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Ginzburg-Landau theory of two-band superconductors: Absence of type-1.5 superconductivity

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It is shown that within the Ginzburg-Landau (GL) approximation the order parameters $\Delta_1(\mathbf{r},T)$ and $\Delta_2(\mathbf{r},T)$ in two-band superconductors vary on the same length scale, the difference in zero-T coherence lengths $\xi_{0\nu} \sim \hbar v_{F\nu}/\Delta_\nu(0)$, $\nu = 1,2$ notwithstanding. This amounts to a single physical GL parameter κ and the classic GL dichotomy: $\kappa < 1/\sqrt{2}$ for type I and $\kappa > 1/\sqrt{2}$ for type II.

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I. INTRODUCTION

The physics of superconductors near their critical temperature, T_c, is based on the Ginzburg-Landau (GL) theory.[1] The smallness of critical fluctuations in most superconductors justifies the usage of this mean-field theory except in a tiny regime close to the transition temperature. This includes multiband superconductors with distinct sheets of the Fermi surface. A number of recent papers deal with two-band materials with coefficients of the GL free energy (for the field-free state),

$$F = \sum_{\nu=1,2} \left(a_\nu \Delta_\nu^2 + b_\nu \Delta_\nu^4/2\right) - 2\gamma \Delta_1 \Delta_2,$$

introduced phenomenologically; see, for example, Ref. 2. Choosing these coefficients in various ways, one could arrive at a number of choice-dependent conclusions.[3,4] However, there are important general constraints on the allowed values. For example, a straightforward analysis of the above free energy yields that the coefficients a_ν do not have the familiar GL form $\alpha(T - T_c)$. Instead, they acquire a constant part, const $+ \alpha(T - T_c)$, that is intimately related to the constant γ of the mixed Josephson-type term to ensure $\Delta_\nu \propto \sqrt{T_c - T}$ near T_c. The coefficients of the GL free energy can furthermore be derived from microscopic theory; they are certain functions of the microscopic coupling constants responsible for superconductivity and temperature T. This was done several decades ago by Tilley[5] and later by Zhitomirsky and Dao,[6] who confirm, as expected, the const $+ \alpha(T - T_c)$ behavior of the a_ν within a weak-coupling model.

We show in this work that, independent of the microscopic origin of the GL coefficients, in the GL domain the ratio of the order parameters is T independent,

$$\Delta_1(\boldsymbol{r},T)/\Delta_2(\boldsymbol{r},T) = \text{const}, \tag{1}$$

with the $\boldsymbol{r}$-independent constant depending on interactions responsible for superconductivity. The one-dimensional (1D) version of Eq. (1) has been obtained while solving the GL problem of the interface energy between superconducting and normal phases relevant for the distinction between type I and type II two-band superconductors.[7] For strong intraband scattering (the dirty limit), the result (1) has been obtained by Koshelev and Golubov provided the interband scattering could be disregarded.[8] Here, we establish this result for *any* problem in the GL domain.

We show that the equations for $\Delta_1(\boldsymbol{r},T)$ and $\Delta_2(\boldsymbol{r},T)$ are reduced to one independent GL equation. In other words, there is a single complex order parameter describing the two-band superconductor in the GL domain and, as a consequence, a single length scale ξ for spatial variation of both $\Delta_1(\boldsymbol{r},T)$ and $\Delta_2(\boldsymbol{r},T)$.

Our results, along with the earlier critique[9] and a comprehensive review by Brandt and Das,[10] question the validity of publications discussing properties of MgB_2 within the GL framework where each band is attributed with its own coherence length and sometimes even with its own penetration depth; see, for example, Ref. 11 and references therein.

We stress that our claim that the gap functions $\Delta_\nu(\mathbf{r},T)$ change on the same length scale relates exclusively to the temperature domain, however narrow it could be, where the GL theory is valid. Out of this domain and at low temperatures in particular, different length scales $\sim \hbar v_{F\nu}/\Delta_\nu(0)$ may enter and result in properties substantially different from those in the GL region. Still, as long as the GL energy functional is used, the assumption of two coherence lengths cannot be justified.

Below, we discuss the phenomenologic two-band GL theory and later confirm our conclusions within a weak-coupling microscopic scheme.

