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Macroscopic quantum tunneling in multigap superconducting Josephson junctions: Enhancement of escape rate via quantum fluctuations of the Josephson-Leggett mode

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We theoretically study the macroscopic quantum tunneling (MQT) in a hetero Josephson junction formed by a conventional single-gap superconductor and a multigap superconductor such as iron-based superconductors and MgB_2. In such a Josephson junction more than one phase difference is defined. We clarify their phase dynamics and construct a theory for the MQT in the multigap Josephson junctions. The dynamics of the phase differences are strongly affected by the Josephson-Leggett mode, which is the out-of-phase oscillation mode of the phase differences. The escape rate is calculated in terms of the effective action renormalized by the Josephson-Leggett mode at zero-temperature limit. We successfully predict drastic enhancement of the escape rate when the frequency of the Josephson-Leggett mode is less than the Josephson-plasma frequency.

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Macroscopic quantum tunneling (MQT)[1–4] is a counter-intuitive phenomenon in quantum mechanics appearing at a macroscopic level, and has been observed in various fields of physics such as condensed matters, nuclei, cosmology, etc. This phenomenon has still attracted great interest in physics communities. In particular, the MQT in Josephson junctions, which is observed in a switching event at low temperature,[3,4] has been intensively studied because it is promising for applications to a Josephson phase qubit.[5–7]

In this Rapid Communication we investigate the physics of MQT in an unexplored type of Josephson junction, which has multiple tunneling channels. Such a Josephson junction can be fabricated by using recently discovered iron-based superconductors[8–10] or MgB_2,[11–13] because these superconductors are multiband ones having more than one disconnected Fermi surfaces and the superconducting gap can be individually well defined on each Fermi surface. In a Josephson junction made of multigap superconductors one may expect that the superconducting tunneling current has multiple channels between the two superconducting electrodes.[14–17] We construct a theory for the quantum switching (i.e., MQT) in Josephson junctions with multiple tunneling channels. We are unaware of any theory that has been formulated for MQTs in multigap systems. The theory predicts that the escape rate, i.e., the rate of quantum tunneling, is drastically enhanced compared with that in conventional single-channel systems.

In multigap superconductors a collective mode called the Leggett's mode[18–20] appears in the low-energy region, which is an out-of-phase oscillation mode of the superconducting phases. In Ref. 14 a theory for the Josephson effect in superconducting hetero junctions made of a single-gap superconductor and a two-gap superconductor is formulated. In such Josephson junctions, because two kinds of gauge-invariant phase differences can be defined, there are two phase oscillation modes, i.e., the in-phase mode and the out-of-phase one, which correspond, respectively, to the Josephson-plasma and the Josephson-Leggett (JL) mode. In this Rapid Communication we construct a theory for the MQT in superconducting hetero-junctions, incorporating the degree of freedom of the JL mode into the quantum switching event from nonvoltage to voltage states. It is shown that the zero-point motion of the JL mode significantly enhances the MQT escape rate when its frequency is less than the Josephson-plasma frequency. We also point out that the ratio E_J/E_{in} in addition to E_J/E_C governs the boundary between the classical and quantum regimes, where E_C, E_J, and E_{in} are, respectively, the charging energy, the Josephson coupling energy between the two superconductors, and the interband Josephson coupling energy in the two-gap superconductor.

Consider a hetero Josephson junction made of a single-gap superconductor and a two-gap superconductor,[14,16,17] as shown schematically in Fig. 1. Such a junction has been already fabricated by using multigap superconductors such as MgB_2 (Ref. 21) or iron-based superconductors.[22,23] In this system one can define two gauge-invariant phase differences, $\theta^{(1)}$ and $\theta^{(2)}$. Then, the Josephson current density between the two superconducting electrodes is given by the sum of the superconducting currents in the two tunneling channels as $j_1 \sin\theta^{(1)} + j_2 \sin\theta^{(2)}$, where j_i is the Josephson critical current density in the ith tunneling channel. When a voltage v appears between the two superconducting electrodes, the gauge-invariant phase differences show a temporal evolution, satisfying the generalized Josephson relation[14]

$$\frac{\alpha_2}{\alpha_1+\alpha_2}\dot{\theta}^{(1)} + \frac{\alpha_1}{\alpha_1+\alpha_2}\dot{\theta}^{(2)} = \frac{2e\Lambda}{\hbar}v, \tag{1}$$

with $\alpha_i = \epsilon\mu_i/d$ and $\Lambda = 1 + \alpha_1\alpha_2/(\alpha_1+\alpha_2)$, where ϵ is the dielectric constant of the insulator with a thickness d and μ_i is the charge screening length owing to the electrons in the ith

