Strong-field magnetotransport in a normal conductor/perfect conductor/insulator disordered composite material: Simulations of a discrete model

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We consider a two-dimensional disordered composite medium made of three constituents: a normal conductor, a perfect conductor, and an insulator, and we examine its macroscopic electrical response in cases where it is subject to a strong magnetic field applied perpendicular to its plane. To this end, we exploit a discrete network model and apply a Monte Carlo procedure for sampling ensembles of finite-size three-constituent networks of this kind. The simulations indicate that when the perfectly conducting and insulating constituents are below the percolation threshold, such that the material has a finite, nonvanishing conductivity tensor, two distinct behaviors of the macroscopic magnetoresistance appear, according to whether the normal conductor *by itself* is below or above the percolation threshold. When the area fraction of the normal conductor is below that threshold, the macroscopic induced magnetoresistance is found to keep increasing with the magnetic field, without any saturation, whereas when that area fraction is above the percolation threshold, the magnetoresistance is found to saturate. Thus, the percolation threshold of the normal conductor is identified as a critical point. This critical phenomenon is associated with *both* a geometrical percolation and the presence of a large Hall effect. Its origin can be qualitatively understood by noticing the surprising fact that, in the strong-field limit, a perfectly conducting inclusion surrounded by a normally conducting neighborhood tends to expel currents almost like an insulating inclusion. The simulations also provide insights into difficulties that arise when simulating finite-size conducting networks at strong magnetic fields.

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I. INTRODUCTION

In a binary composite medium where the two constituents are characterized by an extreme contrast of their electrical conductivities, the behavior of the macroscopic electrical response depends on whether the microstructure is such that the better conductor percolates. This is the case in a normal conductor/insulator or a normal conductor/perfect conductor composite. The critical behavior of the macroscopic electrical response of such binary composites is associated with the phenomenon of geometrical percolation.[1]

A different kind of critical behavior of the macroscopic response can be found in a three-constituent composite where the constituents are, again, characterized by a sharp contrast of their conductivities and when the composite is subject to a strong magnetic field. In such cases, a singular behavior is associated with *both* geometrical percolation and the presence of a large Hall effect due to the applied magnetic field.[2–5]

In this paper, a case of the latter kind is considered. Explicitly, we consider a two-dimensional (2D) disordered composite medium consisting of the following three constituents: a normal conductor (denoted M), a perfect conductor (S) and an insulator (I). A strong magnetic field is applied perpendicular to the plane of the medium, thus inducing a classical Hall effect in the normal conductor. The notation S for a perfect conductor is used due to its association with a superconductor, but it must be emphasized that the term means here simply a classical electrical conductor characterized by a vanishing resistivity, no other properties associated with superconductivity being implied. We note, in this respect, that the theory we employ is irrelevant to recent experiments with disordered superconducting films that exhibit a superconductor-insulator transition, as those reported, for example, in Refs. 6–8. The magnetotransport properties of these films are determined by quantum effects associated with superconductivity, whereas the macroscopic response exhibited by the kind of disordered composites studied in this paper is determined by effects pertaining to the classical regime of electromagnetism and to the phenomenon of geometrical percolation.

The combined area fraction of the normal conductor and perfect conductor constituents is kept above the percolation threshold, such that the material necessarily conducts. The presence of the insulating constituent renders it possible to ensure that the perfectly conducting constituent does not percolate even when the normal conductor is below the percolation threshold. Hence, the macroscopic resistivity tensor is always finite and nonvanishing.

This M/S/I composite was examined in the past using an asymptotic analysis of the self-consistent effective medium approximation (SEMA).[3,5] The SEMA analysis predicts the appearance of a critical point when the area fraction of the M constituent is equal to the percolation threshold. The dependence of the macroscopic induced magnetoresistance on the field changes abruptly from a nonsaturating to a saturating one when this point is crossed. Because SEMA is an uncontrolled approximation it is, naturally, desirable to provide independent evidence for predictions based upon it. In this paper, we report on numerical results obtained by exploiting a discrete model[9,10] for simulating the 2D M/S/I composite. These results lend support to the SEMA prediction mentioned above. This numerical study also provides indications that even rather mild physical conditions would suffice for experimentally observing the critical behavior, relaxing the requirements to use a strict perfect conductor (or superconductor) for the S constituent and to apply an extremely strong magnetic field.

In Sec. II, we describe the M/S/I system in more detail, and in Sec. III we review the discrete model used to simulate it.

Section IV focuses on the characteristic distributions of the macroscopic conductances of the sampled random networks. We explain how certain effects lead to an emergence of double-peaked distributions of the Ohmic conductances. The conductances of the two parts of these distributions differ by orders of magnitude. We note the conditions that must be imposed on both network size and field strength in order for the networks to adequately mimic a real large composite sample. In Sec. V, the numerical results for the field dependence of the macroscopic resistivity tensor are presented. In particular, it is shown that the behavior of the macroscopic Ohmic resistivity indeed exhibits a transition when the fraction of the M constituent crosses the percolation threshold. The results are summarized and discussed in Sec. VI.

II. PHYSICAL CONSIDERATION OF THE M/S/I SYSTEM

The 2D M/S/I composite under consideration is characterized by a disordered microstructure, i.e., its three constituents are distributed randomly and isotropically over the area constituting the medium. We assume that the typical spatial scale of any homogeneous region within that medium is much larger than any microscopic length scale (for example, the mean free path of a charge carrier). Accordingly, the homogeneous regions, as well as the inhomogeneous medium, may be characterized by similar physical concepts and properties which are defined within the scope of continuum electrical transport, particularly, by an electrical conductivity or resistivity tensor of a continuous medium. In addition, we assume that quantum effects are negligible, i.e., that the circumstances are such that classical continuum electromagnetism holds. In particular, the S constituent just has a vanishing resistivity and lacks any of the other special properties of a real superconductor.

