Cotunneling suppression in a hybrid single-electron transistor by a dissipative electromagnetic environment

Vladimir Bubanja

Measurement Standards Laboratory of New Zealand, Industrial Research Ltd, PO Box 31-310, Lower Hutt 5040, Wellington, New Zealand

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We study electron transport through a hybrid single-electron transistor, consisting of a superconducting island and normal-metal electrodes, in the Coulomb blockade regime. We derive analytic expressions for the elastic and inelastic cotunneling currents, which exhibit power law suppression induced by the dissipative electromagnetic environment. The results can be used to improve the accuracy of hybrid devices employed in electrical metrology and for noise measurements in quantum information processing.

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I. INTRODUCTION

The strong electron-electron interactions in a mesoscopic system have a significant effect on its transport properties, therefore enabling detection of quantum states and accurate charge quantization.[1] Interactions with the measurement apparatus introduce further interesting phenomena, and when this environment is suitably designed it can increase the control of the transport properties of the system. We study the effects of the interplay of the Coulomb and Cooper-pairing interactions on the electron transport through a nanometer scale superconducting island embedded in a dissipative electromagnetic environment.

One application of such a system is related to the recently proposed new International System of Units (SI). This quantum SI (Ref. 2) is going to be based on exactly defined values of a set of fundamental constants, from which units should be derived. For example, using the value of the elementary charge and atomic-clock-based frequency standard, the ampere could be realized with a device that controllably transfers individual electrons through an electronic circuit. During the past two decades a number of different materials and circuit designs have been considered for the implementation of such a device. These included metallic turnstiles and pumps,[3–6] superconducting devices,[7] quasi-one-dimensional channels in GaAs/AlGaAs heterostructures,[8] semiconductor quantum dots,[9] Si-wire charge coupled devices,[10] and carbon nanotubes.[11] However, owing to the difficulty in satisfying the combined requirements on accuracy ($\sim 10^{-8}$) and amplitude (~ 100 pA), the establishment of such a standard has so far evaded the efforts of the metrology community.

The effects that limit the accuracy of the operation of single-electron tunneling (SET) devices include thermal and background charge fluctuations, as well as cycles with missed tunneling events, all of which can be reduced by suitable fabrication, filtering of high-frequency noise, and properly chosen ac drives. On the fundamental side, of most interest are cotunneling processes, which persist even in the zero temperature limit, whereby electrons are transferred in either the forward or backward direction through several junctions via intermediate virtual states. The highest accuracy of electron transport has so far been achieved in metallic pumps, where cotunneling processes can be suppressed by increasing the number of junctions.[4] Alternatively, suppression of cotunneling can be achieved by external impedance, as predicted in (Ref. 5), and experimentally confirmed in (Ref. 6). The difficulty of scaling up the currents generated by these pumps is that the dc offset of each island has to be tuned independently, requiring a large number of sources for paralleling the devices.

Recently, very promising experimental results with hybrid normal-metal-superconductor devices[12] have reinvigorated interest in this field. In the case of a transistor whose electrodes and island are made of a normal metal, the neighboring stability regions for the excess number of electrons on the island, in the bias-voltage vs gate-voltage plane, barely touch. In contrast, these regions overlap for the hybrid transistor due to the superconducting energy gap, therefore enabling turnstile operation. The simplicity of such single-island devices has already enabled their paralleling with output current levels of up to 100 pA.[13] The theoretical analysis of the accuracy of both NISIN (normal metal-insulator-superconductor-insulator-normal metal) and SINIS structures was performed in Ref. 14, the conclusion of which was that the NISIN transistor operation is limited by elastic cotunneling to the $10^{-6}-10^{-7}$ level, which is insufficient for metrological applications. Subsequent work has therefore concentrated on the SINIS transistor.[13,15–17] In this paper, we show that a high-Ohmic on-chip environment can be used to suppress these unwanted processes, and therefore offers a possibility for improvement of accuracy of the NISIN transistor.