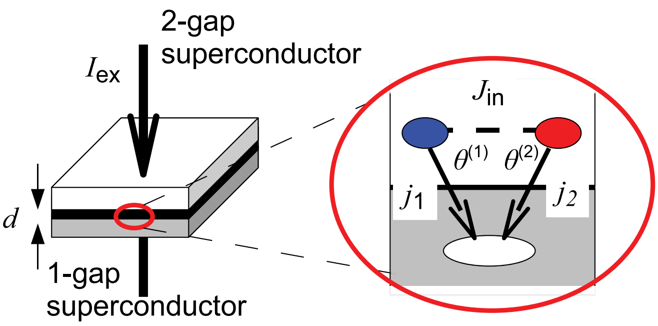

FIG. 1. (Color online) Schematic view of a superconductor-insulator-superconductor (SIS) hetero Josephson junction. We have two tunneling channels between the two superconductors with critical current densities j_1 and j_2 as indicated in the right-hand panel. In the upper two-gap electrode the interband Josephson coupling with the coupling constant $J_{\rm in}$ exists.

band. The constant α_i is related to the charge compressibility in the two-gap superconducting electrode.[16]

As shown in Ref. 14, the Lagrangian in the hetero Josephson junction with an in-plane area W and capacitance $C = \epsilon W/4\pi d$ is expressed as

$$L = \frac{1}{2}\frac{\hbar^2 C}{(2e)^2}\left(\frac{\dot{\theta}^2}{\Lambda} + \frac{\dot{\psi}^2}{\alpha_1+\alpha_2}\right) - V + E_{\rm J}\frac{I_{\rm ex}}{I_{\rm c}}\theta, \quad (2a)$$

$$V = -E_{\rm J1}\cos\theta^{(1)} - E_{\rm J2}\cos\theta^{(2)} - \kappa E_{\rm in}\cos\psi, \quad (2b)$$

under a bias current $I_{\rm ex}$ in the absence of an external magnetic field, where θ and ψ are the center-of-mass phase difference and the relative phase difference defined as

$$\theta = \frac{\alpha_2}{\alpha_1+\alpha_2}\theta^{(1)} + \frac{\alpha_1}{\alpha_1+\alpha_2}\theta^{(2)}, \quad \psi = \theta^{(1)} - \theta^{(2)}.$$

The first two terms in Eq. (2b) are the Josephson coupling energies with the coefficients $E_{{\rm J}i} = \hbar W j_i/2e$ and the third term represents the interband coupling energy, where the coefficient $E_{\rm in}$ is expressed as $E_{\rm in} = \hbar W|J_{\rm in}|/2e$.[14] Because the "interband current" $J_{\rm in}$ can take both signs, depending on the gap symmetry, we introduce the sign factor $\kappa = J_{\rm in}/|J_{\rm in}|$. The total critical Josephson current $I_{\rm c}$ and the coefficient $E_{\rm J}$ in the last term in Eq. (2a) are defined as $I_{\rm c} = W|j_1 + \kappa j_2|$ and $E_{\rm J} = \hbar I_{\rm c}/2e$. We note that the voltage v appearing in the junction is related to only θ, as seen in Eq. (1).

From Eq. (2a) one can derive the Euler-Lagrange equation for the center-of-mass phase difference θ as

$$\Lambda^{-1}\ddot{\theta} + \omega_{\rm P1}^2\sin\theta^{(1)} + \omega_{\rm P2}^2\sin\theta^{(2)} = \omega_{\rm P}^2\frac{I_{\rm ex}}{I_{\rm c}}, \quad (3)$$

with $\hbar\omega_{{\rm P}i} = \sqrt{2E_{\rm C}E_{{\rm J}i}}$ and $\hbar\omega_{\rm P} = \sqrt{2E_{\rm C}E_{\rm J}}$. We note that $\omega_{{\rm P}i}$ is the Josephson-plasma frequency in the ith tunneling channel. From Eq. (2a) we also have the Euler-Lagrange equation for ψ as