II. TWO-BAND GL IN FIELD

The two-band GL functional reads as follows:

$$\mathcal{F} = \int dV \Bigg\{ \sum_{\nu=1,2} \left(a_\nu |\Delta_\nu|^2 + \frac{b_\nu}{2}|\Delta_\nu|^4 + K_\nu |\mathbf{\Pi}\Delta_\nu|^2 \right) - \gamma(\Delta_1\Delta_2^* + \Delta_2\Delta_1^*) + \frac{B^2}{8\pi} \Bigg\}, \tag{2}$$

where $\mathbf{\Pi} = \mathbf{\nabla} + 2\pi i \mathbf{A}/\phi_0$. Explicit expressions for the constant γ along with the coefficients a,b,K for a weak-coupling model will be given later. However, our results do not rely on the validity of the weak-coupling theory and are more general.

The GL equations are minimum conditions for the functional (2). One obtains, by varying $\mathcal{F}$ with respect to Δ_ν^*,

$$a_1\Delta_1 + b_1\Delta_1|\Delta_1|^2 - \gamma\Delta_2 - K_1\mathbf{\Pi}^2\Delta_1 = 0, \tag{3}$$

$$a_2\Delta_2 + b_2\Delta_2|\Delta_2|^2 - \gamma\Delta_1 - K_2\mathbf{\Pi}^2\Delta_2 = 0. \tag{4}$$

We now recall that in the one-band GL equation,

$$a\Delta + b\Delta|\Delta|^2 - K\mathbf{\Pi}^2\Delta = 0,$$

all terms are of the same order, $(1 - T/T_c)^{3/2} = \tau^{3/2}$ ($\Delta \propto \tau^{1/2}$, $a \propto \tau$, and $\mathbf{\Pi}^2 \propto \xi^{-2} \propto \tau$). This is more subtle in the case of Eqs. (3) and (4) because γ is a constant and a_ν contain constant parts. Keeping this in mind, we express Δ_2 in terms of Δ_1 from Eq. (3) and substitute the result in Eq. (4), keeping only terms up to order $\tau^{3/2}$:

$$(a_1a_2-\gamma^2)\Delta_1+\big(b_1a_2+b_2a_1^3/\gamma^2\big)\Delta_1|\Delta_1|^2 - (a_1K_2+a_2K_1)\mathbf{\Pi}^2\Delta_1=0. \quad (5)$$

Similarly, one obtains an equation for Δ_2:

$$(a_1a_2-\gamma^2)\Delta_2+\big(b_2a_1+b_1a_2^3/\gamma^2\big)\Delta_2|\Delta_2|^2 - (a_1K_2+a_2K_1)\mathbf{\Pi}^2\Delta_2=0. \quad (6)$$

In zero field, one has $\Delta_\nu^2\propto(a_1a_2-\gamma^2)$, so that at T_c, $a_1a_2-\gamma^2=0$, and therefore a_ν must contain constant parts,

$$a_\nu=a_{\nu c}-\alpha_\nu\tau,$$

such that $a_{1c}a_{2c}=\gamma^2$.

Equations (5) and (6) for Δ_ν can now be written as

$$-\alpha\tau\Delta_1+\beta_1\Delta_1|\Delta_1|^2-K\mathbf{\Pi}^2\Delta_1=0, \quad (7)$$
$$-\alpha\tau\Delta_2+\beta_2\Delta_2|\Delta_2|^2-K\mathbf{\Pi}^2\Delta_2=0, \quad (8)$$

with

$$\alpha=\alpha_1a_{2c}+\alpha_2a_{1c},\quad K=a_{1c}K_2+a_{2c}K_1,$$
$$\beta_1=b_1a_{2c}+b_2a_{1c}^3/\gamma^2,\quad \beta_2=b_2a_{1c}+b_1a_{2c}^3/\gamma^2. \quad (9)$$

We note that within the accuracy of the GL theory, up to $\mathcal{O}(\tau^{3/2})$, these equations differ only in coefficients β of the nonlinear terms that determine the overall amplitude of the solutions, whereas the rest of the coefficients are the same. The equations for Δ_1 and Δ_2 are coupled only via the vector potential. In particular, in zero field we have

$$\Delta_{\nu0}^2=\alpha\tau/\beta_\nu,$$

so that the ratio

$$\frac{\Delta_{10}^2(T)}{\Delta_{20}^2(T)}=\frac{\beta_2}{\beta_1} \quad (10)$$

comes out to be T independent in the GL domain.

