Weakly incoherent regime of interlayer conductivity in a magnetic field

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We investigate the electronic conductivity in layered metals in magnetic field in the "weakly incoherent" limit, when the interlayer transfer integral is smaller than the Landau-level separation and broadening by impurities, but the interlayer electron tunneling conserves the intralayer momentum. It is shown that the impurity potential has much stronger effect in this regime, than in the quasi-two-dimensional metals in the coherent limit. The weakly incoherent regime has several unique qualitative features, missed in the previous theoretical approaches. The background interlayer magnetoresistance in this regime monotonically grows with the increase of magnetic field perpendicular to the conducting layers. The Dingle temperature increases with magnetic field, which damps the magnetic quantum oscillations and changes the field dependence of their amplitudes. The angular magnetoresistance oscillations become much smoother, and the positions of magnetoresistance maxima at the Yamaji angles slightly shift. The monotonic part of the angular dependence of magnetoresistance also considerably changes in the weakly incoherent regime.

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I. INTRODUCTION

The crossover between coherent and incoherent electron transport in layered metals attracts great attention, both theoretical[1–8] and experimental,[8–14] for its influence on the properties of high-temperature cuprate superconductors, organic metals, heterostructures, and many other layered materials. This crossover can be driven by temperature T, volume impurity concentration n_i, and external magnetic field $\mathbf{B} = (B_x, B_y, B_z)$. In the magnetic field this crossover in conductivity is very pronounced because it qualitatively changes the magnetoresistance behavior.

The electronic conductivity in the magnetic field is widely used to investigate the electronic structure of various metals. In strongly anisotropic quasi-two-dimensional (quasi-2D) layered metals, when the interlayer transfer integral t_z is much smaller than the Fermi energy E_F, the influence of magnetic field on conductivity has many specific features. One has to separate several different regimes of interlayer magnetotransport, depending on the ratios of three energy parameters: the interlayer transfer integral t_z, the inverse mean free time $\Gamma_0 = \hbar/2\tau_0$ due to the impurity scattering, and the Landau-level (LL) separation $\hbar\omega_c$, where $\omega_c = eB_z/m^*c$ is the cyclotron frequency.

When the interlayer transfer integral is larger than the LL separation and Γ_0, $t_z > \max\{\hbar\omega_c, \Gamma_0\}$, the three-dimensional (3D) electronic dispersion is well defined and given in the tight-binding approximation by

$$\epsilon_{3D}(\mathbf{k}) \approx \epsilon(k_x, k_y) - 2t_z \cos(k_z d), \tag{1}$$

where $\epsilon(k_x, k_y)$ is the in-plane electron dispersion and d is the interlayer spacing. Then the classical magnetoresistance shows the angular Yamaji oscillations,[15,16] which are used to determine the in-plane Fermi momentum. The magnetic quantum oscillations (MQO) of interlayer conductivity in this case have beats of amplitude,[17] and these beats are shifted with respect to the beats of MQO of magnetization or of the other thermodynamic quantities.[18–20] The slow oscillations also appear in the interlayer conductivity, which can be used to separate the contributions of different scattering mechanisms to the electron mean free time.[20,21]

When the interlayer transfer integral is smaller than the LL separation, $t_z < \hbar\omega_c$, the beats of MQO disappear. This limit happens in strong fields in very anisotropic metals. If the interlayer transfer integral is still larger than the LL broadening Γ, $t_z > \Gamma$, the dispersion (1) survives, and the MQO can be described by the "coherent" theory in Refs. 18–24. Note that the LL broadening Γ is larger than Γ_0 in strongly anisotropic metals [see Eq. (20) below].

In the very anisotropic limit, when the interlayer transfer integral is the smallest parameter, $t_z < \hbar\omega_c, \Gamma$, the traditional 3D approach fails to describe the interlayer magnetoresistance. For example, in this limit, the experimentally observed interlayer magnetoresistance grows with the increase of the out-of-plane magnetic field not only in the maxima, but also in the minima of MQO (see, e.g., Refs. 9, 25, and 26). The angular dependence of magnetoresistance also has many unusual features in this regime.[8,12] This change of the magnetoresistance behavior when the magnetic-field strength or the impurity concentration increase was called the "coherence-to-incoherence crossover." It was observed in various compounds[8–14] and has attracted considerable theoretical attention.[1,6,8] The term "weakly incoherent" has been introduced[1] to separate this regime from the coherent 3D limit $t_z > \hbar\omega_c, \Gamma_0$, and from the completely incoherent regime, where the electron tunneling to the adjacent layers does not conserve the in-plane electron momentum. The completely incoherent interlayer electron tunneling happens when it goes via the resonance impurities,[2,7,8] due to the interaction with phonons[3,5] and in some other models.

The theory of weakly incoherent magnetoresistance in Ref. 1 is based on the phenomenological Green's function [see Eq. (53) of Ref. 1], which is equivalent to

$$G_R^0(\boldsymbol{r}_1, \boldsymbol{r}_2, j, \varepsilon) = \sum_{n,k_y} \frac{\Psi^{0*}_{n,k_y,j}(x_2, y_2)\Psi^0_{n,k_y,j}(x_1, y_1)}{\varepsilon - \varepsilon_n - i\Gamma_0}. \tag{2}$$

Here j is the number of conducting layer, related to the z coordinate as $z = jd$. The LL number n and the momentum

component k_y form the complete set of quantum numbers of the 2D electrons in magnetic field with free-electron dispersion

$$\varepsilon_n = \hbar\omega_c(n + 1/2). \tag{3}$$

In magnetic field $\boldsymbol{B} = (B_x, 0, B_z)$ (we may choose $B_y = 0$ without loss of generality because the in-plane dispersion is uniform) the electromagnetic potential in the Landau gauge is $\boldsymbol{A} = (zB_y, xB_z - zB_x, 0)$. Then the 2D electron wave functions are

$$\Psi_{n,k_y,j}(x,y) = \Psi_n\left(x - l_{Hz}^2\left[k_y + jd/l_{Hx}^2\right]\right)e^{ik_y y}, \tag{4}$$

where

$$\Psi_n(x) = \frac{\exp\left(-x^2/2l_{Hz}^2\right)H_n(x/l_{Hz})}{\left(\pi l_{Hz}^2\right)^{1/4}2^{n/2}\sqrt{n!}}, \tag{5}$$

$H_n(x/l_{Hz})$ is the Hermite polynomial and, for brevity, we introduced the notation of magnetic length components $l_{Hx} = \sqrt{\hbar c/eB_x}$ and $l_{Hz} = \sqrt{\hbar c/eB_z}$.

In the Green's function in Eq. (2) the impurity scattering produces only the imaginary part of the self-energy $i\Gamma_0$, which is independent of the quantum numbers $\{n, k_y, j\}$ and of the magnetic-field strength B. This approximation is incorrect in the weakly incoherent regime, as will be shown in Sec. II below. Therefore, the main conclusion of Ref. 1, that the interlayer conductivity in the weakly incoherent regime does not differ from the coherent 3D regime, is also incorrect. Below we will show that both the angular and magnetic-field dependence of interlayer magnetoresistance in the weakly incoherent regime differs from that in the 3D coherent limit.

