Photonic analog of a spin-polarized system with Rashba spin-orbit coupling

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(Received 24 November 2010; published 10 March 2011)

We show that a gyrotropic (chiral) medium supporting a longitudinal-wave excitation exhibits a Dirac point in the corresponding photon dispersion lines. By breaking the time-reversal symmetry in such a medium, the dispersion relation resembles the energy dispersion of a spin-polarized two-dimensional electron gas with Rashba spin-orbit coupling. The resulting split bands of the dispersion relation correspond to nonzero Chern numbers implying the existence of nontrivial topological states of the electromagnetic field.

DOI: 10.1103/PhysRevB.83.113101 PACS number(s): 42.70.Qs, 42.25.Bs, 42.50.Gy, 78.67.Pt

Photonic crystals (PCs) are man-made structures originally introduced as the electromagnetic (EM) counterpart of atomic solids. To this end, PCs with absolute frequency band gaps have been proposed and realized, mimicking the electronic band gap of atomic semiconductors.[1,2] The analogy between photonic and atomic semiconductors went further with the study of deviations from perfect periodicity such as point and linear defects, interfaces/heterostructures of different PCs, photonic wells, wires, and dots (cavities).[3] Perhaps the only topic of semiconductor physics which had not found analogy to the physics of PCs until recently has been the quantum Hall effect. It was the advent of a new allotropic form of carbon, graphene,[4] which provided the photonic analog of the quantum Hall effect. Namely, the presence of a Dirac (conical) singularity in the electronic band structure of graphene (gapless transition from the valence to the conduction band) and the opening of the singularity via time-reversal symmetry (TRS) breaking leads to chiral edge states in graphene ribbons. These states are topological electronic states similar to the states emerging in a two-dimensional (2D) electron gas confined within a semiconductor heterostructure and under the influence of a strong magnetic field (integer quantum Hall effect). Adopting the lattice symmetry of graphene (honeycomb) in the photonic problem, 2D PCs with honeycomb symmetry were shown to exhibit a Dirac singularity in the corresponding photonic band structure. The inclusion of a Faraday term in either the permittivity or permeability tensor of the unit cell of the PC leads to TRS breaking and the emergence of topological states of the EM field[5] which allow light propagation along a sole direction at the edges of a finite slab of the PC.[6]

In the anomalous quantum Hall effect, the Hall conductivity may rise in the absence of an external magnetic field. Namely, the presence of an exchange field may suffice for breaking TRS in the spin space. The simultaneous presence of a strong spin-orbit (SO) coupling introduces a TRS-breaking-induced band gap in the band structure and the generation of topological electron states. In this Brief Report, we present an analogy between the energy spectrum of such an electron system and the frequency spectrum of a Faraday-active gyrotropic (chiral) medium exhibiting a strong resonant behavior within a given frequency window. We show that such a medium can be realized as a chiral lattice of resonant spheres.

A spin-polarized 2D electron gas with Rashba SO coupling in the presence of an exchange field is described by the following Hamiltonian:[7,8]

$$H = \frac{\hbar^2 k^2}{2m} + \lambda(\vec{k} \times \vec{\sigma}) \cdot \hat{\mathbf{z}} - \Delta\sigma_z, \tag{1}$$

where $\vec{\sigma}$ is the vector of Pauli spin matrices, λ the SO coupling strength, and Δ the exchange field. The third term in Eq. (1) breaks the TRS in spin space rendering the electron gas ferromagnetic. The TRS breaking in real space is mediated by the SO coupling [second term of Eq. (1)]. The energy dispersion relation corresponding to Eq. (1) is given by

$$\epsilon_\pm = \hbar^2 k^2/2m \pm \sqrt{\lambda^2 k^2 + \Delta^2}. \tag{2}$$

The above energy dispersion relation, which also holds for the SO split bands in magnetic semiconductors[9] and in graphene doped with magnetic atoms,[10] exhibits an energy gap of width 2Δ between the two energy bands (the graph of the dispersion relation is qualitatively the same as that depicted in Fig. 1). When the Fermi energy lies within the gap, the Hall conductivity is resonantly enhanced giving rise to the so-called anomalous Hall effect. We note that the anomalous Hall effect is evident even in the absence of an external magnetic field since the necessary TRS breaking is provided by the (intrinsic) exchange field [last term of Eq. (1)]. As it will be shown below, EM modes within a resonant Faraday-active chiral medium are analogous to the electron states described by Eq. (1).