The normal conductor constituent is assumed to be isotropic and to exhibit a linear electrical response. When a magnetic field is applied perpendicular to the composite, a classical Hall effect is induced in the M constituent. Its resistivity tensor $\widehat{\rho}_{\mathrm{M}}$ then has antisymmetric off-diagonal components and is given by

$$\widehat{\rho}_{\mathrm{M}} = \rho_0 \begin{pmatrix} \alpha_{\mathrm{M}} & -\beta_{\mathrm{M}} \\ \beta_{\mathrm{M}} & \alpha_{\mathrm{M}} \end{pmatrix} \equiv \rho_0 \alpha_{\mathrm{M}} \begin{pmatrix} 1 & -H \\ H & 1 \end{pmatrix}. \tag{1}$$

Here, ρ_0 is a physical resistivity factor, α_{M} is the dimensionless Ohmic resistivity of the M constituent, and β_{M} is its dimensionless Hall resistivity. The parameter $H \equiv \beta_{\mathrm{M}}/\alpha_{\mathrm{M}}$ denotes the Hall-to-Ohmic resistivity ratio, which can also be expressed in terms of the Hall mobility of the M constituent, μ_{H}, and the physical magnetic field, $\mathbf{B}$: $H = \mu_{\mathrm{H}}|\mathbf{B}|$. Thus, H is a dimensionless measure of the strength of the externally applied magnetic field, and the regime of strong magnetic fields translates into the condition $|H| \gg 1$. In a simple free-carrier conductor, neither α_{M} nor μ_{H} depends on $\mathbf{B}$. However, the results to be presented are not restricted to this case and remain valid even when the M constituent is a more complicated isotropic conductor, provided α_{M} saturates with increasing $\mathbf{B}$. The resistivity tensor of the perfect conductor vanishes: $\widehat{\rho}_{\mathrm{S}} = \rho_0 \alpha_{\mathrm{S}} \widehat{I}$, $\alpha_{\mathrm{S}} = 0$; and that of the insulator diverges: $\widehat{\rho}_{\mathrm{I}} = \rho_0 \alpha_{\mathrm{I}} \widehat{I}$, $\alpha_{\mathrm{I}} \to \infty$ (here $\widehat{I}$ is the unit tensor).

The area fractions of the three constituents are assumed to satisfy the conditions $p_{\mathrm{I}} \neq 0$, $p_{\mathrm{S}} < p_{\mathrm{c}}$, and $p_{\mathrm{M}} + p_{\mathrm{S}} \geqslant p_{\mathrm{c}}$, p_{c} being the percolation threshold. The macroscopic (or bulk-effective) resistivity tensor is then finite and nonvanishing. Because the microstructure is statistically isotropic, this tensor is isotropic and may be written in the form

$$\widehat{\rho}_{\mathrm{e}} = \rho_0 \begin{pmatrix} \alpha_{\mathrm{e}} & -\beta_{\mathrm{e}} \\ \beta_{\mathrm{e}} & \alpha_{\mathrm{e}} \end{pmatrix}. \tag{2}$$

Here, α_{e} is the dimensionless macroscopic Ohmic resistivity and β_{e} is the dimensionless macroscopic Hall resistivity. For simplicity, we will adhere to these dimensionless representations of resistivities throughout the paper.

At first thought, one might expect that in such a composite the behavior of the macroscopic response would depend on the combined area fraction of the M and S constituents, $p_{\mathrm{M}} + p_{\mathrm{S}}$, in a qualitatively similar fashion to the behavior in an M/I composite, namely, a vanishing macroscopic conductivity for $p_{\mathrm{M}} + p_{\mathrm{S}} < p_{\mathrm{c}}$ and a single type of behavior of the nonvanishing, finite conductivity for $p_{\mathrm{M}} + p_{\mathrm{S}} > p_{\mathrm{c}}$. However, a more careful reflection shows that this is not the case in the regime of a strong applied magnetic field. In order to understand why, consider a section of the composite where a perfectly conducting inclusion is entirely surrounded by the normal conductor constituent. The current density and electric field at any point of the interface between the M and S phases can be decomposed into components normal and tangential to the interface. Because the M conductor is isotropic, the tangential component of the electric field on the M side of the interface, $E_{\mathrm{t}}^{(\mathrm{M})}$, can then be related to the tangential and normal components of the current density, $J_{\mathrm{t}}^{(\mathrm{M})}$ and $J_{\mathrm{n}}^{(\mathrm{M})}$, by the resistivity tensor given in Eq. (1), namely, $E_{\mathrm{t}}^{(\mathrm{M})} = \rho_0(\alpha_{\mathrm{M}} J_{\mathrm{t}}^{(\mathrm{M})} - \beta_{\mathrm{M}} J_{\mathrm{n}}^{(\mathrm{M})})$. But because the electric field in the perfect conductor vanishes, and the tangential component of the electric field at the interface between the phases is continuous, it follows that $E_{\mathrm{t}}^{(\mathrm{M})} = 0$, and one is led to the result

$$\frac{|J_{\mathrm{n}}^{(\mathrm{M})}|}{|J_{\mathrm{t}}^{(\mathrm{M})}|} = \frac{\alpha_{\mathrm{M}}}{|\beta_{\mathrm{M}}|} = \frac{1}{|H|} \quad \text{at an M/S interface.} \tag{3}$$

