Direction dependence of spin relaxation in confined two-dimensional systems

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The dependence of spin relaxation on the direction of the quantum wire under Rashba and Dresselhaus (linear and cubic) spin-orbit coupling is studied. Comprising the dimensional reduction of the wire in the diffusive regime, the lowest spin relaxation and dephasing rates for (001) and (110) systems are found. The analysis of spin relaxation reduction is then extended to nondiffusive wires where it is shown that, in contrast to the theory of dimensional crossover from weak localization to weak antilocalization in diffusive wires, the relaxation due to cubic Dresselhaus spin-orbit coupling is reduced and the linear part is shifted with the number of transverse channels.

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I. INTRODUCTION

Spin dynamics in semiconductors have been studied for decades, but still the prime condition for building spintronic devices; namely, the understanding of spin relaxation, is not satisfactorily fulfilled. In the following we focus on materials where the dominant mechanism for spin relaxation is governed by the D'yakonov-Perel spin relaxation (DPR).[1] This mechanism results from lifting the spin degeneracy, which is due to time-inversion symmetry and spatial-inversion symmetry and leads to the effect of slower spin dephasing the faster the momentum relaxes (motional narrowing[2–4]). In one of the most-studied systems (GaAs/AlGaAs), DPR is the most relevant mechanism in the metallic regime.[5]

Preserving time-inversion symmetry, the spin splitting can be due to bulk-inversion asymmetry (BIA)[6] and also due to the asymmetry arising from the structure of the quantum well (QW); the structure-inversion asymmetry (SIA).[7] In Refs. 8 and 9 it was shown how the spin relaxes in a quasi-one-dimensional (quasi-1D) electron system in a QW grown in the [001] direction, depending on the width of the wire, where the normal of the boundary was pointing in the [010] direction. It is already known that, in a (001) two-dimensional (2D) system with BIA and SIA, we get an anisotropic spin relaxation.[10–12] This has also been studied numerically in quasi-1D GaAs wires.[13] In this work, in Sec. II, we present analytical results concerning this anisotropy for the 2D case as well as the case of a QW with spin- and charge-conserving boundaries.

We also extend our analysis to other growth directions (see Sec. III). Searching for long spin decoherence times at room temperature, the (110) QW attracted attention.[14,15] The properties of spin relaxation in systems with this growth direction have also been related to weak localization (WL) measurements.[16] We present analytical explanations for dimensional spin relaxation reduction and discuss the crossover from WL to weak antilocalization (WAL); see Sec. IV.

As we will show in the following sections, the cubic Dresselhaus spin-orbit coupling (SOC) always gives rise to a limitation of the spin relaxation time in the diffusive case $W \gg l_e$, with the wire width W and the elastic mean-free path l_e. However, some of the experiments are done on ballistic wires, and we need to modify the theory used in Refs. 8,9 to enable us to study the crossover from diffusive to ballistic wires. In Sec. V we show how the spin relaxation, which is due to cubic Dresselhaus SOC, reduces with the number of channels in the QW. In the following we set $\hbar = 1$.

We consider the following Hamiltonian with SOC:

$$H = \frac{1}{2m_e}(\mathbf{p} + e\mathbf{A})^2 + V(\mathbf{x}) - \frac{1}{2}\gamma\boldsymbol{\sigma}[\mathbf{B} + \mathbf{B}_{\mathrm{SO}}(\mathbf{p})], \quad (1)$$

where m_e is the effective electron mass, $\mathbf{A}$ is the vector potential due to the external magnetic field $\mathbf{B}$, $\mathbf{B}_{\mathrm{SO}}^T = (B_{\mathrm{SO}x}, B_{\mathrm{SO}y})$ is the momentum-dependent SO field, $\boldsymbol{\sigma}$ is a vector with components $\boldsymbol{\sigma}_i$, $i = x,y,z$ being the Pauli matrices, γ is the gyromagnetic ratio with $\gamma = g\mu_{\mathrm{B}}$ with the effective g factor of the material, and $\mu_{\mathrm{B}} = e/2m_e$ is the Bohr magneton constant. For example, III-V and II-VI semiconductors such as GaAs and InSb have the zinc-blend structure. This BIA causes an SO interaction which, to lowest order in the wave vector $\mathbf{k}$, is given by[6]

$$-\frac{1}{2}\gamma\mathbf{B}_{\mathrm{SO,D}} = \gamma_D \sum_i \hat{e}_i k_i \left(k_{i+1}^2 - k_{i+2}^2\right), \quad (2)$$

where the principal crystal axes are given by $i \in \{x,y,z\}, i \to [(i-1)\bmod 3] + 1$ and the spin-orbit coefficient for the bulk semiconductor γ_D. We consider the standard white-noise model for the impurity potential $V(\mathbf{x})$, which vanishes on average $[\langle V(\mathbf{x})\rangle = 0]$, is uncorrelated $[\langle V(\mathbf{x})V(\mathbf{x}')\rangle = \delta(\mathbf{x}-\mathbf{x}')/(2\pi\nu\tau)]$, and is weak $[\epsilon_F\tau \gg 1]$. Here, $\nu = m_e/(2\pi)$ is the average density of states per spin channel, ϵ_F is the Fermi energy, and τ is the elastic scattering time. To address both the WL corrections as well as the spin relaxation rates in the system, we analyze the Cooperon[17]

$$\hat{C}(\mathbf{Q} = \mathbf{p} + \mathbf{p}')^{-1} = \frac{1}{\tau}\left(1 - \int \frac{d\varphi}{2\pi} \frac{1}{1 + I\tau[\mathbf{v}(\mathbf{Q} + 2e\mathbf{A} + 2m_e\hat{a}\mathbf{S}) + H_{\sigma'} + H_Z]}\right), \quad (3)$$

where the integral is performed over all angles of velocity $\mathbf{v}$ on the Fermi surface, $H_{\sigma'} = -(\mathbf{Q} + 2e\mathbf{A})\hat{a}\boldsymbol{\sigma}'$, and the Zeeman coupling to the external magnetic field yields

$$H_Z = -\tfrac{1}{2}\gamma(\boldsymbol{\sigma}' - \boldsymbol{\sigma})\mathbf{B}. \quad (4)$$

