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Anomalous Cherenkov spin-orbit sound

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The Cherenkov effect is a well-known phenomenon in the electrodynamics of fast charged particles passing through transparent media. If the particle is faster than the light in a given medium, the medium emits a forward light cone. This beautiful phenomenon has an acoustic counterpart where the role of photons is played by phonons and the role of the speed of light is played by the sound velocity. In this case the medium emits a forward sound cone. Here, we show that in a system with spin-orbit interactions in addition to this *normal* Cherenkov sound there appears an *anomalous* Cherenkov sound with forward and *backward* sound propagation. Furthermore, we demonstrate that the transition from the normal to anomalous Cherenkov sound happens in a *singular* way at the Cherenkov cone angle. The detection of this acoustic singularity therefore represents an alternative experimental tool for the measurement of the spin-orbit coupling strength.

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Purely electric manipulation of the electron spin is, no doubt, the core idea of modern spintronics.[1,2] Systems with spin-orbit interaction (SOI) represent a distinct platform for the practical implementation of this idea by way of concrete spintronic devices such as a spin transistor.[3] In two-dimensional semiconductor heterostructures usually the Bychkov-Rashba SOI (BRSOI),[4] resulting from the structure inversion asymmetry, and linear Dresselhaus[5] SOI (DSOI), resulting from the inversion asymmetry of crystal structure of the bulk host material, are of most practical importance in the spin dynamics. The cubic Dresselhaus term is less significant but in exotic situations, when the coupling strengths of BRSOI and DSOI are equal, it becomes the main term violating the SU(2) symmetry and thus plays a crucial role in limiting the electron spin lifetime as has been recently demonstrated in the fascinating experiments on the persistent spin helix.[6,7]

Most works on systems with SOI focus on electron spin and charge dynamics, in particular, on pure spin current generation: by means of the intrinsic spin-Hall effect,[8,9] polarized light,[10] or spin ratchets.[11] Also fundamental experimental research on the spin-orbit coupling (SOC) strength mainly addresses the electron degrees of freedom, through Shubnikov–de Haas oscillations,[12,13] photocurrents,[14] or optical monitoring of the angular dependence of the electron spin precession on the electron motion direction with respect to the crystal lattice.[15]

This trend, which puts the electron degrees of freedom in the center of the research, is clear. From one side it is explained by the fact that exactly electron dynamics represents the source of SOC. SOI is an outcome of special relativity where in the reference frame of a moving electron electric fields transform into magnetic ones. These magnetic fields interact with the electron spin, removing the spin degeneracy. From the other side it is explained by significant advances in experimental techniques dealing with electrons.

However, in real systems the electron degrees of freedom may interact with degrees of freedom of a different nature. It is therefore challenging to study the traces of SOC on the particles interacting with electrons. One possibility is provided by the lattice vibrations of the heterostructure host crystal. Indeed, the electron orbital degrees of freedom are electrostatically coupled to the orbital degrees of freedom of the crystal lattice. Since the electron orbital dynamics in systems with SOI depends on spin, it is clear that the lattice dynamics will be in a certain way modified by electron SOI.

The interaction between electrons and the crystal lattice is described in terms of quantized lattice vibrations, referred to as phonons, and it has been widely studied in systems with SOI. However, the research mostly centered again on the impact of phonons on the electron degrees of freedom, either charge or spin. Examples date back to research on spin relaxation due to the Dyakonov-Perel'[16] mechanism involving electron scattering on phonons with the corresponding momentum relaxation time. More recent examples are phonon-induced decay of the electron spin in quantum dots[17,18] or phonon-limited mobility in a two-dimensional electron gas (2DEG) with SOC.[19] Among other examples is the use of coherent acoustic phonons to create dynamic quantum dots in SOC systems.[20]

Here we take a different viewpoint on systems with SOI and instead of electrons focus on phonons, in particular on the consequences and fingerprints of SOI which could be observed in the phonon dynamics. There are obviously various aspects of the phonon dynamics in solids. In the present investigation we will study one of them, an important and beautiful phenomenon of the Cherenkov radiation.

