Inhomogeneity and transverse voltage in superconductors

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Voltages parallel and transverse to electric current in slightly inhomogeneous superconductors can contain components proportional to the field and temperature derivatives of the longitudinal and Hall resistivities. We show that these anomalous contributions can be the origin of the zero field and even-in-field transverse voltage occasionally observed at the superconductor-to-normal-state transition. The same mechanism can also cause an anomaly in the odd-in-field transverse voltage, interfering with the Hall-effect signal.

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I. INTRODUCTION

The Hall effect in the superconducting state is one of the major tools for studying vortex dynamics and, as such, has enjoyed much attention over the past two decades. Two puzzling features observed in the vicinity of the superconductor-normal phase transition have attracted particular interest, but nevertheless have remained elusive until now: (i) development of an excess transverse voltage at zero applied magnetic field[1,2] and an even-in-field transverse voltage (ETV) under applied field,[3–5] and (ii) reversal of the Hall coefficient polarity.[6–8]

The appearance of a transverse voltage at zero applied magnetic field was attributed to attraction among vortices and antivortices, generated at two opposite edges of a film by the self-field of the applied electrical current.[2,9] Attractive vortex-antivortex interaction modifies the vortex trajectory by providing a velocity component parallel or antiparallel to the current, thus generating a local electric field transverse to the current direction. Polarity of this local field depends on the trajectory of individual vortices; therefore, development of nonzero transverse voltage across macroscopic samples can take place only if the symmetry of the interaction is broken along the current line, a condition that is hard to justify. Breaking of time-reversal symmetry due to fractional statistics in two-dimensional, high-temperature superconductors was suggested[10] as an alternative interpretation, although such a mechanism cannot explain the presence of the effect in conventional three-dimensional superconductors.

The development of the ETV under applied magnetic field has been discussed most often in terms of guided vortex motion.[11–18] In this scenario the pinning landscape plays a crucial role. Nonzero ETV can be generated if vortices are forced to move along tracks not normal to current direction over a length comparable to the distance between voltage probes. Such guided motion can be achieved in materials with orientational pinning anisotropy, for example, single crystals with oriented twin boundaries,[11,12] ordered arrays of Josephson junctions,[13] foils treated by mechanical rolling,[6,14] films with particular lithographic patterning,[15–17] or materials deposited onto faceted substrates.[18] Surprisingly, both ETV and zero-field transverse voltage were also observed in a variety of untreated superconductors with no orientational pinning,[1–4,19] where transport properties should be isotropic.

In general, the magnitude and details of anomalous transverse voltage effects are unpredictable and hardly reproducible. Samples produced and measured under the same experimental conditions can give a different ETV response.[5] Similarly, reports on the sign reversal of the Hall coefficient are inconsistent. While many groups reported the effect in different low- and high-temperature superconductors,[6–8] others detected no sign reversal in identical materials.[20]

A nonuniform transport current due to inhomogeneity of material was suggested[1,21,22] as a possible explanation for at least part of the anomalous behavior. Against this it was argued[10] that the effect was observed both in high-quality samples with a narrow superconducting transition and in disordered samples with a wide transition.

An interesting feature noted in several cases[3,10] was a correlation between the anomalous temperature-dependent transverse voltage and the temperature derivative of the longitudinal resistivity $\partial R_{xx}/\partial T$. A similar correlation was also found[5] between the ETV and the field derivative of the longitudinal resistance $\partial R_{xx}/\partial H$.

In this paper we report on a systematic study of the transverse voltage in several low-T_c and high-T_c superconducting materials. We shall demonstrate that the anomalous transverse voltage in untreated superconductors can be consistently explained by the presence of a minor asymmetric spatial inhomogeneity in the material, and the correlation of the longitudinal and Hall resistivities with their derivatives is the trademark of this transverse voltage's origin.

