Magnetic field enhancement at optical frequencies through diffraction coupling of magnetic plasmon resonances in metamaterials

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We studied theoretically the diffraction coupling of magnetic resonances in metamaterials consisting of a planar rectangular array of metal rod pairs with a dielectric spacer. A narrow-band mixed mode was observed due to a strong interaction between magnetic resonances in individual pairs of metal rods and an in-plane propagating collective surface mode arising from the array periodicity. Upon the excitation of this mixed mode, a five-fold enhancement of magnetic field in the dielectric spacer was achieved as compared with the purely magnetic resonance. It was also found that only a collective surface mode with its magnetic field parallel to the array plane could mediate the excitation of such a mixed mode.

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Metallic nanoparticles exhibit rich optical properties and have many applications because they support surface plasmon (SP) resonances associated with a huge enhancement of electric fields in their vicinity.¹ Due to radiative or nonradiative (absorptive) damping, however, the particle SP resonances usually have a broad bandwidth or short lifetime, which will limit their applications in some cases. Now recent studies have shown that the SP bandwidth could be tuned through interparticle near-field interaction^{2,3} or far-field diffraction coupling⁴⁻⁸ in periodic arrays of metallic nanoparticles. In the later case, a narrow-band plasmon mode near particle SPs was predicted^{4,5} and observed experimentally, $^{6-8}$ when the array period is close to the particle resonance wavelength. It is well known that the arrangement of metallic nanoparticles into a periodic lattice can give rise to an in-plane propagating collective surface mode (also referred to as lattice resonance), which appears near the Wood anomaly.⁹ The narrow-band mode steps from the interaction between a particle SP mode and this type of collective surface mode.⁴⁻⁹ Because of a narrow bandwidth, such a hybridized plasmon mode is anticipated to find potential applications. For example, it has been used to enhance and direct fluorescent emission in periodic plasmonic nanoantenna arrays.⁸

In metamaterials, specially shaped metallic nanoparticles, like split-ring resonators (SRRs)¹⁰ and paired rods^{11,12} or nanodisks,13 can induce a magnetic moment counteracting the external magnetic field and thus produce diamagnetic responses, termed magnetic SP resonances to differentiate them from electric SP resonances observed in usual metallic nanoparticles. These metallic nanostructures have been widely employed as artificial magnetic atoms to fabricate negative-permeability or negative-refractive-index metamaterials¹⁴ with peculiar electromagnetic properties.¹⁵ Although the interactions of electric SP resonances in metallic nanostructures have been extensively studied and well understood, less is known about the interactions of magnetic SP resonances which could result in interesting physical phenomena.¹⁶ Very recently, a classical analog for electromagnetic induced transparency (EIT) observed in a three-level atomic system¹⁷ has been demonstrated in metamaterials made of coupled SRRs.¹⁸

On the other hand, achieving magnetic field enhancement at optical frequencies is now drawing increasing attention,¹⁹ due to its potential applications such as magnetic nonlinearity²⁰ and magnetic sensors. However, in light-matter interactions, the magnetic component of light generally plays a negligible role since it is very weak.²¹ Therefore, seeking new mechanisms to enhance the magnetic field becomes quite important. It is well known that a metallic nanogap can provide huge electric field enhancement at electric SP resonances.²² Very recently, its complementary structure, namely, the metallic nanowire, was proposed to achieve extraordinary enhancement of magnetic field at terahertz (THz) frequencies based on Babinet's principple.²³ But, it is still a challenge to achieve huge magnetic field enhancement in the region of optical frequencies.

In this Rapid Communication, we propose an approach to enhancing magnetic fields at optical frequencies via the diffraction coupling of magnetic SP resonances in metamaterials consisting of two-dimensional (2D) periodic arrays of paired metal rods. We found a narrow-band mixed mode due to the strong interaction between magnetic SP resonances and an in-plane propagating collective surface mode of the array. At the optical resonance of this mixed mode, the maximum of magnetic field intensity is about 450 times of the incident field. The magnetic fields in the metal rod pairs are enhanced to nearly 5 times larger than those at the magnetic resonance of individual pairs of metal rods. More interestingly, we found that only a collective surface mode with its electric (magnetic) field perpendicular (parallel) to the array plane could mediate such a coupling.

Figure 1(a) schematically shows the metamaterials to be studied, which are 2D arrays of two parallel Ag rods and a glass spacer between them. The length, width, and height of the Ag rods are, respectively, l = 150 nm, w = 100 nm, and h = 50 nm. The dielectric spacers have identical dimensions and a refractive index n = 1.45. The relative permittivity of Ag is described by a Drude model: $\varepsilon = 1 - \omega_p^2 / [\omega(\omega + i\tau^{-1})]$, where ω_p is the plasma frequency and τ is the relaxation time related to energy loss. The parameters are taken to be $\hbar\omega_p = 9.2$ eV and $\hbar\tau^{-1} = 0.02$ eV.²⁴ The coordinates are chosen such that the Ag rod pairs lie on the *xy* plane, with its origin located at the center of one of the rod pairs. The array periods along