One then concludes that in the strong-field limit, when $|H| \gg 1$, the components of the current at the M/S interface satisfy $|J_{\mathrm{n}}^{(\mathrm{M})}|/|J_{\mathrm{t}}^{(\mathrm{M})}| \ll 1$; to leading order, currents tend *not* to flow through S inclusions embedded in M surroundings, and the behavior of these S inclusions resembles that of insulating ones. In such a state of affairs, currents will flow mainly through regions occupied by the M conductor, avoiding any isolated S inclusions. Consequently, it is of great significance whether a percolating backbone entirely consisting of the normally conducting phase exists in the system. Accordingly, the behavior of the macroscopic response is expected to depend not on the combined area fraction $p_{\mathrm{M}} + p_{\mathrm{S}}$, but only on p_{M} itself. In the strong-field limit it is thus expected that, when $p_{\mathrm{M}} < p_{\mathrm{c}}$, α_{e} will be much greater than when $p_{\mathrm{M}} > p_{\mathrm{c}}$, because in the former case all current paths necessarily pass through both M and S regions, where the S inclusions tend to *increase* the resistance to current flow! This suggests that when $p_{\mathrm{M}} = p_{\mathrm{c}}$, there should appear an abrupt change in the behavior of α_{e}, i.e., that $p_{\mathrm{M}} = p_{\mathrm{c}}$ is a critical point. The objective of the presented numerical study was to test this conjecture.

III. DISCRETE NETWORK MODEL

A common numerical approach employed for investigating a disordered composite conductor is the introduction of a discrete model that consists of a lattice of sites or bonds, upon which resistor-like discrete circuit elements are positioned randomly and independently, forming a random network. It should be emphasized that one cannot expect such a discrete model to produce the same value of the macroscopic resistivity tensor as that of a continuum composite with the same electrical and microstructural characteristics. It *is* expected, however, that the model and the real continuum composite which it is supposed to represent belong to the same universality class of the investigated phenomenon. That is, it is expected that, provided the model is properly designed, it will exhibit the same types of macroscopic behavior as the actual physical composite. In particular, if one considers critical phenomena we expect that both systems will be characterized by the same critical exponents. Finding an appropriate discrete model for a composite with a spatially varying nonscalar resistivity tensor is nontrivial. In fact, some models suggested in the past turned out to be inadequate for representing systems with a Hall effect.[11–13]

The model used in this study was first proposed by Sarychev and co-workers.[9,10] In this model, the discrete circuit element is a cross-like resistor, as shown in Fig. 1. To each of the four terminals of this circuit element there corresponds an electric potential, and currents flow into or out of these terminals, along the four bonds. The four potentials and currents are linearly related by a resistance matrix that mimics the relation between the local electric field and current density in a continuous, isotropic homogeneous conductor that exhibits a linear response. For an element representing the M constituent, the resistance matrix $\widehat{r}_{\rm M}$ is given by

$$\widehat{r}_{\rm M} = \rho_0\alpha_{\rm M}\left[\frac{1}{2}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} + \frac{H}{4}\begin{pmatrix}0&-1&0&1\\1&0&-1&0\\0&1&0&-1\\-1&0&1&0\end{pmatrix}\right]. \tag{4}$$

The matrix $\widehat{r}_{\rm M}$ is thus a sum of two terms. The first term represents Ohm's law. The second term imitates the Hall effect by requiring that a current along a bond of a given M element induces a potential drop across the perpendicular bond of that element. A perfect conductor is modeled by a circuit element having an extremely low resistance relative to that of the element modeling the normal conductor, the Hall term being excluded:

$$\widehat{r}_{\rm S} = \rho_0\alpha_{\rm S}\frac{1}{2}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}, \quad \alpha_{\rm S} \ll \alpha_{\rm M}. \tag{5}$$

As illustrated in Fig. 1, the network is formed by randomly placing the centers of the three kinds of circuit elements at the sites of a simple square lattice and electrically connecting the adjacent terminals of neighboring elements. The percolation threshold for this independent random-sites square lattice is $p_c = 0.592\,746$.[1]

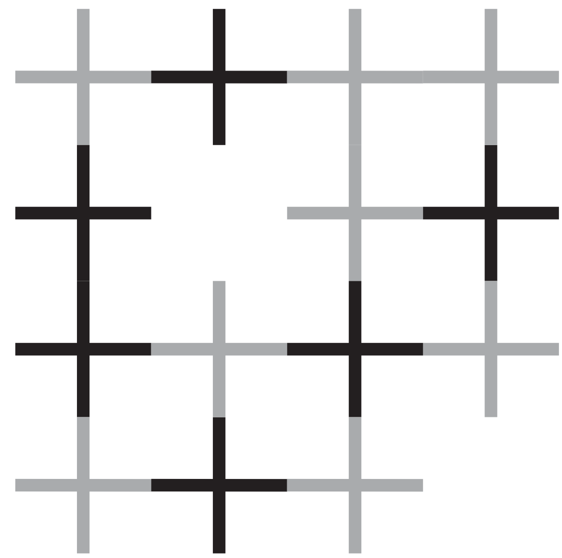

FIG. 1. A typical 4 × 4 section of the random network used for modeling the 2D M/S/I composite. The centers of discrete circuit elements with a cross-like shape are randomly placed at the sites of a simple square lattice, adjacent terminals of neighboring elements being electrically connected. Black crosses represent S elements; gray crosses, M elements; and voids, I elements. The potentials at the four terminals of a conducting element and the currents flowing into each of those terminals are related to each other by a resistance matrix given by Eq. (4) for an M element and by Eq. (5) for an S element. This kind of network has the advantage of enabling the construction of a dual network with the same microstructure. Such a network will satisfy a discrete analog of the duality relation of local electrical responses that is automatically satisfied in any real 2D continuous conducting medium. This is required to ensure that the network and the physical medium it models are members of the same universality class of critical behavior; see Ref. 10 for a more detailed discussion of this property.

