Proposal for nonlocal electron-hole turnstile in the quantum Hall regime

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We present a theory for a mesoscopic turnstile that produces spatially separated streams of electrons and holes along edge states in the quantum Hall regime. For a broad range of frequencies in the nonadiabatic regime the turnstile operation is found to be ideal, producing one electron and one hole per cycle. The accuracy of the turnstile operation is characterized by the fluctuations of the transferred charge per cycle. The fluctuations are found to be negligibly small in the ideal regime.

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I. INTRODUCTION

Transport along edge states in the integer quantum Hall regime has recently attracted large interest. The unidirectional transport properties of the edge states together with the possibility of using quantum point contacts as beam splitters has motivated a number of experiments on electronic analogs of optical interferometers, such as single-particle Mach-Zehnder[1] and two-particle Hanbury-Brown-Twiss[2] interferometers. In the experiment by Altimiras *et al.*[3] the electronic-optic analog was supported by probing the nonequilibrium electronic distribution along the edge. Moreover, the prospect of entanglement generation in electronic two-particle interferometers[4] has provided a connection between quantum information processing and edge state transport.

Another important aspect of edge state transport is the high-frequency properties. The experiment of Gabelli *et al.*[5] confirmed the quantization of the charge relaxation resistance, predicted in Ref. 6. In a pioneering experiment Fève *et al.*[7] demonstrated that a mesoscopic capacitor coupled to an edge state can serve as an on-demand source for electrons and holes, operating at gigahertz frequencies. The experiment[7] was followed by a number of theoretical works investigating the accuracy of the on-demand source[8,9] and, e.g., particle colliders with two synchronized sources.[10] The successful realization of the electronic on-demand source also motivated new work[11] on entanglement generation on demand in the quantum Hall regime.[12] A key feature of Ref. 7 is that the on-demand source produces a single stream with alternating electrons and holes; the current has no dc component, only ac components. For quantum information tasks it would be desirable to have an on-demand source that produces two separate streams, one with electrons and one with holes. Such a source implemented in edge states and operating at gigahertz frequencies would also be of interest for metrological applications.

In this work we propose such an on-demand source. It comes as a nonlocal electron-hole turnstile (see Fig. 1) consisting of a double barrier (DB) formed by two quantum point contacts modulated periodically in time. A bias voltage is applied between the two sides of the turnstile, to have one resonant level of the DB in the bias window. An ideal operation cycle of the turnstile is shown in Fig. 1: (i) Contact A is opened and one electron is transmitted into the region inside the DB, leaving a hole behind in the filled stream of electrons continuing toward terminal 3. (ii) Contact A closes and subsequently (iii) B opens and the electron trapped inside the DB is transmitted out through B and (iv) continues to terminal 2. Thus during the cycle exactly one hole and one electron are emitted into spatially separate terminals.

Since the early turnstile experiments[13] there has been large progress in operation speed and accuracy. Recent turnstiles or single electron pumps have demonstrated operation at gigahertz frequencies[14] and single parameter pumping.[15,16] The observed trend with increasing accuracy at large magnetic fields[17] provides additional motivation for our quantum Hall turnstile.

Our proposal has a number of key features which have not been addressed together in earlier theoretical[18] or experimental[14–17] works. First and foremost, the four-terminal edge state geometry gives spatially separated streams of electrons and holes. This can be investigated by independent measurements of electron and hole currents as well