The Green's function in Eq. (2) is inapplicable to describe the MQO even in the coherent limit, because the MQO of the electron density of state (DOS) at the Fermi level lead to similar oscillations of the electron self-energy, which must be taken into account in the theory of MQO.[19,20,22–24] In metals with well-defined 3D electron dispersion given by Eq. (1) with $t_z > \hbar\omega_c, \Gamma$, one can apply the Born approximation. This gives the imaginary part Γ of the electron self-energy to be proportional to the DOS, i.e. it oscillates around the field- and energy-independent value Γ_0:

$$\Gamma = \Gamma(\varepsilon) = \Gamma_0[1 + \rho(\varepsilon, B)/\rho_0], \tag{6}$$

where $\rho(\varepsilon)$ and ρ_0 are the electron DOS with and without magnetic field. The electron Green's function of the form

$$G_R^0(\boldsymbol{r}_1, \boldsymbol{r}_2, j, \varepsilon) = \sum_{n,k_y,k_z} \frac{\Psi^{0*}_{n,k_y,j}(x_2,y_2)\Psi^0_{n,k_y,j}(x_1,y_1)e^{ik_z(z_1-z_2)}}{\varepsilon - \varepsilon_{2D}(n,k_y) + 2t_z\cos(k_z d) - i\Gamma(\varepsilon)}, \tag{7}$$

with $\Gamma(\varepsilon)$ from Eq. (6), has been substituted to the Kubo formula in the calculation of MQO of interlayer conductivity σ_{zz} in quasi-2D metals in Refs. 19, 20, and 22–24. Equations (6) and (7) can only be applied to the metals with well-defined 3D electron dispersion, as in Eq. (1) at $t_z \gg \hbar\omega_c, \Gamma_0$. In this case the beats of MQO must be observed, as in Refs. 18–21. However, the Born approximation and Eqs. (6) and (7) are inapplicable to the very anisotropic almost 2D case, when $t_z < \hbar\omega_c, \Gamma_0$. Therefore, the conclusions of Refs. 22–24 about the field dependence of interlayer magnetoresistance in the nearly 2D limit $t_z \ll \hbar\omega_c, \Gamma_0$ are incorrect. In this limit, in addition to the oscillating field dependence in Eq. (6), the LL broadening Γ has a strong monotonic field dependence [see Eq. (20) below], which changes both the angular and field dependence of interlayer magnetoresistance.

The completely incoherent variable-range hopping mechanism of the interlayer magnetotransport, which does not conserve the in-plane electron momentum during the interlayer hopping, has also been suggested[6] to explain the exponential growth in interlayer magnetoresistance with decreasing temperature. However, all these approaches cannot explain the monotonic increase of magnetoresistance with increasing B in the minima of MQO, observed in β''-(BEDT-TTF)$_2$SF$_5$CH$_2$CF$_2$SO$_3$.[9,10]

Below we reexamine the calculation of interlayer conductivity in the weakly incoherent limit $\hbar\omega_c > \Gamma_0 > t_z$. We argue that Eqs. (2), (6), and (7) do not hold in this limit, and we derive the different formula for the electron Green's function. Then we calculate the interlayer conductivity with the obtained Green's function, which gives the considerably different result as compared to the coherent theory in Refs. 1 and 18–24. This explains several unique qualitative features of MQO and of the angular dependence of interlayer magnetoresistance observed in the weakly incoherent limit.

II. THE MODEL

The electron Hamiltonian in layered compounds with small interlayer coupling consists of the three terms

$$\hat{H} = \hat{H}_0 + \hat{H}_t + \hat{H}_I. \tag{8}$$

The first term $\hat{H}_0$ is the 2D free-electron Hamiltonian summed over all layers:

$$\hat{H}_0 = \sum_{m,j} \varepsilon_{2D}(m) c^+_{m,j} c_{m,j},$$

where $\{m\}$ is the set of quantum numbers of electrons in magnetic field on a 2D conducting layer, $\varepsilon_{2D}(m)$ is the corresponding free-electron dispersion given by Eq. (3), and $c^+_m(c_m)$ are the electron creation (annihilation) operators in the state $\{m\}$. The second term in Eq. (8) gives the coherent electron tunneling between two adjacent layers:

$$\hat{H}_t = 2t_z \sum_j \int dx\, dy [\Psi^\dagger_j(x,y)\Psi_{j-1}(x,y) + \Psi^\dagger_{j-1}(x,y)\Psi_j(x,y)], \tag{9}$$

where $\Psi_j(x,y)$ and $\Psi^\dagger_j(x,y)$ are the creation (annihilation) operators of an electron on the layer j at the point (x,y) . We call this interlayer tunneling Hamiltonian "coherent" because it conserves the in-layer coordinate dependence of the electron wave function (in other words, it conserves the in-plane electron momentum) after the interlayer tunneling. The last term

$$\hat{H}_I = \sum_i V_i(r) \tag{10}$$

is the impurity potential. The impurities are taken to be pointlike and randomly distributed on the layers. The impurity

distributions on any two adjacent layers are uncorrelated. The potential $V_i(r)$ of any impurity located at point r_i is given by

$$V_i(r) = U\delta^3(r - r_i). \tag{11}$$

In the 3D limit, when the interlayer transfer integral t_z is much larger than the electron level broadening Γ due to the impurity scattering, the impurity potential $\hat{H}_I$ can be considered as the small perturbation for the electrons with dispersion (1). In the Born approximation this gives $\Gamma = \pi n_i U^2 \rho(E_F)$, in agreement with Eq. (6), where n_i is the volume impurity concentration, and $\rho(E_F)$ is the DOS at the Fermi level. This leads to the standard theory of magnetic quantum oscillations in quasi-2D metals.[17,20] In the opposite limit, $t_z \ll \Gamma, \hbar\omega_c$, the interlayer hopping t_z must be considered as a perturbation for the disordered uncoupled stack of 2D metallic layers, where Eq. (6) is no longer valid.[27–33]

The 2D metallic electron system in magnetic field in the pointlike impurity potential has been extensively studied.[27–33] The pointlike impurity potential leads to the broadening of the LLs, which is described by the DOS distribution function $D(E)$. Since each LL without disorder is strongly degenerate, even a weak impurity potential lifts this degeneracy and leads to considerable broadening of the LLs. The electron Green's functions acquire a cut instead of the pole as in Eq. (2). In the self-consistent single-site approximation[27] the Green's function is given by

$$G(\boldsymbol{r}_1,\boldsymbol{r}_2,\varepsilon) = \sum_{n,k_y} \Psi^{0*}_{n,k_y}(r_2)\Psi^{0}_{n,k_y}(r_1)G(\varepsilon,n), \tag{12}$$

where

$$G(E,n) = \frac{E + E_g(1 - c_i) - \sqrt{(E - E_1)(E - E_2)}}{2EE_g}, \tag{13}$$

and the DOS $D(E) = (-1/\pi)\mathrm{Im}\, G_R(E)$ on each LL is described by the domelike function[27]

$$D(E) = \frac{\sqrt{(E - E_1)(E_2 - E)}}{2\pi|E|E_g}, \tag{14}$$

where the electron energy E is counted from the last occupied LL, $E = \varepsilon - \varepsilon_{2D}(n,k_y)$, and