Ensembles of finite-size networks with prescribed values of the electrical properties of circuit elements and area fractions were constructed by a Monte Carlo sampling procedure. The conductance properties of each network were obtained using the transfer matrix method. This method was originally devised by Derrida and co-workers for computing effective electrical properties of networks consisting of ordinary resistors.[14–16] We modified the original algorithm and adapted it to the structure of the discrete model used in this study and to the presence of a Hall effect. For the application of the method, the resistance matrices (4) and (5) of the M and S circuit elements were transformed into corresponding conductance matrices. By setting values that satisfy the condition $\alpha_{\rm S} \ll \alpha_{\rm M}$, the resulting finite conductance of the S element is much greater than the resulting Ohmic and Hall conductances of the M element. The conductance of the I element is set to zero. To calculate the conductance properties of a given network, the following boundary conditions were imposed: all external terminals at one edge of the square $L \times L$ network were grounded, whereas all external terminals at the opposite edge were maintained at the same fixed potential. This is equivalent to applying a prescribed average voltage between these opposite edges.

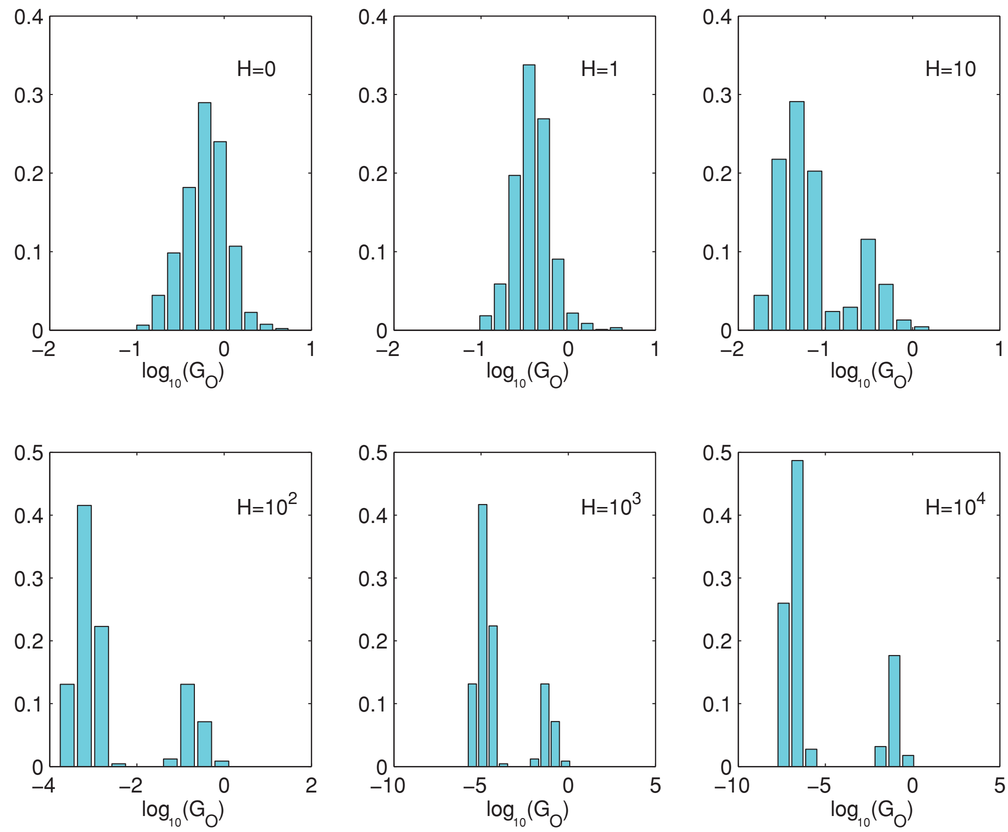

FIG. 2. (Color online) Distributions of Ohmic conductances G_O in ensembles of M/S/I networks simulated at various magnetic fields: $H = 0,1,10,10^2,10^3,10^4$. The size of these networks was $L = 10$, and they were simulated at $p_M = 0.4$, $p_S = 0.3$. The histogram columns represent the relative occurrence frequency of the different sampled values. When $|H|$ becomes large compared to L, the sampled Ohmic conductances split into two sets differing by orders of magnitude. (Note that the abscissa shows the base 10 logarithm of G_O.) A double-peaked distribution is then formed in which the location of the right peak is field independent, while the location of the left peak decreases with the field. Compare to the distributions shown in Figs. 3 and 4.

Periodic boundary conditions were imposed on the external terminals at the other two edges of the network. The total current flowing between each pair of opposite edges was calculated and used to obtain the macroscopic Ohmic and Hall conductances of the network. The distributions of these conductances for ensembles of networks having the same finite size were recorded and used to calculate average values as functions of magnetic field and network size.

IV. RESULTS FOR THE DISTRIBUTIONS OF SYSTEM-SPANNING PATHS AND CONDUCTANCES

Three kinds of percolating M/S/I finite-size networks with a finite conductance can be distinguished according to their system-spanning paths: (a) a network in which all system-spanning paths consist entirely of M circuit elements (to be referred to as an M network), (b) a network in which all system-spanning paths consist of both M and S elements (an MS network), and (c) a network in which both kinds of paths exist (an M&MS network).

Simulations performed with networks of size $L = 60$, where the M constituent was below, at, and above p_c and the S constituent was not too close to p_c, yielded the following distribution of system-spanning paths. For $p_M < p_c$, all the networks were MS networks. For $p_M = p_c$, about 40% were MS networks while the rest were M&MS networks. For $p_M > p_c$, about 0.5% were MS networks, the rest being M&MS networks. No perfectly conducting networks were found, nor were M networks found.

