## Thermodynamic identification of tungsten borides

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(Received 22 April 2011; revised manuscript received 29 May 2011; published 21 June 2011)

Combining first-principles calculations with thermodynamics, we evaluate the structure stability of the borontungsten system ( $WB_x$ ), which is a promising superhard material. The highest boride of tungsten, previously denoted  $WB_4$  with a three-dimensional boron network, is identified as  $WB_3$  with two-dimensional boron nets. Furthermore, it is revealed that the mechanical properties of  $WB_x$  correlate with their formation energies, which justifies the thermodynamic considerations in the design of intrinsically hard materials.

DOI: 10.1103/PhysRevB.83.220102

PACS number(s): 61.50.Ah, 61.50.Ks, 62.20.-x

Transition metal borides have attracted much attention for a long time since they are refractory, chemically inert, and hard substances with high thermal and electrical conductivity.<sup>1</sup> Nowadays, several experimental researches (OsB<sub>2</sub>, ReB<sub>2</sub>,  $RhB_{1,1}$ , and  $IrB_{1,35}$ ) (Refs. 2–4) have reinvigorated the fundamental question of whether transition metal borides become intrinsically superhard materials. However, common issues on the ambiguity of crystal structures still remain in transition metal borides. Because of the large mass difference between transition metal atoms and boron atoms, only the positions of the metal atoms can be directly determined by x-ray diffraction, but the arrangements of the boron atoms are speculated from space considerations, assuming that boron atoms occupy the largest holes of the metal lattice. Moreover, transition metal borides have rich phase diagrams and many phases coexist. Chemical analysis may not distinguish between free and chemically bound boron. These technological difficulties contribute to the uncertainty of the boron architecture. It is therefore highly desirable to develop a theoretical method to identify the exact composition and structure and, in particular, to provide the essential guidance in the design of superhard materials.

Recently,  $WB_x$  has raised many expectations for future superhard materials. The Vickers hardness of WB<sub>2</sub> was reported to be quite high with a value of 43.8–47.4 GPa (Ref. 5). The micro-indentation test has revealed that the claimed WB<sub>4</sub> has extremely high hardness (Vickers hardness 46.2 GPa) (Ref. 6). Compared to boron-poor borides, boron-rich borides not only reduce the weight and cost of materials, but also increase covalent bonding, and thus the particular interest lies in WB<sub>4</sub>. Instead, the agreements are far from complete on the composition and structure of  $WB_x$ . Recent work<sup>7</sup> confirmed that the earliest established  $W_2B_5$  (Ref. 8) should be  $W_2B_4$ . The AlB2-type WB2 has been reported as the experimental ground state,<sup>9</sup> but theoretical calculations<sup>5,10</sup> recently showed that it is actually a high-pressure phase. However, the ReB<sub>2</sub> type becomes the ground state phase. For WB<sub>4</sub>, it seems as if the experimental and theoretical studies<sup>6,11</sup> reached the consensus that the three-dimensional boron network composed of an in-plane honeycomb B sublattice and an out-of-plane B<sub>2</sub> dimer is responsible for its high hardness. Nevertheless, it is a bit surprising that such a hard material has so low a shear modulus (103.6–129.1 GPa) (Ref. 11).

In this Rapid Communication, a thermodynamic approach combining first-principles calculations with chemical potentials is applied to investigate reasonable structures of  $WB_x$ at pressures up to 80 GPa. Seventeen candidate structures were chosen for  $WB_x$ , namely, tetragonal  $W_2B$ , tetragonal WB, orthorhombic CrB and FeB, hexagonal PtB and OsB, hexagonal ReB<sub>2</sub>, orthorhombic OsB<sub>2</sub>, rhombohedral MoB<sub>2</sub>, hexagonal WB<sub>2</sub> and AlB<sub>2</sub>, hexagonal W<sub>2</sub>B<sub>5</sub>, hexagonal MoB<sub>3</sub>, orthorhombic CrB<sub>4</sub>, monoclinic MnB<sub>4</sub>, hexagonal WB<sub>4</sub>, and tetragonal ThB<sub>4</sub> type.<sup>12,13</sup> We have identified that the long assumed WB<sub>4</sub> is, in fact, WB<sub>3</sub> with two-dimensional boron nets, which reformulates the concept of the three-dimensional boron network responsible for its high hardness. Furthermore, our results indicate that the ReB<sub>2</sub>-type structure, which is corroborated to be the ground state for WB<sub>2</sub>, transforms into the  $MoB_2$  type above 9.2 GPa.