The results also have significance for the studies of mobile Einstein-Podolsky-Rosen (EPR) pairs, as a resource for secure quantum communication protocols in solid-state devices based on the hybrid design. In this case, two electrons from the same Cooper pair are injected from a superconductor into two normal-metal leads by the crossed Andreev reflection (CAR),[18,19] leading to a nonlocal spin singlet state in the leads. The degree of entanglement can be deduced from current-current correlations in the normal leads,[20] or by performing Bell inequality violation experiments.[21] For the asymmetrical potential of the leads, another mechanism of transport is provided by elastic cotunneling (EC),[22,23] where our results show the environmental effect on this channel of transport.

II. MODEL AND METHOD

We consider the NISIN structure schematically represented in Fig. 1. The Hamiltonian of the system, containing the essential physics, is given by

$$H = H_0 + H_T, \tag{1}$$

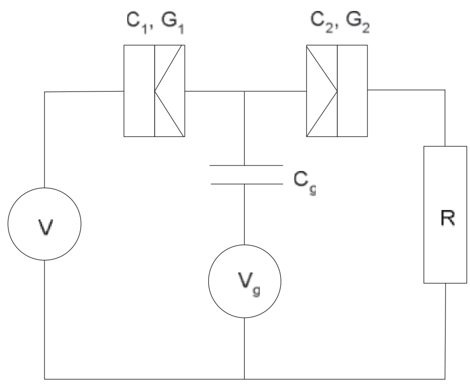

FIG. 1. The electrical circuit model of the system.

where H_0 is the unperturbed part:

$$H_0 = \sum_{i=S,I,D} H_i + H_{\rm env} + F, \tag{2}$$

where the terms describing the source and the drain electrodes are given by $H_i = \sum_k \epsilon_{i,k} c_{i,k}^\dagger c_{i,k}$ $(i = S,D)$. The term corresponding to the superconducting island is given by the Bardeen-Cooper-Schrieffer (BCS) Hamiltonian, $H_I = \sum_\alpha E_\alpha(\gamma_\alpha^\dagger\gamma_\alpha + \gamma_{-\alpha}^\dagger\gamma_{-\alpha})$, where the Bogoliubons are introduced by the usual transformation, $c_{\alpha,\sigma} = u_\alpha\gamma_{\alpha,\sigma} + \sigma v_\alpha \gamma_{-\alpha,-\sigma}^\dagger$, with the coherence factors given by $u_\alpha^2 = 1 - v_\alpha^2 = (1+\epsilon_\alpha/E_\alpha)/2$, and with a spectrum given by $E_\alpha = \sqrt{\epsilon_\alpha^2 + \Delta^2}$. The electromagnetic environment is described by the term $H_{\rm env} = \sum_q \hbar\omega_q b_q^\dagger b_q$. The last term in (2) consists of the Legendre transform of the charging energy and the energy of a quasiparticle on the island:

$$F = E_c\left\{\left(n - \frac{Q}{e}\right)^2 - \frac{2V}{e}\left[n_1\left(C_2 + \frac{C_g}{2}\right) + n_2\left(C_1 + \frac{C_g}{2}\right)\right]\right\} + \frac{1-(-1)^{\tilde n}}{2}\Delta, \tag{3}$$

where $E_c = e^2/2C_\Sigma$, with C_Σ being the total capacitance of the island $C_\Sigma = C_1 + C_2 + C_g$, and V the source-drain voltage. The charge Q includes the gate-induced and the random background charge, $Q = C_g V_g - Q_0$. The total number of free electrons on the island is $\tilde n = N_0 + n$, where N_0 is their initial number, and $n = n_1 - n_2$, where n_i is the number of electrons that have tunneled through the junction i.