Originally the Cherenkov effect was discovered by Cherenkov in the electrodynamics of fast charged particles passing through transparent media[21] and it was later theoretically explained by Tamm and Frank.[22] It consists in the appearance of a forward light cone emitted by a given medium under the impact of a charged particle moving with a velocity larger than the speed of light in this medium. This Cherenkov effect, also referred to as the normal optical Cherenkov effect, is qualitatively different from the well-known deceleration radiation because in the latter case the radiation is emitted by the particle itself.[23]

With the appearance of photonic crystals the optical Cherenkov effect was rediscovered and an anomalous optical Cherenkov radiation with a backward-pointing cone was predicted.[24] Here the anomalous radiation is the result of strong inhomogeneity of the medium, leading to complex Bragg scattering.

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It turns out that the normal Cherenkov effect has an acoustic counterpart. In a three-dimensional (3D) medium an electron whose velocity is larger than the sound velocity of the medium excites a forward sound Cherenkov cone. The sound intensity $I_s^{(3D)}$ is located inside a three-dimensional cone and its azimuthal angular distribution[25] is $I_s^{(3D)}(\theta) = \Theta(\theta_c - \theta)[(v/c)\cos(\theta) - 1]^2 \sin(\theta)$, where $0 < \theta < \pi$ is the azimuthal angle, the angle between the incident electron momentum $\mathbf{p}$ (chosen as the direction of the z axis; $\mathbf{p} = m\mathbf{v}$, where m is the particle mass and $|\mathbf{v}| = v$) and the phonon momentum $\mathbf{q}$, c is the sound velocity ($c < v$), $\theta_c = \arccos(c/v)$ is the Cherenkov cone angle, and $\Theta(x)$ is the Heaviside step function.

In two dimensions (2D) the sound intensity $I_s^{(2D)}$ is located inside a two-dimensional Cherenkov cone with the polar angular distribution $I_s^{(2D)}(\phi) = \Theta(\phi_c - |\phi|)[(v/c)\cos(\phi) - 1)]$, where $-\pi < \phi < \pi$ is the polar angle (here $\mathbf{p}$ is the direction of the x axis) and the Cherenkov angle ϕ_c of the two-dimensional cone is given by the same expression as θ_c in three dimensions.

The essential difference between $I_s^{(3D)}$ and $I_s^{(2D)}$ is that in 2D the strictly forward ($\phi = 0$) sound emission is allowed while in 3D the phonons with $\theta = 0$ are dimensionally forbidden.

The characteristic feature of the normal acoustic Cherenkov effect (ACE) is that in both the 3D and 2D cases the sound does not exist out of the Cherenkov cone. However, as we demonstrate below, this situation radically changes as soon as the spin and orbital degrees of freedom of the incident electron, exciting the Cherenkov sound, are coupled by a certain SOI mechanism. In this case it is shown that one obtains an acoustic counterpart of the anomalous optical Cherenkov radiation. However, what is remarkable is that this anomalous ACE does not require any inhomogeneity of the medium at all, in contrast with the optical version of the effect in photonic crystals.[24]

To study the essential physics we consider a 2DEG with BRSOI. The Bychkov-Rashba Hamiltonian is $\hat{H}_0 = \hat{\mathbf{p}}^2/2m + (\alpha/\hbar)[\hat{\sigma} \times \hat{\mathbf{p}}] \cdot \mathbf{n}$. Here $\mathbf{n}$ is the unit vector perpendicular to the 2DEG plane, m is the electron effective mass, $\hat{\mathbf{p}}$ is the momentum operator, $\hat{\sigma}$ is the Pauli matrix vector, and $\alpha \equiv \hbar p_{\rm so}/m$ is the SOC strength. In the following we choose the direction of $\mathbf{n}$ to be the direction of the z axis.

The Hamiltonian $\hat{H}_0$ lifts the twofold spin degeneracy at momenta $\mathbf{p} \neq \mathbf{0}$ and results in a spin-orbit band splitting. The spin is not a good quantum number anymore. It is well known that the chirality operator $\hat{R} \equiv [\hat{\sigma} \times \hat{\mathbf{e}}\,] \cdot \mathbf{n}$, with $\hat{\mathbf{e}} \equiv \hat{\mathbf{p}}/|\hat{\mathbf{p}}|$ being the operator of the momentum direction, commutes with the Hamiltonian $\hat{H}_0$ and momentum operator. Its eigenvalues $\lambda = \pm 1$ characterize the electron energy spectrum $\epsilon_{\mathbf{p}\lambda} = \mathbf{p}^2/2m + \lambda p_{\rm so}|\mathbf{p}|/m$, and the normalized eigenspinors of $\hat{H}_0$, $\langle \mathbf{r}\sigma|\mathbf{p}\lambda\rangle = \exp(i\mathbf{pr}/\hbar)(1/\sqrt{2})[1, \lambda i \exp(-i\varphi_{\mathbf{p}})]$, where $\varphi_{\mathbf{p}}$ is the angle between the electron momentum $\mathbf{p}$ and the x axis and $\mathbf{r}$ is the 2D coordinate.