II. EXPERIMENTAL RESULTS AND DISCUSSION

The primary mechanism responsible for depinning and motion of vortices in type II superconductors is the Lorentz force, $\vec{F}_L = \vec{J} \times \vec{B}$, produced by the transport current $\vec{J}$ and magnetic induction $\vec{B}$ per unit volume of the vortex lattice. The force is largest when the magnetic field is applied normal to the electrical current, and it diminishes to zero when the field is aligned precisely along the current line. As such, any phenomenon related to flux motion is expected to depend strongly on the presence and orientation of the applied field relative to electrical current. Figure 1(a) presents the transverse voltage V_{xy} measured as a function of temperature in a plain 200-nm-thick Pb film in a zero applied magnetic field. The V_{xy} signal shown in this and the following figures was obtained by subtracting the normalized mismatch voltage corresponding to an unavoidable misalignment of the transverse contact pads. The longitudinal voltage measured simultaneously is marked

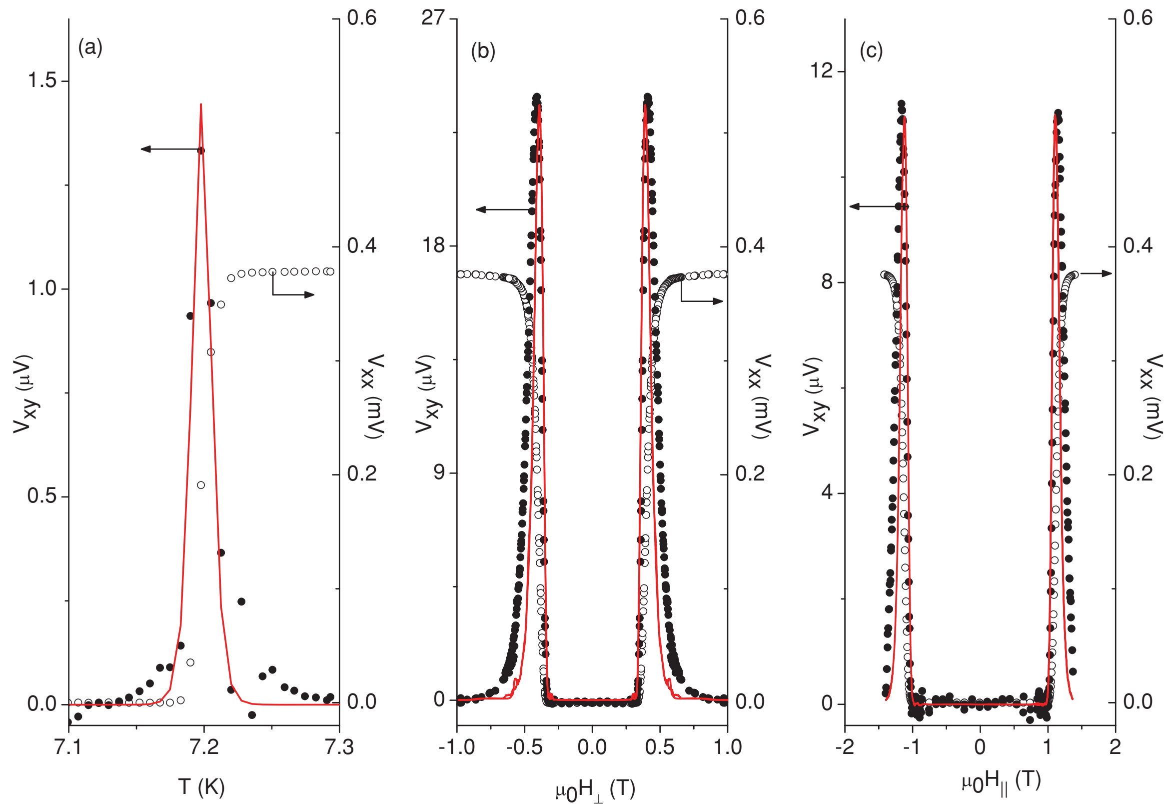

FIG. 1. (Color online) V_{xy} (•) and V_{xx} (○) measured in a 200-nm-thick Pb film as (a) a function of temperature at zero applied magnetic field, (b) a function of field applied normal to the film plane, and (c) in-plane parallel to electric current at $T = 4.2$ K. Solid lines (red) are fits calculated according to Eqs. (3) and (4).

by V_{xx}. It can be seen that V_{xy}appears at the onset of the transition and disappears when resistivity reaches its normal state value. Figure 1(b) presents the longitudinal voltage and the ETV measured in the same sample at 4.2 K as a function of field applied normal to the film plane. The data are shown for both magnetic-field polarities. Nonzero ETV appears at the onset of the resistive transition and disappears in the normal state. ETV is also observed in the Lorentz force-free configuration, with field applied in-plane parallel to the current contact line, as shown in Fig. 1(c). Misalignment of the field orientation from the film plane, estimated by the Hall resistance slopes at temperatures above T_c, does not exceed 1°–2°, which is far too small to justify the observed effect. The magnitude of V_{xy} in both field orientations is very close, implying that the anomalous effect does not depend on the orientation and the actual presence of the applied magnetic field. These findings contradict any direct correlation between the transverse voltage and vortex motion.

