Spin waves in diluted magnetic quantum wells

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We study collective spin excitations in two-dimensional diluted magnetic semiconductors, placed in an external magnetic field. Two coupled modes of the spin waves (the electron and ion modes) are found to exist in the system along with a number of the ion-spin excitations decoupled from the electron system. We calculate analytically the spectrum of the waves, taking into account the exchange interaction of itinerant electrons both with each other and with electrons localized on the magnetic ions. The interplay of these interactions leads to a number of intriguing phenomena, including tunable anticrossing of the modes and a field-induced change in a sign of the group velocity of the ion mode.

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Diluted magnetic semiconductors (DMS) have recently been the subject of great interest[1,2] due to their potential in combining magnetic and semiconductor properties in a single material. The DMS are formed by replacing cations in ordinary semiconductors with magnetic ions, typically Mn ions. Strong exchange interaction (EI) between the itinerant electrons and the electrons localized on d shells of the magnetic ions leads to a number of remarkable features of the DMS. In particular, it results in the effective indirect interaction between the ion spins and is thus promising for creating room-temperature ferromagnetic systems that may offer the advantages of semiconductors. It also dramatically enhances the effective coupling of the itinerant electrons with the external magnetic field. In contrast to conventional GaAs/GaAlAs systems, where small values of the g factor prevent manipulation of the spin degree of freedom, the giant electron Zeeman splitting arising in the DMS as a manifestation of the EI can be on the order of the Fermi energy,[3,4] offering a wide range of spintronics applications.

Here we discuss spin excitations in the two-dimensional DMS. Our studies are motivated by recent experiments[5–9] and a theoretical discussion[9–11] focused on the spin dynamics in diluted magnetic $Cd_{1-x}Mn_xTe$ quantum wells placed into the magnetic field B.[12] In Refs. 5–7, the spectrum of the spin waves $\omega(k)$ was measured. Only *one* excitation mode was observed. It was demonstrated that the excitations exist in a finite range of wavelengths, $k < k_m$, and their group velocity is negative: $d\omega(k)/dk < 0$. The experimental data were interpreted[5–7,10] in terms of conventional spin waves in the Fermi liquids,[13] while k_m was attributed to the edge of the Stoner continuum (SC) of the single-particle spin excitations (see Ref. 14 for discussion of the Stoner excitations). Such interpretation implies that the only effect of the magnetic ions on the electron-spin waves is the strong renormalization of the electron Zeeman splitting. However, more recent experimental observations[8,9] supported by theoretical studies[9,11] appear to be in disagreement with this conclusion. Indeed, in Refs. 8 and 9, *two* modes of the collective homogeneous ($k = 0$) spin excitations were observed in $Cd_{1-x}Mn_xTe$ wells. The modes were identified[8,9,11] as the spin excitations of delocalized electrons (the electron mode) and the electrons on d shells of Mn ions (the ion mode). The dependencies of the frequencies $\omega_{1,2}(0)$ of observed modes on B are shown schematically in Fig. 1. The most important observation is the anticrossing (AC) of the modes, which occurs at a certain "resonant" field $B = B_{\rm res}$. As was also shown,[9] other types of spin modes may exist in the system, corresponding to excitations of the ion spins decoupled from the spins of the itinerant electrons.

In this paper, we develop a theory of the spin waves in diluted magnetic quantum wells placed into a magnetic field. We study analytically two collective modes that correspond to coupled propagation of the electron and ion-spin excitations. We also discuss the ion modes decoupled from the electron system. To describe the homogeneous spin oscillations ($k = 0$), it is sufficient to take into account only one type of the EI: the interaction of the itinerant electrons with electrons localized on the Mn ions. The thus-obtained results coincide with those presented in Refs. 8 and 9. For $k \neq 0$, the EI between delocalized electrons comes into play.[15] Our main purpose is to demonstrate that the simultaneous presence of two types of EI give rise to interesting phenomena; the most remarkable one is the magnetic-field-driven AC of the spin modes. In contrast to the case $k = 0$, the AC can take place in a wide range of B and may be tuned by the field to occur at a certain value of k (see Fig. 2).