The physics of the ACE comes from the electron coupling to acoustic phonons. The phonon Hamiltonian has the standard second quantized expression[26] $\hat{H}_{\rm ph} = \sum_{\mathbf{q}} \hbar\omega(\mathbf{q})(b_{\mathbf{q}}^\dagger b_{\mathbf{q}} + 1/2)$, where $b_{\mathbf{q}}^\dagger$ and $b_{\mathbf{q}}$ are the phonon creation and annihilation operators, and the acoustic phonon spectrum is $\hbar\omega(\mathbf{q}) = c|\mathbf{q}|$ with c being the sound velocity. The Hamiltonian of the electron-phonon interaction reads[27] $\hat{H}_{\text{el-ph}} = g\sum_\sigma \int d\mathbf{r}\hat{\psi}_\sigma^\dagger(\mathbf{r})\hat{\psi}_\sigma(\mathbf{r})\hat{\varphi}(\mathbf{r})$, where g is the electron-phonon coupling strength, $\hat{\psi}_\sigma^\dagger(\mathbf{r})$ and $\hat{\psi}_\sigma(\mathbf{r})$ are the electronic field operators, and $\hat{\varphi}(\mathbf{r}) = i\sum_{\mathbf{q}} \sqrt{\hbar\omega(\mathbf{q})/2V}[\exp(i\mathbf{qr}/\hbar)b_{\mathbf{q}} - \exp(-i\mathbf{qr}/\hbar)b_{\mathbf{q}}^\dagger]$.

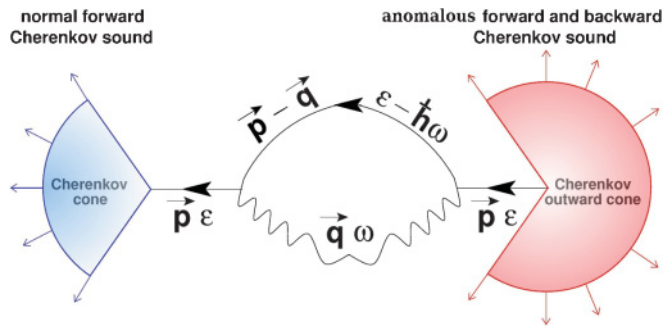

FIG. 1. (Color online) The Feynman diagram of ACE describing the process of the acoustic phonon excitation by an incident electron with momentum $\mathbf{p}$ and energy ε. The phonons with momentum $\mathbf{q}$ are emitted in the normal forward sound cone and, for SOC systems, in the anomalous forward and backward outward sound cones.

Since the ACE is an effect of the electron propagation through a medium, the natural mathematical language to describe this physical phenomenon is the language of the Feynman propagators or the Green's functions. In the present case the Green's functions containing the physics of the ACE are defined with respect to physical vacuum.[25]

The corresponding self-energy diagram is shown in Fig. 1. The analytic expression corresponding to this diagram is obtained according to the standard analytic reading rules[27] and leads to the following expression:

$$\Sigma_{\mathbf{p}\lambda}(\varepsilon) = \frac{ig^2}{16\pi^3\hbar^2}\sum_{\lambda'}\int\left[\frac{c^2\mathbf{q}^2}{\hbar^2\omega^2 - c^2\mathbf{q}^2 + i\eta} \times \frac{1 + \lambda\lambda'\cos(\varphi_{\mathbf{p}-\mathbf{q}} - \varphi_{\mathbf{p}})}{\varepsilon - \hbar\omega - \epsilon_{\mathbf{p}-\mathbf{q}\lambda'} + i\eta}\right]d\omega d\mathbf{q}, \quad (1)$$