The photonic analog of the above electron system will be clear below. We consider an isotropic gyrotropic (chiral) medium described by a permittivity tensor $\hat{\epsilon}(\omega)$. The electric displacement $\mathbf{D}(\omega,\mathbf{k})$ is related to the electric field $\mathbf{E}$ through[11]

$$\mathbf{D} = \hat{\epsilon}\mathbf{E} + \mathrm{i}\gamma\mathbf{E} \times \mathbf{k}, \tag{3}$$

where $\mathbf{k}$ is the wavevector and γ is the gyrotropic parameter. The second term of equation (3) describes the spatial dispersion of the chiral medium. Because of this term, it is not necessary to write down the corresponding expression for the magnetic field $\mathbf{H}$.[11,12]

In general, the permittivity tensor $\hat{\epsilon}$ will contain a Faraday term $i\eta$ which explicitly breaks TRS;[5] i.e.,

$$\hat{\epsilon} = \begin{pmatrix} \epsilon(\omega) & 0 & i\eta \\ 0 & \epsilon(\omega) & 0 \\ -i\eta & 0 & \epsilon(\omega) \end{pmatrix}. \tag{4}$$

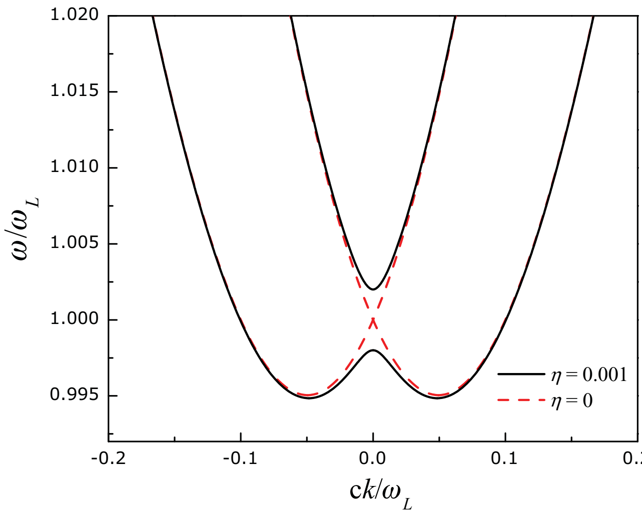

FIG. 1. (Color online) Dispersion relation for a gyrotropic medium with (solid line) and without (dashed line) Faraday term η.

For plane waves, Maxwell equations yield

$$\frac{\mathbf{D}}{\epsilon_0} = \frac{k^2c^2}{\omega^2}[\mathbf{E} - \mathbf{k}(\mathbf{k}\cdot\mathbf{E})/k^2]. \tag{5}$$

Considering transverse waves ($\mathbf{k}\cdot\mathbf{E} = 0$), Eq. (5) becomes

$$\frac{\mathbf{D}}{\epsilon_0} = \frac{k^2c^2}{\omega^2}\mathbf{E}. \tag{6}$$

From Eqs. (3), (4), and (6) we obtain the following homogeneous systems of equations:

$$\begin{aligned}
[\epsilon(\omega) - k^2c^2/\omega^2]E_x + i\gamma k_z E_y + i(\eta - \gamma k_y)E_z = 0,\\
-i\gamma k_z E_x + [\epsilon(\omega) - k^2c^2/\omega^2]E_y + i\gamma k_x E_z = 0,\\
-i(\eta - \gamma k_y)E_x - i\gamma k_x E_y + [\epsilon(\omega) - k^2c^2/\omega^2]E_z = 0.
\end{aligned} \tag{7}$$

For nontrivial solutions for the unknowns E_x, E_y, E_z, the corresponding determinant of Eqs. (7) must vanish; i.e.,

$$\begin{vmatrix} [\epsilon(\omega) - k^2c^2/\omega^2] & i\gamma k_z & i(\eta - \gamma k_y) \\ -i\gamma k_z & [\epsilon(\omega) - k^2c^2/\omega^2] & i\gamma k_x \\ -i(\eta - \gamma k_y) & -i\gamma k_x & [\epsilon(\omega) - k^2c^2/\omega^2] \end{vmatrix} = 0 \tag{8}$$