We consider the two-dimensional (2D) degenerate electron gas interacting with randomly placed magnetic ions. The electrons are located in the $\mathbf{r} = (x,y)$ plane and occupy the lowest level in the well. The ions are distributed homogeneously with the 2D concentration n_J, which is assumed to be much higher than the electron concentration n_e. The magnetic field is applied parallel to the well plane ($\mathbf{B} \parallel \mathbf{e}_x$). The field leads to the Zeeman splitting of the electron- and ion-spin levels with energies $\hbar\omega_e$ and $\hbar\Omega_J$, respectively, while the orbital motion remains intact. The Hamiltonian of electron-ion EI reads $\hat{H}_{Je} = -\alpha/2 \sum_k \hat{\boldsymbol{\sigma}} \hat{\mathbf{J}}_k \delta(\mathbf{r} - \mathbf{r}_k)|\Psi(z_k)|^2$, where $\hat{\boldsymbol{\sigma}}$ is the Pauli matrix vector, $\hat{\mathbf{J}}_k$ are the spin operators of the ions located at the points $\mathbf{R}_k = (\mathbf{r}_k, z_k)$, and $\Psi(z) = \sqrt{2/a}\sin(\pi z/a)$ is a wave function of the lowest level in a rectangular well of width a. Since $n_J \gg n_e$, the distance between the ions is much smaller than the electron wave length, and the mean-field approximation is applicable. In this approximation, we first replace $\hat{\boldsymbol{\sigma}}\hat{\mathbf{J}}_k$ with $\langle\hat{\boldsymbol{\sigma}}\rangle\hat{\mathbf{J}}_k + \hat{\boldsymbol{\sigma}}\langle\hat{\mathbf{J}}_k\rangle$, where averaging is taken

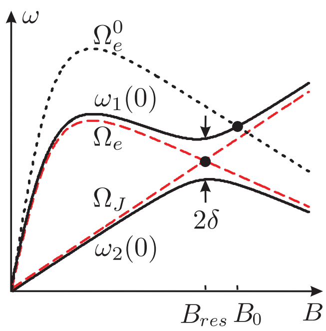

FIG. 1. (Color online) AC of the electron- and ion-spin precession frequencies Ω_e and Ω_J at resonant magnetic field $B = B_{\mathrm{res}}$; Ω_e^0 is the edge of the SC.

over density matrix of the system $\hat{\rho}$. After such decoupling, one may search the solution of the quantum Liouville equation for $\hat{\rho}$ as a product of the electron and ion density matrices: $\hat{\rho} = \hat{\rho}_e(\mathbf{r},\mathbf{r}',t)\prod_k \hat{\rho}_k(t)$. The average spins of the electrons and ions are given by $\mathbf{s}_0(\mathbf{r},t) = n_e^{-1}\int \mathbf{s}(\mathbf{r},\mathbf{p},t)d^2\mathbf{p}/(2\pi\hbar)^2$, $\mathbf{J}_k(t) = \mathrm{Tr}[\hat{\rho}_k(t)\hat{\mathbf{J}}_k]$, where $\mathbf{s}(\mathbf{r},\mathbf{p},t) = \mathrm{Tr}(\hat{\boldsymbol{\sigma}}\hat{f})/2$ is the electron-spin density and $\hat{f} = \hat{f}(\mathbf{r},\mathbf{p},t)$ is the Wigner function corresponding to $\hat{\rho}_e$. Next, we replace $\mathbf{J}_k(t)$ with a smooth function $\mathbf{J}(\mathbf{r},z,t)$. Doing so, one finds the electron-spin precession frequency in the ion-induced exchange field,

$$\boldsymbol{\omega}_{eJ}(\mathbf{r},t) = \alpha n_J \bar{\mathbf{J}}(\mathbf{r},t)/\hbar a \tag{1}$$

[here $\bar{\mathbf{J}}(\mathbf{r},t) = \int dz|\Psi(z)|^2\mathbf{J}(\mathbf{r},z,t)$], and the local frequency of the ion-spin precession,

$$\boldsymbol{\omega}_{Je}(\mathbf{r},z,t) = \alpha n_e \mathbf{s}_0(\mathbf{r},t)|\Psi(z)|^2/\hbar. \tag{2}$$

In addition to the electron-ion EI, we take into account the isotropic ferromagnetic electron-electron EI by adding the term[13]

$$\boldsymbol{\omega}_{ee}(\mathbf{r},t) = -2Gn_e\nu^{-1}\mathbf{s}_0(\mathbf{r},t) \tag{3}$$

to the electron-spin precession frequency. Here $G < 0$ is the interaction constant, $\nu = m/2\pi\hbar^2$, and m is the electron effective mass.