$$\ddot{\psi} + \kappa\omega_{\rm JL}^2\sin\psi = -\alpha_1\omega_{\rm J1}^2\sin\theta^{(1)} + \alpha_2\omega_{\rm J2}^2\sin\theta^{(2)}, \quad (4)$$

where $\omega_{\rm JL}$ is the frequency of the JL mode[14] given as $\hbar\omega_{\rm JL} = \sqrt{2(\alpha_1+\alpha_2)E_{\rm C}E_{\rm in}}$. The above two equations are coupled because $\theta^{(1)}$ and $\theta^{(2)}$ are functions of θ and ψ. We note that the bias current is the source for the time evolution of θ but not for ψ, which is consistent with the generalized Josephson relation (1). It should be noted also that we have two characteristic energy scales, the Josephson-plasma frequency $\omega_{{\rm J}i}$ and the JL one $\omega_{\rm JL}$, in this system.

Let us now study the macroscopic quantum effects in the Josephson junction with multiple tunneling channels on the basis of the Lagrangian (2a) and evaluate the MQT escape rate. In the following we assume $\kappa > 0$, because the case of $\kappa < 0$ shows qualitatively no difference.

Suppose that the switching to the voltage state is induced by the quantum tunneling of the phase differences $\theta^{(1)}$ and $\theta^{(2)}$ that are confined inside a potential well. When both $\theta^{(1)}$ and $\theta^{(2)}$ show the tunneling at the switching, its transition probability is given by the expectation value of the time evolution operator with respect to the state $|\theta^{(1)} = 0,\theta^{(2)} = 0\rangle (= |\theta = 0,\psi = 0\rangle)$,[3] which yields the formula for the MQT escape rate as

$$\Gamma = \frac{2}{\hbar\beta}{\rm Im}\,K(\{0\},\{0\};\beta). \quad (5)$$

Here, the symbol $\{0\}$ means $(\theta,\psi) = (0,0)$, and β is the inverse temperature, $\beta = 1/k_{\rm B}T$. The propagator $K(X,X';\beta)$ in Eq. (5) is expressed in terms of the imaginary time path integral

$$K(X,X';\beta) = \int_{X(0)=X'}^{X(\hbar\beta)=X}\mathcal{D}\theta\,\mathcal{D}\psi\, e^{-\int_0^{\hbar\beta}d\tau\, L^{\rm E}/\hbar},$$

where $X = (\theta,\psi)$ and $L^{\rm E}$ is the Euclidean version of the Lagrangian (2a). Let us assume that ψ is confined in a small region around $\psi = 0$ at the tunneling, which is justified when the interband coupling is not so strong. In this case one can utilize the expansion with respect to ψ. Then, up to the order of ψ^2, the Euclidean Lagrangian $L^{\rm E}$ is approximated as $L^{\rm E} = L^{\rm E}_{\rm cm} + L^{\rm E}_{\rm rlt} + L^{\rm E}_{\rm int}$, where

$$L^{\rm E}_{\rm cm} = \frac{\hbar^2}{4E_{\rm C}}\left(\frac{d\theta}{d\tau}\right)^2 - E_{\rm J}\left(\cos\theta + \frac{I_{\rm ex}}{I_{\rm c}}\theta\right), \quad (6a)$$

$$L^{\rm E}_{\rm rlt} = \frac{\hbar^2}{4(\alpha_1+\alpha_2)E_{\rm C}}\left(\frac{d\psi}{d\tau}\right)^2 + \frac{1}{2}E_{\rm in}\psi^2, \quad (6b)$$

$$L^{\rm E}_{\rm int} = g_+E_{\rm J}\psi^2\cos\theta - g_-E_{\rm J}\psi\sin\theta. \quad (6c)$$

Here, $E_{\rm J} = E_{\rm J1} + E_{\rm J2}$ and $\Lambda \approx 1$ is assumed. The coupling constants g_+ and g_- in Eq. (6c) are defined as $g_+ = (E_{\rm J1}/2E_{\rm J})[\alpha_1/(\alpha_1+\alpha_2)]^2 + (E_{\rm J2}/2E_{\rm J})[\alpha_2/(\alpha_1+\alpha_2)]^2$ and $g_- = (E_{\rm J1}/E_{\rm J})[\alpha_1/(\alpha_1+\alpha_2)] - (E_{\rm J2}/E_{\rm J})[\alpha_2/(\alpha_1+\alpha_2)]$. We note that in the fully quantum case we have the discrete energy levels as schematically illustrated in Fig. 2. To calculate the escape rate Γ in Eq. (5) we employ the mean field approximation for ψ, that is, ψ^2 and ψ in Eq. (6c) are approximated with their expectation values. Then, at zero temperature we find $\langle\psi\rangle_{\rm th} = 0$ and