Eq. (8) yields the following dispersion relations:

$$\epsilon(\omega) = k^2c^2/\omega^2 \tag{9}$$

and

$$[\epsilon(\omega) - k^2c^2]^2 - \gamma^2\left(k_x^2 + k_z^2\right) + (\eta - \gamma k_y)^2 = 0. \tag{10}$$

Obviously, Eq. (9) is a dispersion relation describing wave propagation in an isotropic homogeneous medium (no chirality or time-reversal symmetry breaking is present) and, as such, it will not concern us here. Since our system is effectively two-dimensional, we may set $k_y = 0$ in Eq. (10) and obtain a simpler dispersion relation:

$$\omega^2 = \frac{c^2k^2}{\epsilon(\omega) \pm \sqrt{\eta^2 + \gamma^2k^2}} \tag{11}$$

with $k^2 = k_x^2 + k_z^2$. Solving for k we obtain

$$k_\pm^2 = \frac{\omega\{2c^2\epsilon(\omega) + \gamma^2\omega^2 \pm \sqrt{4c^4\eta^2 + \gamma^2\omega^2[4c^2\epsilon(\omega) + \gamma^2\omega^2]}\}}{\sqrt{2}c^2}. \tag{12}$$

We consider a gyrotropic medium where the diagonal element ϵ of the permittivity tensor supports a longitudinal mode at frequency ω_L; i.e., $\epsilon(\omega_L) = 0$. Such types of permittivity are offered by the lossless Drude model,

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \tag{13}$$

or the single-oscillator model, i.e.,

$$\epsilon(\omega) = \epsilon_b\left(\frac{\omega^2 - \omega_L^2}{\omega_L^2 - \omega_T^2}\right). \tag{14}$$

In the vicinity of ω_L the permittivity can be written as

$$\epsilon(\omega) \simeq D_L(\omega - \omega_L). \tag{15}$$

Substituting Eq. (15) in Eq. (12) we obtain the graph of the dispersion relation (see Fig. 1) which refers to a gyrotropic medium with $\gamma\omega_L/c = 0.1$, $\eta = 0.001$, and $D_L\omega_L = 1/2$ (solid lines). The broken lines refer to a gyrotropic medium with TRS, i.e., $\eta = 0$, and, as can be seen, the dispersion relation exhibits a Dirac point at $\omega = \omega_L$. The inclusion of the off-diagonal Faraday terms in the permittivity tensor breaks TRS and a frequency gap is formed. The dispersion relation of Eq. (11) depicted in Fig. 1 is analogous to that of Eq. (2). Also, an analogy can be seen between the Hamiltonian of Eq. (1) and the constitutive relations of the EM field, namely Eqs. (3) and (4). The diagonal part of the permittivity tensor of Eq. (4) describes free-space propagation similarly to the term $\hbar^2k^2/2m$ of Eq. (1). The term $i\gamma\mathbf{E}\times\mathbf{k}$ describing spatial dispersion is analogous to the SO coupling term $\lambda(\vec{k}\times\vec{\sigma})\cdot\hat{\mathbf{z}}$ while the off-diagonal Faraday term $i\eta$ of the permittivity tensor is the EM counterpart of the exchange-field term $\Delta\sigma_z$. We note that an analogy has been experimentally demonstrated between a system exhibiting anomalous quantum Hall effect via SO coupling and a light beam following a curved trajectory within a graded-index medium (ray-optics regime).[13]

In order to establish the existence of topologically nontrivial phases of the EM field, we need to calculate the Chern number for both branches of the dispersion relation of Eq. (12). The Chern number is defined as

$$C^\pm = \frac{1}{2\pi i}\int d^2\mathbf{k}\Omega_y^\pm, \tag{16}$$

where $\Omega_y^\pm$ is the Berry curvature. By employing the effective-Hamiltonian description for a frequency gap generated by TRS breaking at a Dirac point,[5] the Berry curvature reads as

$$\Omega_y^\pm = \pm\frac{2\gamma^2\eta}{(\eta^2 + \gamma^2k^2)^{3/2}}. \tag{17}$$

We note that an effective-medium approach for quadratic degeneracies (a Dirac point is a linear degeneracy) also