The equilibrium electron-spin density, $\mathbf{s}^{\mathrm{eq}}(\mathbf{p}) = [n_\uparrow(\epsilon) - n_\downarrow(\epsilon)]\mathbf{e}_x/2$ (here $n_{\uparrow\downarrow}(\epsilon) = \{\exp[(\epsilon \mp \hbar\Omega_e^0/2 - E_F)/T] + 1\}^{-1}$ and $\epsilon = p^2/2m$), is expressed via the effective Zeeman splitting $\hbar\Omega_e^0$. The average electron spin reads $\mathbf{s}_0^{\mathrm{eq}} = \mathbf{e}_x\xi/2$,

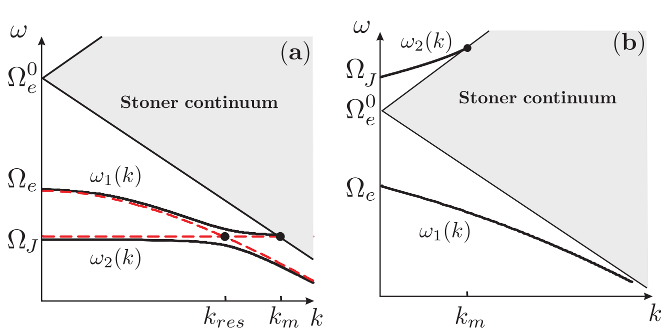

FIG. 2. (Color online) (a) AC of the two collective spin modes for $B < B_{\mathrm{res}}$. (b) Sign inversion of the group velocity of the ion mode for $B > B_0$.

where $\xi = \hbar\Omega_e^0/2E_F$. The frequency $\Omega_e^0 = \omega_e + \omega_{eJ} + \omega_{ee}$ is found self-consistently from Eqs. (1) and (3):

$$\Omega_e^0 = \Omega_e/(1+G), \tag{4}$$

where $\Omega_e = \omega_e + \alpha n_J J_x^{\mathrm{eq}}/a\hbar$ is the effective electron Zeeman splitting renormalized by EI with the ions. In deriving these equations, $\mathbf{J}(\mathbf{r},z)$ was substituted with the equilibrium ion polarization, $\mathbf{J}^{\mathrm{eq}} = J_x^{\mathrm{eq}}\mathbf{e}_x = \frac{5}{2}B_{5/2}(\hbar\Omega_J/T)\mathbf{e}_x$, where $B_J(x)$ is the Brillouin function. We also assumed that the equilibrium exchange field acting on the ions is small, $\alpha n_e\xi/2a \ll \hbar\Omega_J$, which implies that the equilibrium ion polarization is not affected by EI. In contrast, the electron Zeeman splitting is strongly enhanced due to high ion concentration, so that $\Omega_e \gg |\omega_e|$.[16]

The out-of-equilibrium spin dynamics can be described by the Landau-Silin equation[17] for the electrons and an equation describing the dynamics of the local ion-spin density:

$$\frac{\partial \hat{f}}{\partial t} + \frac{\mathbf{p}}{m}\nabla\hat{f} - \frac{1}{2}\left\{\frac{\partial \hat{f}}{\partial \mathbf{p}},\frac{\partial \hat{\varepsilon}}{\partial \mathbf{r}}\right\} + \frac{i}{\hbar}[\hat{\varepsilon},\hat{f}] = 0, \tag{5}$$

$$\frac{\partial \mathbf{J}}{\partial t} + [\Omega_J\mathbf{e}_x + \boldsymbol{\omega}_{Je}] \times \mathbf{J} = 0. \tag{6}$$