A phenomenon closely related to the subject of this work is the occasional observation of an excess voltage in longitudinal resistivity across the superconductor-normal-state transition.[1,23] Vaglio *et al.*[24] suggested that this effect could be explained by inhomogeneity of the material and described by a simple current distribution model. Here we adapt and extend this model to treat the development of the transverse voltage. The sample is represented by a four-resistor network, shown in Fig. 2(a), where resistors R_{a-d} represent four quarters of a superconducting sample and thus depend on field and temperature. Longitudinal voltage V_{xx} is measured between contacts A and E or between B and F, and the transverse voltage V_{xy} is measured between C and D. We are interested in V_{xx} and V_{xy} in the transition range, where conductivity of any macroscopic section of the sample is finite. For simplicity we assume all resistors to be Ohmic. V_{xx} and V_{xy} are calculated by Kirchhoff's laws as

$$V_{xx} = I\frac{(R_a + R_c)(R_b + R_d)}{R_a + R_b + R_c + R_d} \quad (1)$$

and

$$V_{xy} = I\frac{R_a R_d - R_b R_c}{R_a + R_b + R_c + R_d}, \quad (2)$$

where I is the total electric current. A nonzero transverse voltage V_{xy} will be generated in any case of diagonal inequality, i.e., $R_a R_d \neq R_b R_c$. We assume the simplest case, in which three resistors are identical,

$$R_a(T,H) = R_b(T,H) = R_c(T,H) \equiv R^*(T,H),$$

and the fourth, R_d, passes from the superconducting to the normal state with a small delay in temperature ΔT_t and/or field ΔH_t, which we will denote as the transverse delays. In the transition range, R_d is given by

$$R_d(T,H) = R^*(T + \Delta T_t, H + \Delta H_t) \approx R^*(T,H) + \Delta T_t \frac{\partial R^*(T,H)}{\partial T} + \Delta H_t \frac{\partial R^*(T,H)}{\partial H}.$$

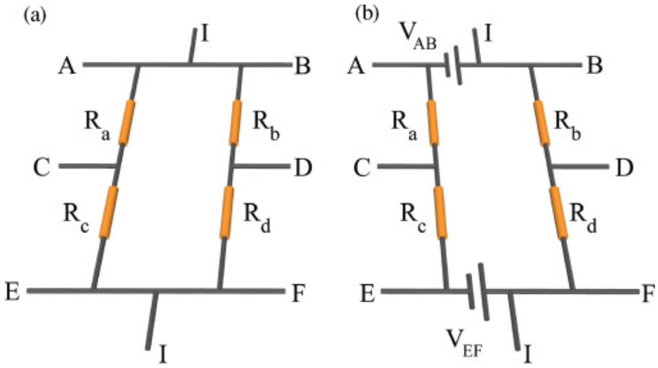

FIG. 2. (Color online) Model circuits representing a superconducting film (a) without Hall voltage and (b) with Hall voltage. Each resistor represents a quarter of the sample.

When the superconducting transition is crossed as a function of temperature in zero or constant magnetic field, the transverse voltage V_{xy} will develop according to

$$V_{xy} = \frac{\Delta T_t}{4}\frac{\partial V_{xx}(T,H)}{\partial T}. \tag{3}$$

Similarly, when the magnetic field is varied at constant temperature, the model predicts the transverse voltage will be given by

$$V_{xy} = \frac{\Delta H_t}{4}\frac{\partial V_{xx}(T,H)}{\partial H}. \tag{4}$$

ΔH_t changes sign at the negative field polarity, so the resulting $V_{xy}(H)$ is an even function of the magnetic field. $V_{xy}(T,H)$ is proportional to the temperature or/and field derivative of the longitudinal resistance and becomes significant at the superconducting transition due to a sharp variation of resistance. Solid lines in Figs. 1(a)–1(c) are fits to Eqs. (3) and (4), calculated using the measured longitudinal resistance and one fitting parameter, ΔT_t or ΔH_t, only. A perfect fit in Fig. 1(a) was obtained with $\Delta T_t = 2.8 \times 10^{-4}$ K, which is more than two orders of magnitude smaller than the width of the superconducting transition $\delta T = 5 \times 10^{-2}$ K, the latter being the temperature span over which resistivity changes from 10% to 90% of its normal value. Fits of $V_{xy}(H)$ drawn in Figs. 1(b) and 1(c) were calculated with $\Delta H_t = 230$G and $\Delta H_t = 170$G for the perpendicular and parallel field orientations, respectively. The transition widths in these orientations are larger by one order of magnitude: $\delta H_{\perp} = 2000$G and $\delta H_{||} = 3000$G.