Furthermore, one easily checks that for any solution $\Delta_1(\mathbf{r},T)$ of Eq. (7), Eq. (8) is satisfied by

$$\Delta_2(\boldsymbol{r},T)=\Delta_1(\boldsymbol{r},T)\sqrt{\beta_1/\beta_2}. \quad (11)$$

In particular, this implies that in equilibrium $\Delta_1(\boldsymbol{r},T)$ and $\Delta_2(\boldsymbol{r},T)$ must have either the same phases or phases differing by π.[12] It is found in Ref. 6 that for small γ the ratio Δ_2/Δ_1 changes away from T_c; we note, however, that this deviation is beyond the GL accuracy. Reliable results beyond GL can be obtained only within microscopic approaches like the Gor'kov or Bogolyubov–de Gennes theories.

Moreover, introducing the order parameters normalized on their zero-field values,

$$\frac{\Delta_1}{\Delta_{10}(T)}=\frac{\Delta_2}{\Delta_{20}(T)}=\Psi, \quad (12)$$

both Eqs. (7) and (8) are reduced to one:

$$\Psi(1-|\Psi|^2)=-\frac{K}{\alpha\tau}\mathbf{\Pi}^2\Psi. \quad (13)$$

Thus, the length scale of the space variation of both Δ_1 and Δ_2, the coherence length, is given by

$$\xi^2=K/\alpha\tau. \quad (14)$$

III. MICROSCOPIC WEAK-COUPLING TWO-BAND MODEL NEAR T_c

To establish a connection of GL equations with the two-band microscopic theory we turn to a weak-coupling model for clean and isotropic materials (not because these restrictions are unavoidable, but rather due to the model simplicity).

Perhaps the simplest formally weak-coupling approach is based on the Eilenberger quasiclassical formulation of the Gor'kov equations valid for general anisotropic order parameters and Fermi surfaces.[13] Eilenberger functions f,g for clean materials in zero field obey the system:

$$0=\Delta g-\hbar\omega f, \quad (15)$$
$$g^2=1-f^2, \quad (16)$$
$$\Delta(\boldsymbol{k})=2\pi TN(0)\sum_{\omega>0}^{\omega_D}\langle V(\boldsymbol{k},\boldsymbol{k}')f(\boldsymbol{k}',\omega)\rangle_{\boldsymbol{k}'}. \quad (17)$$

Here, $\boldsymbol{k}$ is the Fermi momentum; Δ is the order parameter that may depend on the position $\boldsymbol{k}$ at the Fermi surface. Further, $N(0)$ is the total density of states (DOS) at the Fermi level per spin; the Matsubara frequencies are given by $\hbar\omega=\pi T(2n+1)$ with an integer n, and ω_D is the Debye frequency; $\langle\ldots\rangle$ stands for averages over the Fermi surface.

Consider a model material with the gap given by

$$\Delta(\mathbf{k})=\Delta_{1,2},\quad \mathbf{k}\in F_{1,2}, \quad (18)$$

where F_1,F_2 are two sheets of the Fermi surface. The gaps are assumed constant at each band. Denoting DOS on the two parts as $N_{1,2}$, we have for a quantity X constant at each Fermi sheet,

$$\langle X\rangle=(X_1N_1+X_2N_2)/N(0)=n_1X_1+n_2X_2, \quad (19)$$

where $n_{1,2}=N_{1,2}/N(0)$; clearly, $n_1+n_2=1$.

Equations (15) and (16) are easily solved:

$$f_\nu=\Delta_\nu/\beta_\nu,\quad g_\nu=\hbar\omega/\beta_\nu,\quad \beta_\nu^2=\Delta_\nu^2+\hbar^2\omega^2, \quad (20)$$

where $\nu=1,2$ is the band index. The self-consistency Eq. (17) takes the form

$$\Delta_\nu=\sum_{\mu=1,2}n_\mu\lambda_{\nu\mu}\Delta_\mu\sum_{\omega}^{\omega_D}\frac{2\pi T}{\beta_\mu}, \quad (21)$$

where $\lambda_{\nu\mu}=N(0)V_{\nu\mu}$ are dimensionless effective interaction constants. The notation commonly used in literature, $\lambda_{\nu\mu}^{(lit)}=n_\mu\lambda_{\nu\mu}$, includes DOS. We find our notation convenient since, being related to the coupling potential, our coupling matrix is symmetric: $\lambda_{\nu\mu}=\lambda_{\mu\nu}$.