FIG. 1. (Color online) (a) Schematic of a 2D rectangular array of pairs of parallel Ag rods with length l = 150 nm, width w = 100 nm, and height h = 50 nm, separated by a glass layer with a refractive index n = 1.45. (b) Normal-incidence transmission spectra of two 2D rectangular arrays of Ag rod pairs for different periods along the x axis, $p_x = 550$ (black dotted line) and 770 nm (red solid line), but the same period along the y axis, $p_y = 200$ nm. (c) Electric field vectors mapped on the xoz plane across the center of one rod pair in the array for dip 1 in (b). Arrows represent field direction and colors show field strength with red larger and black smaller. Two green rectangles outline the regions of two metal rods. Blue signs "+" and "–" stand for positive and negative charges, respectively.

the *x* and *y* axes are p_x and p_y , respectively. The electric field E_{in} , magnetic field H_{in} , and wave vector K_{in} of the incident light are along the *x*, *y*, and *z* axes, respectively.

Figure 1(b) presents the normal-incidence transmission spectra of two typical 2D rectangular arrays of Ag rod pairs, calculated with the commercial software package COMSOL MULTIPHYSICS. The two arrays have the same period in the y direction ($p_y = 200$ nm) but different periods in the x direction ($p_x = 550$ nm and 770 nm). A broad transmission dip (labeled as dip 1) centered at $\lambda_1 = 740$ nm is observed for both arrays. However, for the array with $p_x = 770$ nm, as shown by the red solid line in Fig. 1(b), a relatively narrow transmission dip (labeled as dip 2) appears at $\lambda_2 = 780$ nm.

The position of the broad dip is almost independent of the period p_x , implying that this transmission dip arises from the excitation of resonance in individual Ag rod pairs. In fact, such a resonance state is a magnetic SP mode. To show this, Fig. 1(c)

PHYSICAL REVIEW B 83, 041402(R) (2011)



FIG. 2. (Color online) Normalized electric field intensity $(E/E_{\rm in})^2$ on the plane of z = 0 (a) and on the plane y = 0 (b) for dip 1. (c) and (d) The same as (a) and (b), respectively, but for normalized magnetic field intensity $(H/H_{\rm in})^2$. Gray rectangles outline the regions of glass layers in (a) and (c), and those of Ag rods in (b) and (d).

maps the electric field vectors on the xoz plane across the center of one rod pair for dip 1, at a time when the incident field reaches its maximum on the xoy plane. It is evident that at this individual particle resonance the electric fields in the upper and lower metal rods oscillate out of phase and produce antiparallel currents, accompanied by an antisymmetric charge distribution on the pair rod ends. The antiparallel ohmic currents, together with the displacement currents in the dielectric layer, form a current loop that induces a magnetic moment.¹²

In order to better understand the properties of magnetic SP resonance, in Fig. 2 we plot the electric and magnetic field distributions at the resonance $\lambda_1 = 740$ nm. It is clear that the electric fields are mainly concentrated near the end points of metal rods [see Figs. 2(a) and 2(b)] and the magnetic fields are highly confined between the metal rods [see Figs. 2(c) and 2(d)], which are characteristics of a magnetic SP resonance of individual pairs of metal rods.¹² The formation of such a magnetic resonance can be well understood through plasmon hybridization.¹³

In Fig. 3, we plot the corresponding electromagnetic field distributions for the dip 2 resonance located at $\lambda_2 = 780$ nm. Although the field patterns are almost the same as the cases shown in Fig. 2, the magnetic fields in the region between the Ag rods and the electric fields near the rod end points become much stronger, with a nearly 5 times enhancement. In particular, the maximum magnetic field is enhanced to be about 450 times of the incident field [please see Fig. 3(d)].

In the following, we will show that such a huge enhancement of magnetic fields at optical frequency results from

MAGNETIC FIELD ENHANCEMENT AT OPTICAL ...



FIG. 3. (Color online) The same as Fig. 2 but for dip 2 marked in Fig. 1(b).

the strong coupling between the magnetic SP resonances in metal rod pairs and an in-plane propagating collective surface mode signaled by the Wood anomaly.^{4–9} This strong coupling leads to the formation of two mixed modes, i.e., the highand low-energy states, whose energies can be calculated with a coupled oscillator model²⁵: $E_{+,-} = (E_{SP} + E_{Wood})/2 \pm$ $\sqrt{\Delta/2 + (E_{\rm SP} - E_{\rm Wood})^2/4}$. Here, $E_{\rm SP}$ and $E_{\rm Wood}$ are the energies of the magnetic SP and Wood anomalies, respectively; and Δ stands for the coupling strength. The wavelength of the Wood anomaly can be calculated by matching the wave vector K_{in} of incident light in the surrounding medium (air) with the reciprocal vector $G_{m,n}$ of a 2D rectangular lattice under normal incidence²⁶: $\lambda_{\text{wood}}^{m,n} = 1/\sqrt{(m/p_x)^2 + (n/p_y)^2}$, where m and n are integers related to different diffraction orders. E_{SP} is determined by the geometrical and material parameters of individual pairs of metal rods, but independent of the array periods. Through numerical simulations, we found that for individual pairs of metal rods E_{SP} is about 1.647 eV, corresponding a wavelength of 753 nm. Figure 4(a) shows the transmission spectra of a series of 2D rectangular arrays of Ag rod pairs, with p_x being varied from 650 nm to 850 nm in steps of 20 nm. The period along the y axis is kept constant $p_v = 200$ nm. In each transmission spectrum, two transmission dips are observed: one is narrow and the other is broad. The open black circles in Fig. 4(b) summarize the dependence of the positions of these transmission dips on p_x . By taking the coupling strength to be $\Delta = 310 \text{ meV}$ in the above coupled oscillator model, we can predict the positions of transmission dips for different periods p_x . The two branches of red lines in Fig. 4(b) give the predicted results. Clearly, they are in a good agreement with the locations dictated from the transmission spectra. The black line and the horizontal green line in Fig. 4(b) show the positions of the