For $E_c < \Delta$, it follows from (3) that $\tilde n$ can only be even, while for $E_c > \Delta$, there is a quasiparticle on the island if the gate voltage is adjusted so that

$$\frac{Q}{e} + N_0 \bmod 2 - 2k \in \left[\frac{E_c+\Delta}{2E_c}, \frac{3E_c - \Delta}{2E_c}\right], \quad k = 0, \pm 1, \ldots . \tag{4}$$

Tunneling is treated as a perturbation and the corresponding term in the Hamiltonian is given by

$$H_T = H_{T1} + H_{T2}, \quad H_{Ti} = H_i^+ + H_i^-, \quad H_1^+ = \sum_{p,\alpha} T_{p\alpha}(u_\alpha\gamma_\alpha^\dagger + v_\alpha\gamma_{-\alpha})c_p e^{-i\varphi_1} \quad H_i^- = (H_i^+)^\dagger, \tag{5}$$

where the phase φ_i is the conjugate variable to the charge on the junction i, and is bilinearly coupled to the coordinates of the bath of harmonic oscillators;[24] in order to simplify the notation we suppress the spin index.

In the resolvent formalism,[25] the current through the system can be expressed as

$$I = -\frac{2e}{\hbar}\,{\rm Im}\langle i|R(E_i + i\eta)|i\rangle, \tag{6}$$

where the level shift operator satisfies the integral equation

$$R(z) = H_T + H_T\frac{\bar\Lambda_i}{z - H_0}R, \tag{7}$$

where $\bar\Lambda_i$ is the projector out of the initial state. By expressing the resolvent as a Laplace transform of the propagator $\exp(-itH_0)$, and moving to the interaction picture in (6), we can write the average value of the current operator in terms of the products of the tunnel Hamiltonian terms averaged over the equilibrium electron subsystems and electromagnetic environment. In the fourth order in the tunneling matrix elements, we obtain from (6)

$$I = \frac{2e}{\hbar^4}\sum_{\substack{p,q=1,2\\(p\neq q)}} {\rm Re}\left[\int_{-\infty}^t dt_1 \int_{-\infty}^t dt_2 \times \int_{-\infty}^{t_2} dt_3 \langle H_p^-(t_3)H_q^-(t_2)H_1^+(t)H_2^+(t_1)\rangle - \int_{-\infty}^t dt_1 \times \int_{-\infty}^{t_1} dt_2 \int_{-\infty}^{t_2} dt_3 \langle H_p^-(t_3)H_q^-(t_2)H_2^+(t_1)H_1^+(t)\rangle\right]. \tag{8}$$

We assume adiabatic switching of the perturbation and express the expectation values of products of $\exp[\pm i\varphi_i(t)]$ in terms of the equilibrium correlation functions. These are given at zero temperature by $J_{ij} = (2/R_K)\int_0^\infty d\omega[{\rm Re}Z_{ij}(\omega)/\omega][\exp(-i\omega t) - 1]$, where Z_{ij} are the self- $(i=j)$, and cross-impedances $(i \neq j)$, and where $R_K = h/e^2$ denotes the quantum resistance.[24] We separate the current into the inelastic cotunneling (IC) part, which corresponds to simultaneous tunneling of two electrons (one through each junction) and the elastic cotunneling (EC) part, which corresponds to tunneling of one electron from the source to the drain electrode, $I = I_{\rm IC} + I_{\rm EC}$. As we are interested in transport at low voltages $(eV < 2\Delta)$, for the inelastic part we take into account only those processes that involve the quasiparticle already present on the island. For gate voltages outside the region in (4), there are no quasiparticles on the island in the lowest energy level of the system, and the inelastic processes are suppressed. We consider the weak cotunneling regime,[25] that is, we assume that the IC rate is much lower than the intrinsic relaxation rate on the island, and therefore at the start of each cotunneling event the existing quasiparticle is at the bottom of the energy spectrum. There are four channels that add coherently to IC: two of them have the electron tunneling first from the source electrode onto the island, and the other two have the electron tunneling first from the island onto the drain electrode. For example, after exchanging the energy with the environment, the tunneling electron in one channel lands at the same level of the existing quasiparticle to form a Cooper pair that drops into the Fermi sea, with another pair being disassociated when tunneling occurs through the second junction, also with energy exchange with the environment, leaving a quasiparticle on the island. There are three other similar processes. In contrast to IC, the EC current is not affected by the presence of a quasiparticle on the island.