These results are in accordance with what one would expect to find in a real macroscopic M/S/I composite for the cases where $p_M + p_S > p_c$ and $p_S < p_c$, i.e., that the sample necessarily has a nonvanishing finite conductance, and that system-spanning paths consisting of both M and S regions necessarily exist in the sample. Such results are invariably found when the simulated networks are large enough. Relying on these findings for $L = 60$ networks, the M/S/I networks of much greater size $L = 250$, which were eventually simulated for studying the strong-field response in the different regimes of p_M, were all assumed to be of the MS or M&MS kind and not of the M kind (networks of the latter kind, if they appeared, would have to be excluded from the calculations).

To investigate the distributions of network conductances, simulations were performed of networks at given constituent compositions for different values of network size L and magnetic field strength H. Figures 2, 3, and 4 depict some of the data collected from these simulations. The distributions of Ohmic conductances G_O in each figure correspond to networks with a given size L simulated at six different values of H, ranging from $H = 0$ to $H = 10^4$.

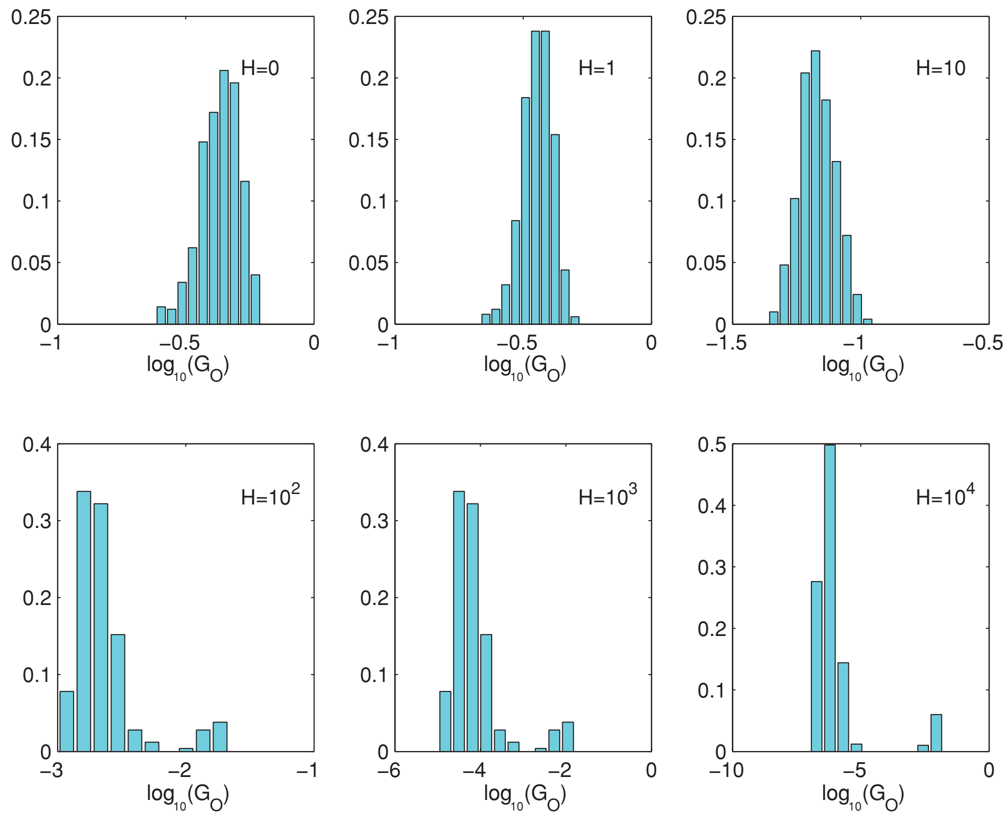

FIG. 3. (Color online) Distributions of Ohmic conductances $G_{\rm O}$ obtained from simulations similar to those described in Fig. 2, but where the size of the networks was $L = 50$.