The coupling between the orbital motion and the spin $\mathbf{S} = (\boldsymbol{\sigma} + \boldsymbol{\sigma}')/2$ is described by the SOC operator $\hat{a}$. The spin quantum number is 1 instead of 1/2 due to the electron-hole excitation. It follows that, for weak disorder and without Zeeman coupling, the Cooperon depends only on the total momentum $\mathbf{Q}$ and the total spin $\mathbf{S}$. Expanding the Cooperon to second order in $(\mathbf{Q} + 2e\mathbf{A} + 2m_e\hat{a}\mathbf{S})$ and performing the angular integral which is, for 2D diffusion (elastic mean-free path l_e smaller than wire width W) continuous from 0 to 2π, yields

$$\hat{C}(\mathbf{Q}) = \frac{1}{D_e(\mathbf{Q} + 2e\mathbf{A} + 2e\mathbf{A_S})^2 + H_{\gamma_D}}. \quad (5)$$

The effective vector potential due to SO interaction is $\mathbf{A_S} = m_e\hat{\alpha}\mathbf{S}/e$, where $\hat{\alpha} = \langle\hat{a}\rangle$ is averaged over angle. The SO term H_{γ_D}, which cannot be rewritten as a vector potential, is in our case due to the appearance of cubic Dresselhaus SOC.

A. Example

To get an idea of the procedure we recall the situation presented in Refs. 8 and 9. For a (001) quasi-1D wire in [100] direction the Dresselhaus term, Eq. (2), is given by[6]

$$-\frac{1}{2}\gamma\mathbf{B}_{\text{SO,D}} = \alpha_1(-\hat{e}_x k_x + \hat{e}_y k_y) + \gamma_D\left(\hat{e}_x k_x k_y^2 - \hat{e}_y k_y k_x^2\right). \quad (6)$$

Here, $\alpha_1 = \gamma_D\langle k_z^2\rangle$ is the linear Dresselhaus parameter, which measures the strength of the term linear in wave vectors k_x and k_y in the plane of the 2D electron system (2DES). When $\langle k_z^2\rangle \sim 1/a^2 \geqslant k_F^2$ (a is the thickness of the 2DES, k_F is the Fermi wave number), that term exceeds the cubic Dresselhaus terms which have coupling strength γ_D. Asymmetric confinement of the 2DES, an SIA, yields the Rashba term which does not depend on the growth direction:

$$-\tfrac{1}{2}\gamma\mathbf{B}_{\text{SO,R}} = \alpha_2(\hat{e}_x k_y - \hat{e}_y k_x), \quad (7)$$

with α_2 being the Rashba parameter.[7,18] Therefore, the Cooperon Hamiltonian, in the case of Rashba and linear and cubic Dresselhaus SOC is given by

$$H_c := \frac{\hat{C}^{-1}}{D_e} = (\mathbf{Q} + 2e\mathbf{A_S})^2 + \left(m_e^2\epsilon_F\gamma_D\right)^2\left(S_x^2 + S_y^2\right), \quad (8)$$

with the effective vector potential

$$\mathbf{A_S} = \frac{m_e}{e}\hat{\alpha}\mathbf{S} = \frac{m_e}{e}\begin{pmatrix} -\tilde{\alpha}_1 & -\alpha_2 & 0 \\ \alpha_2 & \tilde{\alpha}_1 & 0 \end{pmatrix}\begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}, \quad (9)$$

with $\tilde{\alpha}_1 = \alpha_1 - m_e\gamma_D\epsilon_F/2$.

It can be easily shown that the Hamiltonian Eq. (8) has only nonvanishing eigenvalues due to $(m_e^2\epsilon_F\gamma_D)^2$ in the 2D case.

The term with $(S_x^2 + S_y^2)$, which is due to cubic Dresselhaus SOC, is not reduced by reason of the boundary in the diffusive case. However, two triplet eigenvalues of this term depend on the wire width:

$$E_{QD1} = \frac{q_{s3}^2}{2}, \quad (10)$$

$$E_{QD2,3} = \frac{q_{s3}^2}{2}\left(\frac{3}{2} \pm \frac{\sin(Q_{\text{SO}}W)}{2Q_{\text{SO}}W}\right), \quad (11)$$

with $q_{s3}^2/2 = (m_e^2\epsilon_F\gamma_D)^2$. In the following we are going to diagonalize the whole Hamiltonian and change the direction of the wire in the (001) plane.

II. SPIN RELAXATION ANISOTROPY IN THE (001) SYSTEM

A. Two-dimensional system

We rotate the system in-plane through the angle θ (the angle $\theta = \pi/4$ is equivalent to [110]). This does not effect the Rashba term but changes the Dresselhaus term to[11,12]

$$\begin{aligned}\frac{1}{\gamma_D}H_{D[001]} = &\ \sigma_y k_y \cos(2\theta)\left(\langle k_z^2\rangle - k_x^2\right) \\ &- \sigma_x k_x \cos(2\theta)\left(\langle k_z^2\rangle - k_y^2\right) \\ &- \sigma_y k_x \frac{1}{2}\sin(2\theta)\left(k_x^2 - k_y^2 - 2\langle k_z^2\rangle\right) \\ &+ \sigma_x k_y \frac{1}{2}\sin(2\theta)\left(k_x^2 - k_y^2 + 2\langle k_z^2\rangle\right),\end{aligned} \quad (12)$$

with the wave vectors k_i. The resulting Cooperon Hamiltonian, including Rashba and Dresselhaus SOC, then reads

$$\begin{aligned}H_c = &\ [Q_x + \alpha_{x1}S_x + (\alpha_{x2} - q_2)S_y]^2 \\ &+ [Q_y + (\alpha_{x2} + q_2)S_x - \alpha_{x1}S_y]^2 + \frac{q_{s3}^2}{2}\left(S_x^2 + S_y^2\right),\end{aligned} \quad (13)$$

where we set

$$\frac{q_{s3}^2}{2} = \left(m_e^2\epsilon_F\gamma_D\right)^2, \quad (14)$$

$$\alpha_{x1} = \frac{1}{2}m_e\gamma_D\cos(2\theta)\left[(m_e v)^2 - 4\langle k_z^2\rangle\right], \quad (15)$$

$$\begin{aligned}\alpha_{x2} &= -\frac{1}{2}m_e\gamma_D\sin(2\theta)\left[(m_e v)^2 - 4\langle k_z^2\rangle\right] \quad (16)\\ &= \left(q_1 - \sqrt{\frac{q_{s3}^2}{2}}\right)\sin(2\theta) \quad (17)\\ &= 2m_e\tilde{\alpha}_1\sin(2\theta), \quad (18)\end{aligned}$$

with $q_1 = 2m_e\alpha_1$ and $q_2 = 2m_e\alpha_2$. We see that the part of the Hamiltonian which cannot be written as a vector field and is due to cubic Dresselhaus SOC does not depend on the wire direction in the (001) plane.