Here $[\cdots]$ and $\{\cdots\}$ stand for the commutator and the anticommutator, respectively, and $\hat{\varepsilon} = -\hbar[\omega_e\mathbf{e}_x + \boldsymbol{\omega}_{eJ} + \boldsymbol{\omega}_{ee}]\hat{\boldsymbol{\sigma}}/2$. For $\xi \ll 1$, Eqs. (5) and (6) give a system of coupled equations for the perpendicular (with respect to $\mathbf{B}$) components of the electron and ion spins:

$$\frac{\partial s}{\partial t} + \left(v_F\mathbf{n}\nabla + i\Omega_e^0\right)(s + Gs_0) = \delta_1(i\bar{J} + \eta\mathbf{n}\nabla\bar{J}),$$
$$\frac{\partial \bar{J}}{\partial t} + i\Omega_J\bar{J} = i\delta_2 s_0. \tag{7}$$

Here $s = s_y + is_z$, $\bar{J} = \bar{J}_y + i\bar{J}_z$, v_F is Fermi velocity, $\mathbf{n} = (\cos\varphi, \sin\varphi)$, φ is the velocity angle in the well plane, $\delta_1 = \alpha n_J\xi/2\hbar a$, $\delta_2 = 3\alpha n_e J_x^{\mathrm{eq}}/2\hbar a$, $\eta = v_F/\Omega_e^0$, and $s_0 = \int_0^{2\pi} s\,d\varphi/2\pi$.[18] From Eqs. (7) we find the dispersion equation for the collective modes,

$$\sqrt{1 - \frac{v_F^2k^2}{\left(\omega - \Omega_e^0\right)^2}} = \frac{\omega}{\omega - \Omega_e^0}\,\frac{\delta^2 + G\Omega_e^0(\omega - \Omega_J)}{\delta^2 + \Omega_e(\omega - \Omega_J)},$$

where $\delta = \sqrt{\delta_1\delta_2}$. For $k = 0$, we get[9,11] $\omega_{1,2}(0) = (\Omega_e + \Omega_J)/2 \pm \sqrt{(\Omega_e - \Omega_J)^2/4 + \delta^2}$. The AC occurs when $\Omega_e(B) = \Omega_J(B)$. We see that the constant G drops out from $\omega_{1,2}(0)$. In contrast, the dispersion of the collective modes strongly depends on the relation between $|G|$ and the dimensionless parameter δ/Ω_e. For $\delta/\Omega_e \ll |G|$, the anticrossing occurs for $B < B_{\mathrm{res}}$ when $\Omega_e > \Omega_J$ [see Fig. 2(a)]. To see this, one may consider the case $\delta = 0$ (coupling between the transverse components of electron and ion spins is turned off) as a first approximation. This approximation was implicitly used in Ref. 10. For $\delta = 0$, there are two branches of the spectrum [dashed lines in Fig. 2(a)], corresponding to the Fermi-liquid spin waves with negative dispersion and the dispersionless excitations of the ion spins. Importantly, for $B < B_{\mathrm{res}}$ these two branches intersect each other. Turning on a finite coupling, $\delta \neq 0$, results in the AC, which, for B close to $B_{\rm res}$, occurs at the point

$$k_{\rm res} = \sqrt{2|G|\Omega_e(\Omega_e - \Omega_J)}/v_F(1+G). \tag{8}$$

Remarkably, $k_{\rm res}$ depends on B, so that the AC position may be tuned by the external field. The splitting between the modes for $k \approx k_{\rm res}$ is given by $\omega_1(k_{\rm res}) - \omega_2(k_{\rm res}) \approx 2\delta$.

As seen from Fig. 2(a), the upper branch of the spectrum at a certain wave vector k_m reaches the SC, which is defined by inequality $|\omega - \Omega_e^0| \leqslant v_F k$. For $k > k_m$ the corresponding ion-type excitations slowly decay in time due to weak exchange coupling with the system of itinerant electrons. This decay is similar to the well-known Landau damping in plasma,[17] so the decay rate γ is calculated in a quite analogous way, yielding

$$\gamma \approx \frac{\delta^2 \Omega_J v_F \sqrt{k^2 - k_m^2}}{\Omega_e^0\left[(1+G)^2 v_F^2\left(k^2 - k_m^2\right) + G^2\Omega_J^2\right]}. \tag{9}$$

As a function of k, γ has a maximum. The maximal value is given by $\gamma_{\max} = \delta^2/2|G|\Omega_e \approx 3\alpha n_e \xi/8\hbar|G|a$. Using the data of Ref. 9 ($n_e = 0.7 \times 10^{11}$ cm^{-2}, $a = 80$ Å, $\alpha = 1.5 \times 10^{-23}$ eV cm^3, $|G| \approx 0.2$, $\xi \approx 0.2$), we find $\gamma_{\max} \approx 10^9$ s^{-1}.