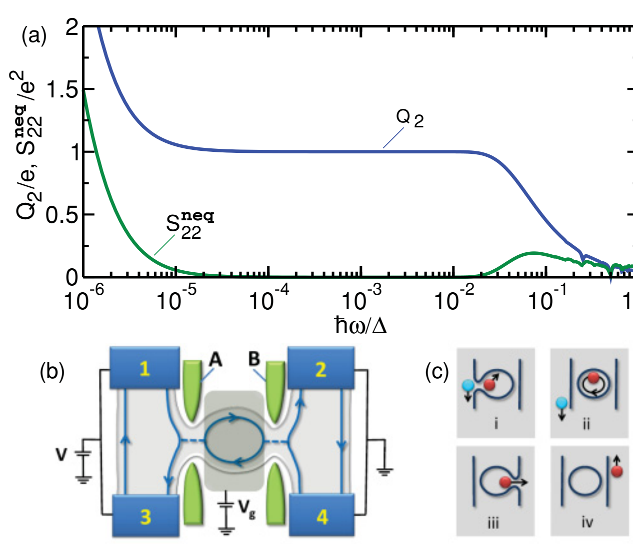

FIG. 1. (Color online) (a) Transferred charge Q_2 and charge fluctuations S_{22}^{neq} per period as a function of frequency with $T_A(t), T_B(t)$ shown in Fig. 2(a) for $kT \ll \Delta$ (see text). (b) Schematic of the four-terminal turnstile with quantum point contacts A and B subjected to time-dependent voltages. The top gate (transparent) is kept at a constant voltage V_g and a bias V is applied between the two sides. Direction of edge state transport shown with arrows. (c) Steps in ideal turnstile cycle, transporting one hole (blue/light gray) to terminal 3 and one electron (red/gray) to 2.

as current auto and cross correlations. Second, we do not restrict ourselves to the tunnel limit but consider operation at arbitrary contact transparencies, allowing for ideal operation at higher drive frequencies. Finally, by taking a time-dependent scattering approach we can analyze both charge currents and correlations at arbitrary drive frequencies within the same framework. In particular, we fully account for the fluctuations caused by the drive of the quantum point contacts, found to only marginally affect the ideal turnstile operation.

In the following we first present the turnstile system and discuss the time-dependent current and the charge transferred per cycle in different driving frequency regimes. Thereafter the fluctuations of the charge transfer are investigated.

II. TURNSTILE MODEL

We consider a DB turnstile implemented in a four-terminal conductor in the quantum Hall regime; see Fig. 1. Terminals 1,3 are biased at eV while 2,4 are grounded. Transport takes place along a single spin-polarized edge state. Scattering between the edges occurs at the two quantum point contacts A and B. The contacts A,B are created by electrostatic gates subjected to time-periodic voltages $V_A(t)=V_A^{\mathrm{dc}}-V_A^{\mathrm{ac}}\sin(\omega t)$ and $V_B(t)=V_B^{\mathrm{dc}}+V_B^{\mathrm{ac}}\sin(\omega t)$, π out of phase for optimal turnstile operation and with a period $\mathcal{T}=2\pi/\omega$.

The transport through the system is conveniently described within the Floquet scattering approach,[19] applied to a DB system in Refs. 19 and 20 with the focus on the quantum pumping effect. The time-dependent current flowing into terminal 2 is naturally parted into two components, $I_2(t)=I_2^{\mathrm{bias}}(t)+I_2^{\mathrm{pump}}(t)$. The $I_2^{\mathrm{bias}}(t)$ part is given by

$$I_2^{\mathrm{bias}}(t)=\frac{e}{h}\int dE|t_{21}(t,E)|^2[f_V(E)-f_0(E)] \qquad (1)$$