where η is a positive infinitesimal. The frequency integration is performed using the residue theorem. There is one pole in the lower half plane, $\omega_0 = c|\mathbf{q}|/\hbar - i\tilde{\eta}$. Thus

$$\Sigma_{\mathbf{p}\lambda}(\varepsilon) = \frac{g^2c}{16\pi^2\hbar^3}\sum_{\lambda'}\int\frac{1 + \lambda\lambda'\cos(\varphi_{\mathbf{p}-\mathbf{q}} - \varphi_{\mathbf{p}})}{\varepsilon - c|\mathbf{q}| - \epsilon_{\mathbf{p}-\mathbf{q}\lambda'} + i\eta}|\mathbf{q}|d\mathbf{q}. \quad (2)$$

The sound intensity is obtained from the imaginary part of Eq. (2) taken on the mass surface, $\varepsilon = \epsilon_{\mathbf{p}\lambda}$,

$$\begin{aligned}\mathrm{Im}\Sigma_{\mathbf{p}\lambda}(\varepsilon = \epsilon_{\mathbf{p}\lambda}) &= -\frac{g^2c}{16\pi\hbar^3}\sum_{\lambda'}\int_0^{k_D}\int_0^{2\pi}[1 + \lambda\lambda'\cos(\varphi_{\mathbf{p}-\mathbf{q}} - \varphi_{\mathbf{p}})] \\ &\quad \times \delta\big(E_{\mathbf{p}\lambda;\mathbf{q}\lambda'}\big)|\mathbf{q}|^2 d|\mathbf{q}|d\phi, \end{aligned} \quad (3)$$

where k_D is the Debye momentum and $E_{\mathbf{p}\lambda;\mathbf{q}\lambda'} \equiv \epsilon_{\mathbf{p}\lambda} - \epsilon_{\mathbf{p}-\mathbf{q}\lambda'} - c|\mathbf{q}|$. It is enough to consider the Cherenkov sound excited only by an electron in the chiral branch with $\lambda = +1$ since it contains all the essential physics of the ACE while the Cherenkov sound excited by an electron with $\lambda = -1$ may be obtained in a similar way and does not lead to new phenomena. Using the expression $\delta[f(x)] = \sum_n [1/|f'(r_n)|]\delta(x - r_n)$,

where r_n are the roots of the equation $f(x) = 0$, we find $\mathrm{Im}\,\Sigma_{\mathbf{p}+}(\varepsilon = \epsilon_{\mathbf{p}+}) = -(g^2 m^2 c^2/2\pi\hbar^3)\int_{-\pi}^{\pi} I_s(\phi)d\phi$. Here the dimensionless Cherenkov sound intensity, $I_s(\phi) = I_s^{(\mathrm{intra})}(\phi) + I_s^{(\mathrm{inter})}(\phi)$, consists of two contributions coming from intra- and interchiral electronic transitions,

$$I_s^{(\mathrm{intra})}(\phi) = \Theta(\phi_c - |\phi|) \left[1 + \frac{1 - (c/v)q_1(\phi)\cos(\phi)}{\sqrt{1 + (c/v)^2 q_1^2(\phi) - 2(c/v)q_1(\phi)\cos(\phi)}}\right]\frac{q_1^2(\phi)}{|h_1(\phi)|}, \tag{4}$$

$$I_s^{(\mathrm{inter})}(\phi) = \left[1 - \frac{1 - (c/v)q_2(\phi)\cos(\phi)}{\sqrt{1 + (c/v)^2 q_2^2(\phi) - 2(c/v)q_2(\phi)\cos(\phi)}}\right]\frac{q_2^2(\phi)}{|h_2(\phi)|}, \tag{5}$$

where $q_{1,2}(\phi)$ are the positive solutions of the equations

$$2\frac{v_{\mathrm{so}}v}{c^2} + 2\left(\frac{v}{c}\cos(\phi) - 1\right)q_{1,2} - q_{1,2}^2 \mp 2\frac{v_{\mathrm{so}}v}{c^2}\sqrt{1 + \left(\frac{c}{v}\right)^2 q_{1,2}^2 - 2\cos(\phi)\frac{c}{v}q_{1,2}} = 0, \tag{6}$$

with $v_{\mathrm{so}} \equiv p_{\mathrm{so}}/m$, and $h_{1,2}(\phi)$ are the following functions:

$$h_{1,2}(\phi) = 4\left[2\left(\frac{v}{c}\cos(\phi) - 1\right) - 2q_{1,2}(\phi) \mp \frac{v_{\mathrm{so}}v}{c^2} \times \frac{2q_{1,2}(\phi)(c/v)^2 - 2(c/v)\cos(\phi)}{\sqrt{1 + (c/v)^2 q_{1,2}^2(\phi) - 2\cos(\phi)(c/v)q_{1,2}(\phi)}}\right]. \tag{7}$$