$$\langle\psi^2\rangle_{\rm th}(T=0) = \frac{\hbar}{2m_{\rm rlt}\omega_{\rm JL}}, \qquad m_{\rm rlt} = \frac{\hbar^2}{2(\alpha_1+\alpha_2)E_{\rm C}}.$$

The finite value of $\langle\psi^2\rangle$ originates from the zero-point motion of the "quantized" JL mode. Under this approximation we

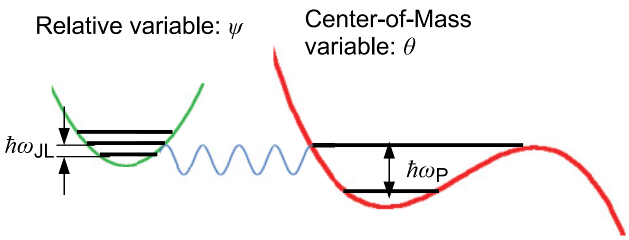

FIG. 2. (Color online) Schematic energy diagram for the fully "quantized" system with two quantum variables θ and ψ. In the case where ψ is weakly oscillating within a potential well, the energy levels of ψ coincide with those of a harmonic oscillator with frequency ω_{JL}. The energy levels of θ are corrected by the quantum oscillations of ψ.

find the effective Lagrangian of single degree of freedom as follows:

$$L^{E}_{cm,eff} = \frac{\hbar^2}{4E_C}\left(\frac{d\theta}{d\tau}\right)^2 + V_{cm,eff}, \tag{7a}$$

where $V_{cm,eff}$ is the renormalized potential

$$V_{cm,eff} = -E_J\left[(1-\varepsilon)\cos\theta + \frac{I_{ex}}{I_c}\theta\right], \tag{7b}$$

$$\varepsilon = g_+\langle\psi^2\rangle_{th} \approx \frac{g_+}{\sqrt{2}}(\alpha_1+\alpha_2)\frac{\omega_P}{\omega_{JL}}\sqrt{\frac{E_C}{E_J}}. \tag{7c}$$

Then, in this approximation the expectation value $K(\{0\},\{0\};\beta)$ in Eq. (5) is reduced to $K(\{0\},\{0\};\beta=\infty) \approx \int_{\theta(0)=0}^{\theta(\hbar\beta=\infty)=0}\mathcal{D}\theta\,\exp(-\hbar^{-1}\int_0^{\hbar\beta=\infty}L^{E}_{cm,eff}\,d\tau)$, which can be evaluated in the standard instanton approximation.[3] Hence, the MQT escape rate corrected by the zero-point motion of ψ is

$$\Gamma = 12\omega_P(I)\sqrt{\frac{3V_0}{2\pi\hbar\omega_P(I)}}\exp\left(-\frac{36V_0}{5\hbar\omega_P(I)}\right), \tag{8}$$

where $\omega_P(I) = \omega_P[(1-\varepsilon)^2 - I^2]^{1/4}$, $V_0 = \hbar^2[\omega_P(I)]^2\cot^2\theta_0/3E_C$, $(1-\varepsilon)\sin\theta_0 = I$, and $I = I_{ex}/I_c$.

Figure 3 shows a contour map of the ratio Γ/Γ_0 in the $(I_{ex}/I_c$ vs $\omega_P/\omega_{JL})$ plane with Γ_0 being the escape rate without correction, i.e., $\varepsilon = 0$. It is seen that the escape rate is drastically enhanced in a wide parameter region. In particular, the enhancement is pronounced in the region of large ω_P/ω_{JL}. As seen in Eqs. (7a) and (7b), the Josephson coupling energy is renormalized by the zero-point motion of ψ and the renormalized one is decreased from the bare one because $\varepsilon > 0$. As a result, the tunneling barrier for θ is lowered as schematically shown in Fig. 4, which causes the strong enhancement of the escape rate. In fact, $R(\varepsilon) \equiv V_0/\hbar\omega_P(I)$ is smaller than $R(\varepsilon=0)$ for fixed I when $0<\varepsilon<1$, that is, the exponent in Eq. (8) is decreased. Thus, the renormalization increases Γ. Also, it should be noted that the zero-point fluctuation becomes larger as the frequency of the JL mode decreases. Thus, the considerable enhancement of Γ occurs for the system with a lower value of ω_{JL}. The MQT in the conventional systems is subject to the ratio E_J/E_C, which is an important parameter for designing a superconducting Josephson qubit.[5] In the system with multiple tunneling channels the ratio $\omega_P/\omega_{JL}(\propto E_J/E_{in})$ also affects the characteristics of the MQT.