The bridge between first-principles calculations and thermodynamics was established to analyze the structural stability of complex materials that can involve intrinsic nonstoichiometries of the chemical compositions.<sup>14</sup> We utilize this theory to explore the stable structures for stoichiometric borides. For simplicity, only the binary system  $WB_x$  is considered. Based on the chemical reaction  $W + xB = WB_x$ , our strategy is to seek the state with the lowest Gibbs energy  $G_{\rm S}$ :  $G_S = G_0 - \mu_W - x\mu_B$ . Here  $G_0$  means the free energy of an ensemble WB<sub>x</sub>, and  $\mu_W$  ( $\mu_B$ ) is the chemical potential of the component tungsten (boron) at a specified thermodynamic environment with temperature T and pressure p. Although chemical potentials can be defined for defects and defect complexes,<sup>14</sup> they are not included in the above equation since we focus on the stoichiometric phases of  $WB_x$ . For an ensemble WB<sub>x</sub>, its free energy  $G_0$  can be written as  $G_0 = E(T, p) + E(T, p)$ pV - TS, where E(T, p) and S denote its internal energy and entropy, respectively. It is well known that temperature T and pressure p are thermodynamically equivalent. Particularly, the contribution of the term TS to the free energy  $G_0$  is normally weak for other systems<sup>15,16</sup> at room temperature. Thus we only consider the free energy  $G_0$  of an ensemble WB<sub>x</sub> at the zero temperature:  $G_0 = E(0K, p) + pV$ . For clarity, the firstprinciples effective chemical potential difference is defined as  $\Delta \mu_i = \mu_i - \mu_i^0(0K, p)$ , where  $\mu_i^0(0K, p) = E_i(0K, p) + pV_i$ (i = W or B) is the chemical potential of the body-centered cubic tungsten or alpha rhombohedral boron at the temperature



FIG. 1. (Color online) The stabilities of various WB<sub>x</sub> relative to W<sub>2</sub>B as a function of the chemical potential difference obtained by first-principles calculations at (a) p = 0 GPa and (b) p = 80 GPa (b).

of T = 0 K and the pressure of p. Therefore, the lowest Gibbs energy is rewritten as  $G_S = \Delta H - \Delta \mu_W - x \Delta \mu_B$ . Here  $\Delta H$  is the formation enthalpy  $\Delta H = E(0K, p) + pV - \mu_W^0(0K, p) - x \mu_B^0(0K, p)$  of WB<sub>x</sub> at the temperature of T = 0 K and the pressure of p. The quantities E(0K, p) and  $\mu_i^0(0K, p)$  are determined by performing density functional calculations within the generalized gradient approximation (GGA).<sup>17</sup> We use the VASP code that implements the projector augmented wave (PAW) method.<sup>18</sup> In reality, this method can be generalized to polynary systems of transition metal borides, carbides, and nitrides. Especially, it takes into account the phase competition of diverse compositions because of the inclusion of the chemical potentials of the constituents.

Figure 1 presents Gibbs energy differences of various candidate WB<sub>x</sub> relative to W<sub>2</sub>B as a function of the chemical potential difference  $\Delta \mu_B$  at pressures of 0 and 80 GPa. In good agreement with experimental observations<sup>8,19</sup> we find that W<sub>2</sub>B and WB are thermodynamically stable at zero and high pressure, while the PtB and OsBtype WB are unstable because of their high relative Gibbs energies. The CrB- and FeB-type WB are energetically very close to the WB structure, and thus they may be high-temperature phases. It can be seen that W<sub>2</sub>B<sub>5</sub> cannot exist at zero and high pressures, but the similar structure WB<sub>2</sub> is far more favorable in Gibbs energy, which supports the recent theoretical and experimental conclusions<sup>7,10</sup> that W<sub>2</sub>B<sub>5</sub> should be WB<sub>2</sub>. These reproduced results substantiate the validity of our thermodynamic considerations from the viewpoint of chemical potentials.