$$E_g = V_0/2\pi l^2_{Hz}.$$

Here $V_0 = U|\psi(z_i)|^2 \approx U/d$ is the 2D analog of the strength U of the pointlike impurity potential

$$V_i(x,y) = V_0\delta(x - x_i)\delta(y - y_i), \tag{15}$$

and $\psi(z)$ is the out-of-plane electron wave function. The boundaries of the DOS dome in Eq. (14) are

$$E_1 = E_g(\sqrt{c_i} - 1)^2, \quad E_2 = E_g(\sqrt{c_i} + 1)^2, \tag{16}$$

where c_i is the ratio of the 2D impurity concentration, $N_i = n_i d$, to the 2D DOS on one LL, $N_{\mathrm{LL}} = 1/2\pi l^2_{Hz}$:

$$c_i = N_i/N_{\mathrm{LL}} = 2\pi l^2_{Hz} n_i d. \tag{17}$$

The function $D(E)$ in Eq. (14) is nonzero in the interval $0 < E_1 < E < E_2$ and normalized to unity: $\int D(E)dE = 1$. The LL broadening is

$$\Gamma_B \equiv (E_2 - E_1)/2 = 2E_g\sqrt{c_i} \propto \sqrt{B}. \tag{18}$$

The ratio

$$\frac{\Gamma_B}{\Gamma_0} = \frac{2V_0\sqrt{n_i d/2\pi l^2_{Hz}}}{\pi n_i U^2\rho(E_F)} \approx \frac{2U\sqrt{n_i N_{LL}/d}}{\pi n_i U^2\rho(E_F)} = \frac{2\sqrt{n_i U^2\rho(E_F)\hbar\omega_c}}{\pi n_i U^2\rho(E_F)} = \sqrt{\frac{4\hbar\omega_c}{\pi\Gamma_0}} \tag{19}$$

grows as $\sqrt{B}$ in high magnetic field. Equations (18) and (19) give the correct asymptote for the LL broadening in a strong magnetic field. In a weak magnetic field, when $\hbar\omega_c \ll \Gamma_0$, the mean scattering time τ_B related to level broadening as $\Gamma_B = \hbar/2\tau_B$, and entering the Drude formula, does not depend on the value of magnetic field along the conductivity: $\tau_B = \tau_0$.[34] To get the correct asymptotic behavior for Γ_B both in strong magnetic field and at $B = 0$, one can take the simple function

$$\Gamma_B \approx \Gamma_0[(4\hbar\omega_c/\pi\Gamma_0)^2 + 1]^{1/4}. \tag{20}$$

More realistic models of the finite-range impurity potential, and a more accurate calculation of the DOS, including the many-site corrections, lead only to the small tails of the DOS dome.[28,30,33] The number of electron states in these tails is much less than the number of states in the DOS dome and can be neglected. However, to take these tails into account and to simplify the subsequent calculation, one can take the Lorentzian DOS distribution with the same broadening:

$$D(E) \approx \frac{\Gamma_B}{\pi\left(E^2 + \Gamma_B^2\right)} = -\frac{\mathrm{Im}\, G_R(E)}{\pi}. \tag{21}$$

Combining Eqs. (12), (A1), and (21), we obtain

$$G(\boldsymbol{r}_1,\boldsymbol{r}_2,\varepsilon) = \sum_{n,k_y} \frac{\Psi^{0*}_{n,k_y}(r_2)\Psi^{0}_{n,k_y}(r_1)}{\varepsilon - \varepsilon_n - i\Gamma_B}, \tag{22}$$

This Green's function will be used in the next section to calculate the interlayer conductivity.

III. CALCULATION OF CONDUCTIVITY

The interlayer conductivity σ_{zz}, associated with the Hamiltonian (9), can be calculated using the Kubo formula and the formalism, developed for the metal-insulator-metal junctions.[35] In analogy to Eq. (44) of Ref. 1,

$$\sigma_{zz} = \frac{e^2 t_z^2 d}{\hbar L_x L_y}\left\langle \int d^2\boldsymbol{r}\, d^2\boldsymbol{r}' \times \int \frac{d\varepsilon}{2\pi} A(\boldsymbol{r},\boldsymbol{r}',j,\varepsilon)A(\boldsymbol{r}',\boldsymbol{r},j+1,\varepsilon)[-n'_F(\varepsilon)]\right\rangle, \tag{23}$$

where the electron spectral functions

$$A(\boldsymbol{r},\boldsymbol{r}',j,\varepsilon) = i[G_A(\boldsymbol{r},\boldsymbol{r}',j,\varepsilon) - G_R(\boldsymbol{r},\boldsymbol{r}',j,\varepsilon)], \tag{24}$$

and the advanced (retarded) Green's functions $G_{A(R)}(\boldsymbol{r},\boldsymbol{r}',j,\varepsilon)$ on each layer j include the interaction with impurities. The product of two spectral functions in Eq. (23) is rewritten as

$$\begin{aligned}\Pi &\equiv A(\boldsymbol{r},\boldsymbol{r}',j,\varepsilon)A(\boldsymbol{r}',\boldsymbol{r},j+1,\varepsilon)\\ &= G_A(\boldsymbol{r},\boldsymbol{r}',j,\varepsilon)G_R(\boldsymbol{r}',\boldsymbol{r},j+1,\varepsilon)\\ &\quad +G_R(\boldsymbol{r},\boldsymbol{r}',j,\varepsilon)G_A(\boldsymbol{r}',\boldsymbol{r},j+1,\varepsilon)\\ &\quad -2\,\mathrm{Re}\,G_R(\boldsymbol{r},\boldsymbol{r}',j,\varepsilon)G_R(\boldsymbol{r}',\boldsymbol{r},j+1,\varepsilon),\end{aligned} \tag{25}$$

In addition to the terms with the product of Green's functions G_AG_R, the expression for conductivity also contains the term $-\mathrm{Re}\,G_RG_R$, which becomes important when MQO are considered.[20,22] Equation (46) in Ref. 1 and subsequent formulas, where this term is omitted, can be applied only when MQO are disregarded. In a strong magnetic field, especially in the layered metals, on the contrary, the MQO are very strong.

The angular brackets in Eq. (23) mean averaging over impurity configurations. Since the impurity distributions on each layer are uncorrelated with other layers, one can perform this averaging separately for each spectral function independently,[36] which gives

$$\begin{aligned}\sigma_{zz} &= \frac{e^2t_z^2d}{\hbar L_xL_y}\int d^2\boldsymbol{r}\,d^2\boldsymbol{r}'\\ &\quad\times\int\frac{d\varepsilon}{2\pi}\langle A(\boldsymbol{r},\boldsymbol{r}',j,\varepsilon)\rangle\langle A(\boldsymbol{r}',\boldsymbol{r},j+1,\varepsilon)\rangle[-n_F'(\varepsilon)].\end{aligned} \tag{26}$$

The averaged Green's (or spectral) functions are translational invariant: $\langle G_R(\boldsymbol{r},\boldsymbol{r}',j,\varepsilon)\rangle = \langle G_R(\boldsymbol{r}-\boldsymbol{r}',j,\varepsilon)\rangle$.