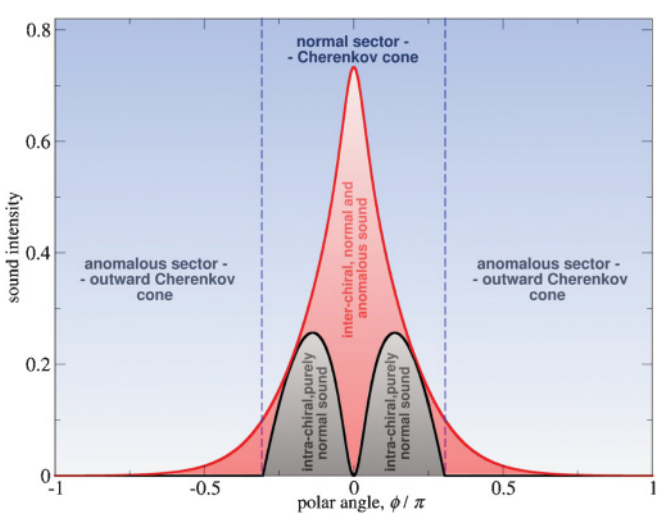

FIG. 2. (Color online) The intra- and interchiral contributions to ACE for $v = 5 \times 10^3$ m/s in a 2DEG with BRSOI in InAs structures with the following parameters: $c = 4.2 \times 10^3$ m/s, $\alpha = 0.15 \times 10^{-11}$ eV m.

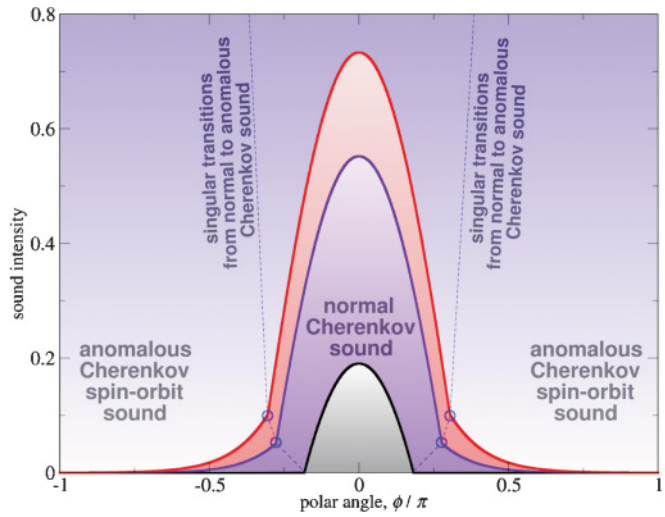

FIG. 3. (Color online) The total intensity of the Cherenkov sound for $v = 5 \times 10^3$ m/s in a 2DEG with BRSOI in InAs structures with $c = 4.2 \times 10^3$ m/s. The black (lower) curve corresponds to the absence of SOI, $\alpha = 0.0$ eV m. The lilac (intermediate) curve shows the situation for $\alpha = 0.1 \times 10^{-11}$ eV m, while the red (upper) one is for $\alpha = 0.15 \times 10^{-11}$ eV m.

The solution of Eqs. (6) may be obtained numerically. It follows that regardless of the existence of SOI the positive solutions $q_1(\phi)$ are located inside the Cherenkov cone with an angle ϕ_c. This is reflected by the presence of the Heaviside step function in Eq. (4). Therefore, the intrachiral contribution contains only the normal ACE. For $v_{\mathrm{so}} \lesssim c$ the Cherenkov cone angle is $\phi_c = \arccos[c/(v + v_{\mathrm{so}})]$, which assumes that the inequality $c < v + v_{\mathrm{so}}$ must be satisfied. In the limiting case $v_{\mathrm{so}} = 0$ one obtains from Eqs. (6) $q_{1,2} = 2[(v/c)\cos(\phi) - 1]$. This leads to the condition $(v/c)\cos(\phi) - 1 > 0$ (requiring $v > c$). Therefore the well-known result $I_s^{(2\mathrm{D})}$ is reproduced.