where $f_V(E)$ and $f_0(E)$ are the Fermi distributions of the biased and grounded terminals, respectively. In the absence of an applied bias $I_2^{\mathrm{bias}}(t)$ is thus zero. The dynamical scattering amplitude[20] $t_{21}(t,E)=t_B(t)\sum_{q=0}^{\infty}e^{i(2q+1)\phi(E)}L_q(t)t_A(t-[2q+1]\tau)$, with $L_q(t)=\prod_{p=1}^{q}r_A(t-[2p-1]\tau)r_B(t-2p\tau)$ for $q\geqslant 1$ and 1 for $q=0$, is the total amplitude for an electron injected from terminal 1 at energy E to be emitted into terminal 2 at a later time t. Here $\tau=L/v_D$ is the time of flight along the edge from A to B (and B to A), with v_D the drift velocity and L the length. The phase $\phi(E)=\phi_0+\pi E/\Delta$ where $\Delta=\pi\hbar v_D/L$ the resonant level spacing in the DB and ϕ_0 a constant phase, controlled by the top-gate potential V_g, determining the level positions. The component $I_2^{\mathrm{pump}}(t)$ is the pumped current, independent on bias. It is found to be negligibly small compared to $I_2^{\mathrm{bias}}(t)$ for $\omega\ll\Delta$, with zero dc component for all ω, and is only discussed in the context of the noise below. The current at terminal 3 is found similarly, with $I_3^{\mathrm{bias}}=-I_2^{\mathrm{bias}}(t+\mathcal{T}/2)$ and the transferred charge per cycle is $Q_2=-Q_3=\int_0^{\mathcal{T}}I_2(t)dt$.

The point contact scattering amplitudes $t_{A/B}(t)=i\sqrt{T_{A/B}(t)}$ and $r_{A/B}(t)=\sqrt{1-T_{A/B}(t)}$ are taken energy independent on the scale $\max\{kT,eV,\hbar\omega\}$, with T the temperature. Motivated by the successful modeling in Ref. 5, we describe the contacts A,B with saddle-point potentials[21] where $T_{A/B}(t)=[1+\exp(\{[V_{A/B}(t)-V_{A/B}^1]/V_{A/B}^0\})]^{-1}$ with $V_{A/B}^{0/1}$ properties of the potential. Throughout the paper it is assumed that the product $T_A(t)T_B(t)\ll 1$. A typical driving scheme is shown in Fig. 2(a). The top gate suppresses charging effects,[5,7] supporting our noninteracting approximation.

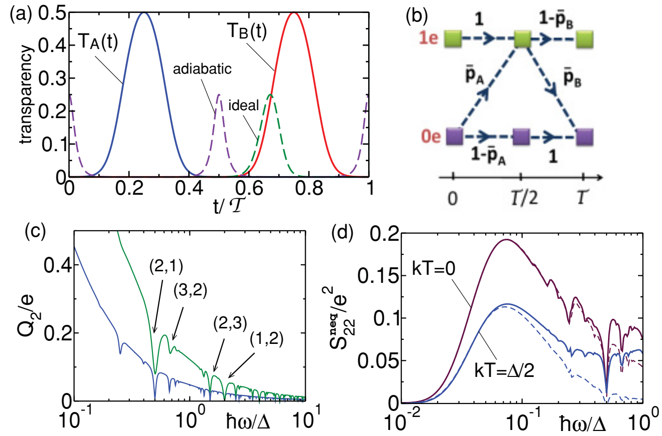

FIG. 2. (Color online) (a) Transparency $T_A(t)$ and $T_B(t)$ for saddle-point potential parameters $V_{A/B}^{\mathrm{dc}}-V_{A/B}^1=V_{A/B}^{\mathrm{ac}}=10V_{A/B}^0$. The time-dependent current $I_2(t)$ (arb. units) in the adiabatic and ideal turnstile regimes is shown (dashed lines). (b) Illustration of the discrete time model of Eq. (4) with probabilities $\bar{p}_A,\bar{p}_B$ and directions for transfer between states with 0 and 1 electrons in the DB region shown. (c) Charge Q_2 for high frequencies, displaying dips described by Eq. (7). Number of (laps, cycles) shown for four dips. Saddle-point parameters as in (a) (lower curve) and $V_{A/B}^{\mathrm{dc}}-V_{A/B}^1=5V_{A/B}^{\mathrm{ac}}/6=5V_{A/B}^0$ (upper curve). (d) Correlations S_{22}^{neq} with (solid line) and without (dashed line) pumping contribution for high frequencies at $kT=0,\Delta/2$ and saddle-point parameters as in (a).