Unexpectedly, the MoB<sub>3</sub>-type WB<sub>3</sub> has the lowest Gibbs energy among the 17 candidate structures under the boron-rich condition and should be thermodynamically stable [Fig. 1(a)]. However, the highest boride of tungsten WB<sub>4</sub> is found to be highly unstable because of its high relative Gibbs energy. The claimed WB<sub>4</sub> was originally synthesized in 1961,<sup>20</sup> and confirmed by several experiments.<sup>21,22</sup> Moreover, its high hardness inspired recent experimental and theoretical studies.<sup>6,10</sup> Surprisingly, our conclusion on its instability completely conflicts with these.<sup>6,10,20–22</sup> For the AlB<sub>2</sub>-type

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WB<sub>2</sub>, it is unlikely that it occurs at ambient condition, which is at odds with the experimental report of Woods *et al.*<sup>9</sup>

At p = 0 GPa, the Gibbs energy differences of the ReB<sub>2</sub>-, RuB<sub>2</sub>-, MoB<sub>2</sub>-type WB<sub>2</sub> and WB<sub>2</sub> phases are within the range of 0.1 eV. Among the four phases, the ReB<sub>2</sub> type has the lowest Gibbs energy as the ground state for WB<sub>2</sub> [Fig. 1(a)]. At p = 80 GPa, the MoB<sub>2</sub> type becomes more stable than the ReB<sub>2</sub> type, and the MoB<sub>3</sub>-type WB<sub>3</sub> becomes thermodynamically unstable [Fig. 1(b)]. To reveal the pressure effect on the stability for five structures of WB<sub>2</sub>, we calculate their enthalpies with the range of 0–80 GPa. As shown in Fig. 2, there are crossings between competing structures signifying phase transitions under pressure. We can see that a structural transformation from ReB<sub>2</sub> type to MoB<sub>2</sub> type takes place at p = 9.2 GPa. Although the ReB<sub>2</sub> type may transform to the AlB<sub>2</sub> type at p = 65 GPa, this latter type has a higher enthalpy than the MoB<sub>2</sub> type. Hence, the AlB<sub>2</sub> type should not be the high-pressure phase for WB<sub>2</sub> below 100 GPa, in contrast with recent calculations.<sup>5,10</sup> As shown in Fig. 1, the boron-rich phases (the CrB<sub>4</sub>-, MnB<sub>4</sub>-, and ThB<sub>4</sub>-type WB<sub>4</sub>) have higher Gibbs energies than the boron-poor phases at zero and high pressure, signifying that it is not easy to prepare more boron-rich WB<sub>x</sub> (x > 3) experimentally.

To solve the contradictions between our and the experimental results on WB<sub>4</sub>, further structural analyses are required. In the  $WB_4$  structure (top left inset of Fig. 3), there are four formula units per unit cell with W at 2c(1/3, 2/3, 1/4) and 2b(0, 0, 1/4) and B at 12i(x, 0, 0) with x = 1/3 and 4f(1/3, 2/3, z)with z = 0.615, labeled as W1, W2, B1, and B2, respectively.<sup>21</sup> The B2-B2 distance (1.716 Å) is shorter than that of any known WB<sub>x</sub> phases except  $W_2B_5$  (1.72 Å). However, recent studies<sup>7,10</sup> and our results confirm that  $W_2B_5$  is  $W_2B_4$ , and such a short B-B distance is improbable. We attempt to substitute vacancies for B2 atoms and calculate the formation energy by  $\Delta E = E(WB_x) - E(W) - xE(B)$ . The formation energy of unit cell  $W_4B_{16-y}$  in the WB<sub>4</sub> structure as a function of vacancy number y is showed in Fig. 3. To our surprise, the formation energy sharply decreases from 1.976 to  $-1.156 \text{ eV/WB}_x$  as the vacancy number y increases from 0 to 4, which confirms that the B2 atoms are unfavorable to the stability. At y = 4, corresponding to no occurrence of B2 atoms, the structure of WB<sub>3</sub> is isostructural with MoB<sub>3</sub>. As a matter of fact, W<sub>2</sub>B and WB are isomorphic with Mo<sub>2</sub>B and MoB, respectively.



FIG. 2. (Color online) Enthalpy differences of different structures relative to  $WB_2$  as a function of pressure.



FIG. 3. (Color online) Formation energies for  $WB_4$  and the  $AlB_2$ type  $WB_2$  as a function of the vacancy number. The crystal structures of  $WB_4$ ,  $WB_3$ , and the  $AlB_2$ -type  $WB_2$  are inset. In the  $WB_4$  structure, large red, large blue, small green, and small black spheres represent W1, W2, B1, and B2, respectively.