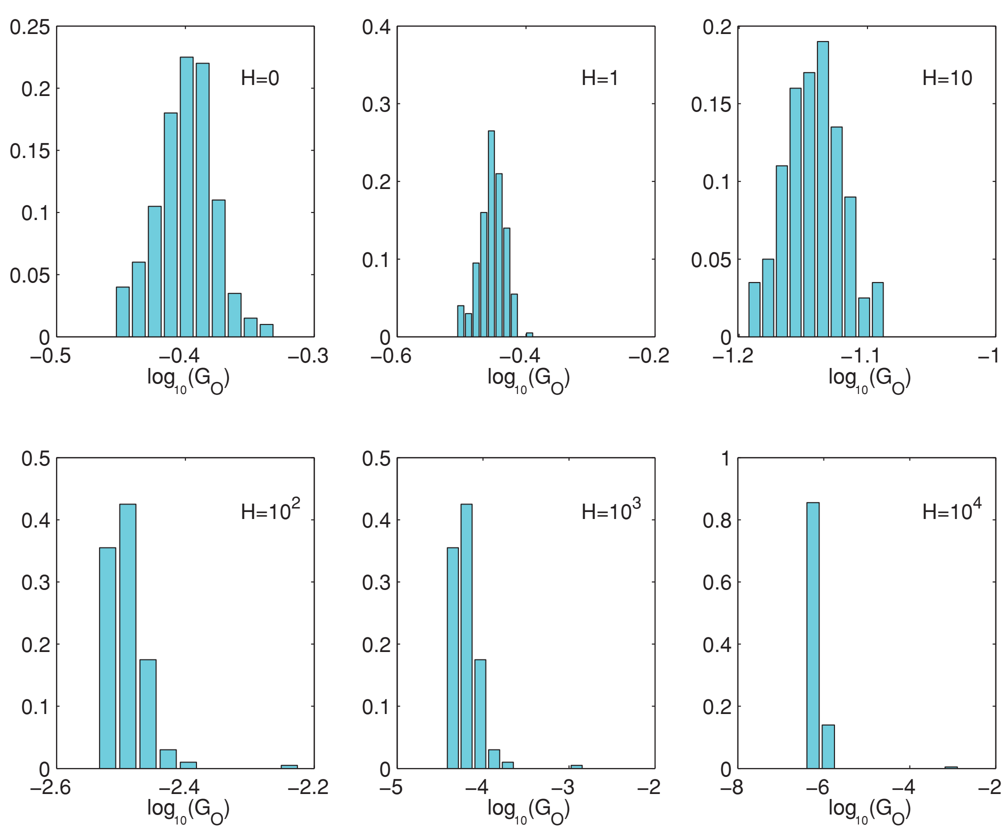

FIG. 4. (Color online) Distributions of Ohmic conductances $G_{\rm O}$ obtained from simulations similar to those described in Fig. 2, but where the size of the networks was $L = 200$.

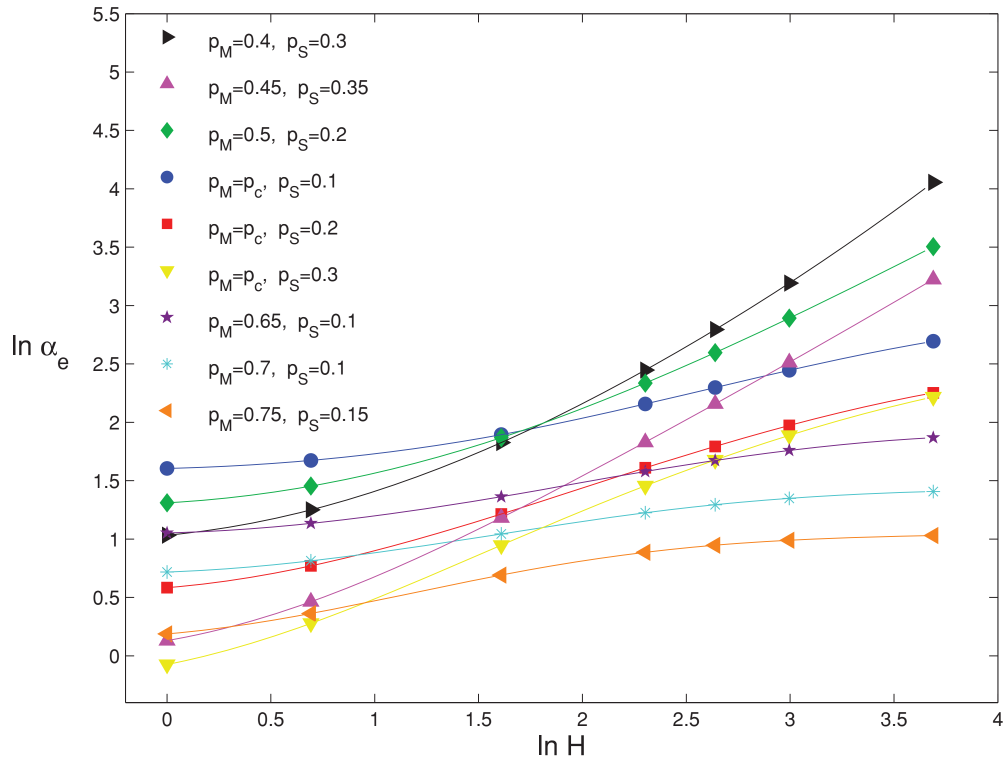

FIG. 5. (Color online) Logarithmic plot of the macroscopic Ohmic resistivity α_e as a function of the field strength H for M/S/I networks of size $L = 250$ and various constituent compositions. The networks were simulated at $H = 1,2,5,10,14,20,40$. The relative statistical errors of the effective Ohmic resistivities themselves are of the order of 1% (error bars are not shown). The curves are merely guides to the eye. The numerical results that correspond to the restricted range of moderate fields, $10 \leqslant H \leqslant 40$, imply that, in the asymptotic limit, α_e saturates when $p_M > p_c$; that α_e increases as a function of the magnetic field when $p_M \leqslant p_c$; and that this increase is more rapid when $p_M < p_c$ than when $p_M = p_c$.

It is clear that when $|H|$ becomes large compared to L, the distribution of Ohmic conductances is double peaked, with the two peaks characterized by distinct orders of magnitude. The larger conductances are distributed around a peak which is found to be field independent, whereas the smaller conductances are distributed around a peak which decreases with the field. When L is relatively small compared to $|H|$, neither of the two peaks has negligible weight. Obviously, if one had to consider such a double-peaked distribution, in which two parts of comparable weights differ by orders of magnitude, it would be unreasonable to regard the mean of the distribution as representing the physical properties of a real macroscopic sample. One would naturally like to consider only one of the two peaks, hoping that it represents the real system behavior in the limit $L \to \infty$, and that the other peak does not appear in that limit. The data suggest that this is indeed the case. For a given value of $|H|$, the weight of the conductances distributed around the field-independent peak diminishes considerably with increasing L, as can be seen in the figures. In fact, the M/S/I networks of size $L = 250$, which were simulated for the examination of the assumed critical behavior, did not exhibit a splitting of the ensemble Ohmic conductances into a double-peaked distribution.