III. CHARGE TRANSPORT REGIMES

In the rest of the paper we consider the case with $eV=\Delta$ giving one DB level inside the bias window, optimal for the ideal turnstile operation shown in Fig. 1. We can then perform the energy integral in Eq. (1) giving

$$\begin{aligned} I_2(t)&=(\Delta e/h)T_B(t)F(t-\tau),\\ F(t)&=T_A(t)+R_A(t)R_B(t-\tau)F(t-2\tau). \end{aligned} \qquad (2)$$

Quite remarkably, the current $I_2(t)$ depends only on the scattering probabilities $T_{A/B}(t)=1-R_{A/B}(t)$ of the contacts A/B at times earlier than t. The result is independent on temperature and holds for arbitrary driving frequency. The recursively defined $0\leqslant F(t)\leqslant 1$ is the probability that an electron injected in the bias window from terminal 1 at a time $t-2n\tau$ ($n\geqslant 0$ integer) is propagating away from A toward B at time t.

In the *adiabatic* transport regime, the dwell time of the particles in the DB is much shorter than the drive period $\mathcal{T}$. The maximum dwell time for particles injected in the bias window is $\sim\hbar/\{\Delta\min[T_A(t)+T_B(t)]\}$, the inverse of the minimum resonant level width (taken over one period). Thus at frequencies $\omega\ll\Delta\min[T_A(t)+T_B(t)]/\hbar$ the transport is adiabatic. The current is found by taking $\tau \to 0$ in Eq. (2), giving

$$I_2^{\text{ad}}(t) = (e\Delta/h)T_B(t)T_A(t)/[1 - R_A(t)R_B(t)]. \quad (3)$$

This is simply the instantaneous DB current. Importantly, the corresponding transferred charge per period $Q_2^{\text{ad}} \gg e$ (see Fig. 2), i.e., many particles traverse the DB during one period. From Eq. (3) and Figs. 1 and 2(a) it is clear that $Q_2^{\text{ad}} \propto 1/\omega$ and that $I_2^{\text{ad}}(t)$ flows around times when $T_A(t)T_B(t)$ is maximal. Consequently, for a driving where contacts A and B are never both open at the same time there is no adiabatic current flow, or equivalently the adiabatic frequency limit $\Delta\min[T_A(t) + T_B(t)]/\hbar \to 0$.

From this reasoning it follows equally that for frequencies in the *nonadiabatic* regime, $\omega \gg \Delta\min[T_A(t) + T_B(t)]/\hbar$, we can neglect the current flow during times when both contacts are open. This leads to the standard physical picture in terms of charging and discharging of the DB region: for the cycle $0 < t < \mathcal{T}$ (mod $\mathcal{T}$), (i) at times $0 < t < \mathcal{T}/2$ contact B is closed and charge is flowing into the DB region through A, (ii) at times $\mathcal{T}/2 < t < \mathcal{T}$ contact A is closed and charge is flowing out through B.

Focusing first on nonadiabatic frequencies much smaller than the level spacing, $\hbar\omega \ll \Delta$, the charge density inside the DB region is uniform and a calculation of the charge in the DB region, injected in the bias window, gives $Q(t) = eF(t)$. Thus $F(t)$ is just the probability to find an electron inside the DB. The time development of the charge is found from Eq. (2),

$$Q(t) = \begin{cases} p_A(t) + [1 - p_A(t)]Q(0) & \text{charging} \\ [1 - p_B(t)]\, Q(\mathcal{T}/2) & \text{discharging,} \end{cases} \quad (4)$$

where, e.g., $p_B(t) = 1 - \prod_{p=0}^{P_B} R_B(t - 2\tau p)$ is the probability that an electron inside the DB at time $\mathcal{T}/2$ has been transmitted out through contact B at time t, with $P_B = \text{int}[(t - \mathcal{T}/2)/(2\tau)]$. $p_A(t)$ and P_A are given analogously. The charge at the opening/closing is $Q(\mathcal{T}) = Q(0) = e\bar{p}_A(1 - \bar{p}_B)/(\bar{p}_A + \bar{p}_B - \bar{p}_A\bar{p}_B)$ and $Q(\mathcal{T}/2) = Q(0)/(1 - \bar{p}_B)$ where $\bar{p}_A = p_A(\mathcal{T}/2)$, $\bar{p}_B = p_B(\mathcal{T})$. Since $\tau \ll \mathcal{T}$, $Q(t - \tau) \approx Q(t)$ and the current is $I_2(t) = (\Delta/h)T_B(t)Q(t)$, shown in Fig. 2(a).