1. Special case: Only linear Dresselhaus SOC equal to Rashba SOC

As a special example for the 2D case, we set $q_{s3} = 0$ and $q_1 = q_2$. To simplify the search for vanishing spin relaxation we go to polar coordinates. Applying free wave functions (with

k_x and k_y) to H_c [Eq. (13)], we end up with (singlet part left out)

$$\frac{H_c}{q_2^2} = \begin{pmatrix} 2+Q^2 & f_{\theta\phi} & -2I\exp(2I\theta) \\ & 4+Q^2 & f_{\theta\phi} \\ \text{c.c.} & & 2+Q^2 \end{pmatrix}, \qquad (19)$$

with $k_x/q_2 = Q\cos(\phi)$, $k_y/q_2 = Q\sin(\phi)$, and

$$f_{\theta\phi} = (I-1)\sqrt{2}\exp(I\theta)[\cos(\phi+\theta) - \sin(\phi+\theta)]Q. \qquad (20)$$

Vanishing spin relaxation is found at $Q = 0$ for arbitrary values of θ (the spin with vanishing spin relaxation is pointing along the [110] direction[19]). Another solution is found at $Q = 2$ with the condition $\theta + \phi = 3\pi/4$, which is equivalent to the $[\bar{1}10]$ crystallographic direction.[11]

B. Quasi-one-dimensional wire

In the following we consider spin- and charge-conserving boundaries. Due to the SOC we have the following modified Neumann condition:[9]

$$\left(-\frac{\tau}{D_e}\mathbf{n}\cdot\langle\mathbf{v}_F[\gamma\mathbf{B}_{\text{SO}}(\mathbf{k})\cdot\mathbf{S}]\rangle - I\partial_{\mathbf{n}}\right)C|_{\partial S} = 0, \qquad (21)$$

where $\langle\ldots\rangle$ denotes the average over the direction of $\mathbf{v}_F$ and $\mathbf{k}$, which we rewrite for the rotated x-y system as

$$[-I\partial_y + 2e(\mathbf{A_S})_y]C\left(x, y = \pm\frac{W}{2}\right) = 0 \ \ \forall x, \qquad (22)$$

where $\mathbf{n}$ is the unit vector normal to the boundary ∂S and x is the coordinate along the wire. In order to do a diagonalization taking only the zero-mode into account, we have to simplify the boundary condition. A transformation acting in the transverse direction is needed according to Eq. (13): $\hat{C} \to \tilde{\hat{C}} = U_A\hat{C}U_A^\dagger$, by using the transformation

$$U = \mathbb{1}_4 - I\sin(q_s y)\frac{1}{q_s}A_y + [\cos(q_s y) - 1]\frac{1}{q_s^2}A_y^2, \qquad (23)$$

with $A_y = (\alpha_{x2} + q_2)S_x - \alpha_{x1}S_y$ and $q_s = \sqrt{(\alpha_{x2}+q_2)^2 + \alpha_{x1}^2}$.

1. Spin relaxation

We diagonalize the Hamiltonian, Eq. (13), after applying the transformation U, taking only the lowest mode into account. The spectrum of the Hamiltonian for small wire width, $Wq_s < 1$, is given by

$$\begin{aligned} E_{1/2}(Q_x > 0) = {} & Q_x^2 \pm Q_x\left(2q_{sm} - \frac{\left(\alpha_{x1}^2 + \alpha_{x2}^2 - q_2^2\right)^2}{12q_{sm}}W^2\right) \\ & + \frac{3}{2}\frac{q_{s3}^2}{2} + q_{sm}^2 \mp \frac{q_{s3}^2}{2Q_x}\frac{\left(\alpha_{x1}^2+\alpha_{x2}^2-q_2^2\right)^2W^2}{96q_{sm}} \\ & - \frac{\left(\frac{q_{s3}^2}{2} + q_{sm}^2\right)\left(\alpha_{x1}^2 + \alpha_{x2}^2 - q_2^2\right)^2}{24q_{sm}^2}W^2, \end{aligned} \qquad (24)$$

$$E_1(Q_x = 0) = q_{s3}^2 + q_{sm}^2 - \frac{\left(\alpha_{x1}^2 + \alpha_{x2}^2 - q_2^2\right)^2 + \frac{q_{s3}^2}{2}q_s^2}{12}W^2, \qquad (25)$$

$$E_2(Q_x = 0) = \frac{q_{s3}^2}{2} + q_{sm}^2 + \frac{q_{s3}^2}{2}\frac{q_2^2\alpha_{x1}^2}{3q_{sm}^2}W^2, \qquad (26)$$

$$E_3 = Q_x^2 + \frac{q_{s3}^2}{2} + \frac{\left(\frac{q_{s3}^2}{2} + q_{sm}^2\right)\left(\alpha_{x1}^2 + \alpha_{x2}^2 - q_2^2\right)^2}{12q_{sm}^2}W^2, \qquad (27)$$

with $q_{sm} = \sqrt{(\alpha_{x2} - q_2)^2 + \alpha_{x1}^2}$. First we notice that the only θ dependence is in the term q_{sm}, which disappears if the Dresselhaus SOC strength $\tilde{\alpha}_1$, which is shifted due to the cubic term, equals the Rashba SOC strength α_2 and the angle of the boundary is $\theta = (1/4 + n)\pi,\ n \in \mathbb{Z}$. Assuming the term proportional to W^2/Q_x to be small, the absolute minimum can be found at

$$E_{1/2,\min} = \frac{3}{2}\frac{q_{s3}^2}{2} + \frac{\left(q_{sm}^2 - \frac{q_{s3}^2}{2}\right)\left(\alpha_{x1}^2 + \alpha_{x2}^2 - q_2^2\right)^2}{24q_{sm}^2}W^2, \qquad (28)$$

which is independent of the width W if $\alpha_{x1}(\theta = 0) = -q_2$ and/or the direction of the wire is pointing in

$$\theta = \frac{1}{2}\arcsin\left(\frac{2\langle k_z^2\rangle(m_e\gamma_D)^2\left[(m_e v)^2 - 2\langle k_z^2\rangle\right] - q_2^2}{\left(m_e^3v^2\gamma_D - 4\langle k_z^2\rangle m_e\gamma_D\right)q_2}\right). \qquad (29)$$