III. RESULTS

We now apply the formalism to obtain the analytic expressions for $I_{\rm IC}$ and $I_{\rm EC}$. Within the Coulomb blockade region, the change in the charging energy due to electron tunneling through the ith junction is $E_i = E_c - eV_i > 0$, where V_i is the voltage across that junction, $V_i = [(C_\Sigma - C_i - C_g/2)V + (-1)^i(Q_0 - C_gV_g)]/C_\Sigma$. To simplify the formulas below, we assume symmetrical circuit layout, $C_1 = C_2 \equiv C$, $C_g \ll C$, with purely Ohmic external impedance of total resistance R. After performing the time integrals, we obtain from the first term in (8):

$$
\begin{aligned}
I_{\rm IC}^{(1)} = {} & \frac{G_1G_2\delta}{4\pi\Gamma(2z)e^3\hbar^2\Omega_0^3}\int d\epsilon_p d\epsilon_k d\epsilon_m f(\epsilon_p)[1-f(\epsilon_m)]\left[u_k^2u_0^2U\left(1,2+\frac{z}{2},\frac{E_1+\sqrt{\epsilon_k^2+\Delta^2}-\epsilon_p}{\hbar\Omega_0}\right)U\left(1,2+\frac{z}{2},\frac{E_2-\Delta+\epsilon_m}{\hbar\Omega_0}\right)\right.\\
& \left.+v_k^2v_0^2U\left(1,2+\frac{z}{2},\frac{E_1-\Delta-\epsilon_p}{\hbar\Omega_0}\right)U\left(1,2+\frac{z}{2},\frac{E_2+\sqrt{\epsilon_k^2+\Delta^2}+\epsilon_m}{\hbar\Omega_0}\right)\right]\left(\frac{\hbar\Omega_0}{eV+\epsilon_p-\sqrt{\epsilon_k^2+\Delta_2}+\Delta-\epsilon_m}\right)^{1-2z}\\
& \times\exp\left(-\frac{eV+\epsilon_p-\sqrt{\epsilon_k^2+\Delta^2}+\Delta-\epsilon_m}{\hbar\Omega_0}\right)\Theta(eV+\epsilon_p-\sqrt{\epsilon_k^2+\Delta^2}+\Delta-\epsilon_m),
\end{aligned}
\tag{9}
$$

where $z = R/R_K$, G_1, and G_2 are the tunnel junction conductances, δ is the average spin-degenerate level spacing on the island, $\Theta(x)$ is the unit step function, and $U(a,b,z)$ is the Tricomi confluent hypergeometric function. While performing the integrals in phase-phase self- and cross-correlations, we approximate the Lorentzian in $\mathrm{Re}Z_{ij}(\omega)$ by the exponential $\exp(-|\omega|/\Omega_0)$, which is valid for $\omega \ll \Omega_0 = 2/RC$.[5] In taking the averages over the island's creation and annihilation operators, we neglect contributions of the terms involving Gorkov's Green's functions as we consider transport in the Coulomb blockade region. In integrating over the energy of the virtual state, we assume that the distribution of the island's excitations can be described by the Fermi function $f_{\delta\mu}(\epsilon) = [1 + e^{(\epsilon-\mu)/k_BT}]^{-1}$, but with a shifted chemical potential $\mu = \mu_S + \delta\mu$, relative to the condensate μ_S, and subject to the constraint of one extra electron on the island, $\int d\epsilon N(\epsilon)[f_{\delta\mu}(\epsilon) - f_0(\epsilon)] = 1$.[26] By integrating over energies in (9), and performing similar calculations in the other terms in (8), we finally obtain