For times t not close to the opening times of A and B, i.e., $P_A, P_B \gg 1$, we can write, e.g., $1 - p_B(t) = e^{\sum_{p=0}^{P_B} \ln[R_B(t-2\tau p)]} \approx e^{(1/2\tau)\int_{\mathcal{T}/2}^{t} \ln[R_B(t')]dt'}$. It is instructive to compare this with the charging and discharging of a classical RC circuit with a capacitance C and a slowly time-varying resistance $\mathcal{R}(t)$, for which $e^{-\int_{\mathcal{T}/2}^{t}[C\mathcal{R}(t')]^{-1}dt'}$ corresponds to $1 - p_B(t)$. This gives a capacitance $C = e^2/\Delta$ and a resistance $\mathcal{R}(t) = (h/e^2)/\ln[1/R_B(t)]$, providing a turnstile analogy of the models for the on-demand source discussed in Refs. 7 and 8.

The transferred charge per cycle, $Q(\mathcal{T}/2) - Q(\mathcal{T})$, is

$$Q_2 = -Q_3 = e\frac{\bar{p}_A\bar{p}_B}{\bar{p}_A + \bar{p}_B - \bar{p}_A\bar{p}_B}. \quad (5)$$

This gives that for $\omega \ll \omega_A^{\text{max}}, \omega_B^{\text{max}}$ with $\hbar\omega_{A/B}^{\text{max}} = \Delta\min\{1, \int_0^{\mathcal{T}}(dt/\mathcal{T})\ln[1/R_{A/B}(t)]\}$, we have $\bar{p}_A, \bar{p}_B = 1$ and $Q_2 = -Q_3 = e$, i.e., exactly one electron and one hole are transferred. This yields a frequency interval $\Delta\min[T_A(t) + T_B(t)]/\hbar \ll \omega \ll \omega_A^{\text{max}}, \omega_B^{\text{max}}$ for the ideal turnstile cycle shown in Fig. 1. For higher frequencies electrons do not have time to completely charge or discharge the DB region and $Q_2 < e$.

Importantly, for tunneling contacts $T_A(t), T_B(t) \ll 1$ and $\omega \ll \Delta/\hbar$ we can directly expand $F(t - 2\tau) = F(t) - 2\tau dF(t)/dt$ in Eq. (2) and arrive at

$$\frac{d}{dt}\begin{pmatrix} P_1 \\ P_0 \end{pmatrix} = \begin{pmatrix} -\Gamma_A(t) & \Gamma_B(t) \\ \Gamma_A(t) & -\Gamma_B(t) \end{pmatrix}\begin{pmatrix} P_1 \\ P_0 \end{pmatrix}, \quad (6)$$

where $P_1(t) = F(t) = 1 - P_0(t)$ and $\Gamma_{A/B}(t) = T_{A/B}(t)\Delta/h$. This is a master equation with time-dependent tunneling rates, investigated in, e.g., Refs. 22, 23, and 16.

At frequencies $\omega \sim \Delta/\hbar$ the expressions in Eqs. (4) and (6) break down and transport through higher-/lower-lying resonances becomes visible, manifested as sharp dips in the transferred charge as a function of frequency; see Fig. 2(c). The most pronounced set of dips, at frequencies

$$\hbar\omega = \Delta(2n + 1 \pm 1/m), \quad (7)$$

results from electrons, which after being injected at A at maximal $T_A(t)$, circulate around the DB region m times during $m(2n + 1) \pm 1$ periods before escaping back out at A [at maximal $T_A(t)$], not transferring any charge.