Although MoB<sub>2</sub> and WB<sub>2</sub> have slightly different structures at ambient condition, the MoB<sub>2</sub>-type WB<sub>2</sub> becomes more stable than WB<sub>2</sub> above p = 9.2 GPa. The analogous nominal MoB<sub>4</sub> was investigated using powder diffractometry in conjunction with the electron microprobe and chemical analysis.<sup>23</sup> There is no experimental evidence that the B2 atoms exist and its correct crystallographic formula is Mo<sub>1-x</sub>B<sub>3</sub>. However, due to the heavy W atoms, nominal WB<sub>4</sub> offers considerably bad experimental conditions to restrict the accuracy. Hence, the overall trend of their isomorphism is unmistakable, and we rationally believe that the claimed WB<sub>4</sub> should be WB<sub>3</sub> isostructural with stoichiometric MoB<sub>3</sub>. In contrast to the three-dimensional boron network of WB<sub>4</sub>, WB<sub>3</sub> is stacked by the two-dimensional boron nets and W layers (top middle inset to Fig. 3).

The WB<sub>3</sub> structure is closely related to the AlB<sub>2</sub> structure, which consists of close packed layers of W atoms located directly above one another with B atoms in the interstices (top right inset to Fig. 3). Hence, the WB<sub>3</sub> phase can be derived from the AlB<sub>2</sub>-type WB<sub>2</sub> by replacing one-third of the W atoms with vacancy systematically so that the remaining W atoms form layers of open hexagons with alternate layers displaced by one atom. We regularly substitute vacancies for W atoms of a supercell of W<sub>6-y</sub>B<sub>12</sub> in the AlB<sub>2</sub> structure, and calculate their formation energies. It is found from Fig. 3 that the AlB<sub>2</sub>type W<sub>6-y</sub>B<sub>12</sub> has the lowest formation energy at y = 2. This case corresponds to WB<sub>3</sub>. Combined with the thermodynamic instability of the AlB<sub>2</sub>-type WB<sub>2</sub> discussed above, we provide direct evidence that the experimental assumed AlB<sub>2</sub>-type WB<sub>2</sub> (Ref. 9) is, in fact, the WB<sub>3</sub> phase.

The conclusion that the claimed WB<sub>4</sub> and the AlB<sub>2</sub>-type WB<sub>2</sub> are WB<sub>3</sub> are supported by our calculated crystal parameters. The measured lattice constants and volume ( $a_0 = 5.159$  Å,  $c_0 = 6.332$  Å,  $V_0 = 36.99$  Å<sup>3</sup>) for WB<sub>4</sub> (Ref. 6) are in very good agreement with the values ( $a_0 = 5.209$  Å,  $c_0 = 6.312$  Å,  $V_0 = 36.97$  Å<sup>3</sup>) we calculated for WB<sub>3</sub>. In contrast, the calculated results for WB<sub>4</sub> ( $a_0 = 5.362$  Å,  $c_0 = 6.452$  Å,  $V_0 = 40.16$  Å<sup>3</sup>) are much larger. For the AlB<sub>2</sub>-type WB<sub>2</sub>,

the experimental lattice constants ( $a_0 = 3.02$  Å,  $c_0 = 3.05$  Å) of the AlB<sub>2</sub> symmetry<sup>9</sup> are well compatible with our values ( $a_0/\sqrt{3} = 3.01$  Å,  $c_0/2 = 3.156$  Å) of WB<sub>3</sub>. We relax WB<sub>2</sub> restricted to the AlB<sub>2</sub> symmetry. The results show that the *c* axis (10.2% larger than the experimental value) becomes much longer due to the "extra" W atoms that want to "come off" from their initial positions.

The thermodynamic stability will most probably be reflected in the variation of mechanical properties owing to the correlation that normally exists between two effects. The mechanical properties (bulk modulus, elastic constants, shear modulus, Young's modulus, and Poisson's ratio) are obtained through our previous methods.<sup>24</sup> Figure 4 shows the trends of formation energy, bulk modulus, and shear modulus with the increase of boron content for the 17 structures of  $WB_{r}$ . It is observed that the shear modulus increases with the decrease of the formation energy. Among the phases of the same composition, the phase with the lowest formation energy has the largest shear modulus. However, the bulk modulus has little direct connection with its formation energy. As we know, the formation energy is closely associated with the bonding nature. This explains why the shear modulus is a significantly better qualitative predictor of hardness than the bulk modulus. Therefore, a thermodynamic consideration of stability is warranted for the design of superhard materials.

