

Resistance oscillations induced by the Hall field in tilted magnetic fields

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We have studied the effect of an in-plane magnetic field on Hall field-induced resistance oscillations in high-mobility two-dimensional electron systems. We have found that the oscillation frequency depends only on the perpendicular component of the magnetic field, but the oscillation amplitude decays exponentially with an in-plane component. While these findings cannot be accounted for by existing theories of nonlinear transport, our analysis suggests that the decay can be explained by an in-plane magnetic-field-induced modification of the quantum scattering rate.

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High mobility two-dimensional electron systems (2DESs) exhibit an array of fascinating transport phenomena occurring in very high Landau levels where the Shubnikov-de Haas oscillations (SdHOs) are not yet resolved. Among these are several classes of magnetoresistance oscillations, such as microwave-induced resistance oscillations (MIROs),¹⁻⁵ phonon induced resistance oscillations (PIROs),⁶ and Hall field-induced resistance oscillations (HIROs),^{7,8} as well as their combinations.^{9,10} In a very clean 2DES, MIROs and HIROs can lead to exotic zero-resistance¹¹ and zero-differential resistance states,¹² respectively, which can be explained in terms of instabilities and the formation of current domains.¹³

MIROs are observed in linear-response magnetoresistivity when a 2DES is irradiated by microwaves and can be understood in terms of microwave-induced transitions between the Landau levels, which lead to oscillatory photoresistivity:⁵

$$\frac{\delta\rho_\omega}{\rho_0} \simeq -\eta\pi\epsilon_{ac}\mathcal{P}_\omega\lambda^2 \sin 2\pi\epsilon_{ac}. \quad (1)$$

Here, ρ_0 is the resistivity at $B = 0$, $\epsilon_{ac} = \omega/\omega_c$, $\omega = 2\pi f$ is the microwave frequency, $\omega_c = eB_\perp/m^*$ is the cyclotron frequency, B_\perp is the magnetic field normal to the 2DES, m^* is the effective mass, $\lambda = \exp(-\pi/\omega_c\tau_q)$ is the Dingle factor, τ_q is the quantum lifetime, \mathcal{P}_ω is the dimensionless microwave power, and η is the scattering parameter, which depends on temperature and type of disorder in the 2DES.⁵

HIROs are observed in differential resistivity $r \equiv dV/dI$ when a direct current I is passed through a 2DES and the perpendicular magnetic field B_\perp is varied. HIROs originate from short-range impurity-mediated transitions between Landau levels tilted by the Hall electric field, $\mathcal{E}_{dc} = \rho_H I/w$ (ρ_H is the Hall resistivity, w is the sample width).^{8,10} In this scenario, a characteristic scattering event involves an electron that is *backscattered* off of an impurity. The guiding center of such an electron is displaced by the cyclotron diameter, $2R_c$, and when $2R_c$ matches an integral multiple of the real-space Landau level separation, $\hbar\omega_c/e\mathcal{E}_{dc}$, the probability of such events is enhanced. This enhancement gives rise to a maximum in the differential resistivity occurring whenever

$\epsilon_{dc} \equiv e\mathcal{E}_{dc}(2R_c)/\hbar\omega_c$ is equal to an integer. At $2\pi\epsilon_{dc} \gg 1$, HIROs are described by^{8,10}

$$\frac{\delta r}{\rho_0} \simeq \frac{16}{\pi} \frac{\tau_{tr}}{\tau_\pi} \lambda^2 \cos 2\pi\epsilon_{dc}, \quad (2)$$

where τ_{tr} is the transport lifetime and τ_π is the time describing electron backscattering off of impurities.^{8,10}

The effect of an in-plane magnetic field, B_\parallel , on the MIROs was investigated recently in two independent experiments.^{2,3} In Ref. 2, the magnetic field B was tilted away from the normal to the 2DES by an angle θ , and in Ref. 3, B_\parallel was applied independently of B_\perp . While both experiments agreed that the MIRO frequency is governed by B_\perp , Ref. 3 found that MIROs are strongly suppressed under $B_\parallel \simeq 0.5$ T while in Ref. 2 MIROs were essentially unchanged up to $\theta \simeq 80^\circ$ ($B_\parallel \lesssim 1.2$ T). This controversy and the very fact that the suppression of MIROs observed in Ref. 3 was left unexplained indicates that the role of B_\parallel is not understood. It is interesting and timely, therefore, to examine how other classes of resistance oscillations, for example, HIROs, evolve with B_\parallel .