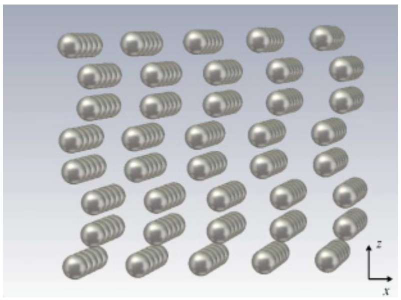

FIG. 2. (Color online) A gyrotropic metamaterial: tetragonal crystal consisting of four nonprimitive planes of spheres parallel to the (001) surface at positions (0,0,0), $(b,0,d/4)$, $(b,b,d/2)$, and $(0,b,3d/4)$, with $b = 0.3a$ and $d = 2a$.

exists.[14] From Eqs. (16) and (17) the Chern number for each branch of Eq. (12) is

$$C^{\pm} = \int_0^{\infty} \Omega_y^{\pm} k dk = \pm 2\gamma^2 \eta \int_0^{\infty} \frac{k dk}{(\eta^2 + \gamma^2 k^2)^{3/2}} = \pm 2. \tag{18}$$

The nonzero values of the Chern numbers for each branch of the dispersion indicate that the corresponding EM states are true topological states.

Next we present an artificial EM structure (chiral metamaterial) which mimics the resonant chiral medium examined above. The structure is shown in Fig. 2 and can be described as a tetragonal crystal with a four-point basis. The crystal is viewed as a succession of planes of spheres parallel to the xy plane. Each plane possesses the same 2D periodicity defined by the primitive vectors $\mathbf{a}_1 = (a,0,0)$ and $\mathbf{a}_2 = (0,a,0)$ [(001) crystallographic surface]. A unit layer of the crystal consists of four nonprimitive planes of spheres at (0,0,0), $(b,0,d/4)$, $(b,b,d/2)$, and $(0,b,3d/4)$. The $(n+1)$-th unit layer is obtained from the n-th layer by the primitive translation $\mathbf{a}_3 = (0,0,d)$. Such a structure (one with dielectric spheres) was originally suggested[15] as an artificial chiral material which can be used for rotating the plane of polarization of linearly polarized light. The spheres of the crystal of Fig. 2 are metallic having radius $S = 0.2a$. Their electric response is described by Eq. (13). The lattice constant a of the 2D square lattice is taken to be $a = c/\omega_p$. For example, for gold $\hbar\omega_p = 8.99$ eV and therefore the lattice constant is $a = 22$ nm and the radius of the spheres $S = 4.4$ nm.

Figure 3 shows the frequency band structure (solid lines), for $\mathbf{k}_{\parallel} = \mathbf{0}$ [normal to the (001) surface], of the crystal of Fig. 2 with $b = 0.3a$ and $d = 2a$. The calculation was based on the layer-multiple-scattering method.[16] All bands appearing in Fig. 3 are nondegenerate due to the chiral symmetry of the crystal. We identify a Dirac point at $\omega/\omega_p = 0.558$. Although the structure of Fig. 2 is three-dimensional, for $\mathbf{k}_{\parallel} = \mathbf{0}$, it can be described by the 2D effective-medium of Eq. (11). The dashed lines in Fig. 3 are obtained by fitting the dispersion relation of Eq. (11) for $\eta = 0, \omega_L/\omega_p = 0.558, D_L = 125, \gamma = 1$ to the actual frequency bands (solid lines).

By introducing a Faraday term with $\eta = 0.1$ we break TRS and obtain the dispersion lines (solid lines) of Fig. 4. Again, the frequency dispersion is similar to that of a spin-polarized electron gas with SO coupling.[7] The dashed line refers to the