What is surprising is that for nonvanishing SOI ($v_{\mathrm{so}} \neq 0$) the positive solutions $q_2(\phi)$ exist in the whole interval $[-\pi, \pi]$. Therefore, the interchiral contribution to the Cherenkov

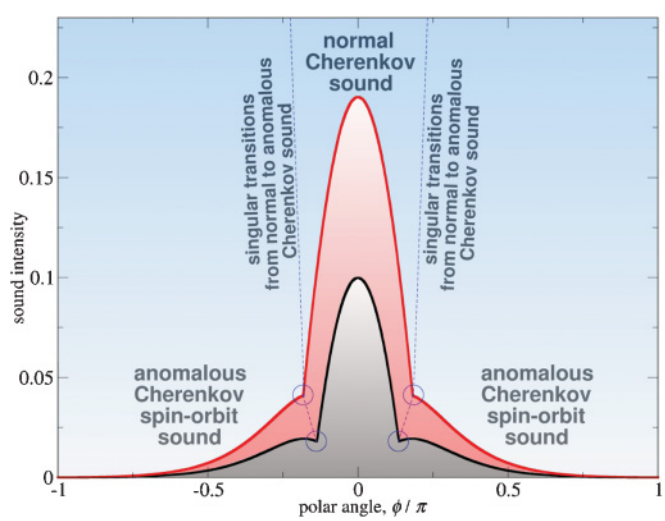

FIG. 4. (Color online) The same as in Fig. 3 but for $v = 3.1 \times 10^3$ m/s and for $\alpha_1 = 0.1 \times 10^{-11}$ eV m (black, lower curve) and $\alpha_2 = 0.125 \times 10^{-11}$ eV m (red, upper curve).

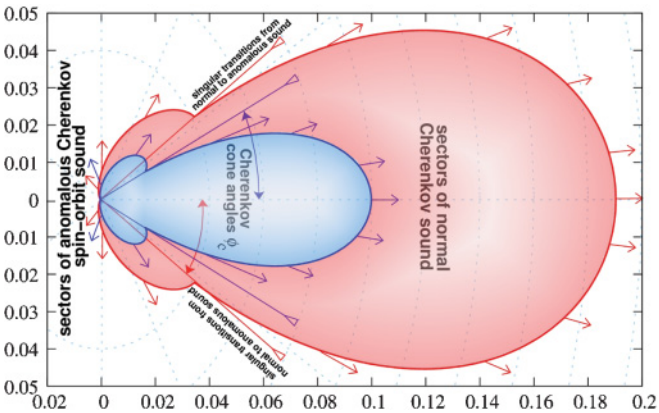

FIG. 5. (Color online) The curves from Fig. 4 in polar coordinates: blue (internal) for α_1, red (external) for α_2.

sound contains both normal (inside the Cherenkov cone) and anomalous (outside the Cherenkov cone) ACE as is shown in Fig. 2.

One can see from Eqs. (4) and (5) that because of the Heaviside step function the first derivative of the total sound intensity $I_s(\phi)$ has a singularity at the Cherenkov angle ϕ_c. This singularity can be seen in Fig. 3 where $v > c$ and in a more striking form in Fig. 4 where $v < c$. The two curves in Fig. 4 are plotted in Fig. 5 using the polar coordinates in the 2DEG plane.

Clearly, the detection of those acoustic singularities is an experimental challenge because it gives ϕ_c in a SOC system, $\cos(\phi_c) = c/(v + v_{so})$, and thus the SOI strength.

In conclusion, the acoustic Cherenkov effect in a two-dimensional electron gas with the Bychkov-Rashba spin-orbit interaction has been considered. It has been shown that in this system in addition to the normal Cherenkov sound inside the Cherenkov cone there also exists an anomalous Cherenkov sound outside this cone. The singular transition from the normal to anomalous sound at the Cherenkov angle provides an alternative experimental measurement of the spin-orbit coupling strength.

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