Presence of the spatial inhomogeneity can be tested explicitly by, e.g., simultaneous probing of different parts of a sample. Figure 3 presents two magnetoresistance measurements taken simultaneously along two opposite edges of Pb film, with field applied normal to the film plane. Both sets of data are normalized by their respective normal-state values at the field of 1 T, denoted by V_n. The inset of Fig. 3 shows a sketch of the sample and location of the voltage probes. V_1 in Fig. 3 is identical to V_{xx} in Fig. 1(b). The relative shift of the transition is ∼250 G, in a fair agreement with the value $\Delta H_t = 230$ G found by the fitting.

Following the model, the sign of the transverse voltage depends on the sign of ΔH_t and ΔT_t, i.e., whether region *d* has higher or lower critical field and/or temperature than the rest of the sample. To test the relevance of the model we fabricated two samples with artificial "diagonal" nonuniformity. Two Pb samples were deposited through two identical Hall bar masks onto glass substrates tilted by 45° from the target direction. Masks were rotated in plane of the substrates by +45° and –45°, as shown in the sketch in Fig. 4. In terms of the labels of Fig. 4, corner B of sample S1 was closer to the target than corner C, and corner A of sample S2 was closer than corner D. As a result, the thickness of the opposite corners (B, C in S1, and A, D in S2) varied by ∼25% for an average thickness of 200 nm, and the thickness gradient was oriented by ∼+45° and –45° from the current line direction. Both samples were prepared simultaneously, under the same conditions. For a sample with a variable thickness, one expects a lower critical field in the thinnest region due to a higher current density. Figure 4 presents the transverse voltage V_{xy} measured in the two samples at $T = 4.2$ K as a function of field applied normal to the films. The transverse voltage peaks are almost identical but have opposite polarity, consistent with the expected: positive for sample S1 and negative for S2. The result was consistently reproduced in a number of similarly fabricated pairs.

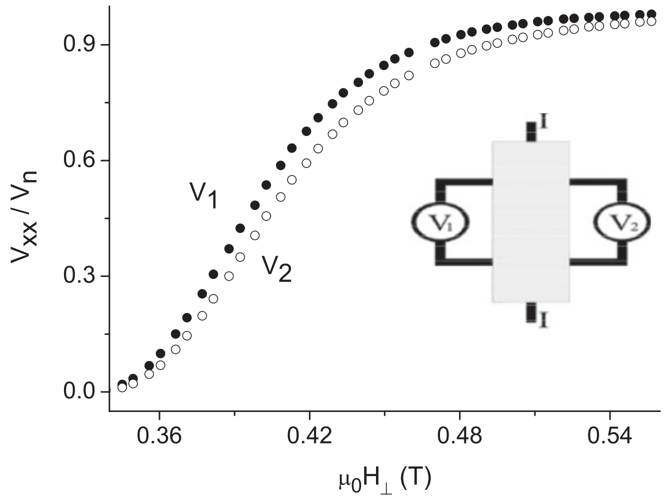

FIG. 3. The normalized longitudinal voltage V_{xx}/V_n measured at two opposite edges of the Pb film as a function of magnetic field applied normal to the film. V_n are the normal-state values at 1 T. The inset shows a sketch of the sample and location of the voltage probes.

Applicability of the model to high-temperature superconductors was tested with YBaCuO films grown by off-axis dc magnetron sputtering onto yttrium-stabilized, ZrO_2-covered sapphire substrates.[25] Figures 5(a) and 5(b) present, respectively, the temperature-dependence of V_{xy} in zero field and the even V_{xy} measured as a function of field applied normal to *ab* planes at $T = 81.8$ K. The anomalous transverse voltages in YBaCuO have the same characteristic features as those found in Pb. The solid lines in Figs. 5(a) and 5(b) were calculated by Eqs. (3) and (4) using the measured $R(T)$ and $R(H)$ and fitting parameters $\Delta T_t = 0.6$ K and $\Delta H_t = 1.17$ T. Both fitting parameters correspond to ∼20% of the respective transition widths. The same model seems therefore to describe consistently the anomalous transverse voltage both in Pb and in YBaCuO.