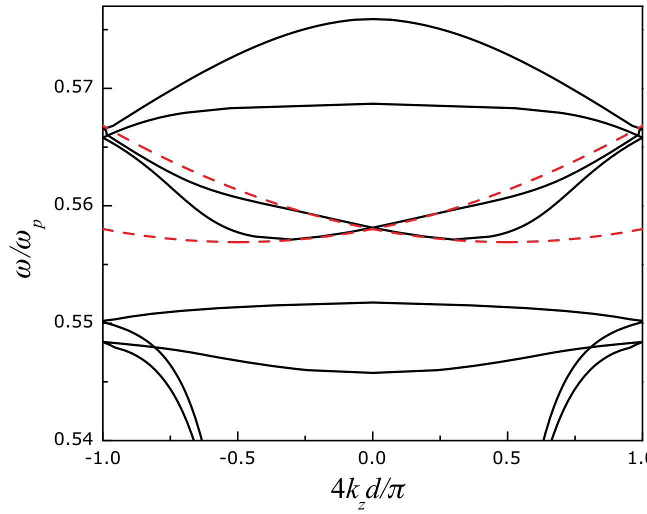

FIG. 3. (Color online) Frequency band structure (solid lines) for $\mathbf{k}_{\parallel} = (k_x,k_y) = \mathbf{0}$ corresponding to the structure of Fig. 2. The dashed lines correspond to Eq. (12) with $\gamma\omega_p/c = 1$ and Eq. (15) with $D_L\omega_p = 125$ and $\omega_L/\omega_p = 0.558$.

corresponding Berry curvature which seems to be significant around the gap ($\mathbf{k}_{\parallel} = \mathbf{0}$). Besides the chiral metamaterial studied here, other types of chiral metamaterials such as a 2D lattice of chiral "Swiss rolls"[17] exhibit a Dirac point in the dispersion lines and are thus able to present nontrivial topological states of the EM field with the inclusion of a Faraday active material.

The presence of Berry curvature in the studied systems betokens the presence of one-way states[5,6] at the edges/surfaces of a finite slab of a gyrotropic medium with a Faraday term. If ones employs the effective-medium approach presented above without a reference to a specific structure (e.g., such as that of Fig. 2), the EM scattering or transfer matrix of a finite slab is needed in order to search for edges states. Since we are dealing with a spiral structure, one needs to employ the Berreman method where the unit cell of a chiral structure is piecewisely approximated as a succession of optically thin

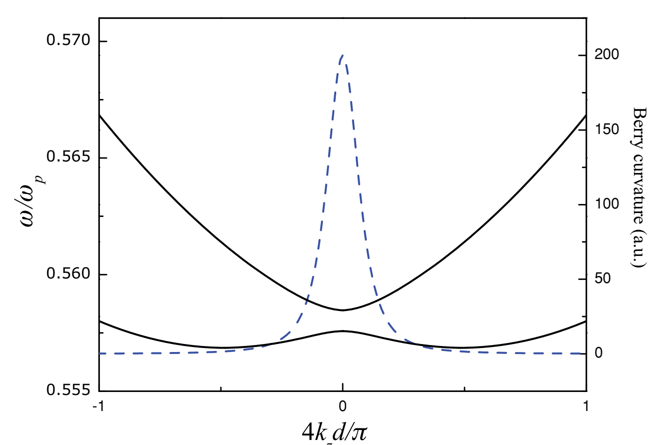

FIG. 4. (Color online) Frequency band structure (solid lines, left axis) for the structure of Fig. 2 including a Faraday term in the permittivity tensor ($\eta = 0.1$). The dashed lines (right axis) refer to the corresponding Berry curvature.

layers with a homogeneous dielectric tensor.[18,19] Then one couples the corresponding matrices of the successive layers either by the scattering or by the transfer matrix technique in order to obtain the final matrix of the entire slab. On the other hand, if one departs from the effective-medium treatment and explores the appearance of edge states for a specific structure, e.g., the crystal of Fig. 2, then it is necessary to incorporate within the layer-multiple-scattering method the scattering *T*-matrix from a Faraday active spherical particle.[20] The existing layer-multiple-scattering code provides the scattering matrix for a finite number of crystal layers which can reveal the existence of edge states. However, the calculation of the *T*-matrix for a Faraday active sphere is much more involved and computationally cumbersome than for the case of a homogeneous sphere and it is left for a future publication.

In conclusion we have presented a generic model of a resonant, Faraday-active chiral medium which is the EM analog of a spin-polarized system with Rashba SO coupling exhibiting intrinsic (anomalous) Hall conductivity. Such a chiral medium possesses a frequency gap stemming from TRS breaking and supports nontrivial topological EM modes with nonzero Chern numbers. A helical lattice of lossless Drude-type spheres models the behavior of the presented photonic analog.

This work has been supported by the European Community's Seventh Framework Programme (FP7/2007-2013) under Grant Agreement No. 228455-NANOGOLD (Self-organized nanomaterials for tailored optical and electrical properties).

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