The distributions of Hall conductances do not exhibit a double-peaked character. However, at very large values of $|H|$, notably when $p_M < p_c$, the widths of these distributions broaden out, sometimes ranging over two orders of magnitude.

A similar splitting of Ohmic conductances of random-resistor networks sampled at strong magnetic fields was found in past simulations of three-dimensional (3D) M/I percolating systems.[17] This is most probably the result of boundary effects related to the intersection of conducting paths in the form of loops and blobs with the external equipotential boundaries. Such intersections are known to enhance the typical conductivity of regions adjacent to the boundary in comparison to the typical conductivity evolving in the rest of the network at strong magnetic fields.[17]

When simulating finite-size networks at strong magnetic fields, one encounters another problematic effect. It is known from theoretical considerations (see, for example, Refs. 18 and 19) that in an inhomogeneous conductor where Hall effects exist, they induce strong distortions of current flows. The linear scale of these distortions may greatly increase as $|H|$ increases to very large values. Furthermore, the study of 3D M/I random-resistor networks reported in Ref. 17 suggested that the current fluctuations are governed by a typical magnetic correlation length $|H|^{\nu_H}$ (ν_H is a characteristic exponent), and that the macroscopic response of a finite network is governed by a scaling function of the scaling variable $L/|H|^{\nu_H}$. If $|H|^{\nu_H}$ is very large compared to L, particular local microstructures may strongly affect the macroscopic behavior despite the random distribution of constituents. Particular distorted current flows of this kind might then extend over relatively large spatial scales which can exceed the percolation correlation length

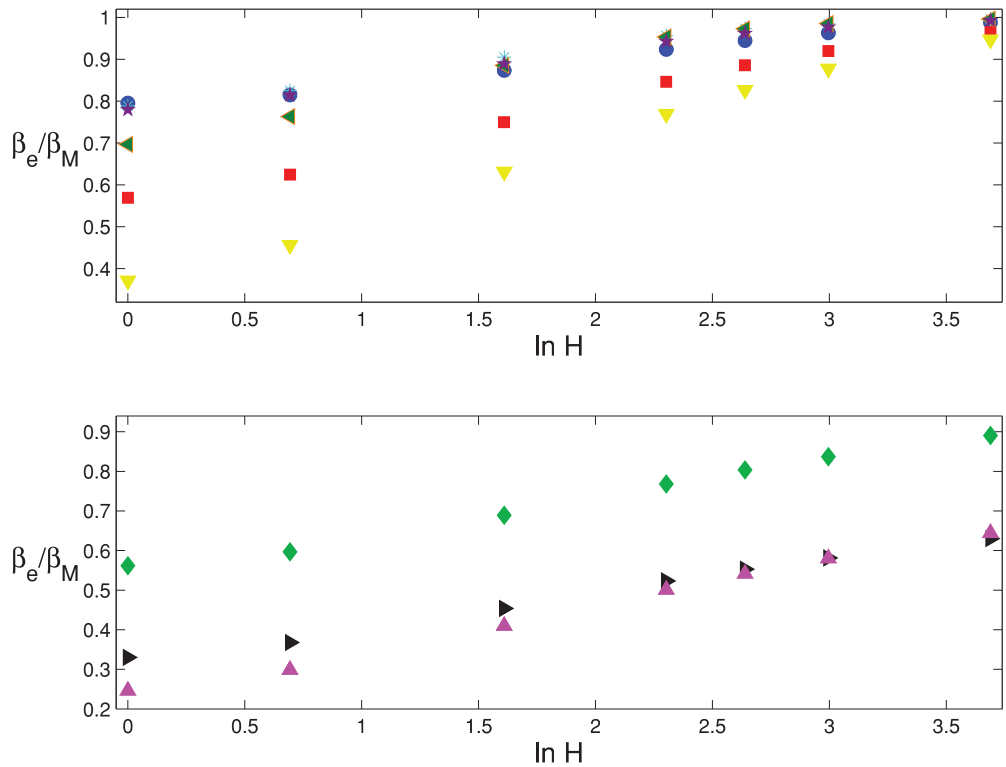

FIG. 6. (Color online) Semilogarithmic plot of β_e/β_M as a function of H at $p_M \geqslant p_c$ (top) and $p_M < p_c$ (bottom) for $H = 1,2,5,10,20,40$, as obtained from the simulations described in Fig. 5. The symbols used to mark the points refer to the same constituent compositions that were shown in Fig. 5. The relative statistical errors of the effective Hall resistivities are of the order of 1% for $p_M < p_c$ and of the order of 0.1% for $p_M \geqslant p_c$. The numerical results imply that, in the asymptotic limit, one should expect that $\beta_e \approx \beta_M$ for $p_M \geqslant p_c$ and $\beta_e \approx k\beta_M$ with $0 < k < 1$ for $p_M < p_c$.

or might even extend over the entire network, significantly affecting its effective resistivity. Such occurrences result in a network that is not self-similar and make it inappropriate for representing a real material, where all typical length scales are much smaller than the total system size.

We could detect quantitatively the undesirable consequences of this effect by exploiting the same model for simulating a 2D composite made of two normal conductors and comparing the numerical results to the known exact results for the special case where these two conductors have equal area fractions.[20,21] The comparison revealed deviations of the effective resistivity tensor of finite networks from the true macroscopic properties already at a field corresponding to $H \approx 400$, in spite of the network size being as large as $L = 250$. The deviations were found to be even greater for binary networks of smaller size $L = 60$, where they appeared already at a field corresponding to $H \approx 100$. It can be expected that, for M/S/I networks, the value of $|H|$ beyond which such undesirable effects occur will be smaller than the corresponding one for binary networks. In a network where all the circuit elements conduct, effects ensuing from few large-scale current distortions might be compensated by other currents flowing in the system. But in an M/S/I network there are fewer current-carrying paths, and therefore a fluctuation of this kind appearing in one of them will have a much greater effect on the overall electrical response.