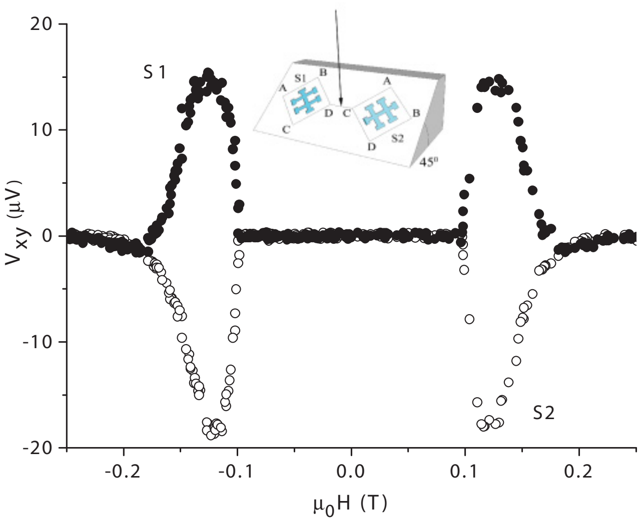

FIG. 4. (Color online) Transverse voltage measurements of two Pb samples with thickness gradients oriented at +45° (•) and –45° (○) to the current line. A sketch of the sample deposition method is shown in the inset. The arrow indicates direction to the target.

The question that needs to be addressed is whether the simple effective four-resistor circuit of Fig. 2(a) models reliably the current and electric potential distribution seen in a real sample. In order to test this, we performed a numerical calculation of current flow in inhomogeneous samples. The discrete form of the continuum transport equations is equivalent to current flow in a resistor grid; therefore, a natural extension of the four-resistor model is its replacement by a large resistor network. We conducted the calculation on a grid consisting of 100×300 resistors (as shown in Fig. 6 inset, with a grid of 4×12 resistors). Magnetic field-dependence of the black resistors is shifted by ΔH_t relative to the gray resistors. In order to compare the calculation to the experimental results, the field dependence of the gray resistors was taken to be proportional to the measured $V_{xx}(H)$ shown in Fig. 1(b): $R(H) = R_0 V_{xx}(H)/V_{xx}(1\ \mathrm{T})$, where $V_{xx}(1\ \mathrm{T})$ is the normal-state value of V_{xx}, measured at a field of 1 T. Coefficient R_0 was determined by stipulating the calculated normal state V_{xx} be equal to the experimental value. The result of the numerical calculation is shown in Fig. 6 by the solid line, with $\Delta H_t = 270$ G. The numerically calculated curve is close to the one obtained by use of Eq. (4) (dashed line in Fig. 6) with similar values of ΔH_t [270 G for the numerical calculation compared to 230 G for Eq.(4)]. One can therefore conclude that the simple four-resistor model captures the essential details of the current and electric potential distribution of a macroscopic sample.

Evidently, the inhomogeneity of arbitrary samples can be more complex than what is modeled above and generates a variety of transverse voltage patterns. One example is the variable-polarity ETV found in a granular Ni-Pb mixture with 15% volume of Ni, shown in Fig. 7. Such a pattern can be explained if, for example, the transition is wider in one-quarter of the sample than in others, while the critical field, defined at the midtransition, is roughly the same everywhere. In this case the difference between resistance of the selected region and the rest of the sample is positive at one stage of