If the magnetic field is tilted by angle θ with respect to the normal to the conducting planes, $B = (B_x,0,B_z) = (B\sin\theta,0,B\cos\theta)$, the Green's functions on two adjacent layers acquire the phase shift [see Eq. (49) of Ref. 1]

$$\begin{aligned}&G_R(\boldsymbol{r},\boldsymbol{r}',j+1,\varepsilon)\\ &\quad= G_R(\boldsymbol{r},\boldsymbol{r}',j,\varepsilon)\exp\{ie[\Lambda(\boldsymbol{r})-\Lambda(\boldsymbol{r}')]/\hbar\},\end{aligned} \tag{27}$$

where

$$\Lambda(\boldsymbol{r}) = -yB_xd = -yBd\sin\theta.$$

Substituting Eq. (27) into Eq. (25) we obtain

$$\begin{aligned}\Pi &= 2G_A(\boldsymbol{r},\boldsymbol{r}',j,\varepsilon)G_R(\boldsymbol{r}',\boldsymbol{r},j,\varepsilon)\cos\{e[\Lambda(\boldsymbol{r})-\Lambda(\boldsymbol{r}')]/\hbar\}\\ &\quad -2\,\mathrm{Re}[G_R(\boldsymbol{r},\boldsymbol{r}',j,\varepsilon)G_R(\boldsymbol{r}',\boldsymbol{r},j,\varepsilon)\\ &\quad\times\exp\{-ie[\Lambda(\boldsymbol{r})-\Lambda(\boldsymbol{r}')]/\hbar\}]\end{aligned} \tag{28}$$

and

$$\begin{aligned}\sigma_{zz} &= \frac{2e^2t_z^2d}{\hbar}\int d^2\boldsymbol{r}\int\frac{d\varepsilon}{2\pi}[-n_F'(\varepsilon)]\\ &\quad\times\left\{|\langle G_R(\boldsymbol{r},\varepsilon)\rangle|^2\cos\left(\frac{eByd}{\hbar}\sin\theta\right)\right.\\ &\quad\left.-\,\mathrm{Re}\left[\langle G_R(\boldsymbol{r},\varepsilon)\rangle^2\exp\left(\frac{ieByd}{\hbar}\sin\theta\right)\right]\right\}.\end{aligned} \tag{29}$$

The term in the third line of Eq. (29) is absent in Eq. (50) of Ref. 1. This term mostly affects the MQO of interlayer conductivity.

In the magnetic field perpendicular to the conducting layers

$$\begin{aligned}\sigma_{zz} &= \frac{2e^2t_z^2d}{\hbar}\int d^2\boldsymbol{r}\int\frac{d\varepsilon}{2\pi}[-n_F'(\varepsilon)][|\langle G_R(\boldsymbol{r},\varepsilon)\rangle|^2\\ &\quad-\mathrm{Re}\langle G_R(\boldsymbol{r},\varepsilon)\rangle^2].\end{aligned} \tag{30}$$

The integration over $\boldsymbol{r}$ for the Green's function of the form (12) is very simple and gives

$$\begin{aligned}\sigma_{zz} &= \frac{2e^2t_z^2dN_{\mathrm{LL}}}{\hbar}\int\frac{d\varepsilon}{2\pi}[-n_F'(\varepsilon)]\sum_n[|\langle G_R(\varepsilon,n)\rangle|^2\\ &\quad-\mathrm{Re}\langle G_R(\varepsilon,n)\rangle^2].\end{aligned} \tag{31}$$

With the approximate Green's function, given by Eq. (22), Eq. (31) becomes

$$\sigma_{zz} = \frac{2e^2t_z^2dN_{\mathrm{LL}}}{\hbar}\int\frac{d\varepsilon}{2\pi}\sum_n\frac{[-n_F'(\varepsilon)]2\Gamma_B^2}{\left[(\varepsilon-\varepsilon_n)^2+\Gamma_B^2\right]^2}. \tag{32}$$

The sum and integral in Eq. (32) is calculated in a standard way, transforming the sum over LL into the harmonic sum by applying the Poisson summation formula,[37]

$$\sum_{n=n_0}^{\infty}f(n) = \sum_{k=-\infty}^{\infty}\int_a^{\infty}e^{2\pi ikn}f(n)\,dn, \tag{33}$$

where $a\in(n_0-1;n_0)$. Then, performing the integrations, we obtain

$$\begin{aligned}\sigma_{zz} &= \sigma_0(B)\sum_{k=-\infty}^{\infty}(-1)^k\exp\left[\frac{2\pi(ik\mu-|k|\Gamma_B)}{\hbar\omega_c}\right]\\ &\quad\times\frac{2k\pi^2T/\hbar\omega_c}{\sinh(2k\pi^2T/\hbar\omega_c)}\left[1+\frac{2\pi|k|\Gamma_B}{\hbar\omega_c}\right],\end{aligned} \tag{34}$$

where

$$\sigma_0(B) = \frac{e^2t_z^2\nu_Fd}{\hbar\Gamma_B}, \tag{35}$$

$\nu_F = N_{\mathrm{LL}}/\hbar\omega_c$ is the DOS at the Fermi level in the absence of magnetic field. Equation (34) outwardly resembles Eqs. (17)–(21) of Ref. 22, derived in the Born approximation in the limit of well-defined 3D electron dispersion. It also resembles Eqs. (12) and(15) of Ref. 19 and Eqs. (10) and(15) of Ref. 20 in the limiting case of $t_z\to 0$. However, there is an important difference between Eqs. (34) and (35) and the previous results, which comes out because Eqs. (34) and (35) are derived beyond the traditionally used Born approximation. This difference consists in the replacement of the field-independent quantities Γ_0 and Γ_ε by Γ_B, which has the strong monotonic dependence on magnetic field given by Eq. (20).

Let us now compare how strongly the field dependence of interlayer conductivity given by Eqs. (34) and (35) differs from the previous results [see, e.g., Eqs. (17)– (21) of Ref. 22]. In Fig. 1 we compare the field dependence of magnetoresistance $R_{zz}(B) = 1/\sigma_{zz}$, calculated using Eq. (34) with Γ_B given by Eq. (20) (solid blue line, corresponding to the result obtained) and with $\Gamma_B = \Gamma_0$ (dashed red line, corresponding to Ref. 22). The strong difference is evident. First, the interlayer magnetoresistance shows the monotonic growth with the increase of magnetic field, directed along conductivity and perpendicular to the conducting layers. This