In this Rapid Communication, we report on the effect of an in-plane magnetic field on HIROs in a high-mobility 2DES. We employ a tilted-field setup and observe that, similar to MIROs,^{2,3} the HIRO frequency depends only on the perpendicular component of the magnetic field, B_\perp . We find further that with increasing tilt angle θ , HIROs are strongly suppressed by modest $B_\parallel \simeq 1$ T. The observed suppression is nonuniform and depends on the oscillation order; the lower-order oscillations decay faster than the higher-order oscillations with increasing θ . While the suppression cannot be readily explained by existing theories, we discuss our findings in the context of Eq. (2) and show that the suppression of the HIRO amplitude by B_\parallel can be understood in terms of a B_\parallel -induced reduction of the quantum lifetime. However, identifying the origin of such modification remains the subject of future theoretical and experimental studies.

While similar results have been obtained from a variety of samples,¹⁴ the data presented here were obtained from a Hall bar ($w = 100 \mu\text{m}$) cleaved from a GaAs/Al_{0.24}Ga_{0.76}As 300-Å-wide quantum well grown by molecular-beam epitaxy. After a brief low-temperature illumination with a red light emitting diode, this sample had electron density $n_e \simeq 3.6 \times 10^{11} \text{ cm}^{-2}$ and mobility $\mu \simeq 1.0 \times 10^7 \text{ cm}^2/\text{V s}$. The

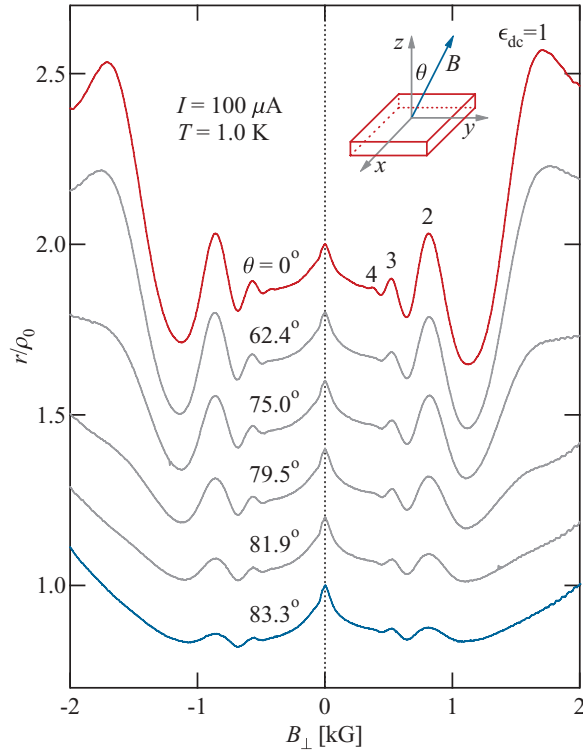


FIG. 1. (Color online) Normalized differential resistivity r/ρ_0 vs B_{\perp} for tilt angles $\theta \simeq 0^\circ, 62.4^\circ, 75.0^\circ, 79.5^\circ, 81.9^\circ$, and 83.3° . The data are vertically offset for clarity in steps of 0.2 starting from the highest θ . HIRO maxima are marked by $\epsilon_{dc} = 1, 2, 3, 4$. Inset: direction of B with respect to the 2DES.

experiment was performed in a ^3He cryostat at $T \simeq 1.0$ K and $I = 100 \mu\text{A}$ using a standard low-frequency lock-in detection scheme.