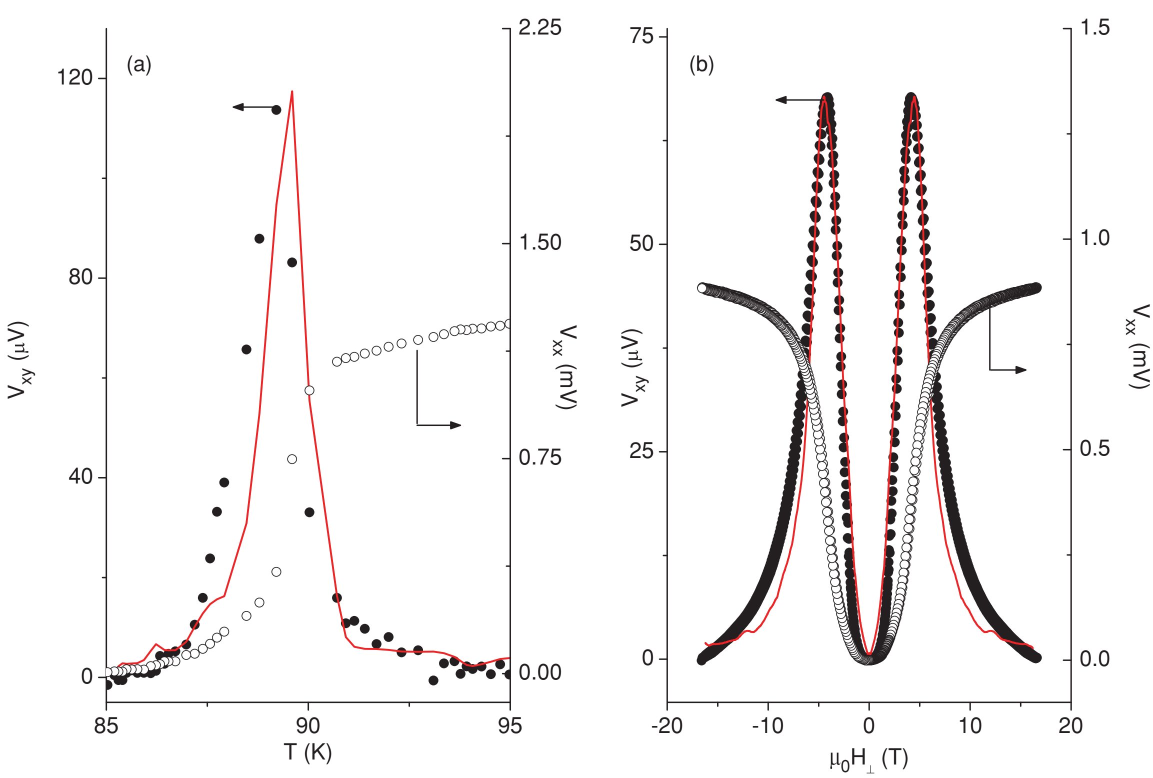

FIG. 5. (Color online)Even V_{xy} (•) and V_{xx} (○) measured in a YBaCuO film as a function of (a) temperature at zero applied magnetic field and (b) field applied normal to the film plane at $T = 81.8$ K. Solid lines (red) are fits calculated according to Eqs. (3) and (4).

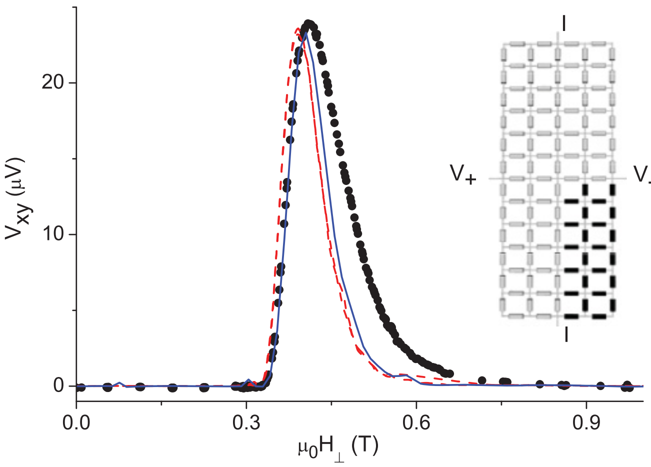

FIG. 6. (Color online) Even V_{xy} in a Pb sample as function of field applied normal to the film. Full circles represent experimental data, the solid line (blue) is the numerically calculated transverse voltage, and the dashed line (red) is the voltage calculated according to Eq. (4). The inset shows a sketch of the resistor grid used for the numerical calculation.

the transition and negative at the other. The ETV, which is determined by the difference between the diagonal resistances, will therefore change sign during the transition. A similar signal was observed in superconducting thin films with magnetic vortex arrays.[22]