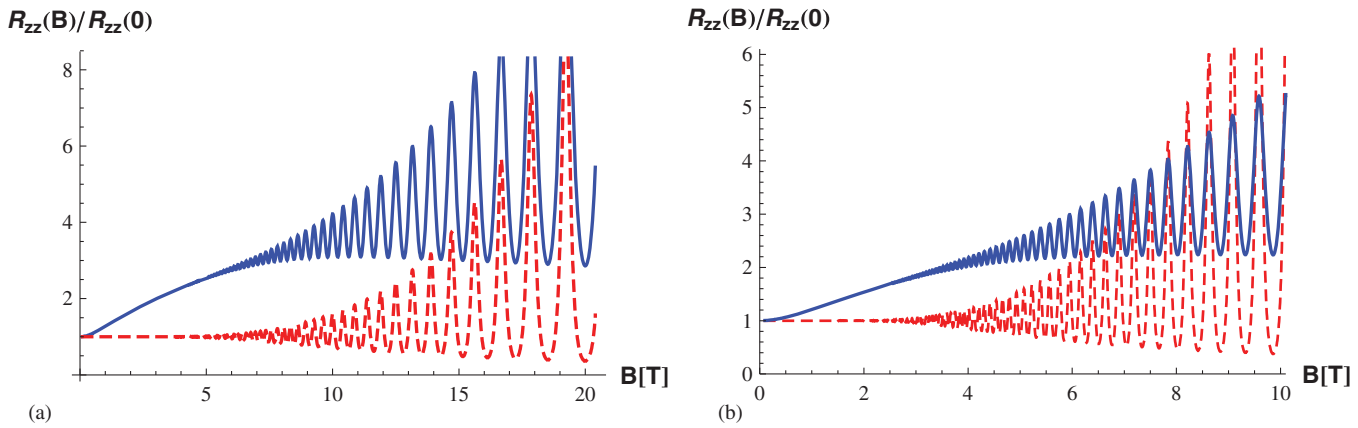

FIG. 1. (Color online) The MQO of resistivity $R_{zz}(B) = 1/\sigma_{zz}$, calculated using Eq. (34) with Γ_B given by Eq. (20) (solid blue line) and with $\Gamma_B = \Gamma_0$, which corresponds to Ref. 22 (dashed red line). The parameters used to plot (a) are $m^* = 1.45m_e$, $\Gamma_0 = 0.58$ K, and $T = 1$ K. The parameters in(b) are $m^* = m_e$, $\Gamma_0 = 1$ K, and $T = 0.6$ K.

monotonic growth is observed in all experiments on interlayer magnetoresistance in strongly anisotropic metals (see, e.g., Refs. 21 and 25) but, to the best of our knowledge, was not explained before and therefore was not used to extract any information about the compounds. Equations (20) and (35) allow to use this dependence for an alternative estimate of the LL broadening $\Gamma(B)$ from the experimental data. Second, the result obtained in Eqs. (34) and (20) takes into account the magnetic-field dependence of the Dingle temperature, which leads to the weaker field dependence of the MQO amplitude and predicts a smaller amplitude of the MQO as compared to the result of Refs. 22 and 23. This difference is most clearly illustrated in Fig. 1(b).

The angular dependence of interlayer magnetoresistance is also considerably modified by the replacement $\Gamma_0 \to \Gamma_B$. In tilted magnetic field the calculation of the integral in Eq. (29), performed in Ref. 1, can be applied with the different magnetic-field-dependent value Γ_B instead of Γ_0, which gives [compare to Eq. (1) of Ref. 1]

$$\sigma_{zz} = \sigma_0(B_z)\left\{[J_0(\kappa)]^2 + 2\sum_{\nu=1}^{\infty}\frac{[J_\nu(\kappa)]^2}{1+(\nu\omega_c\tau_B)}\right\}, \qquad (36)$$

where $\kappa \equiv k_F d\tan\theta$ is the same as in Ref. 1, but $\sigma_0(B_z)$ and τ_B acquire the field and angular dependence: $\sigma_0(B_z)$ is given by Eq. (35) and

$$\tau_B = \tau_B(B_z) = \hbar/2\Gamma_B = \tau_0(\Gamma_0/\Gamma_B). \qquad (37)$$

In the high magnetic field $\omega_c\tau \gg 1$ and in the weakly incoherent limit $t_z < \Gamma_B$, $\sigma_0(B_z) \propto \tau_B \propto 1/\sqrt{B\cos\theta}$.

There are two main differences between the obtained angular dependence of magnetoresistance, given by Eq. (36), and that given by Eq. (1) of Ref. 1. First, the sharp peaks of magnetoresistance at the Yamaji angles become smoother in the obtained formula. This change is due to the higher harmonics $[J_\nu(\kappa)]^2$ in angular magnetoresistance oscillations (AMROs), which are less damped in Eq. (36) as compared to Eq. (1) in Ref. 1 because of the smaller value of $\tau_B = \tau_0(\Gamma_0/\Gamma_B) \propto 1/\sqrt{B_z}$. These higher harmonics also slightly shift the positions of the Yamaji angles. Second, the monotonic part of the angular dependence of magnetoresistance changes because of the additional angular dependence of the prefactor $\sigma_0(B_z)$ given by Eq. (35), which in a strong field $\propto 1/\sqrt{B\cos\theta}$.

To illustrate these differences, in Figs. 2 and 3 we plot the angular dependence of conductivity $\sigma_{zz}(\theta)$ and of magnetoresistance $R_{zz}(\theta) = 1/\sigma_{zz}(\theta)$ given by Eq. (36) with $\tau_B = \hbar/2\Gamma_B = \tau_0(\Gamma_0/\Gamma_B)$ (solid blue line) and $\tau_B = \tau_0$ (dotted red line). For simplicity, we take the axially symmetric case, i.e., the symmetric in-plane electron dispersion. One can see that in the minima of conductivity, i.e., at the Yamaji angles, the replacement $\tau_0 \to \tau_B$ is very important. The predicted value of conductivity at the Yamaji angles with τ_B given by Eq. (37) is much larger than with $\tau_B = \tau_0$ [see Figs. 2(a) and 3(a)]. This difference grows with the increase of magnetic field. It is even more evident on the resistivity plots in Figs. 2(b) and 3(b). The maxima of magnetoresistance, given by Eq. (36), are much lower than according to Ref. 1. The positions of the conductivity minima, i.e., the Yamaji angles, also slightly shift after the replacement $\tau_0 \to \tau_B$ in Eq. (36) (see Figs. 2 and 3). For the first Yamaji angle at $B = 3$ T this shift $\Delta\theta_{\rm Yam} \approx 1^\circ$ [see Fig. 2(a)]. The angular-dependent prefactor $\sigma_0(B_z)$ in Eq. (36) considerably changes the ratio $\sigma_{zz}(\theta = \pi/2)/\sigma_{zz}(\theta = 0)$, which, according to Eqs. (35) and (36), becomes larger by the factor Γ_B/Γ_0 [see Eq. (19) and Figs. 2 and 3].