$$
\begin{aligned}
I_{\rm IC} &= \frac{\hbar G_1G_2\delta}{8\pi e^3}\left(\frac{\Delta}{\hbar\Omega_0}\right)^{2+2z}\frac{\sqrt{\pi}}{4^{1+z}}\frac{\left[\frac{eV}{\Delta}\left(2+\frac{eV}{\Delta}\right)\right]^{\frac{3}{2}+2z}}{\left(1+\frac{eV}{\Delta}\right)^{1+2z}}\,{}_2\tilde{F}_1\left[\frac{1}{2}+z,1+z;\frac{5}{2}+2z;\frac{\frac{eV}{\Delta}\left(2+\frac{eV}{\Delta}\right)}{\left(1+\frac{eV}{\Delta}\right)^2}\right]\mathcal{F},\\
\mathcal{F} &= \sum_{i,j,k=1,2}U\left[1,2+\frac{z}{2},\frac{E_i+(-1)^k\Delta}{\hbar\Omega_0}\right]U\left[1,2+\frac{z}{2},\frac{E_j+(-1)^{i+j+k}\Delta}{\hbar\Omega_0}\right],
\end{aligned}
\tag{10}
$$

where ${}_2\tilde{F}_1(a,b;c;z)$ is the regularized hypergeometric function. As the source-drain voltage increases and the energy gain of the intermediate state vanishes, $\min(E_i - \Delta) \to 0$, the above IC rate diverges. This can be removed by the partial resummation in (6) of the infinite number of the most divergent Feynman diagrams, similar to the NININ transistor case.[27] In the low-impedance limit, $z \ll 1$, and for $E_i + \Delta \ll \hbar\Omega_0$, the above formula reduces to

$$
\begin{aligned}
I_{\rm IC} = {} & \frac{\hbar G_1G_2\delta}{8\pi e^3}\sqrt{\frac{\pi}{2}}\frac{\Gamma^2(1+z/2)}{\Gamma(5/2+2z)}\left(\frac{\Delta}{\hbar\Omega_0}\right)^z\left(\frac{eV}{\Delta}\right)^{2z}\left\{\left[\left(\frac{1}{E_1/\Delta-1}\right)^{1+z/2}+\left(\frac{1}{E_2/\Delta+1}\right)^{1+z/2}\right]^2\right.\\
& \left.+\left[\left(\frac{1}{E_1/\Delta+1}\right)^{1+z/2}+\left(\frac{1}{E_2/\Delta-1}\right)^{1+z/2}\right]^2\right\}.
\end{aligned}
\tag{11}
$$

In the high-impedance limit, $z \gg 1$, and for $E_i - \Delta \gg \hbar\Omega_0$, we have

$$
I_{\rm IC} = \frac{\hbar G_1G_2\delta}{8\pi e^3}\sqrt{\frac{\pi}{2}}\frac{1}{\Gamma(5/2+2z)}\left(\frac{eV}{\Delta}\right)^{\frac{3}{2}}\left(\frac{eV}{\hbar\Omega_0}\right)^{2z}\left[\left(\frac{1}{\bar{E}_1/\Delta-1}+\frac{1}{\bar{E}_2/\Delta+1}\right)^2+\left(\frac{1}{\bar{E}_1/\Delta+1}+\frac{1}{\bar{E}_2/\Delta-1}\right)^2\right], \tag{12}
$$

where $\bar{E}_i = E_i - 2E_c/\pi$.

The EC component $I_{\rm EC}$ accounts for the coherence between the tunneling events.[28] By performing similar calculations to those described in the Ref. 29, we obtain

$$I_{\rm EC} = \frac{2e}{\hbar^4}\frac{\pi}{\Gamma(2z)}\frac{1}{\Omega_0^3}\sum_{p,\alpha,\alpha',m} f(\epsilon_p)[1-f(\epsilon_m)]\,F(\epsilon_\alpha,\epsilon_p,\epsilon_m)$$
$$\times F(\epsilon_{\alpha'},\epsilon_p,\epsilon_m)\left(\frac{\hbar\Omega_0}{eV+\epsilon_p-\epsilon_m}\right)^{1-2z} e^{-\frac{eV+\epsilon_p-\epsilon_m}{\hbar\Omega_0}}$$
$$\times\Theta(eV+\epsilon_p-\epsilon_m){\rm Re}\,(T^*_{\alpha p}T^*_{m\alpha}T_{\alpha' p}T_{m\alpha'}), \tag{13}$$