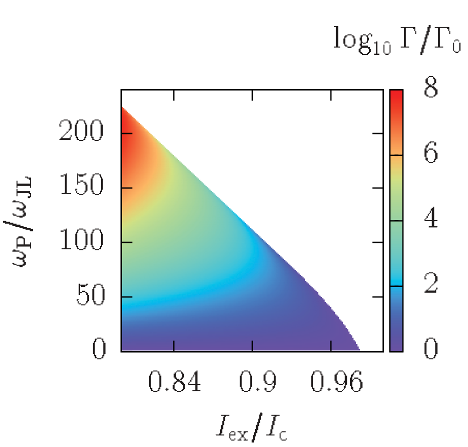

FIG. 3. (Color online) Ratio Γ/Γ_0 in the case of $E_J/E_C = 10^2$. We assume that $\alpha_1 = \alpha_2 = 0.1$ and $j_1 = j_2$ for simplicity.

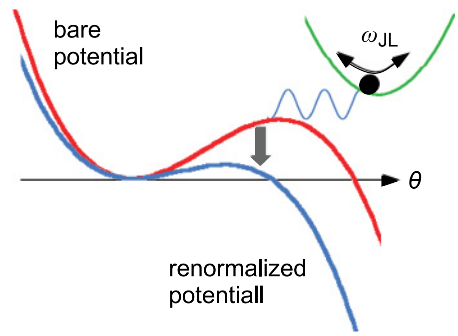

FIG. 4. (Color online) Renormalized potential $V_{cm,eff}$. The potential for θ is modified by the zero-point fluctuation of the JL mode.

In this Rapid Communication we have focused on the tunneling process, $|\theta=0,\psi=0\rangle \to |0,0\rangle$, and clarified the effect of the JL mode on the MQT. We mention that such a process is not the unique one that contributes to the MQT rate in this system, because a system with two degrees of freedom generally has many tunneling routes. For example, the tunneling process in which the quantum switching in the $\theta^{(1)}$ channel takes place successively after the switching in the $\theta^{(2)}$ channel will be also possible in the present system.[24] In this case the escape rate can be calculated from the transition process $|\theta^{(1)}=0\rangle \to |0\rangle$ with $\theta^{(2)} = f(t)$, where $f(t)$ is a time-dependent c-number function. This tunneling process is analogous to the MQT under a periodically time-dependent perturbation.[25] It is also noted that the relative phase difference ψ might play the role of an environmental variable for θ through a term that is linear in ψ in Eq. (6c). The MQT rate in this process can be evaluated by using the influential functional integral method.[1,26] The competition between the zero-point fluctuation and the "dissipation" occurs in this case. The enhancement via the JL mode may be superior to the reduction from such a dissipation when $g_+ > |g_-|$.

We also mention that our theory for the MQT in the hetero Josephson junctions can be extended to the case of intrinsic Josephson junctions (IJJs) with multiple tunneling channels.[15] The MQT in such systems will be observed in several recently discovered highly anisotropic layered iron-based superconductors.[27–29] In the IJJs, correction owing to the JL mode for the corporative MQT among the junctions[30,31] will be expected.

Finally, we remark that the present theoretical prediction relies on the coexistence of the Josephson-plasma and JL modes. Because the observation of Leggett's mode in a bulk MgB_2 sample has been reported[32] and in junctions with MgB_2,[33,34] we expect that such a collective mode can be detected in a junction system and the theory will be verified experimentally.

In summary, we have constructed a theory of the MQT in hetero Josephson junctions with multiple tunneling channels. We have clarified that the zero-point fluctuation of the relative phase differences brings about a drastic enhancement of the MQT escape rate. The enhancement is large when the JL mode has a lighter mass than that of the Josephson plasma.

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