IV. DISCUSSION

Let us formulate the main difference of the present approach to the calculation of interlayer conductivity in the weakly incoherent regime as compared to the previous methods, developed in Refs. 19, 20, and 22–24 to calculate the MQO of conductivity. In these papers the impurity potential is considered as a small perturbation on the background of a free-electron gas with well-defined 3D electron dispersion given by Eq. (1). Hence, the impurity scattering was taken into account only by the imaginary part of the electron self-energy given by Eq. (6), which was calculated in the Born approximation. The impurities are treated even less accurately in Ref. 1, where the constant electron mean free time has been used to include the interaction with impurities. The Born approximation can be applied only in the 3D coherent limit, when the interlayer

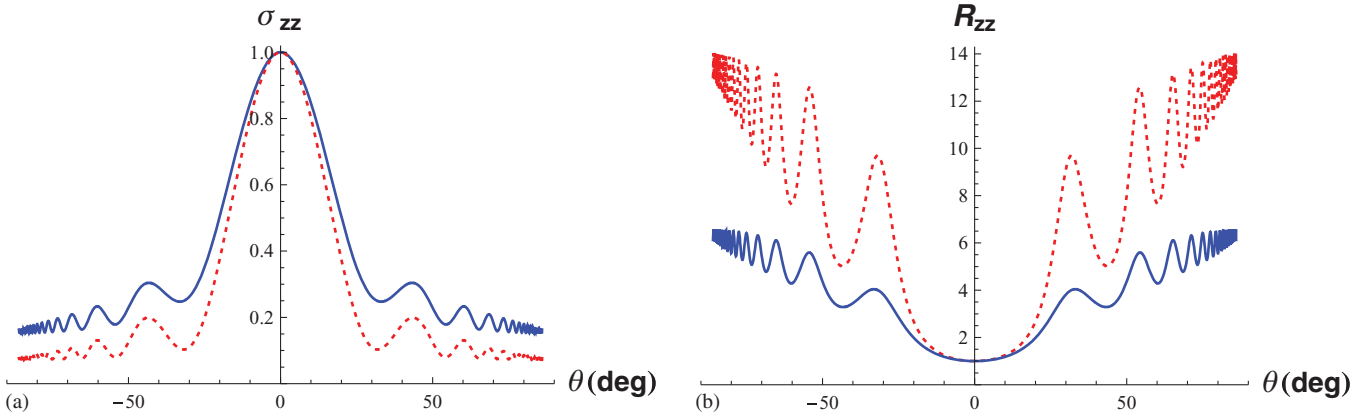

FIG. 2. (Color online) The angular dependence of conductivity $\sigma_{zz}(\theta)/\sigma_{zz}(0)$ (a) and of magnetoresistance $R_{zz}(\theta)/R_{zz}(0)$ (b), calculated using Eq. (36) with τ_B given by Eq. (37) (solid blue line) and with $\tau_B = \tau_0$ (dotted red line). The parameters for this plot are $k_F d = 4$, $m^* = m_e$, $B = 3$ T, $\Gamma_0 = 1$ K, which gives $\omega_c\tau \approx 1.74$.

transfer integral is much larger than the LL broadening. In the weakly incoherent regime, when $t_z < \Gamma, \hbar\omega_c$, this is incorrect, because for a 2D electron system in magnetic field the impurity potential has a much stronger effect than in 3D. Qualitatively, in a 3D electron system the electrons after scattering by an impurity move away in the interlayer direction and never return to this impurity. Therefore, this impurity only leads to single scattering of this electron into some other state, which is well described by the Born approximation or even by a constant electron mean free time τ_0, or, equivalently, by the constant imaginary part Γ_0 of the electron self-energy. In a 2D electron system in magnetic field, after scattering the electrons return to the same impurity after the cyclotron period. Therefore, the impurity has a permanent influence on the electron state, considerably shifting the electron energy and modifying the electron states. Hence, in the weakly incoherent regime, when $t_z < \Gamma, \hbar\omega_c$, the interlayer hopping term (9) in the Hamiltonian (8), rather the impurity potential (10), must be considered as a small perturbation. Therefore, to calculate the interlayer conductivity, we start from the stack of isolated 2D disordered conducting layers in the magnetic field, where the effect of the impurity potential is considered much more accurately, at least in the self-consistent single-site ("noncrossing") approximation. This allows us to go beyond the Born approximation, incorrectly applied in Refs. 22–24. Then we substitute the obtained electron Green's functions to the Kubo formula for the tunneling conductivity between adjacent conducting layers, which does not require the 3D electron dispersion. The effect of impurities in the final results turned out to be much stronger than in the previous approaches. Phenomenologically, this difference can be taken into account by the replacement of the initial level broadening Γ_0 in Eq. (2) by the larger value given by Eq. (20).

One can also obtain Eq. (22) with the value of Γ given by Eq. (20) using the alternative arguments. The physical origin of large DOS broadening in Eq. (14) is not the finite lifetime τ of electron states, which is mathematically described by the imaginary part of the self-energy $\mathrm{Im}\,\Sigma = \Gamma_0 = \hbar/2\tau$, as in the 3D limit. On the 2D layers the LL broadening comes from the energy shift of each electron state, which is described by the state-dependent real part of the electron self-energy $\mathrm{Re}\,\Sigma$. The averaging of the electron Green's function in Eq. (2) over the impurity configurations is independent on each conducting layer, since the impurity distribution is assumed to be uncorrelated. Then, the coordinate part of the Green's function remains of the form (2) with the bare electron wave

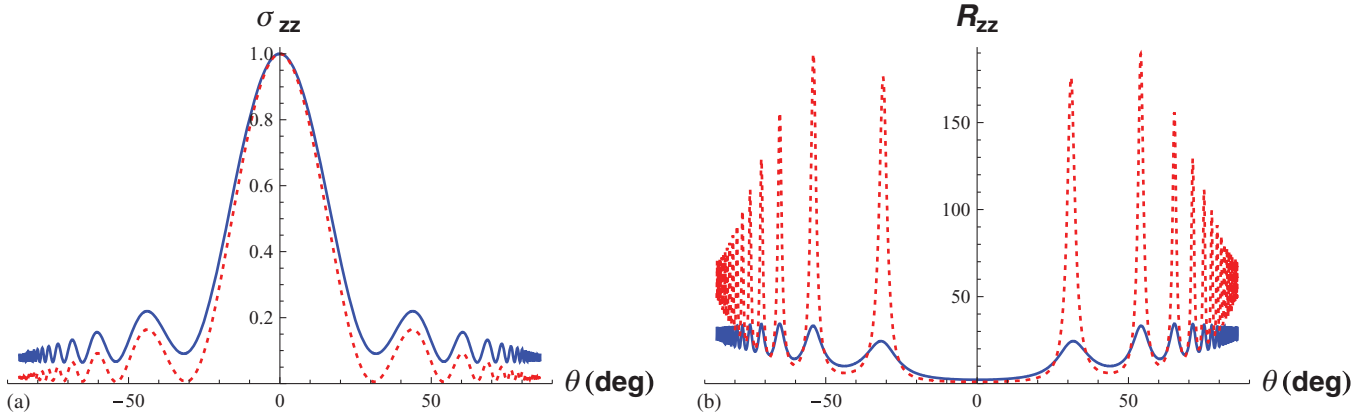

FIG. 3. (Color online) The same as in Fig. 2 but at a higher magnetic field value $B = 15$ T, which corresponds to $\omega_c\tau \approx 8.7$.

functions in the numerator [see Eq. (12) and the Appendix], but the denominator acquires the real part of electron self-energy, which is distributed with the DOS function $D(E)$:

$$\left\langle G_R^0(\boldsymbol{r}_1,\boldsymbol{r}_2,j,\varepsilon)\right\rangle = \int dE\, D(E) \sum_{n,k_y} \frac{\Psi^{0*}_{n,k_y,j}(x_2,y_2)\Psi^{0}_{n,k_y,j}(x_1,y_1)}{\varepsilon - E - \hbar\omega_c(n+1/2) - i0}.$$

The angle brackets indicate averaging over impurity configurations. Substituting the approximate Lorentzian DOS distribution, given by Eq. (21), one can easily perform the integration over E and obtain

$$\left\langle G_R^0(\boldsymbol{r}_1,\boldsymbol{r}_2,j,\varepsilon)\right\rangle \approx \sum_{n,k_y} \frac{\Psi^{0*}_{n,k_y,j}(x_2,y_2)\Psi^{0}_{n,k_y,j}(x_1,y_1)}{\varepsilon - \hbar\omega_c(n+1/2) - i\Gamma_B}. \quad (38)$$

This Green's function is equivalent to Eq. (22) and differs from Eq. (2) by the increase of the imaginary self-energy part, $\Gamma_0 \to \Gamma_B$, with Γ_B given by Eq. (20).