The second possible absolute minimum, which dominates for sufficiently small width W and $q_{sm} \neq 0$ [compare with $E_2(Q_x = 0)$], is found at

$$E_{3,\min} = \frac{q_{s3}^2}{2} + \frac{\left(\frac{q_{s3}^2}{2} + q_{sm}^2\right)\left(\alpha_{x1}^2 + \alpha_{x2}^2 - q_2^2\right)^2}{12q_{sm}^2}W^2. \qquad (30)$$

The minimal spin-relaxation rate is found by analyzing the prefactor of W^2 in Eq. (30), see Fig. 1. We see immediately that, in the case of vanishing cubic Dresselhaus or in the case where $\alpha_{x1}(\theta = 0) = -q_2$, we have no direction dependence of the minimal spin relaxation. Notice the shift of the absolute minimum away from $q_1 = q_2$ due to $q_{s3} \neq 0$. In the case of $q_1 < (q_{s3}/\sqrt{2})$ we find the minimum at $\theta = (1/4 + n)\pi,\ n \in \mathbb{Z}$, or else at $\theta = (3/4 + n)\pi,\ n \in \mathbb{Z}$, which is indicated by the dashed line in Fig. 1.

2. Spin dephasing

Concerning spintronic devices it is interesting to know how an ensemble of spins initially oriented along the [001] direction dephases in a wire of different orientation θ. To do this analysis we only have to know that the eigenvector for the eigenvalue E_1 at $Q_x = 0$ [Eq. (25)], is the triplet state $|S = 1; m = 0\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2} \equiv |\rightarrow\rangle \hat{=} (0,1,0)^T$. This is equal to the z component of the spin density whose evolution is described by the spin diffusion equation.[9] As an example we assume the case where the cubic Dresselhaus term can be neglected and where the Rashba and linear Dresselhaus SOC are equal. We notice that the spin dephasing is then width independent, given by

$$\frac{1}{\tau_s(W)} = 2D_e q_2^2[1 - \sin(2\theta)] \qquad (31)$$

which is plotted in Fig. 2. At definite angles the spin dephasing time diverges—as for the in-plane polarized states with eigenvalue $E_2(Q_x = 0)$. We have longest spin dephasing time at $\theta = (1/4 + n)\pi,\ n \in \mathbb{Z}$. For $\theta = (3/4 + n)\pi,\ n \in \mathbb{Z}$

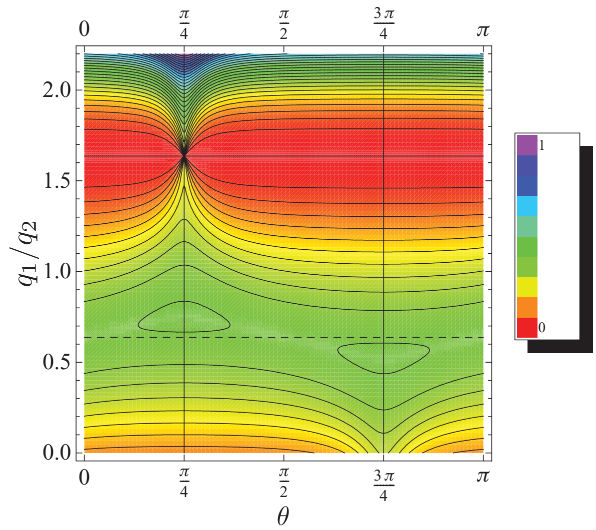

FIG. 1. (Color online) Dependence of the W^2 coefficient in Eq. (30) on the lateral rotation (θ). The absolute minimum is found for $\alpha_{x1}(\theta=0)=-q_2$ (here: $q_1/q_2=1.63$) and for different SO strength we find the minimum at $\theta=(1/4+n)\pi,\ n\in\mathbb{Z}$ if $q_1<(qs3/\sqrt{2})$ [dashed line: $q_1=(qs3/\sqrt{2})$] and at $\theta=(3/4+n)\pi,\ n\in\mathbb{Z}$ otherwise. Here we set $q_{s3}=0.9$. The scaling is arbitrary.

we get the 2D result, $T_2=1/(4q_2^2D_e)$, which is given by the eigenvalue of the spin relaxation tensor:[1,9,20]

$$\frac{1}{\tau_{sij}}=\tau\gamma^2[\langle \mathbf{B}_{\rm SO}(\mathbf{k})^2\rangle\delta_{ij}-\langle B_{\rm SO}(\mathbf{k})_i B_{\rm SO}(\mathbf{k})_j\rangle] \tag{32}$$

to the triplet state $|S=1;m=0\rangle$.

This gives an analytical description of the numerical calculation done by J.Liu *et al.*[13]

Switching on cubic Dresselhaus SOC leads to finite spin dephasing time for all angles θ. In addition, T_2 is than width dependent. In the case of strong cubic Dresselhaus SOC, where $q_{s3}^2/2=q_1^2=q_2^2$, the dephasing time T_2 is angle-independent and, for $q_{s3}^2/2>q_1^2=q_2^2$, the minima in $T_2(\theta)$ change to maxima and vice versa.