The same inhomogeneity mechanism can also be responsible for the generation of an anomalous odd-in-field transverse signal. If the superconducting transition is not identical along the sample, the Hall voltage will also differ at different cross sections. The representing circuit is shown in Fig. 2(b). V_{AB} and V_{EF} indicate the Hall voltages at two cross sections, while resistors R_{a-d} model the sample's longitudinal resistance. Assuming that $V_{\mathrm{AB}}(H) = V_{\mathrm{EF}}(H + \Delta H_l)$ and $R_d = R^*(H + \Delta H_t)$, where ΔH_l is a small longitudinal delay in the critical field, one can calculate V_{xx} between points F and B and V_{xy} between points C and D as, respectively,

$$V_{xx} = IR + \frac{\Delta H_l}{2}\frac{\partial V_{xy}}{\partial H} + \frac{\Delta H_t}{4}\frac{\partial V_{xx}}{\partial H}, \tag{5}$$

$$V_{xy} = V_{\mathrm{EF}} + \frac{\Delta H_l}{2}\frac{\partial V_{xy}}{\partial H} + \frac{\Delta H_t}{4}\frac{\partial V_{xx}}{\partial H}. \tag{6}$$

Here, in addition to the regular signals IR and V_{EF}, both the longitudinal and transverse voltages contain two additional terms: an odd-in-field term proportional to the field derivative of the Hall voltage and an even-in-field term proportional to the field derivative of resistivity. These extra "inhomogeneity" terms can be significant when resistivity or Hall voltage vary sharply. Polarity of the odd term $\frac{\Delta H_l}{2}\frac{\partial V_{xy}}{\partial H}$ can be the same as or opposite to V_{EF}; however, its magnitude does not exceed the latter. Therefore, the superposition of two odd terms can result in an anomaly of the Hall-effect signal, although not in a reversal of its polarity. In our samples we did not identify an anomalous odd signal exceeding the experimental accuracy.

The effect of a minor asymmetric inhomogeneity discussed here is relevant not only for superconducting transitions but also for other systems where the longitudinal or Hall resistivity varies sharply as a function of any external parameter such as pressure, temperature, or magnetic or electric field. Noteworthy examples are reversal of magnetization in ferromagnetic films with perpendicular magnetic anisotropy[26] and development of fractional quantum Hall steps in a two-dimensional electron gas.[27] A general warning: Straightforward determination of the longitudinal and Hall resistivities as the even- and odd-in-magnetic field components respectively of the measured data can lead to erroneous conclusions.

III. CONCLUSIONS

In summary, we tested and found no evidence of a direct correlation between the vortex dynamics and development of an anomalous zero field and even-in-field transverse voltage in untreated low-T_c and high-T_c superconductors. On the other hand, the effect can be consistently explained by the presence of a minor asymmetric inhomogeneity of the material. In this case, a simple circuit model predicts an appearance of additional voltage signals proportional to the temperature and field derivatives of the longitudinal resistivity in excellent agreement with the experiment. The same mechanism can also cause an anomaly in the odd-in-field transverse voltage, interfering with the Hall-effect signal, although not causing a reversal of its polarity. The effect can be present both in high-quality samples with a narrow and sharp transition and in disordered samples with a wide transition, since only a minor relative transverse inhomogeneity is sufficient for its development.

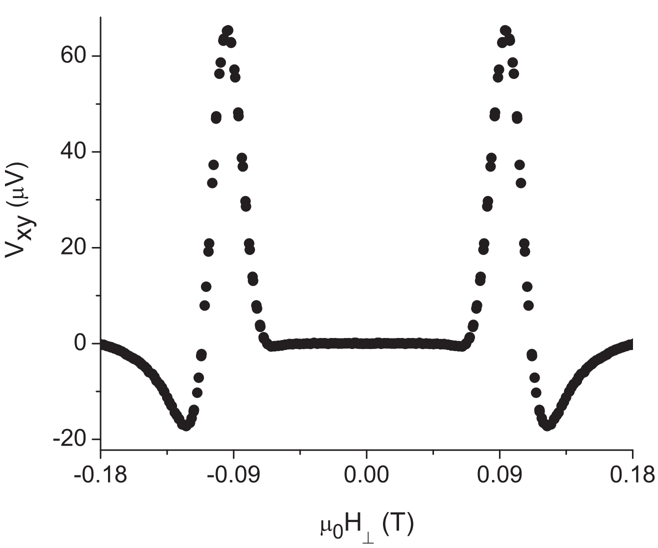

FIG. 7. Even transverse voltage measured in a 200-nm-thick NiPb sample as a function of field normal to the film plane at $T = 4.2$ K.

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