The proposed analysis considers only the limiting case $\Gamma_B \gg t_z$, when Γ_B is given by Eqs. (18) or (20), but it is not accurate for the intermediate case $t_z \sim \Gamma_B$, where the crossover from the coherent to the weakly incoherent regime takes place. The phenomenological formula (20) gives only a qualitative dependence $\Gamma_B(B_z)$ in this region. The crossover from the coherent to the weakly incoherent regime may be driven by the disorder (impurity concentration) or by magnetic field B_z. The latter happens because, with the increase of the magnetic field, the effective LL broadening Γ_B also increases [see Eq. (18)] and at some crossover field $B_c \sim t_z^2 m_e^* c/\Gamma_0 e\hbar$ it becomes greater than the interlayer transfer integral t_z. To calculate the exact value B_c of the crossover field and to describe the behavior of interlayer conductivity in this region, one needs to calculate the electron Green's function in layered metals with impurities and the magnetic field in the crossover region $t_z \sim \Gamma_B$. This is an interesting and still an open problem.

Above, we did not study the MQO in a tilted magnetic field. The second term in the curly braces in Eq. (29) does not contribute to the background magnetoresistance, but it affects the MQO. This term amplifies the MQO by the last factor in Eq. (34) and modifies the angular dependence of the MQO amplitude. The latter is a fine effect, which is harder to measure because the angular dependence of the MQO amplitude is also affected by the Zeeman splitting and by possible magnetic ordering.

If the normalized pointlike impurity concentration $c_i < 1$, the $N_{\mathrm{LL}} - N_i$ electron states on each LL left degenerate, and besides the DOS dome the sharp $\delta(E)$ term in the DOS survives.[29] However, as has been shown in Ref. 32, the numerous weak defects and the impurities, situated far from the conducting layers, are important for the lifting of the LL degeneracy in all layered materials. For an achievable magnetic field even in the pulsed magnets $B < 100$ T, $l_{Hz} > 25$ Å. Therefore, the typical normalized impurity concentration is greater than unity, $c_i > 1$, and one can use the one-maximum DOS distribution as in Eq. (14).

The oscillations of the chemical potential, which appear to be strong in the artificial layered compounds as heterostructures[38] and are also observed in some other materials,[39] may considerably affect the MQO of thermodynamic and transport quantities.[40,41] For example, they may lead to the mixing of the MQO frequencies even in the de Haas–van Alphen effect.[40] However, in the natural layered compound, even in the extremely anisotropic almost 2D layered system as β''-(BEDT-TTF)$_2$SF$_5$CH$_2$CF$_2$SO$_3$, the oscillations of the chemical potential turned out to be negligibly small, as was experimentally confirmed by analyzing the shape of the magnetization oscillations.[42] This shape turned out to be the same as in the 2D theory of magnetization oscillations with a fixed chemical potential (see Fig. 3 of Ref. 42). In anisotropic 3D metals such as beryllium, where the magnetic quantum oscillations are very strong, the oscillations of the chemical potential are also damped by more than ten times.[43] The absence of the chemical potential oscillations was explained by the observation of the MQO of the sample volume, which leads to oscillations of the electron concentration and cancels the oscillations of the chemical potential.[43] The observation of a strong MQO of the metallic sample volume is not surprising, because the delocalized electrons give the main contribution to the modulus of elasticity of metals.[44] The role of this magnetostriction on the damping of the MQO of the chemical potential is somewhat analogous to the electron reservoir, which can be simply included in the theory of MQO.[22,41] The main result of the present paper, that the monotonic field dependence of the LL broadening strongly affects the magnetic field and angular dependence of the interlayer magnetoresistance, is not sensitive to the oscillations of the chemical potential.

The electron-electron interaction, neglected in the above calculations, is more important in the layered, strongly anisotropic compounds than in usual 3D metals and may modify the quantitative behavior of magnetoresistance. For example, the superconducting fluctuations may considerably change the properties of cuprate high-temperature superconductors even above the superconducting transition temperature.[45] The quantum corrections to conductivity are small, but may also become detectable when the magnetic field is tilted toward the conducting plane.[46] However, all these effects are much weaker than the gross qualitative effects obtained in the present work.

The term "weakly incoherent transport" was introduced in Ref. 1, where this limit was treated incorrectly because of neglecting the magnetic-field dependence of the electron self-energy part Γ_0 in the in-plane Green's function. A similar mistake has been made in later theoretical papers, considering this limit.[22–24] Nevertheless, the term weakly incoherent became widely used both in the experimental and theoretical works (see, e.g., Refs. 8, 12, 23, 24, and 46). Strictly speaking, the term weakly incoherent is not exact, because the Hamiltonian of the model in Eq. (8) contains the tunneling term given by Eq. (9), which conserves the in-plane electron momentum. This Hamiltonian does not contain any additional coherence-breaking terms as compared to the standard "coherent" theory. The only difference is the extreme anisotropy. The more accurate term for this regime of interlayer transport would be "extremely anisotropic," "conditionally coherent," or "weakly coherent." However, we keep the original term weakly incoherent to avoid confusion and to emphasize the relation with previous publications.

In Sec. III we have shown that the weakly incoherent regime strongly differs from the coherent limit. It also differs from the completely incoherent limit, where the different mechanisms of the interlayer electron transport, including that via resonance impurities[2,7,8] and the hopping conductivity between completely localized states,[6] play an important role. One difference of the weakly incoherent regime from the completely incoherent one is that the AMROs are not damped, being almost of the same amplitude as in the coherent regime. Only the higher harmonics in AMROs increase, making the AMRO maxima less pronounced [see Figs. 2(b) and 3(b)]. The second difference is that the temperature dependence of conductivity in the weakly incoherent regime is the same as in the coherent limit (usually metallic), while the temperature dependence of the hopping conductivity[6] is exponential. Therefore, the weakly incoherent regime of interlayer magnetotransport is a separate regime, which should be distinguished from the coherent and completely incoherent limits.