Let us now comment on experimental results.[5–9] References 8 and 9 were mostly focused on the resonant case, $B \approx B_{\rm res}$. In the vicinity of the resonance, the electron and ion collective modes are strongly coupled and, consequently, are equally excited by the light impulse. This allowed the authors of Refs. 8 and 9 to identify both modes and to observe the AC at $k = 0$. In contrast, for $B \ll B_{\rm res}$ (which corresponds to the experimental situation in Refs. 5–7), the coupling is very weak for small k, so the ion mode is difficult to excite. The coupling is enhanced in the AC point. One can see, however, that k_m becomes smaller than $k_{\rm res}$ for $\Omega_J < \Omega_e^0(\kappa + \xi)/(1 + \kappa + \sqrt{1+2\kappa})$ [here $\kappa = \xi(1+2G)/G^2$], and the AC disappears.[19] This inequality was satisfied in Refs. 5–7, which may explain why the ion-type mode and the AC were not observed in those experiments.

Next, we focus on another interesting phenomenon arising due to the interplay of two types of interaction, namely, a change in a sign of the group velocity of the ion mode. It can by understood by analyzing the spectrum in the limit $k \to 0$ when $\omega_{1,2}(k) \approx \omega_{1,2}(0) + v_F^2 k^2/2\beta$. Here $\beta = \Omega_e^0[\omega_{1,2}(0) - \Omega_e^0][(\omega_{1,2}(0) - \Omega_e)^2 + \delta^2]/\omega_{1,2}(0)\delta^2$, so that the dispersions of the modes are controlled by signs of $\omega_1(0) - \Omega_e^0$ and $\omega_2(0) - \Omega_e^0$, respectively. As seen from Fig. 1, there is a critical field $B_0 \approx B_{\rm res}/(1+G)$, at which $\omega_1(0) = \Omega_e^0$ (for $G \approx -0.2$ and $B_{\rm res} = 6$ T,[8,9] one gets $B_0 = 7.5$ T). For $B < B_0$, both branches are below the SC and have negative dispersions. While B increases, the ion branch becomes shorter ($k_m \to 0$) and disappears when $B = B_0$. For $B > B_0$, this branch appears again above the SC and has a positive dispersion [see Fig. 2(b)]. The dispersion of the ion mode can also change sign for $\Omega_e > \Omega_J$, provided that $\delta/\Omega_e \gg |G|$.

Above we discussed the coupled collective modes. Now we notice that Eqs. (7) also have a solution $s = 0$ and $\bar{J} = 0$. Importantly, the latter equation, apart from the trivial solution $\mathbf{J}(\mathbf{r},z,t) = 0$, also has nonzero solutions obeying the constraint $\int dz|\Psi(z)|^2\mathbf{J}(\mathbf{r},z,t) = 0$. Such solutions were called "decoupled" modes.[9] To find a number of such modes, one should go beyond continuous approximation and replace integration over dz in all of the above equations with the sum over N atomic layers. This yields $N-1$ decoupled modes, corresponding to independent solutions of the equation $\sum_{m=1}^{m=N} J_m|\Psi(z_m)|^2 = 0$.[9] All these modes are, indeed, decoupled from electron system, provided that one neglects the equilibrium electron exchange field acting on the ion spins. In this approximation, we find from Eq. (6) that the modes have no dispersion and their frequencies coincide and are equal to Ω_J. In fact, weak interaction with the electrons gives rise to a small splitting of the ion Zeeman energies, which become dependent on m: $\hbar\Omega_{Jm} = \hbar\Omega_J + \alpha n_e \xi|\Psi(z_m)|^2/2$. Taking into account this splitting, one finds that in a symmetric quantum well, which we consider here, the decoupled modes with antisymmetric distribution of ion spins $J_m \propto \delta_{m,m_0} - \delta_{m,-m_0}$ still obey the condition $\bar{J} = 0$, thus having no dispersion. For the m_0th mode, the ion-spin precession frequency is equal to Ω_{Jm_0}. As for the modes with a symmetric distribution, they become weakly coupled to the electron collective mode. However, the corresponding dispersion is very weak, provided that $n_e/n_J \ll 1$. Symmetric modes also become weakly coupled to the single-electron excitations and, consequently, slowly decay in the region of the SC with the characteristic rate $\delta^2 v_F\sqrt{k^2 - k_m^2}/\Omega_e\Omega_J N$.