In Fig. 1, we present the normalized differential resistivity r/ρ_0 versus B_{\perp} for different θ , as marked. The data are vertically offset for clarity in steps of 0.2 starting from the highest θ . At the top curve ($\theta = 0$), the first four HIRO maxima are marked by $\epsilon_{dc} = 1, 2, 3, 4$. Examination of the data reveals that, regardless of θ , the positions of the same-order maxima and minima coincide roughly with each other. This finding provides firm experimental evidence that, similar to MIROs,^{2,3} the frequency of HIROs is controlled by B_{\perp} . At the same time, it is evident that the oscillation amplitude decreases with increasing θ .

The top trace in Fig. 1 obtained at $B_{\parallel} = 0$ shows that the oscillation amplitude grows monotonically with increasing B_{\perp} due to the increase of the Dingle factor λ appearing in Eq. (2). Examination of other data in Fig. 1 reveals that HIROs gradually decay with increasing θ and that the decay is nonuniform; the strongest, fundamental HIRO peak (cf. “ $\epsilon_{dc} = 1$ ”) is considerably more sensitive to θ than the higher-order peaks appearing at lower B_{\perp} . Indeed, it virtually disappears at $\theta \simeq 81.9^\circ$ ($B_{\parallel} \simeq 1.1$ T) while the higher-order peaks are still clearly observed. This finding indicates that at finite θ , the amplitude is no longer a monotonic function of B_{\perp} and thus cannot be described by an exponential dependence of Eq. (2).

Since Eq. (2) dictates that the HIRO amplitude $A \equiv (16/\pi)(\tau_{tr}/\tau_{\pi}) \exp(-2\pi/\omega_c \tau_q)$, one has to examine

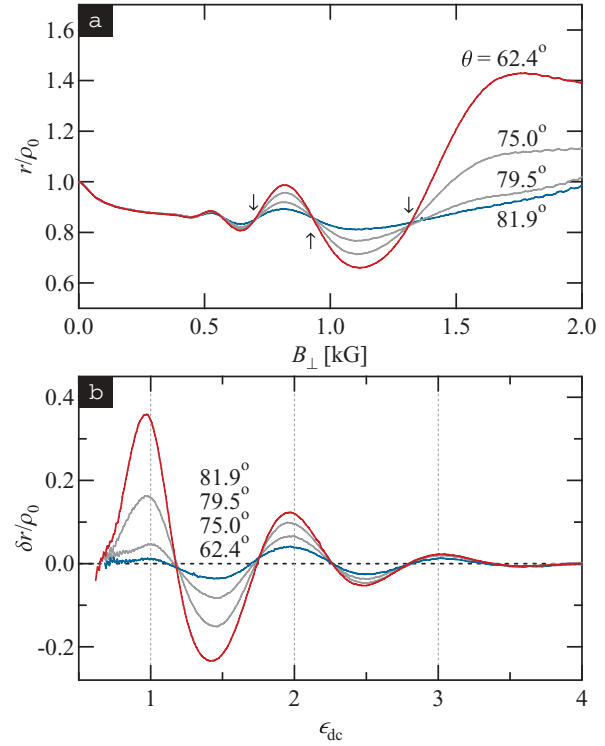


FIG. 2. (Color online) (a) Normalized differential resistivity r/ρ_0 vs B_{\perp} for $\theta = 62.4^\circ, 75.0^\circ, 79.5^\circ$, and 81.9° . The data are *not* vertically offset. (b) Normalized oscillatory part of the differential resistivity $\delta r/\rho_0$ vs ϵ_{dc} for the same tilt angles.

possible effects of B_{\parallel} on the various scattering parameters, that is, on τ_{tr} , τ_{π} , and τ_q . In our 2DES at weak B_{\perp} , $\rho \simeq \rho_0 \propto 1/\tau_{tr}$, and one can estimate the effect of B_{\parallel} on $1/\tau_{tr}$ by investigating the evolution of the background part of r with increasing θ . This is done in Fig. 2(a), showing r/ρ_0 as a function of B_{\perp} for different θ (as marked) *without* a vertical offset. Plotted in such a way, the data clearly show that r oscillates about a smooth, slowly varying background which *does not* change with θ . This conclusion is further supported by the existence of common crossing points (cf. \downarrow, \uparrow) which occur at $\epsilon_{dc} \simeq n \pm 1/4$ ($n = 1, 2, 3, \dots$) and where $\delta\rho \simeq 0$, as prescribed by Eq. (2). We thus conclude that $1/\tau_{tr}$ does not change significantly in our 2DES under $B_{\parallel} \lesssim 1$ T.