where $F(\epsilon,\epsilon_p,\epsilon_m)$ is the amplitude of electron propagation across the island and is at zero temperature given by

$$F(\epsilon_\alpha,\epsilon_p,\epsilon_m) = u_\alpha^2\, U\left[1,2+\frac{z}{2},\frac{E_1+E_\alpha-\epsilon_p}{\hbar\Omega_0}\right]$$
$$-v_\alpha^2\, U\left[1,2+\frac{z}{2},\frac{E_2+E_\alpha+\epsilon_m}{\hbar\Omega_0}\right]. \tag{14}$$

The above formula (13) depends on the character of electron motion accross the island. When this motion is diffusion[28,29] with short characteristic diffusion time, $\tau \ll \hbar/\Delta$, ($\tau = L^2/D$, where L is the length of the island and D the diffusion coefficient) we obtain in the high-impedance limit, $z \gg 1$, and for low voltages, $eV \ll \Delta,\hbar\Omega_0$:

$$I_{\rm EC} = \frac{\hbar G_1G_2\delta}{8\pi e^2}\frac{V\Delta}{\Gamma(2+2z)}\left(\frac{eV}{\hbar\Omega_0}\right)^{2z}\Bigg\{\sum_{\substack{i=1,2\\(j=2-[i/2])}}\frac{1}{\Delta\bar{E}_i(\bar{E}_i^2-\Delta^2)}$$
$$\times\left[(2\bar{E}_i^2-\Delta^2)-\frac{\Delta\left[2\bar{E}_i^3-\Delta^2(\bar{E}_i+\bar{E}_j)\right]}{\bar{E}_i(\bar{E}_i-\bar{E}_j)}\mathcal{I}\left(\frac{\bar{E}_i}{\Delta}\right)\right]$$
$$-\frac{\pi}{2}\left(\frac{1}{\bar{E}_1}+\frac{1}{\bar{E}_2}\right)^2\Bigg\}, \tag{15}$$

where

$$\mathcal{I}(x) = \begin{cases} \frac{2}{\sqrt{1-x^2}}\tan^{-1}\frac{\sqrt{1-x^2}}{1+x}, & |x|<1,\\ \frac{1}{\sqrt{x^2-1}}\ln(x+\sqrt{x^2-1}), & x>1. \end{cases} \tag{16}$$

IV. DISCUSSION

The results, demonstrating strong suppression due to environment, for inelastic, Eq. (10), and elastic cotunneling currents, Eqs. (13)–(14) and (15)–(16), are shown in Fig. 2, in cases of vanishing environmental impedance as well as for $R = 2R_K$. This environmental effect could also be used for cotunneling suppression in more elaborate designs involving hybrid multi-island turnstiles and pumps, where the above formulas for IC and EC can be used to evaluate corresponding rates for the NISIN sequence.

The derived results can also be used for considerations of escape processes in circuits containing the single-electron box. In addition to metrology, parity effects in small superconducting islands have attracted attention recently due to decoherence caused by quasiparticle poisoning of a Cooper pair box forming a charge qubit.[30] The subgap current observed in experiments with NIS junctions is often attributed to the