V. SUMMARY

We reexamine theoretically the conducting properties of layered metals in the so-called weakly incoherent regime, when the interlayer transfer integral t_z is much less than the Landau-level separation $\hbar\omega_c$ and broadening Γ_B due to the interaction with impurities. The angular and field dependence of interlayer conductivity in this regime is calculated. We obtain that both these effects in the weakly incoherent limit considerably differ from the coherent regime. This contradicts previous theoretical results.[1,22] The background interlayer conductivity σ_{zz} decreases with the increase of magnetic field B_z according to Eq. (35), with Γ_B approximately given by Eq. (20), while in the standard coherent theory[22] it remains constant (see Fig. 1). The Dingle temperature of MQO also increases with magnetic field $\propto\Gamma_B$, which modifies the field dependence of the MQO amplitude [see Fig. 1(b) for an illustration). Meanwhile, in the weakly incoherent regime the angular oscillations of background magnetoresistance are not damped as in the completely incoherent mechanisms of the interlayer electron transport, considered, e.g., in Refs. 2–8. However, the angular dependence of interlayer magnetoresistance in the weakly incoherent regime, given by Eqs. (36) and (37), considerably differs from that in the standard coherent theory[47] and from the dependence obtained in Refs. 1 for the same weakly incoherent regime [see Figs. 2 and 3 and the discussion after Eqs. (36) and (37)]. This difference includes both the monotonic and oscillating parts of the angular dependence of magnetoresistance. Phenomenologically, the differences between the coherent and weakly incoherent regimes can be taken into account by the replacement of the electron mean free time τ_0 by the different value $\tau_B \approx \tau_0/[(8\omega_c\tau_0/\pi)^2+1]^{1/4}$ in all formulas for the field and angular dependence of interlayer magnetoresistance.

ACKNOWLEDGMENTS

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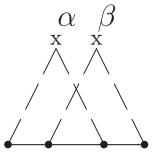

FIG. 4. The first diagram for the electron self-energy with the intersection of the impurity lines.

APPENDIX: THE IN-PLANE ELECTRON GREEN'S FUNCTION IN THE IMPURITY POTENTIAL

Consider the noninteracting 2D electron gas in the potential of randomly distributed pointlike impurity, as given in Eq. (15). The peculiarity of the 2D electron gas in a strong magnetic field in the presence of impurities is that the Born approximation of the scattering amplitude on each impurity is insufficient to describe the system. Physically, this means that an electron scatters many times by one impurity, because in the magnetic field the electrons periodically return to the same point after passing along the cyclotron orbit. In the 3D case, the diagram in Fig. 4 with the intersections of impurity lines is small by the parameter n_i/n_e, where n_i and n_e are the volume impurity and electron concentrations. In the 2D case in the magnetic field there is no general proof that the diagrams with intersections of impurity lines are small. However, the calculations of the DOS in Refs. 28, 30, and 33 show that these diagrams only lead to the small tails of the DOS. Therefore, in our calculation of interlayer conductivity we keep only the diagrams without intersections of impurity lines.

Now we prove by the method of mathematical induction that, if one neglects the diagrams with the intersection of impurity lines, the electron Green's function, averaged over impurity configurations, has the form of Eq. (12) with

$$G(\varepsilon,n) = 1/[\varepsilon - \varepsilon_n - \Sigma_n(\varepsilon)]. \tag{A1}$$

The energy ε_n of the nth LL is given by Eq. (3), and the electron wave functions $\Psi^0_{n,k_y}(r)$ by Eq. (4). The self-energy part $\Sigma_n(\varepsilon)$ for the nth LL must be determined self-consistently, and is given by the set of diagrams shown in Fig. 5. In the self-consistent single-site (noncrossing) approximation the self-energy part is

$$\Sigma_n(\varepsilon) = \frac{E - E_g(1-c_i)}{2} - \frac{\sqrt{(E-E_1)(E_2-E)}}{2}.$$

The restriction given by Eq. (12) is nontrivial because $G(\varepsilon,n)$ does not depend on k_y.

Without impurities, i.e., in the zeroth order of mathematical induction, Eq. (12) holds by definition. Assume it holds for an arbitrary number N of impurities in the electron Green's

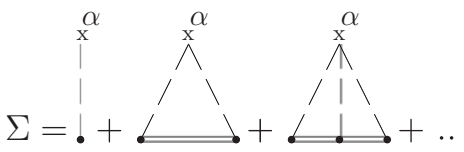

FIG. 5. The set of diagrams for the irreducible self-energy, corresponding to the self-consistent single-site approximation. The double solid line symbolizes the exact electron Green's function.

function $G_N(\boldsymbol{r}_1,\boldsymbol{r}_2,\varepsilon)$. When we add one more impurity center, the Green's function $G_{N+1}(\boldsymbol{r}_1,\boldsymbol{r}_2,\varepsilon)$ is given by

$$G_{N+1}(\boldsymbol{r}_1,\boldsymbol{r}_2,\varepsilon) = \int d\boldsymbol{r}_\alpha\, G_N(\boldsymbol{r}_1,\boldsymbol{r}_\alpha,\varepsilon) G_N(\boldsymbol{r}_\alpha,\boldsymbol{r}_2,\varepsilon)\Sigma(\varepsilon,\boldsymbol{r}_\alpha), \tag{A2}$$

where $\Sigma(\varepsilon)$ is given by the set of diagrams in Fig. 5, with the double line standing for $G_N(\boldsymbol{r}_\alpha,\boldsymbol{r}_\alpha,\varepsilon)$. Performing the integration over k_y in Eq. (12), we find

$$G_N(\boldsymbol{r}_\alpha,\boldsymbol{r}_\alpha,\varepsilon) = \sum_n \frac{N_{\mathrm{LL}}}{\varepsilon - \varepsilon_n - \Sigma_{N,n}(\varepsilon)}.$$

Therefore, $\Sigma(\varepsilon,\boldsymbol{r}_\alpha) = \Sigma(\varepsilon)$, and substituting Eq. (12) into Eq. (A2) and integrating over $\boldsymbol{r}_\alpha$, we obtain

$$\begin{aligned} G_{N+1}(\boldsymbol{r}_1,\boldsymbol{r}_2,\varepsilon) &= \sum_{n,k_y} \frac{\Psi^{0*}_{n,k_y}(r_2)\Psi^0_{n,k_y}(r_1)}{\varepsilon - \varepsilon_n - \Sigma_{N,n}(\varepsilon)} + c_i \int dr_\alpha \sum_{n,k_y} \frac{\Psi^{0*}_{n,k_y}(r_\alpha)\Psi^0_{n,k_y}(r_1)}{\varepsilon - \varepsilon_n - \Sigma_{N,n}(\varepsilon)} \sum_{n',k'_y} \frac{\Psi^{0*}_{n',k'_y}(r_2)\Psi^0_{n',k'_y}(r_\alpha)}{\varepsilon - \varepsilon_{n'} - \Sigma_{N,n'}(\varepsilon)}\Sigma(\varepsilon) + \cdots \\ &= \sum_{n,k_y} \frac{\Psi^{0*}_{n,k_y}(r_2)\Psi^0_{n,k_y}(r_1)}{\varepsilon - \varepsilon_n - \Sigma_{N,n}(\varepsilon)} \sum_{j=0}^{\infty} \left(\frac{c_i\Sigma(\varepsilon)}{\varepsilon - \varepsilon_n - \Sigma_{N,n}(\varepsilon)} \right)^j = \sum_{n,k_y} \frac{\Psi^{0*}_{n,k_y}(r_2)\Psi^0_{n,k_y}(r_1)}{\varepsilon - \varepsilon_n - \Sigma_{N+1,n}(\varepsilon)}, \end{aligned} \tag{A3}$$

where

$$\Sigma_{N+1,n}(\varepsilon) = \Sigma_{N,n}(\varepsilon) + c_i\Sigma(\varepsilon).$$

Equation (A3) has the form (12), which proves our statement.

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