therefore the relaxation terms in (14) take the following forms:

$$I^{\mathrm{rel}}\{\langle\hat{n}_{\vec{k}\uparrow,\downarrow}(t)\rangle\} = -\frac{1}{\tau}(\langle\hat{n}_{\vec{k}\uparrow,\downarrow}(t)\rangle - \overline{\langle\hat{n}_{\vec{k}\uparrow,\downarrow}(t)\rangle}),$$
$$I^{\mathrm{rel}}\{\langle\hat{S}_{\vec{k}_2\downarrow;\vec{k}_1\uparrow}(t)\rangle\} = -\frac{1}{\tau}(\langle\hat{S}_{\vec{k}_2\downarrow;\vec{k}_1\uparrow}(t)\rangle - \overline{\langle\hat{S}_{\vec{k}_2\downarrow;\vec{k}_1\uparrow}(t)\rangle}), \quad (16)$$
$$I^{\mathrm{rel}}\{\langle\hat{S}^{+}_{\vec{k}_1\uparrow;\vec{k}_2\downarrow}(t)\rangle\} = -\frac{1}{\tau}(\langle\hat{S}^{+}_{\vec{k}_1\uparrow;\vec{k}_2\downarrow}(t)\rangle - \overline{\langle\hat{S}^{+}_{\vec{k}_1\uparrow;\vec{k}_2\downarrow}(t)\rangle}).$$

To find the explicit form of Eqs. (14), it is necessary to derive electron wave functions, corresponding to an instantaneous magnetization distribution in the CSW. These functions are eigenfunctions of the Schrodinger equation,

$$\hat{H}(t)\Phi_{\vec{k},\vec{q},\pm}(t) = \varepsilon_{\vec{k},\vec{q},\pm}\Phi_{\vec{k},\vec{q},\pm}(t), \quad (17)$$

where t should be read as a parameter, and eigenvalues $\varepsilon_{\vec{k},\vec{q},\pm}$ are time independent.

The solutions of Eq. (17) are

$$\Phi_{\vec{k},\vec{q},+}(t) = \frac{e^{i\vec{k}\vec{r}}}{V^{1/2}}\begin{pmatrix}(1+N^2)^{-1/2}e^{iqz/2}\\ N(1+N^2)^{-1/2}e^{i(-qz/2+\omega t)}\end{pmatrix}, \quad (18)$$

$$\Phi_{\vec{k},\vec{q},-}(t) = \frac{e^{i\vec{k}\vec{r}}}{V^{1/2}}\begin{pmatrix}N(1+N^2)^{-1/2}e^{iqz/2}\\ -(1+N^2)^{-1/2}e^{i(-qz/2+\omega t)}\end{pmatrix}, \quad (19)$$

where

$$N = (S_\perp A)^{-1}\left\{-\frac{\hbar^2}{2}\left[\frac{k_z q}{m_e} + 2\frac{(S_\parallel A)}{\hbar^2}\right] + \sqrt{\frac{\hbar^4}{4}\left[\frac{k_z q}{m_e} + 2\frac{(S_\parallel A)}{\hbar^2}\right]^2 + (S_\perp A)^2}\right\}.$$

Functions (18) and (19) correspond to the energy bands:

$$\varepsilon_{\vec{k},\vec{q},\pm} = \varepsilon_{\vec{k}} + \frac{\hbar^2 q^2}{8m_e} \pm \sqrt{\frac{\hbar^4}{4}\left[\frac{k_z q}{m_e} + 2\frac{(S_\parallel A)}{\hbar^2}\right]^2 + (S_\perp A)^2}. \quad (20)$$

In the thermodynamic equilibrium state, only the lower band $\varepsilon_{\vec{k},\vec{q},-}$ is occupied. This corresponds to the situation when conduction electrons are completely spin polarized in the initial state (at $t = -\infty$). Based on this fact and using expressions for $\Phi_{\vec{k},\vec{q},-}(t)$ (19) and for $\psi_{\vec{k}\uparrow\downarrow}$ (6), we can easily get an explicit expression for (15). Therefore, using (9), (15) and (16), all the expressions in the system of kinetic equations (14) will get an explicit form.