The correlations between the formation energy and mechanical properties are strongly embodied in the WB<sub>4</sub> and WB<sub>3</sub> phases. Although the bulk moduli of WB<sub>4</sub> (297 GPa) and WB<sub>3</sub> (291 GPa) are in accordance with the experimental result (304 GPa) (Ref. 6), the shear modulus of WB<sub>4</sub> (102 GPa) is only 40% that of WB<sub>3</sub> (252 GPa), even smaller than that of W (150 GPa). According to the linear correlation that exists between the shear modulus and the Vickers hardness for many of the known hard materials,<sup>25</sup> we estimate that WB<sub>3</sub> has a



FIG. 4. (Color online) (a) Formation energies, (b) bulk moduli, and (c) shear moduli for various  $WB_x$  versus the boron composition parameter *x*. In (a), (b), and (c) the same symbol denotes the same phase.

Vickers hardness of 42 GPa, consistent with the experimental value of 46.2 GPa (Ref. 6). On the contrary, the estimated hardness of WB<sub>4</sub> (15 GPa) largely deviates from the experimental result. The Poisson's ratio of WB<sub>3</sub> is 0.168, typical for covalent materials such as cBN (0.124) and ReB<sub>2</sub> (0.171) (Ref. 26), while that of WB<sub>4</sub> is 0.348, even larger than that of W (0.293). In addition, the Young's modulus and elastic constants of WB<sub>3</sub> (E = 588 GPa,  $C_{11} = 656$  GPa,  $C_{33} = 479$  GPa,  $C_{44} = 277$  GPa) are superior to those of WB<sub>4</sub> (E = 274 GPa,  $C_{11} = 399$  GPa,  $C_{33} = 444$  GPa,  $C_{44} = 154$  GPa). The sharp contrasts of their mechanical properties are interrelated with their formation energies. Hence our calculated thermodynamic and mechanical properties provide incontrovertible evidence that the claimed WB<sub>4</sub> is the MoB<sub>3</sub>-type WB<sub>3</sub>.

As we can see from Fig. 4, the boron-poor phases ( $W_2B$ , WB, CrB, and FeB type) have a larger bulk modulus (above 346 GPa), but smaller shear modulus (below 210 GPa), indicating that they are ultra-incompressible but not superhard candidates. The boron-rich phases (ReB<sub>2</sub>, WB<sub>3</sub>, CrB<sub>4</sub>, and MnB<sub>4</sub> type) have a larger shear modulus (above 252 GPa) and thus they are potentially superhard materials. In particular, the CrB<sub>4</sub>- and MnB<sub>4</sub>-type WB<sub>4</sub> with three-dimensional boron networks have negative formation energies. Moreover, our calculated elastic constants confirm that they are mechanically

stable. However, the above analysis shows that the  $CrB_4$ and  $MnB_4$ -type  $WB_4$  are thermodynamically unstable. These results point out that only the negative formation energy is not enough for the thermodynamic stability of a phase and the thermodynamic consideration from the viewpoint of chemical potentials is necessary in the design of intrinsically superhard transition metal borides, carbides, and nitrides.

In summary, we present a comprehensive thermodynamic method to assess the phase stability of transition metal borides. The application to  $WB_x$  proves that the long regarded  $WB_4$ - and  $AlB_2$ -type  $WB_2$  are actually  $WB_3$ . Furthermore, the ReB<sub>2</sub>-type  $WB_2$  is corroborated as the ground state for  $WB_2$  and transforms to the MoB<sub>2</sub> type at p = 9.2 GPa. Interestingly, the correlation between the thermodynamic stabilities and mechanical properties for  $WB_x$  is clarified to underline the importance of the thermodynamic consideration of stability in the design of superhard materials.

This research was supported by the NSFC (No. 51072213), the Local Colleges Faculty Construction of Shanghai MSTC (No. 08210511900), the Innovation Program of Shanghai MEC (No. 11ZZ147), and the Postdoctoral Foundation of China (No. 20090450738) and Shanghai (No. 10R21416700).

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