To conclude, we have developed a theory of the spin waves in the 2D DMS. We have described analytically two collective modes corresponding to the coupled excitations of the electron and ion spins and a large number of decoupled excitations of the ion spins. Our main finding is the tunable AC of the collective modes. We have also predicted a field-induced change in a sign of the group velocity of the ion mode and have calculated the decay of the waves in the SC.

Note added in proof. Recently, we became aware of two papers were published discussing closely related issues.[20,21]

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[1] G. A. Prinz, Science **282**, 1660 (1998).
[2] J. Cibert and D. Scalbert, in *Spin Physics in Semiconductors*, edited by M. I. Dyakonov (Springer, Berlin, 2008), Chap. 13.
[3] J. A. Gaj, R. Planel, and G. Fishman, Solid State Commun. **29**, 435 (1979).
[4] A. Lemaître, C. Testelin, C. Rigaux, T. Wojtowicz, and G. Karczewski, Phys. Rev. B **62**, 5059 (2000).
[5] B. Jusserand, F. Perez, D. R. Richards, G. Karczewski, T. Wojtowicz, C. Testelin, D. Wolverson, and J. J. Davies, Phys. Rev. Lett. **91**, 086802 (2003).

[6]F. Perez, B. Jusserand, D. Richards, and G. Karczewski, Phys. Status Solidi B **243**, 873 (2006).
[7]F. Perez, C. Aku-leh, D. Richards, B. Jusserand, L. C. Smith, D. Wolverson, and G. Karczewski, Phys. Rev. Lett. **99**, 026403 (2007).
[8]F. J. Teran, M. Potemski, D. K. Maude, D. Plantier, A. K. Hassan, A. Sachrajda, Z. Wilamowski, J. Jaroszynski, T. Wojtowicz, and G. Karczewski, Phys. Rev. Lett. **91**, 077201 (2003).
[9]M. Vladimirova, S. Cronenberger, P. Barate, D. Scalbert, F. J. Teran, and A. P. Dmitriev, Phys. Rev. B **78**, 081305(R) (2008).
[10]F. Perez, Phys. Rev. B **79**, 045306 (2009).
[11]J. Konig and A. H. MacDonald, Phys. Rev. Lett. **91**, 077202 (2003).
[12]The zero-field case was also discussed; see D. Frustaglia, J. Konig, and A. H. MacDonald, Phys. Rev. B **70**, 045205 (2004).
[13]P. M. Platzmann and E. Wolf, *Waves and Interaction in Solid State Plasmas* (Academic, New York, 1973).
[14]K. Yosida, *Theory of Magnetism* (Springer, New York, 1998).
[15]For $Cd_{1-x}Mn_xTe$ structures used in experiments[5–9] with x ranging from 0.2% to 0.8%, one can neglect the d-d EI between the ion spins (Ref. 2).
[16]Here $\omega_e < 0$ because of the negative electron g factor, which explains the nonmonotonic dependence of Ω_e on B shown in Fig. 1 (Refs. 8 and 9).
[17]A. I. Akhiezer, V. G. Baryakhtar, and S. V. Peletminskii, *Spin Waves* (North-Holland, Amsterdam, 1968).
[18]For low temperature and $\xi \ll 1$ one can search the electron-spin density in the following form: $\mathbf{s}(\mathbf{p},\mathbf{r},t) = n_e \nu^{-1} \delta(\epsilon - E_F)\mathbf{s}(\mathbf{r},\varphi,t)$.
[19]To obtain this result, one should go beyond the on-shell approximation (Ref. 18).
[20]F. Perez, J. Cibert, M. Vladimirova, and D. Scalbert, Phys. Rev. B **83**, 075311 (2011).
[21]C. Aku-Leh, F. Perez, B. Jusserand, D. Richards, and G. Karczewski, Phys. Rev. B **83**, 035323 (2011).