The backscattering rate $1/\tau_{\pi}$ appearing in Eq. (2) is essentially a scattering rate from sharp disorder potential, for example, residual background impurities and/or interface roughness. Scattering off of background impurities can hardly be affected by B_{\parallel} because of their 3D character. It is also unlikely that B_{\parallel} can *reduce* interface roughness scattering. Finally, a significant decrease in $1/\tau_{\pi}$ should lead to a noticeable *decrease* in $1/\tau_{tr}$, which is not observed. We thus conclude that the decay originates from an increase of the quantum scattering rate $1/\tau_q$ under applied B_{\parallel} .

To examine the effect of B_{\parallel} on the quantum scattering rate $1/\tau_q$, we first extract the oscillatory part $\delta r/\rho_0$ by subtracting the B_{\parallel} -independent background from the data in Fig. 2(a). The result is presented in Fig. 2(b) as a function of ϵ_{dc} showing the expected period and phase with the maxima occurring near integer ϵ_{dc} for all θ . In addition, Fig. 2(b) allows easy extraction

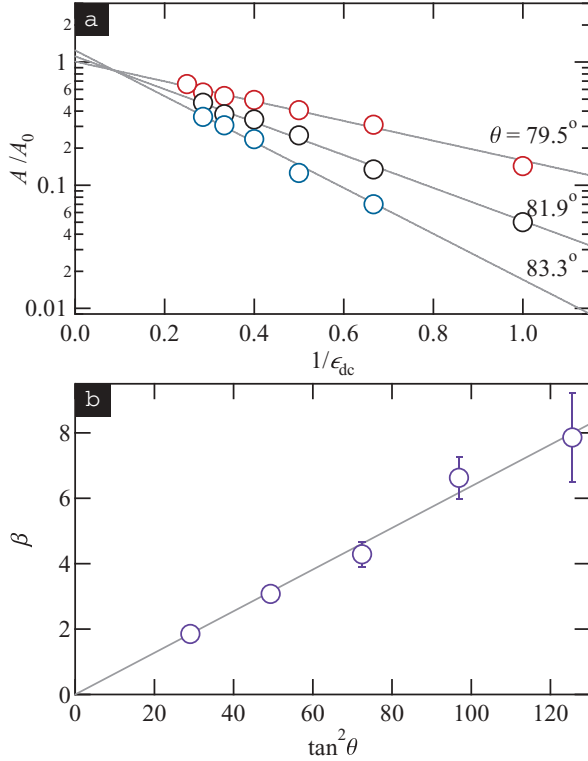


FIG. 3. (Color online) (a) Normalized HIRO amplitude A/A_0 vs $1/\epsilon_{dc}$ for $\theta = 79.5^\circ$, 81.9° , and 83.3° (circles) and fits to $A \exp(-\beta/\epsilon_{dc})$ (lines). (b) Extracted values of β vs $\tan^2 \theta$ (circles) and a fit, $\beta \approx 0.064 \tan^2 \theta$ (line).

of the oscillation amplitude A and shows, again, that the rate at which the oscillations disappear depends strongly on ϵ_{dc} , with the higher-order HIROs persisting to higher θ .

We proceed with the analysis by adopting an empirical relation, $1/\tau_q = 1/\tau_q^0 + \Delta$, where $1/\tau_q^0$ is the scattering rate at $B_{\parallel} = 0$ and Δ is the B_{\parallel} -induced correction. Then it follows that to account for a faster decay at higher B_{\perp} , Δ should increase faster than B_{\parallel} . Indeed, if $\Delta \propto B_{\parallel}$, then the argument of the Dingle factor would acquire a correction $-\pi \Delta/\omega_c \propto -B_{\parallel}/B_{\perp} = -\tan \theta$. As a result, oscillations of all orders would decay with θ at the same rate, in contradiction with our findings. If, however, $\Delta \propto B_{\parallel}^2$, then $-\pi \Delta/\omega_c \propto -B_{\parallel}^2/B_{\perp} \propto -\tan^2 \theta/\epsilon_{dc}$. This correction increases with θ for a given ϵ_{dc} and decreases with ϵ_{dc} for a given θ , consistent with our experimental observations. This result implies that the amplitude should decay with θ as $A = A_0 \exp(-\alpha \tan^2 \theta/\epsilon_{dc})$, where A_0 is the amplitude at $\theta = 0$ and α is a constant independent of B_{\parallel} .