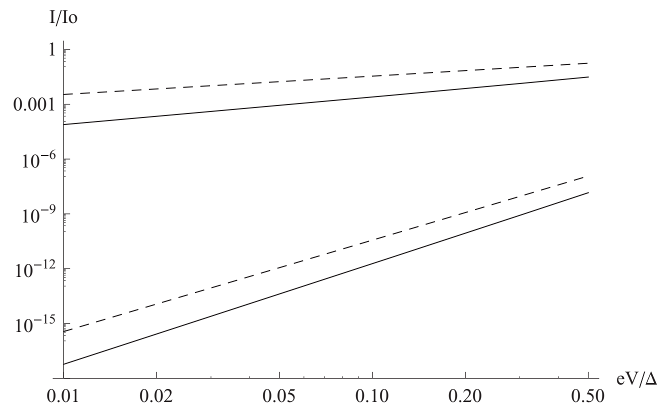

FIG. 2. Cotunneling currents vs the source-drain voltage of the NISIN transistor. Solid lines are the inelastic and dashed the elastic cotunneling currents for $z = 0$ (top) and $z = 2$ (bottom lines). Parameters are $I_0 = \hbar G_1G_2\delta/8\pi e^3$, $E_c = 10\Delta$, $Q = 0$, N_0 even.

Dynes density of states (DOS)[31] based on lifetime broadening of the BCS DOS:

$$\mathcal{N}^D(E) = \left|{\rm Re}\left(\frac{E/\Delta+i\gamma}{\sqrt{(E/\Delta+i\gamma)^2-1}}\right)\right|, \tag{17}$$

where the nonzero parameter γ introduces the states within the gap region, $|E| < \Delta$. It was recently demonstrated[17] that such form of the subgap DOS can be attributed to the high temperature ($k_BT_{\rm env} > \Delta$) in the low-Ohmic environment of effective resistance R, ($R \ll R_K$). In this case the convolution of the BCS DOS with the probability density for an electron to exchange energy E with the environment results in effective DOS given by (17) with $\gamma = 2\pi Rk_BT_{\rm env}/R_K\Delta$. It was previously shown numerically[32] that the increase of coupling to the environment increases the subgap quasiparticle current at finite temperature in small-capacitance Josephson junctions. These considerations are of lowest order (second in tunnel matric elements), and therefore the effects disappear in the limit of zero temperature. Subgap states can also be created by normal-metal inclusions within a superconductor. The rate for cotunneling from a normal-metal electrode into such an inclusion within a superconducting island and in a dissipative environment, can be obtained from the expressions (13)–(14), by taking $E_1 = -eV$, $E_2 = 0$. Since such processes can be viewed as direct tunneling into the superconductor, by taking the derivative of the obtained rate with respect to energy, we get for the subgap density of states at low energies ($\epsilon \ll \Delta,\hbar\Omega_0$),

$$\mathcal{N}_z(\epsilon) = \mathcal{N}_0(0)\frac{1}{\Gamma(2z+2)}\left(\frac{\epsilon}{\hbar\Omega_0}\right)^{2z}, \tag{18}$$

where $\mathcal{N}_0(\epsilon) = [1-(\epsilon/\Delta)^2]^{-3/2}R_KG_2/16\pi\Delta$ gives the subgap DOS for vanishing circuit impedance ($z \to 0$)[28]; in this case the constant for reduced DOS deep in the gap is $\delta\mathcal{N}_0(0)$. The expression (18) shows that the dissipative environment strongly suppresses such DOS.

In the limit $\Delta \to 0$, the above-derived formulas for cotunneling current correspond to the NININ transistor. The expressions (15)–(16) reduce to the formula (25) in Ref. 29, while the IC current (10) has additional terms containing the Fermi distribution for the excitations at the island. We neglect these terms here, but in the normal-metal island case they produce results (25)–(26) in Ref. 5. These terms are also important above the gap, $eV > 2\Delta$, in the case of superconducting island but here we are interested only in low voltages. This correspondence could be tested experimentally by applying a magnetic field to destroy the superconductivity.

It was shown in Ref. 18 that the dynamical Coulomb blockade caused by the resistive leads enhances CAR, compared to the direct Andreev reflection. Using the above results (15)–(16), it would be possible to test experimentally the transition from CAR to EC (similar to the experiment described in Ref. 23).

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