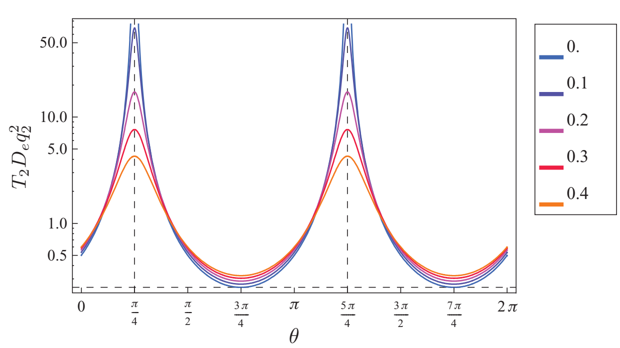

FIG. 2. (Color online) The spin dephasing time T_2 of a spin initially oriented along the [001] direction in units of $(D_eq_2^2)$ for the special case of equal Rashba and linear Dresselhaus SOC. The different curves show different strengths of cubic Dresselhaus in units of q_{s3}/q_2. In the case of finite cubic Dresselhaus SOC, we set $W=0.4/q_2$. If $q_{s3}=0$, T_2 diverges at $\theta=(1/4+n)\pi,\ n\in\mathbb{Z}$ (dashed vertical lines). The horizontal dashed line indicated the 2D spin dephasing time, $T_2=1/(4q_2^2D_e)$.

3. Special case: $\theta = 0$

In this case the longitudinal direction of the wire is [100]. If we neglect the term proportional to W^2/Q_x in Eq. (24), the lowest spin relaxation is found to be

$$\frac{1}{D_e\tau_s}=\frac{q_{s3}^2}{2}+\frac{(\alpha_{x1}^2-q_2^2)^2\left(q_s^2+\frac{q_{s3}^2}{2}\right)W^2}{12q_s^2} \tag{33}$$

or

$$\frac{1}{D_e\tau_s}=\frac{3q_{s3}^2}{4}+\frac{(\alpha_{x1}^2-q_2^2)^2\left(q_s^2-\frac{q_{s3}^2}{2}\right)W^2}{24q_s^2}, \tag{34}$$

depending on whether

$$-\frac{q_{s3}^2}{4}+\frac{(\alpha_{x1}^2-q_2^2)^2\left(q_s^2+3\frac{q_{s3}^2}{2}\right)W^2}{24q_s^2} \tag{35}$$

is negative or positive. This shows that the cubic Dresselhaus term adds not only a constant term to the relaxation rate but is also width dependent. However, this width dependence does not reduce the spin relaxation rate below $q_{s3}^2/2$.

III. SPIN RELAXATION IN QUASI-ONE-DIMENSIONAL WIRE WITH [110] GROWTH DIRECTION

To get the spin-relaxation in a [110] QW with Rashba and Dresselhaus SOC again we have to rotate the spatial coordinate system of the Dresselhaus Hamiltonian Eq. (2), but now with the rotation matrix

$$R=\begin{pmatrix}\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\\ 0 & 1 & 0\end{pmatrix}. \tag{36}$$

We get

$$\frac{H_{D[110]}}{\gamma_D}=\sigma_x\left(-k_x^2k_z-2k_y^2k_z+k_z^3\right)+\sigma_y(4k_xk_yk_z)+\sigma_z\left(k_x^3-2k_xk_y^2-k_xk_z^2\right). \tag{37}$$

The confinement in z direction ($z\equiv$[110]) leads to $\langle k_z\rangle=\langle k_z^3\rangle=0$ and $\langle k_z^2\rangle=\int|\nabla\phi|^2dz$. The Hamiltonian for the QW in the [110] direction then has the following form:[16]

$$H_{[110]}=-\gamma_D\sigma_zk_x\left(\tfrac{1}{2}\langle k_z^2\rangle-\tfrac{1}{2}\left(k_x^2-2k_y^2\right)\right). \tag{38}$$

Including the Rashba SOC (q_2) and noting that its Hamiltonian does not depend on the orientation of the wire,[16] we end up with the following Cooperon Hamiltonian:

$$\frac{C^{-1}}{D_e}=(Q_x-\tilde{q}_1S_z-q_2S_y)^2+(Q_y+q_2S_x)^2+\frac{\tilde{q}_3^2}{2}S_z^2. \tag{39}$$

with $\tilde{q}_1=2m_e\frac{\gamma_D}{2}\langle k_z^2\rangle-\frac{\gamma_D}{2}\frac{m_e\epsilon_F}{2}$, $q_2=2m_e\alpha_2$, and $\tilde{q}_3=[3m_e\epsilon_F^2(\gamma_D/2)]$. We see immediately that, in the 2D case, states polarized in the z direction have vanishing spin relaxation as long as we have no Rashba SOC. Compared with the (001) system, the constant term due to cubic Dresselhaus does not mix spin directions. Here we set the appropriate Neumann boundary condition as follows:

$$(-i\partial_y+2m_e\alpha_2S_x)C\left(x,y=\pm\frac{W}{2}\right)=0\ \ \forall x. \tag{40}$$

The presence of Rashba SOC adds a vector potential proportional to S_x. Applying a non-Abelian gauge transformation as before to simplify the boundary condition, we diagonalize the transformed Hamiltonian [Appendix (A1)] up to second order in $q_2 W$ in the zero-mode approximation.

A. Special case: without cubic Dresselhaus SOC

The spectrum is found to be

$$E_1 = Q_x^2 + \frac{1}{12}\Delta^2(q_2 W)^2, \tag{41}$$

$$E_{2,3} = Q_x^2 + \frac{1}{24}\Delta^2[24-(q_2W)^2] \pm \frac{\Delta}{24}\sqrt{\Delta^2(q_2W)^4 + 4Q_x^2[24-(q_2W)^2]^2}, \tag{42}$$

with the lowest spin relaxation rate found at finite wave vectors $Q_{x_{\min}} = \pm\frac{\Delta}{24}[24-(q_2W)^2]$,

$$\frac{1}{D_e\tau_s} = \frac{\Delta^2}{24}(q_2W)^2. \tag{43}$$

We set $\Delta = \sqrt{\tilde{q}_1^2 + q_2^2}$.