The expression for the electrical current density in the limit $t \gg \tau$ has the form

$$J_z = -\frac{2|e|q\omega}{m_e}(AS_\perp)^2\tau^2\int\left\{1+\left[\left(\frac{\hbar^2 k_z q}{m_e} + 2S_\parallel A - \hbar\omega\right)^2 + 4(S_\perp A)^2\right]\frac{\tau^2}{\hbar^2}\right\}^{-1}\left[\left(\frac{\hbar^2 k_z q}{m_e} + 2S_\parallel A\right)^2 + 4(S_\perp A)^2\right]^{-\frac{1}{2}}\frac{d\vec{k}}{(2\pi)^3}. \quad (21)$$

The expression for $M_{z,el}$ magnetization in the same limit is

$$M_{z,el} = M_{z,el}\,|_{\tau=0} - \frac{2\mu_B m_e}{|e|\hbar q}J_z \equiv \mu_B\int\left(\frac{\hbar^2 k_z q}{2m_e} + S_\parallel A\right)\times\left[\left(\frac{\hbar^2 k_z q}{2m_e} + S_\parallel A\right)^2 + (S_\perp A)^2\right]^{-\frac{1}{2}}\frac{d\vec{k}}{(2\pi)^3} - \frac{2\mu_B m_e}{|e|\hbar q}J_z. \quad (22)$$

In (21) and (22) the domain of integration by $\vec{k}$ is limited by the surface $\varepsilon_{\vec{k},\vec{q},-} = \mathrm{const}$, where the value of const is defined by the electron concentration. The integral term in (22) does not depend on ω and represents the electron gas magnetization $M_{z,el}\,|_{\tau=0}$ corresponding to "frozen" magnetization in CSW.

Expressions (21) and (22) are the solution of the problem. As may be shown, J_z is an odd function of ω and q. It is evident from (21) that $J_{z|_{\tau=0}} = 0$, that is, in the case of "strong" relaxation ($\tau \to 0$), the steady current is 0. Let us note that electron gas magnetization depends directly on the electrical current, (22). This demonstrates an interaction between the orbital motion of the conduction electrons and their spin. It constitutes the physical essence of the effect.

Under the condition

$$\frac{\hbar}{\tau}, \hbar\omega, \frac{\hbar^2 k_z q}{m_e} \ll 2A, \quad (23)$$

the formulas for electrical current (21) and magnetization (22) are simplified:

$$J_z \simeq -|e|n\upsilon_F\left(\frac{q}{k_F}\right)\left(\frac{\hbar\omega}{A}\right)S_\perp^2, \quad (24)$$

$$M_{z,el} \simeq \mu_B n S_\parallel - 2\mu_B n\left(\frac{\hbar\omega}{A}\right)S_\perp^2, \quad (25)$$

where $\upsilon_F = \hbar k_F/m_e$ is the Fermi velocity. We can see that expression (24) corresponds to the first nonzero term in (21) in the case of small values of $\hbar q\omega/(k_F A)$. The physical sense of conditions (23) is discussed in Ref. 18.

The first term on the right-hand side of (25) is $M_{z,el}\,|_{\tau=0}$. The second term on the right-hand side of (25) represents the contribution of $\vec{S}(\vec{r},t)$ vector rotation (with ω frequency around the z axis) to $M_{z,el}$ magnetization. This contribution does not depend on q. It is similar to the well-known contribution to the spin system magnetization under the action of a rotating uniform magnetic field.[25] To avoid ambiguity, we note that the absence of dependence on τ in expressions (24) and (25) is the direct consequence of relations (23). It makes sense to rewrite (24) using the CSW power flux density expression, which we can easily get by using (2):

$$P_z \simeq \left(\frac{2\mu_B k_B^2 T_c^2 S_\perp^2}{a^2 M_0 \hbar}\right)q^3. \quad (26)$$

As a consequence of (26), the current density expression (24) takes the compact form

$$J_z \simeq \left(\frac{|e|\hbar^2 na}{4m_e k_B T_c A}\right)P_z. \quad (27)$$

Let us carry out numerical estimations for the considered effect in a degenerate FMS, $CdCr_2Se_4$, where $2A \simeq 0.5$ eV, $T_c \sim 130$ K, $a = 10^{-7}$ cm, $m_e \lesssim 10^{-27}$ g, and $M_0 \sim 3.5 \times 10^2$ G.[18,27] For an electron concentration $n \sim 10^{20}$ cm^{-3}, k_F is about 10^7 cm^{-1}. A typical value of spin relaxation time τ is 10^{-14} s at $T \sim 10^2$ K and τ increases with decreasing T.[27] As can be observed for a CSW, $q < 10^6$ cm^{-1},[27] we assume that $q \lesssim 10^6$ cm^{-1}; then from (2) we get $\omega \lesssim 10^{10}$ Hz at $q \lesssim 10^6$ cm^{-1}. For such parameters, conditions (23) are true.

We assume that $P_z \sim 1$ W/cm^2, which corresponds to $S_\perp \sim 10^{-1}$ at $q \sim 10^6$ cm^{-1}, and we get $J_z \sim 1$ A/cm^2. Note that $S_\perp \sim 10^{-1}$ corresponds to a sufficiently high amplitude of the CSW, and this justifies the initial consideration of the CSW as the classical object.

Thus the magnitude of the effect is quite sufficient for experimental observation. In conclusion, let us note that our calculation procedure can be easily modified to make it appropriate for calculation of the corresponding effect in ferromagnetic metals, where the Fermi energy exceeds the energy of the s-d exchange.

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