IV. NOISE AND FLUCTUATIONS

For a long measurement time $t_0 = N\mathcal{T}, N \gg 1$, to characterize the accuracy of the turnstile it is important to investigate not only the average charge transferred per cycle, $Q_2 = (1/N)\int_0^{t_0} dt I_2(t)$, but also the fluctuations,[19,20,24] experimentally accessible via current correlations.[25] To this end we first write the current[19] $I_2(t) = \sum_q i_{2,q}\exp(iq\omega t)$, with $i_{2,q} = (e/h)\int dE j_{2,q}(E)$ and $j_{2,q}(E) = j_{2,q}^{\text{bias}}(E) + j_{2,q}^{\text{pump}}(E)$ where

$$j_{2,q}^{\text{pump}}(E) = \sum_n \left[T_{21}^{q,n}(E) + T_{24}^{q,n}(E)\right][f_0(E_n) - f_0(E)],$$
$$j_{2,q}^{\text{bias}}(E) = \sum_n T_{21}^{q,n}(E)[f_V(E_n) - f_0(E_n)]. \quad (8)$$

Here $E_n = E + n\hbar\omega$, $T_{2\alpha}^{q,n}(E) = t_{2\alpha}^*(E, E_n)t_{2\alpha}(E_{-q}, E_n)$, $\alpha = 1,4$, and $t_{2\alpha}(E_m, E) = \int_0^{\mathcal{T}}(dt/\mathcal{T})e^{im\omega t}t_{2\alpha}(t, E)$ with $t_{21}(t, E)$ given above and $t_{24}(t, E) = r_B(t) + t_B(t)\sum_{q=0}^{\infty} e^{i2(q+1)\phi(E)}L_q(t)r_A(t - [2q + 1]\tau)t_B(t - 2[q + 1]\tau)$. The current at terminal 3 is found similarly.

The autocorrelations of transferred charge at terminal 2 is $S_{22} = (1/N^2)\int_0^{t_0}\int_0^{t_0} dt dt'\langle\Delta I_2(t)\Delta I_2(t')\rangle$ where $\Delta I_2(t)$ is the current fluctuations.[26] Calculations following Ref. 19 give $S_{22} = S_{22}^{\text{neq}} + S^{\text{th}}$ with

$$S_{22}^{\text{neq}} = \frac{\mathcal{T}e^2}{h}\int dE\left[j_{2,0}[1 - 2f_0(E)] - \sum_q |j_{2,q}|^2\right] \quad (9)$$

and $S^{\text{th}} = 2\mathcal{T}(e^2/h)kT$ the thermal noise in the absence of both drive and bias. The autocorrelator S_{33} and the cross correlators $S_{32} = S_{23}$ are found similarly.

We first consider the correlations at $\hbar\omega, kT \ll \Delta$. In this regime the fluctuations are minimized for DB levels at energies $\Delta(n + 1/2)$, one level in the middle of the bias window. In particular we find that the pumping components $j_{2,q}^{\mathtt{pump}}(E)$ contribute negligibly to the correlations (dc component $i_{2,0}^{\mathtt{pump}}=0$) and hence $S_{22}^{\mathtt{neq}}=S_{33}^{\mathtt{neq}}\equiv S^{\mathtt{bias}}=(\mathcal{T}e^2/h)\int dE[j_{2,0}^{\mathtt{bias}}-\sum_q|j_{2,q}^{\mathtt{bias}}|^2]$. Importantly, the total cross correlator $S_{23}=-S^{\mathtt{bias}}$, independent on equilibrium thermal fluctuations. This allows for an independent investigation of the turnstile accuracy.