B. With cubic Dresselhaus SOC

If cubic Dresselhaus SOC cannot be neglected, the absolute minimum of spin relaxation can also shift to $Q_{x_{\min}} = 0$. This depends on the ratio of Rashba and linear Dresselhaus SOC. If $q_2/q_1 \ll 1$, we find the absolute minimum at $Q_{x_{\min}} = 0$,

$$E_{\min 1} = \frac{\tilde{q}_3 + \tilde{q}_1^2 + q_2^2}{2} - \Delta_c + \frac{1}{12}\Delta_c(q_2W)^2, \tag{44}$$

with

$$\Delta_c = \tfrac{1}{2}\sqrt{\left(\tilde{q}_3 + \tilde{q}_1^2\right)^2 + 2\left(\tilde{q}_1^2 - \tilde{q}_3\right)q_2^2 + q_2^4}. \tag{45}$$

If $q_2/q_1 \gg 1$, we find the absolute minimum at $Q_{x_{\min}} \approx \pm\frac{\Delta}{24}[24-(q_2W)^2]$,

$$\begin{aligned} E_{\min 2} = {} & Q_{x_{\min}}^2 - Q_{x_{\min}} q_2\left(\frac{\tilde{q}_1^2}{q_2^2} + 2\right) - \frac{\tilde{q}_3^2}{16 Q_{x_{\min}} q_2} \\ & + \tilde{\Delta}^2 + \frac{\tilde{q}_3}{2}\left(\frac{\tilde{q}_1^2}{q_2^2} + 1\right) - \left\{\frac{\tilde{q}_3\tilde{q}_1^2}{12} - \frac{\tilde{q}_3^2 q_2}{3072 Q_{x_{\min}}^3}\right. \\ & - \frac{q_2^2}{24}\left(\tilde{q}_3 - \tilde{q}_1^2\right) + \frac{q_2^4}{24} - \left(\frac{\tilde{q}_1^2}{24} + \frac{q_2^2}{12}\right) q_2 Q_{x_{\min}} \\ & \left. - \frac{q_2}{Q_{x_{\min}}}\left[\left(\frac{\tilde{q}_3^2}{128} + \frac{\tilde{q}_3\tilde{q}_1^2}{192}\right) - \frac{\tilde{q}_3 q_2^2}{96}\right]\right\} W^2. \end{aligned} \tag{46}$$

We can conclude that reducing the wire width W will not cancel the contribution to the spin relaxation rate due to cubic Dresselhaus SOC.

IV. WEAK LOCALIZATION

In Refs. 8 and 9 the crossover from WL to WAL due to changing wire width and SOC strength was explained in the case of a (001) system. Whether WL or WAL is present depends on the suppression of the triplet modes of the Cooperon. The suppression in turn is dominated by the absolute minimum of the spectrum of the Cooperon Hamiltonian H_c. The findings, presented in Sec. II B, therefore point out that, for example, the crossover width at which the system changes from WL to WAL can shift with the wire direction θ. Recently, experimental results on WL and WAL by J. Nitta *et al.*[21] seem to show a strong dependence on growth direction. Our presented results can also support the method proposed in Ref. 22—to determine the relative strength of Rashba and Dresselhaus SOC from WL or WAL measurements without fitting parameters—with inclusion of the cubic Dresselhaus term for wire directions different from [100] and [010] in an analytical manner.

In the (110) system the situation is different. In the 2D case it was shown by Pikus *et al.*[16] that, in the absence of the Rashba terms, the negative magnetoconductivity cannot be observed. In the case of a wire geometry we can conclude from Eqs. (41)–(46) that we have no width dependence if Rashba SOC vanishes. A change of the quantum correction to the static conductivity therefore cannot be achieved in this wire geometry by changing the wire width. The reason is the vector potential in the boundary condition [Eq. (40)], which only depends on Rashba SOC.

V. DIFFUSIVE-BALLISTIC CROSSOVER

In the following we assume a (001) 2D system with both Rashba and linear and cubic Dresselhaus SO coupling.

Experiments measuring WL in diffusive QW with SOC[23,24] are in great agreement with theoretical calculations by S. Kettemann.[8] But considering, for example, the work of Refs. 25 and 26, one realizes that the scope of application of the theory has to be extended to also describe the crossover to the ballistic regime; $l_e > W$. We have shown in Sec. II B that the presence of cubic Dresselhaus SOC in the sample leads to a finite spin relaxation even for wire widths $Q_{SO}W \ll 1$, regardless of the boundary direction in a (001) system. To account for the ballistic case we have to modify the derivation of the Cooperon Hamiltonian, Eq. (8). It has to be noticed that, in the case of only few channels the electron-electron interaction can play a crucial role concerning the spin relaxation.[27] But as long as the quantum wires are in the regime where the spin relaxation is dominated by DPR, the electron-electron and also the electron-phonon interaction are incorporated by substituting $1/\tau$ with the total scattering rate $1/\tau(T)$.[4] In the case of a wire where the mean-free path l_e is comparable to the wire width W, we cannot integrate in Eq. (3) over the Fermi surface in a continuous way. Instead, we assume k_F/W to be finite and sum over the number of discrete channels $N = [k_F W/\pi]$, where $[\ldots]$ is the integer part. Because $H_{\gamma D} \sim \epsilon_F^2$ this constant term due to cubic Dresselhaus should reduce if we reduce the number of channels. If we expand the Cooperon to second order in $(\mathbf{Q} + 2e\mathbf{A} + 2m_e\hat{\alpha}\mathbf{S})$ before averaging over the Fermi surface, $\langle\ldots\rangle$, and use the Matsubara trick, we get

$$\frac{C^{-1}}{D_e} = 2f_1\left(Q_y + 2\alpha_2 S_x + 2\left(\alpha_1 - \gamma_D v^2 \frac{f_3}{f_1}\right) S_y\right)^2 + 2f_2\left(Q_x - 2\alpha_2 S_y - 2\left(\alpha_1 - \gamma_D v^2\frac{f_3}{f_2}\right) S_x\right)^2 + 8\gamma_D^2 v^4\left[\left(f_4 - \frac{f_3^2}{f_2}\right) S_x^2 + \left(f_5 - \frac{f_3^2}{f_1}\right) S_y^2\right], \quad (47)$$

with $m_e = 1$ and functions $f_i(\varphi)$ (Appendix B) which depend on the number N of transverse modes. In the diffusive case we can perform the continuous sum over the angle φ in Eqs. (B3)–(B7), and we recover the old result with $f_1 = f_2 = 1/2$, $f_3 = 1/8$, and $f_4 = f_5 = 1/16$:

$$H_c = \left(Q_y + 2\alpha_2 S_x + 2\left(\alpha_1 - \tfrac{1}{2}\gamma_D\epsilon_F\right) S_y\right)^2 + \left(Q_x - 2\alpha_2 S_y - 2\left(\alpha_1 - \tfrac{1}{2}\gamma_D\epsilon_F\right) S_x\right)^2 + (\gamma_D\epsilon_F)^2\left(S_x^2 + S_y^2\right). \quad (48)$$