In Fig. 3(a), we present normalized HIRO amplitude A/A_0 versus $1/\epsilon_{dc}$ on a semilog scale for three different θ (as marked). We observe that the data are described by $A = A_0 \exp(-\beta/\epsilon_{dc})$ reasonably well and that β increases with θ . In Fig. 3(b), we show that β scales roughly linearly with $\tan^2 \theta$, the dependence that follows from $\Delta \propto B_{\parallel}^2$. We thus conclude that the observed decay of HIROs can be explained by the B_{\parallel} -induced correction to the quantum scattering rate, which scales roughly as B_{\parallel}^2 .

It is interesting to examine the possibility that the B_{\parallel} -induced suppression of MIROs observed in Ref. 3 can also

be explained within the same picture. First, we notice that the Dingle factor enters equally in the description of both MIROs [Eq. (1)] and HIROs [Eq. (2)] and that both MIROs and HIROs are destroyed by similar $B_{\parallel} \sim 1$ T. Second, if B_{\parallel} is used as a parameter (instead of θ), the correction to the argument of the Dingle factor can be written as $-\pi \Delta/\omega_c \propto -\epsilon_{ac} \Delta$. This correction scales with ϵ_{ac} (and not with $1/\epsilon_{ac}$ if $\theta = \text{const}$) and therefore *higher-order* oscillations should be more sensitive to B_{\parallel} . Indeed, direct examination of Fig. 1 in Ref. 3 reveals that (i) the number of oscillations quickly decreases with increasing B_{\parallel} , (ii) this decrease occurs at the expense of higher orders, (iii) the lower-order oscillations persist to a much higher B_{\parallel} than the higher orders, and (iv) the onset of the oscillations increases with B_{\parallel} . All these observations imply that $1/\tau_q$ increases with B_{\parallel} . We believe that a standard Dingle plot analysis (which is not feasible on our data obtained at fixed θ) would readily reveal the exact dependence of $1/\tau_q$ on B_{\parallel} .

Finally, we mention that Ref. 3 observed that the onset of the SdHOs increased by $\simeq 50\%$ under applied $B_{\parallel} \simeq 1$ T, which indicates the same increase in the quantum scattering rate $1/\tau_q'$ that enters the SdHO amplitude. Such a modest increase of quantum scattering rate, indeed, is not sufficient to explain complete quenching of MIROs. However, $1/\tau_q'$ is often considerably larger than $1/\tau_q$ entering Eqs. (1) and (2), which is also the case for the 2DES used in Ref. 3 since the MIRO onset is much smaller than the SdHO onset. The overestimated $1/\tau_q'$ is usually attributed to the fact that the amplitude of the SdHOs ($\propto \lambda^1$) is very sensitive to macroscopic density inhomogeneities, which, however, do not significantly affect the amplitude of “induced” oscillations ($\propto \lambda^2$). Since inhomogeneities can hardly be affected by B_{\parallel} , a modest increase in $1/\tau_q'$ observed in Ref. 3 likely signals a *much larger* increase in $1/\tau_q$, which, in turn, is responsible for the decay of MIROs observed in Ref. 3.

In summary, we have studied the effect of an in-plane magnetic field on Hall field-induced resistance oscillations in a high-mobility 2DES. We have found that while the oscillation frequency remains unchanged, the amplitude quickly decays as the magnetic field is tilted away from the normal to the 2DES. The decay is very sensitive to the oscillation order and cannot be readily explained by existing theories of nonlinear transport. Our analysis shows that the decay can be understood in terms of the B_{\parallel} -induced increase of a single-particle scattering rate. However, the exact mechanism of such an increase remains a subject of future theoretical and experimental studies.

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