The reason for the negligible pumping noise can be understood as follows: the term $j_{2,q}^{\mathtt{pump}}(E)$ describes creation of electron-hole pairs close to the Fermi energy [due to the factor $f_0(E_n)-f_0(E)$ in Eq. (8)]. DB levels at $\Delta(n+1/2)$ imply completely off-resonance Fermi energy transport, strongly suppressing the electron-hole pair creation. Formally, the terms in Eq. (9) containing $j_{2,q}^{\mathtt{pump}}$ are found to be of order $(\hbar\omega/\Delta)\int_0^{\mathcal{T}}(dt/\mathcal{T})T_A(t)T_B(t)$ smaller than terms from $j_{2,q}^{\mathtt{bias}}$. This is supported by the numerics in Fig. 2.

In the *adiabatic* regime the correlations are found by inserting the frozen scattering amplitudes into the expression for $S^{\mathtt{bias}}$, giving the time integral over one period of the instantaneous DB shot noise.[26] In the *nonadiabatic* regime the full distribution of the transferred charge can be found from Eq. (4), describing the time evolution of the probability $F(t)=Q(t)/e$ to have one electron inside the DB. The possible processes taking the DB between charge states with 0 and 1 electrons at times $t=0$ and $t=\mathcal{T}/2$ (mod $\mathcal{T}$) are shown in Fig. 2. The full counting statistics for the charge transfer, described by a time-discrete master equation, is known.[23,27] The generating function for the probability distribution is

$$\xi(\lambda_2,\lambda_3)=N\ln[h+\sqrt{h^2+(1-\bar{p}_A)(1-\bar{p}_B)}],\qquad(10)$$

where $h=1-\{\bar{p}_A+\bar{p}_B-\bar{p}_A\bar{p}_B\exp[i(\lambda_2-\lambda_3)]\}/2$ and λ_2,λ_3 the counting fields. The cumulants are obtained by taking successive derivatives of $\xi(\lambda_2,\lambda_3)$ with respect to λ_2,λ_3. Here we focus on the second cumulant,

$$S^{\mathtt{bias}}=e^2\frac{\bar{p}_A\bar{p}_B[\bar{p}_A^2(1-\bar{p}_B)+\bar{p}_B^2(1-\bar{p}_A)]}{(\bar{p}_A+\bar{p}_B-\bar{p}_A\bar{p}_B)^3}.\qquad(11)$$

In the ideal regime, $\bar{p}_A=\bar{p}_B=1$, the noise is zero. At higher frequencies the noise increases due to the stochastic charging and discharging of the DB region; see Fig. 2. For a concrete estimate of the accuracy, we consider the regime where $1-\bar{p}_{A/B}\equiv\epsilon_{A/B}\ll 1$. The probability that an electron-hole pair is *not* transferred during a cycle is $\epsilon_A+\epsilon_B$ and Eqs. (5) and (11) give $Q_2=e(1-[\epsilon_A+\epsilon_B])$ and $S^{\mathtt{bias}}=e^2(\epsilon_A+\epsilon_B)$. A level spacing $\Delta/\hbar\sim 100$ GHz (e.g., Mahé *et al.* in Ref. 25) and the driving in Figs. 1 and 2(a) then give an accuracy $\epsilon_A+\epsilon_B\sim 10^{-4}$ at a frequency $\omega=10^{-2}\Delta/\hbar\sim 1$ GHz, comparable to existing schemes.[7,14] For large frequencies $\omega\sim\Delta/\hbar$ both the components $j_{2,q}^{\mathtt{pump}}$ and $j_{2,q}^{\mathtt{bias}}$ contribute to the correlations. The correlations are evaluated numerically; the result is plotted in Fig. 2.

V. CONCLUSIONS

In conclusion, we have analyzed a mesoscopic turnstile implemented in a double barrier system in the quantum Hall regime. At ideal operation the turnstile produces one electron and one hole at different locations per driving cycle, making it promising both for quantum information and metrological tasks. The noise due to the driving is found to be negligibly small at frequencies for ideal operation. We note that after the submission of our work an experiment on a two-terminal charge pump in the quantum Hall regime appeared,[28] closely related to our proposal.

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