It is seen from the system (21) that $\Delta_{1,2}$ turns to zero at the same temperature T_c unless $\lambda_{12}=0$ and Eqs. (21) decouple, the property that has been noted in earlier work.[14–16] As $T\to T_c$, $\Delta_{1,2}\to 0$, and $\beta\to\hbar\omega$. The sum over ω in Eq. (21) is readily evaluated:

$$S=\sum_{\omega}^{\omega_D}\frac{2\pi T}{\hbar\omega}\Big|_{T_c}=\ln\frac{2\hbar\omega_D}{T_c\pi e^{-\gamma}}=\ln\frac{2\hbar\omega_D}{1.76T_c}, \tag{22}$$

where $\gamma=0.577$ is the Euler constant. This relation can also be written as

$$1.76T_c=2\hbar\omega_D e^{-S}. \tag{23}$$

The system (21) at T_c is linear and homogeneous:

$$\begin{aligned}\Delta_1&=S(n_1\lambda_{11}\Delta_1+n_2\lambda_{12}\Delta_2),\\ \Delta_2&=S(n_1\lambda_{12}\Delta_1+n_2\lambda_{22}\Delta_2).\end{aligned} \tag{24}$$

The zero determinant gives S and, therefore, T_c:

$$S^2n_1n_2\eta-S(n_1\lambda_{11}+n_2\lambda_{22})+1=0, \tag{25}$$

$$\eta=\lambda_{11}\lambda_{22}-\lambda_{12}^2. \tag{26}$$

The roots of this equation are

$$S=\frac{n_1\lambda_{11}+n_2\lambda_{22}\pm\sqrt{(n_1\lambda_{11}-n_2\lambda_{22})^2+4n_1n_2\lambda_{12}^2}}{2n_1n_2\eta}. \tag{27}$$

Various possibilities that arise depending on values of $\lambda_{\mu\nu}$ are discussed, for example, in Refs. 14–18. Introducing T-independent quantities,

$$S_1=\lambda_{22}-n_1\eta S,\quad S_2=\lambda_{11}-n_2\eta S, \tag{28}$$

we write Eq. (25) as

$$S_1S_2=\lambda_{12}^2, \tag{29}$$

the form useful for manipulations below.

If $\lambda_{12}=0$, Eq. (27) provides two roots: $1/n_1\lambda_{11}$ and $1/n_2\lambda_{22}$. The smallest one gives T_c, whereas the other corresponds to the temperature at which the second gap turns to zero. We note that this situation is unlikely; it implies that the ever-present Coulomb repulsion is exactly compensated by the effective interband attraction.

Since the determinant of the system (24) is zero, the two equations are equivalent and give at T_c

$$\left(\frac{\Delta_2}{\Delta_1}\right)_{T_c}=\frac{1-n_1\lambda_{11}S}{n_2\lambda_{12}S}. \tag{30}$$

When the right-hand side is negative, the Δ's are of opposite signs. Within the one-band BCS, the sign of Δ is a matter of convenience; for two bands, Δ_1 and Δ_2 may have equal or opposite signs.[19]

After simple algebra, Eq. (30) can be manipulated to

$$\left(\frac{\Delta_2}{\Delta_1}\right)_{T_c}^2=\frac{S_1}{S_2}. \tag{31}$$

We thus obtain, by comparing with Eq. (10), the ratio of phenomenological coefficients in terms of microscopic couplings: $\beta_1/\beta_2=S_1/S_2$. We have seen above that within the GL approximation this ratio remains the same at any T in the GL domain not only for a uniform field-free state (or for $\gamma\to\infty$ as in Ref. 20) but for any situation with Δ's depending on coordinates in the presence of magnetic fields.

We note that the proportionality of Δ_1 and Δ_2 has also been shown to hold within microscopic weak-coupling theory in the dirty limit by Koshelev and Golubov.[8] It is also worth mentioning here that the proof of this proportionality in the preceding section based on the GL approach is quite general and holds for any scattering, gap anisotropies, etc.