PHYSICAL REVIEW B 83, 041402(R) (2011)



FIG. 4. (Color online) (a) Normal-incidence transmission spectra of a series of 2D rectangular arrays of Ag rod pairs, with p_x varied from 650 nm (top line) to 850 nm (bottom line) in steps of 20 nm but with p_y kept constant at 200 nm. Individual spectra are vertically offset from one another by 1.0 for clarity. Other structural parameters are the same as those used in Fig. 1(b). (b) The p_x dependences of the positions (open black circles) of transmission dips in (a), and the predicted positions (two red curved lines) using a coupling model of the Wood anomaly and magnetic SP resonance. The Wood wavelength $\lambda_{wood}^{1.0} = p_x$ (black diagonal line) and the position (horizontal green line) of magnetic SP resonance are also shown.

Wood anomaly and magnetic SPs, respectively. At the crossing of these two lines, the positions of transmission dips present an obvious anticrossing, which is a characteristic of the strong coupling between the collective surface mode and magnetic SP. Away from this strong coupling regime, the transmission dip positions follow approximately one of these two lines.

In a series of arrays investigated in Fig. 4(a), the period p_{y} is set to be 200 nm, a value much smaller than the spectral range of interest from 500 nm to 900 nm. As a result, the diffraction channel in the y direction is closed, and only the diffraction channel in the x direction is opened. In this situation, the above formula for the wavelength of Wood anomaly reduces to a simple form: $\lambda_{\text{wood}}^{m,0} = p_x/m$. A first-order diffraction corresponding to $\lambda_{\text{wood}}^{1,0} = p_x (m = 1)$ is predicted, as shown by black line in Fig. 4(b). The other higher order (m = 2, 3, ...) diffraction wavelengths are all smaller than 500 nm. Therefore, our arrays support only a first-order collective surface mode propagating along the x direction. By differentiating the x, y, and z components of the total electromagnetic fields at resonance (not shown here), we found that the electric field of this surface mode is along the z axis, and its magnetic field is along the y axis. Because the surface mode has a magnetic field of the same direction as the induced magnetic moment in the Ag rod pairs, it can strongly interact with the magnetic SP resonance in each pair of metal rods when grazing the metamaterial surface. Such a strong interaction can suppress radiative damping since the electromagnetic fields of the surface mode are trapped in the lattice,^{9,27} thus forming a narrow-band mixed mode at a wavelength slightly larger than the Wood wavelength $\lambda_{wood}^{1,0}$.

By varying the array periods while keeping the other conditions unchanged, we have also studied other situations in which the diffraction channel along the x direction is closed but the one along the y direction is opened (not shown here). A narrow-band mode is also observed when the period p_{y} approaches the magnetic SP resonance wavelength. Nevertheless, in these cases the collective surface mode propagating in the y direction has an electric (magnetic) field parallel to the x(z) axis. Strictly speaking, the narrow-band mode does not arise from the diffraction coupling of magnetic SP resonances, but is more likely a result of the diffraction coupling of electric SP resonances, since the magnetic field of the surface mode is orthogonal to the induced magnetic moment (along the y direction) in the Ag rod pairs, that is, the surface mode cannot interact directly with magnetic SP resonances of individual pairs of Ag rods. In fact, at the narrow-band mode resonance, the magnetic fields in the metal rod pairs become weaker, rather than getting enhanced, as compared at the pure magnetic SP resonance.

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PHYSICAL REVIEW B 83, 041402(R) (2011)

In conclusion, we have shown that in metamaterials consisting of 2D periodic arrays of pairs of metal rods, the interaction between magnetic resonances of the metal rod pairs and an in-plane propagating collective surface mode signaled by the Wood anomaly can form a narrow-band hybrid mode. It is found that only a collective surface mode with its magnetic field parallel to the induced magnetic moment in the rod pairs could mediate such an interaction. The huge magnetic field enhancement at the hybrid mode resonance suggests the metamaterials have a potential to explore optical phenomena based on enhanced magnetic fields, such as second-harmonic generation from magnetic nonlinearity.²⁰

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