V. DEPENDENCE OF THE MACROSCOPIC RESISTIVITY TENSOR ON THE MAGNETIC FIELD

To study the field dependence of the macroscopic resistivity tensor of the M/S/I composite, networks of size $L = 250$ were simulated, in which p_M was below, at, and above p_c, the networks being subject to various magnetic fields. This network size was the largest that still afforded a practical time consumption of the computations. Due to the problematic effects of far-reaching current fluctuations obscuring the macroscopic electrical response, as explained in Sec. IV, we were compelled to regard only networks that were simulated at moderate values of $|H|$. Only then are the length scales of such fluctuations tolerable in comparison with the network size, which is necessary in order that those networks can reliably mimic the behavior of a real macroscopic composite.

Figure 5 depicts the macroscopic Ohmic resistivity α_e as a function of H for various constituent compositions of these simulated networks. As can be seen, the restriction to moderate values of the magnetic field precludes an evaluation of explicit exponents that characterize the asymptotic field dependence of α_e. That is because at such values of H the asymptotic term does not yet dominate, and terms higher than the leading one apparently contribute non-negligibly to the effective response. Nonetheless, even at this range of moderate magnetic fields, three distinct behaviors of α_e can be clearly identified by focusing upon the data in the range $10 \leqslant H \leqslant 40$. For $p_M > p_c$, α_e evidently saturates, and its saturation value increases as p_M approaches p_c from above. By contrast, for $p_M \leqslant p_c$ there appears a nonsaturating dependence on H. An inspection of the slopes of the curves shows that, when $p_M < p_c$, the macroscopic magnetoresistance increases more rapidly as a function of the magnetic field than when $p_M = p_c$. These numerical results, therefore, provide evidence for the expected critical point in the strong-field behavior of α_e which is determined by the percolation threshold of the normal conductor. They corroborate the prediction of the SEMA asymptotic analysis.[3,5]

Figure 6 depicts the ratio of the macroscopic Hall resistivity, β_e, to the Hall resistivity of the M component, β_M, as a function of H for the same simulated networks. Once again, the restriction in the range of fields does not enable an evaluation of the explicit asymptotic field dependence. Nevertheless, it is rather clear from the plotted data that these numerical results, too, are consistent with the asymptotic SEMA predictions.[3,5] The results indicate that for $p_M \geqslant p_c$, $\beta_e \approx \beta_M$, while for $p_M < p_c$, $\beta_e \approx k\beta_M$ with the positive coefficient $k < 1$ depending on p_M and p_S.

VI. SUMMARY AND DISCUSSION

Our study exploited computer simulations of 2D M/S/I random networks of four-terminal cross-like elements satisfying $p_S < p_c$ and $p_M + p_S \geqslant p_c$ in order to test the expectation that the strong-field macroscopic magnetoresistance exhibits a critical behavior that depends on the area fraction of the M constituent by itself.

The major difficulty that was encountered in those simulations is the effect arising from the appearance of large-scale current distortions within a composite conducting medium subject to strong magnetic fields. Because of the relatively small number of current-carrying paths in M/S/I networks where $p_M \leqslant p_c$, this troubling phenomenon seems to have a greater undesirable influence on the electrical characteristics of these networks than on the characteristics of similar networks where $p_M > p_c$ or, more generally, on networks where all the circuit elements conduct.

The encountering of this problem, which compels the consideration of networks at not very strong fields, precluded an explicit quantitative evaluation of the asymptotic field dependences of the elements of the macroscopic resistivity tensor. Nevertheless, even the results corresponding to the restricted range of moderate magnetic fields clearly indicate the existence of the conjectured transition in the behavior of the macroscopic Ohmic resistivity, a transition from a nonsaturating to a saturating dependence on the strong magnetic field as p_M crosses p_c from below.

The results of this study should motivate experimental work aimed to observe the predicted critical phenomenon. The experimental sample would be a thin film with a thickness much smaller than the typical size of the internal homogeneous regions. The most obvious candidate that comes into mind for the S component is a type II superconductor with a high upper critical magnetic field. This would require an experimental design at extremely low temperatures. However, our simulations actually imply that in order to observe the typical behavior, one is not bound to using a strict superconductor as the S constituent. Rather, one may use three constituents with resistivities that satisfy the following more relaxed sequence of inequalities: $0 < \alpha_S, |\beta_S| \ll \alpha_M \ll |\beta_M| \ll \alpha_I$. Accordingly, a doped semiconductor with a large Hall mobility—for example, a silicon-doped GaAs—can serve as the M constituent; while a normal metal with a low Hall mobility—copper, for instance—can serve as the S constituent. Voids etched in the metallic layer would serve as the insulating constituent. Moreover, the simulations indicate that even moderate strengths of the applied magnetic field should suffice for observing the main effects, even if the extreme asymptotic limit will not be reached in such circumstances.

The sample size in experiments of this sort will have to be relatively large for the reasons discussed before. However, it will also be interesting to experiment on small-size samples in order to observe the presumed effects of large-scale current and electric field fluctuations, as well as the presumed boundary effects, leading to the split of Ohmic conductances into two sets differing by orders of magnitude when the magnetic field is strong enough.

We note again that the characteristic spatial scale of the three types of homogeneous regions constituting the M/S/I composite we have studied is macroscopic, and that the particular shape of any of these regions is unimportant. This should make the fabrication of samples appropriate for experimentation relatively simpler than the fabrication of composite media with atomic or microscopic disorder or composites with subtle periodic microstructures.

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