A. Spin Relaxation at $Q_{SO}W \ll 1$

In the first section we analyzed the lowest spin relaxation in wires of different direction in a (001) system. We have shown, that for every direction, there is still a finite spin relaxation at a wire width which fulfills the condition $Q_{SO}W \ll 1$ due to cubic Dresselhaus SOC. It is clear that this finite spin relaxation vanishes when the width is equal to the Fermi wave length λ_F. In the following we show how this finite spin relaxation depends on the number N of transverse channels. We show in Ref. 28 that the findings are consistent with calculations going beyond the perturbative ansatz. This is possible, in a similar manner as has been done previously in Ref. 29, for wires without SOC, which showed the crossover of the magnetic phase shifting rate, which had been known before in the diffusive and ballistic limit only.

To find the spectrum of the Cooperon Hamiltonian with boundary conditions as in Sec. (II B), we stay in the zero-mode approximation in the Q space and proceed as before. According to Eq. (47), the non-Abelian gauge transformation for the transversal direction y is given by

$$U = \exp\left(-I\left[2\alpha_2 S_x + 2\left(\alpha_1 - \gamma_D v^2\frac{f_3}{f_1}\right) S_y\right] y\right). \quad (49)$$

To concentrate on the constant width-independent part of the spectrum we extract the absolute minimum at $Q = 0$; see Figs. 3 and 4. A clear reduction of the absolute minimum is visible. Due to the factor f_3/f_1 in the transformation U, the decrease of the minimal spin relaxation also depends on the ratio of Rashba and linear Dresselhaus SOC.

From Eq. (47) it is clear that, not only is the H_{γ_D} affected by the reduction of the number N of channels, but also the shift of the linear Dresselhaus SOC, α_1, in the orbital part. A model to extract the ratio of Rashba and linear Dresselhaus SOC developed in Ref. 30 by Scheid *et al.* did not show much difference between the strict 1D case and the nondiffusive case with wires of finite width. The results presented here should allow for extending the model to finite cubic Dresselhaus SOC. Deducing from our theory, the direction of the SO field should change with the number of channels due to the mentioned N-dependent shift.

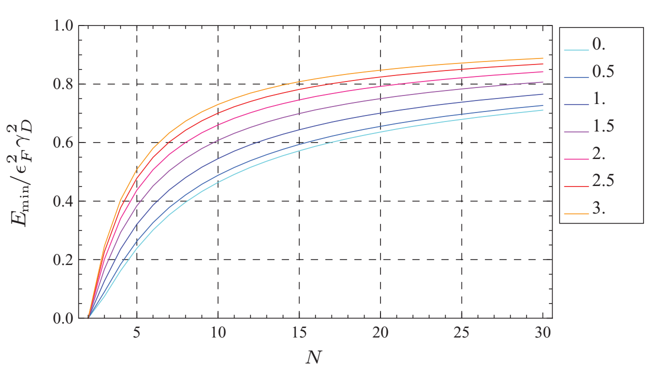

FIG. 3. (Color online) The lowest eigenvalues of the confined Cooperon Hamiltonian Eq. (47), equivalent to the lowest spin relaxation rate, are shown for $Q = 0$ for different numbers of modes, $N = k_F W/\pi$. Different curves correspond to different values of α_2/q_s.

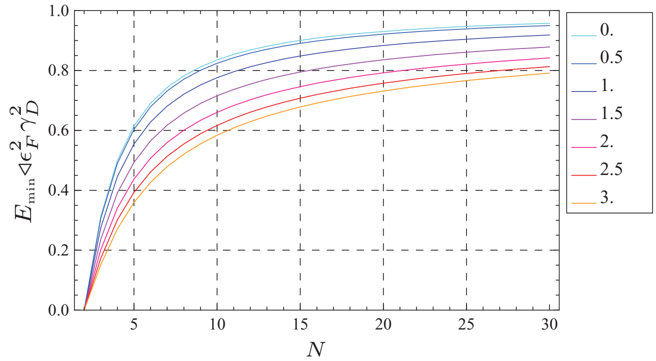

FIG. 4. (Color online) The lowest eigenvalues of the confined Cooperon Hamiltonian Eq. (47), equivalent to the lowest spin relaxation rate, are shown for $Q = 0$ for different numbers of modes, $N = k_F W/\pi$. Different curves correspond to different values of α_1/q_s.

VI. CONCLUSIONS

Summarizing the results, we have characterized the anisotropy and width dependence of spin relaxation in a (001) QW. There are special angles θ which are optimal for spin transport in quantum wires of finite width; namely, the [110] and the [$\bar{1}$10] directions. At [110] we find the the longest spin dephasing time T_2. If the absolute minimum of spin relaxation is found at [110] or [$\bar{1}$10] direction depends on the strength of cubic Dresselhaus and wire width. The findings for the spin dephasing time are in agreement with numerical results. The analytical expression for T_2 allows us to see directly the interplay between the cubic Dresselhaus SOC and the dimensional reduction, having an effect on T_2. In addition we analyzed the special case of a (110) system and found the minimal spin relaxation rates depending on Rashba and linear and cubic Dresselhaus SOC in the presence of boundaries. This results can be used to understand width and direction-dependent WL measurements in QWs. Finally, we have shown how the reduction of channels in the wire reduces the finite spin relaxation rate which is due to cubic Dresselhaus SOC and does not reduce if the wire is small ($Wq_s \ll 1$) and diffusive ($W \gg l_e$). The change in channel number also changes the shift of linear Dresselhaus SOC strength, $\tilde{\alpha}_1$. This has to be considered if extracting SOC strength from wires with only few transverse channels.