In the following we use the GL coefficients obtained in Refs. 5 and 6. In our notation they read

$$a_\nu=\frac{N(0)}{\eta}(S_\nu-n_\nu\eta\tau),\quad b_\nu=\frac{N(0)}{W^2}n_\nu,\quad W^2=\frac{8\pi^2T_c^2}{7\zeta(3)},$$
$$\gamma=\frac{N(0)}{\eta}\lambda_{12},\qquad K_\nu=\frac{N(0)\hbar^2v_\nu^2}{6W^2}n_\nu, \tag{32}$$

where the energy scale $W\sim\pi T_c$ is introduced for brevity and v_ν are the Fermi velocities in two bands which for simplicity is assumed isotropic. We, in fact, confirmed Eqs. (32) of Zhitomirsky and Dao employing different methods (except our b_ν is by a factor of 2 larger than that of Ref. 6). It is worth noting that the microscopically derived a_ν are not proportional to τ as in the standard one-band GL unless one of the parameters S_ν is zero; given the condition (29) this may happen only if $\lambda_{12}=0$. This feature of the two-band GL is sometimes overlooked.[21,22]

As stressed in Ref. 6, the term $K_\nu|\mathbf{\Pi}\Delta_\nu|^2$ with order parameter gradients is the only possible in the GL energy, although the symmetry may allow for other combinations of gradients.

The coefficients entering the GL Eqs. (7) and (8) are

$$\alpha=\frac{N(0)^2C}{\eta},\quad K=\frac{\hbar^2\tilde{v}^2N(0)^2}{6W^2\eta},\quad s\beta_\nu=\frac{N(0)^2DS_\nu}{\eta W^2\lambda_{12}^2}, \tag{33}$$

where

$$\tilde{v}^2=n_1S_2v_1^2+n_2S_1v_2^2 \tag{34}$$

has the dimension of a squared velocity and

$$C=n_1S_2+n_2S_1,\quad D=n_1S_2^2+n_2S_1^2 \tag{35}$$

are constants.

Hence, we can express the length scale (14) of the space variation of both Δ_1 and Δ_2 in the GL domain in terms of microscopic parameters:

$$\xi^2=\frac{\hbar^2\tilde{v}^2}{2W^2C\tau}. \tag{36}$$

The upper critical field follows: $H_{c2}=\phi_0/2\pi\xi^2$. The one-band limit is obtained by setting $n_1=1, n_2=0$ so that $C=S_2$ and $\tilde{v}^2=S_2v^2/3$, which yields $\xi^2=7\zeta(3)\hbar^2v^2/48\pi^2T_c^2\tau$ as it should.

Variation of the free energy $\mathcal{F}$ with respect to the vector potential $\boldsymbol{A}$ gives the current density. Following the standard procedure we obtain for the penetration depth of a weak magnetic field,

$$\frac{1}{\lambda^2}=\frac{32\pi^3}{\phi_0^2}\sum_{\nu=1,2}\Delta_{\nu0}^2K_\nu=\frac{16\pi CN(0)e^2\tilde{v}^2}{c^2D}\tau. \tag{37}$$

In the one-band limit this yields the correct result: $\lambda^{-2}=[16\pi e^2N(0)v^2/3c^2]\tau$.

A straightforward calculation yields the equilibrium zero-field free energy:

$$F_0 = -N(0)W^2 \frac{C^2}{2D}\tau^2. \tag{38}$$

The thermodynamic field H_c follows: $H_c^2/8\pi = -F_0$. One can show that the relative specific heat jump at T_c differs from the one-band value $12/7\zeta(3) = 1.43$ by a factor $C^2/D < 1$.[23]

One can now form the dimensionless GL parameter,

$$\kappa^2 = \frac{\lambda^2}{\xi^2} = \frac{c^2 W^2 D}{8\pi N(0) e^2 \hbar^2 \tilde{v}^4}, \tag{39}$$

and verify the standard relation $H_{c2}/H_c\sqrt{2} = \kappa$.

Finally, the equilibrium energy is evaluated by substituting the solutions of the GL equations to the functional (2):

$$\mathcal{F} = \frac{H_c^2}{4\pi}\int dV \left\{ b^2 - \frac{1}{2}|\Psi|^4 \right\}, \tag{40}$$

where $b = B/H_c\sqrt{2}$ is the dimensionless field. Thus, the theory of a two-band superconductor near T_c is mapped onto the standard one-order parameter GL scheme.

In particular, this mapping means that the GL problem of the interface energy between normal and superconducting phases has the same solution as in the one-band case, that is, $\kappa = 1/\sqrt{2}$ separates type I and type II superconductors. This has been demonstrated in Ref. 7 by solving numerically the nonlinear system of GL Eqs. (3) and (4) without discarding the terms $\mathcal{O}(\tau^2)$ employed here.

A. Remark on boundary conditions

The solution (11) for the two gap functions of the GL Eqs. (7) and (8) holds indeed provided the boundary conditions for Δ_2 are the same as for Δ_1 multiplied by the factor $\sqrt{\beta_1/\beta_2}$. This is clearly the case for the 1D problem of the S-N interface energy discussed in Ref. 7. The same is true for the problem of the single-vortex structure: both Δ's are zero at the vortex center and approach $\Delta_{\nu,0}$ with the correct ratio at infinity. However, for example, for proximity situations with a two-band superconductor on one side of the contact with a normal metal, the condition on the superconducting side far from the boundary is satisfied, whereas the question of boundary conditions at the interface remains open. In this case, one cannot claim that both $\Delta(\boldsymbol{r})$'s are proportional to each other. Nevertheless, as is seen from Eqs. (7) and (8), the length scale $\xi = \sqrt{K/\alpha\tau}$ is still the same for both order parameters.

IV. DISCUSSION

Two-band GL equations have been used in a number of publications where the coefficients in the GL energy functional a_ν, b_ν, K_ν and γ were varied and possible consequences were discussed. Moreover, different ξ's and even λ's were assigned to the two bands along with two different κ's. This led to speculations that situations may exist where one of the bands behaves as a type II superconductor with $\kappa_1 > 1/\sqrt{2}$, while the other may have $\kappa_2 < 1/\sqrt{2}$ and behave near T_c as the type I; the superconductivity in such situations was called "type 1.5." MgB_2 has been suggested as such an example; see, for example, Ref. 11 and references therein.

The present work argues that such situations do not exist. The point is that the GL equations are derived from the microscopic theory within certain approximations that lead to the free energy near T_c being proportional to $(1 - T/T_c)^2$ and the order parameter (or parameters) varying as $(1 - T/T_c)^{1/2}$. Formally, the nonlinear system of GL Eqs. (3) and (4) for two-band materials can be solved with whatever accuracy one chooses. However, physically there is no point in going for accuracy higher than that of equations themselves; whatever results obtained along these lines will be unreliable. To get a near-T_c description more accurate than GL, one should go back to microscopic theory that generates many extra terms in the free energy expansion even for the one-band situation; see, for example, Ref. 24, so that the multiband generalization of such an approach is unlikely to produce a useful theory. It is demonstrated on a one-dimensional problem of Ref. 7 and is shown for a general case in this paper that *within the GL accuracy*, both order parameters of a two-band superconductor vary on the same length scale ξ of Eq. (14) contrary to requirements of "1.5-type superconductivity."

We note that this conclusion holds for the "GL domain" defined as the temperature interval near T_c where the GL expansion can be justified. We do not specify this domain explicitly because its size may vary from one case to another; for example, it is argued in Ref. 8 that for two dirty bands (with no interband scattering) of MgB_2, the domain of GL applicability shrinks practically to zero. However, whatever this size is, within this domain the two order parameters vary on the same length scale. Therefore, attempts to employ the GL functionals, on the one hand, and to assume different length scales, on the other, cannot be justified.

Moreover, we show that—within the GL accuracy—the two GL equations for the two-band case are reduced to a single equation for the normalized order parameter; in other words, the two-band superconductor is described by a single complex order parameter. This excludes possibilities of having "fractional vortices" with exotic properties such as those discussed in Refs. 25 and 26.

Microscopically, our results were derived within a weak-coupling theory of clean superconductors. We believe, however, that our conclusions go beyond that. For our results to hold it is crucial that due to the finite interband Josephson coupling γ, the coefficients a_ν in the GL energy remain finite at T_c. Once this is guaranteed our qualitative conclusions remain unchanged, even if assumptions of the weak coupling, no scattering, and isotropy do not apply.

Note added in proof. In the recent paper by Shanenko *et al.*,[27] our conclusion on a single length scale ξ in two-band superconductors near T_c is confirmed. Extra terms in the GL expansion discussed in this work are, by construction, small corrections and do not change our conclusion that the idea of 1.5-type superconductivity is not warranted by the GL theory.

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