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APPENDIX A: HAMILTONIAN IN [110] GROWTH DIRECTION

The Cooperon Hamiltonian in the zero-mode approximation is given as follows

$$H_{c,0} = \begin{pmatrix} A & B & C \\ B^* & D & E \\ C^* & E^* & F \end{pmatrix} + M_{q3}, \tag{A1}$$

with

$$A = \frac{1}{4q_2 W}\Big\{q_2\big[4Q_x^2 + 3(\tilde{q}_1^2 + q_2^2)\big]W - 16Q_x\tilde{q}_1 \sin\left(\frac{q_2 W}{2}\right) + (\tilde{q}_1^2 - q_2^2)\sin(q_2 W)\Big\}, \tag{A2}$$

$$B = \frac{I\big[4Q_x \sin\left(\frac{q_2 W}{2}\right) - \tilde{q}_1 \sin(q_2 W)\big]}{\sqrt{2}W}, \tag{A3}$$

$$C = -\frac{q_2(\tilde{q}_1^2 + q_2^2)W + (q_2^2 - \tilde{q}_1^2)\sin(q_2 W)}{4q_2 W}, \tag{A4}$$

$$D = \frac{q_2(2Q_x^2 + \tilde{q}_1^2 + q_2^2)W + (q_2^2 - \tilde{q}_1^2)\sin(q_2 W)}{2q_2 W}, \tag{A5}$$

$$E = \frac{I\big[4Q_x \sin\left(\frac{q_2 W}{2}\right) + \tilde{q}_1 \sin(q_2 W)\big]}{\sqrt{2}W}, \tag{A6}$$

$$F = \frac{1}{4q_2 W}\Big\{q_2\big[4Q_x^2 + 3(\tilde{q}_1^2 + q_2^2)\big]W + 16Q_x\tilde{q}_1 \sin\left(\frac{q_2 W}{2}\right) + (\tilde{q}_1^2 - q_2^2)\sin(q_2 W)\Big\}, \tag{A7}$$

and the term due to cubic Dresselhaus SOC

$$M_{q3} = q_3 \begin{pmatrix} \frac{1}{4}\operatorname{sinc}(q_2 W) + \frac{3}{4} & 0 & \frac{1}{4}\operatorname{sinc}(q_2 W) - \frac{1}{4} \\ 0 & \frac{1}{2} - \frac{1}{2}\operatorname{sinc}(q_2 W) & 0 \\ \frac{1}{4}\operatorname{sinc}(q_2 W) - \frac{1}{4} & 0 & \frac{1}{4}\operatorname{sinc}(q_2 W) + \frac{3}{4} \end{pmatrix}. \tag{A8}$$

APPENDIX B: SUMMATION OVER THE FERMI SURFACE

The Cooperon Hamiltonian in the 2D case is given by

$$\begin{aligned} H_c = \tau v^2\big\{&\langle\cos^2(\varphi)\rangle(\mathbf{Q} + 2m_e\hat{\alpha}\mathbf{S})_x^2 \\ &+ \langle\sin^2(\varphi)\rangle(\mathbf{Q} + 2m_e\alpha\mathbf{S})_y^2 \\ &+ 4m_e^2\gamma_D v^2\langle\cos^2(\varphi)\sin^2(\varphi)\rangle(\mathbf{Q} + 2m_e\alpha\mathbf{S})_x.S_x \\ &- 4m_e^2\gamma_D v^2\langle\sin^2(\varphi)\cos^2(\varphi)\rangle(\mathbf{Q} + 2m_e\alpha\mathbf{S})_y.S_y \\ &+ (2m_e^3\gamma_D v^2)^2\big[\langle\cos^2(\varphi)\sin^4(\varphi)\rangle S_x^2 \\ &+ \langle\sin^2(\varphi)\cos^4(\varphi)\rangle S_y^2\big]\big\}, \end{aligned} \tag{B1}$$

with wave vector $\mathbf{Q}$ and SOC matrix $\hat{\alpha}$ as defined in Eq. (9) but here with $\tilde{\alpha}_1 = \alpha_1$. We set

$$m_e \equiv 1, \tag{B2}$$

$$f_1 := \langle\sin^2(\varphi)\rangle, \tag{B3}$$

$$f_2 := \langle\cos^2(\varphi)\rangle, \tag{B4}$$

$$f_3 := \langle\sin^2(\varphi)\cos^2(\varphi)\rangle, \tag{B5}$$

$$f_4 := \langle\sin^4(\varphi)\cos^2(\varphi)\rangle, \tag{B6}$$

$$f_5 := \langle\sin^2(\varphi)\cos^4(\varphi)\rangle. \tag{B7}$$

Using the Matsubara trick we write

$$\int_0^{2\pi}\frac{d\varphi}{2\pi} = \frac{2}{\pi N}\sum_{s=1}^{N}\frac{1}{\sqrt{1 - \left(\frac{s}{N}\right)^2}}. \tag{B8}$$

This gives us

$$f_1 = \frac{2}{\pi N}\sum_{s=1}^{N-1}\frac{s^2}{N^2\sqrt{1 - \left(\frac{s}{N}\right)^2}}, \tag{B9}$$

$$f_2 = \frac{2}{\pi N}\sum_{s=1}^{N}\sqrt{1 - \left(\frac{s}{N}\right)^2}, \tag{B10}$$

$$f_3 = \frac{2}{\pi N}\sum_{s=1}^{N}\left(\frac{s}{N}\right)^2\sqrt{1 - \left(\frac{s}{N}\right)^2}, \tag{B11}$$

$$f_4 = \frac{2}{\pi N}\sum_{s=1}^{N}\left(\frac{s}{N}\right)^4\sqrt{1 - \left(\frac{s}{N}\right)^2}, \tag{B12}$$

$$f_5 = \frac{2}{\pi N}\sum_{s=1}^{N}\left(\frac{s}{N}\right)^2\left[1 - \left(\frac{s}{N}\right)^2\right]^{\frac{3}{2}}. \tag{B13}$$

Writing Eq. (B1) in a compact way gives us Eq. (47).

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