Spectroscopy of electron flows with single- and two-particle emitters

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To analyze the state of injected carrier streams of different electron sources, we propose to use correlation measurements at a quantum point contact with the different sources connected via chiral edge states to the two inputs. In particular, we consider the case of an on-demand single-electron emitter correlated with the carriers incident from a biased normal reservoir, a contact subject to an alternating voltage, and a stochastic single-electron emitter. The correlation can be viewed as a spectroscopic tool to compare the states of injected particles of different sources. If at the quantum point contact the amplitude profiles of electrons overlap, the noise correlation is suppressed. In the absence of an overlap, the noise is roughly the sum of the noise powers generated by the electron streams in each input. We show that the electron state emitted from a (dc- or ac-) biased metallic contact is different from a Lorentzian amplitude electron state emitted by the single-electron emitter (a quantum capacitor driven with slow harmonic potential), because with these inputs the noise correlation is not suppressed. In contrast, if quantized voltage pulses are applied to a metallic contact instead of a dc (ac) bias, then the noise can be suppressed. We find a noise suppression for multielectron pulses and for the case of stochastic electron emitters for which the appearance of an electron at the quantum point contact is probabilistic.

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I. INTRODUCTION

The experimental realization[1] of an on-demand, high-frequency, single-electron source (SES) makes it possible to inject single particles, electrons, and holes into a solid-state circuit, in a controllable way. By using several uncorrelated SESs, mesoscopic circuits were proposed that permit variations in the amount of fermionic correlations[2] and production of controllable orbitally entangled pairs of particles.[3] Similar high-frequency sources of single electrons were realized by using dynamical quantum dots without[4] or with a perpendicular magnetic field.[5] The principal advantage of on-demand SESs over the usually used metallic contacts (MCs) as electron sources is the possibility, in the former case, to switch on and off the quantum correlations between particles initially emitted from uncorrelated sources. An example of correlations generated by normal metallic contacts is the two-particle Aharonov-Bohm effect in the solid-state Hanbury-Brown–Twiss interferometer, which has been discussed theoretically[6–8] and found experimentally.[9] In contrast, with SESs, the two-particle interferometer, as it is discussed in Ref. 3, can show or not show the Aharonov-Bohm effect depending on whether or not sources are driven in synchronism.

The appearance of quantum correlations (fermionic in the case of electrons) between initially uncorrelated particles results from the overlap of wave packets on the wave splitter. For electrons in solid-state circuits the splitter is a quantum point contact (QPC) (see Fig. 1, the QPC labeled C). Such correlations are well known in optics (see, e.g., Ref. 10). The overlap of fermions was discussed in Refs. 11 and 12. The overlap depends on the spatial extent of wave packets and also on the times when they arrive at the wave splitter. Thus the resulting correlations can be used to access information about the space-time extension of quantum states. For MCs working as electron sources, such information is rather hidden because the mentioned correlations are always present. In contrast with on-demand SESs, control of the emission time can be achieved, i.e., the appearance or disappearance of correlations can be controlled. Thus with such sources the space-time extension of quantum states becomes accessible.

In mesoscopics physics shot noise[12] is the natural quantity that can be used to find information on two-particle correlations.[13–15] The shot noise of carriers emitted by two SESs is suppressed if the wave packets overlap at the QPC connecting edge states in which emitted particles propagate.[2] If two sources are identical and they emit particles at the same time, the emitted particles are in identical quantum states and the shot noise is suppressed down to zero. This effect is similar to the Hong-Ou-Mandel effect in optics,[16] with the evident difference that electrons are rather forced into different output channels while photons are bunched into the same output channel.

The aim of this paper is to use the shot-noise suppression (ShNS) as a spectroscopic tool, allowing for a comparison of the quantum states emitted by the different electron sources. As the test state we will use the one emitted by the SES. The SES is composed of a quantum capacitor[1] in the quantum Hall-effect regime. The SES is connected to one of the arms of the mesoscopic electron collider (Fig. 1). Under the action of a potential $U(t) = U\cos(\Omega t)$ periodic in time, the SES emits a sequence of alternating electrons and holes.[17] In a certain range of amplitudes, in the quantized emission regime, the SES emits one electron and one hole. At low driving frequency, in the adiabatic regime, the emitted state by the SES is close to the state generated by voltage pulses of Lorentzian form with a time integral equal to a flux quantum: Such a quantized voltage pulse produces a single-particle state on top of the Fermi sea.[18–20] In the second arm we apply the source of interest and investigate the resulting shot noise.

The paper is organized as follows: In Sec. II we calculate the zero-frequency cross correlator of currents flowing into two outputs in the collider circuit with a SES in one input and a biased metallic contact in the other input. In Sec. III we address

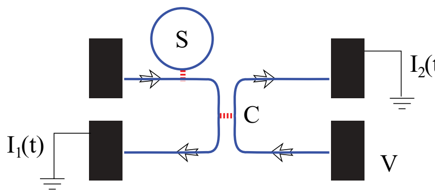

FIG. 1. (Color online) A mesoscopic electron collider circuit with a SES, S, a circular edge state, and a metallic contact source biased with a voltage V. At the quantum point contact C the particles emitted by the two sources can collide if the times of emission are adjusted properly. Solid blue lines are edge states with direction of movement indicated by arrows. Short dashed red lines are quantum point contacts connecting different parts of a circuit. Black rectangles are metallic contacts.

the effect of stochastic single-particle emitters that emit or do not emit a particle in a given period that arrives at the QPC. We demonstrate the irrelevance of such stochastic emission to the ShNS effect. In Sec. IV the ShNS effect is found for colliding single- and two-electron pulses. A discussion of our results is given in Sec. V. Much of the analysis is grouped into three appendices. In Appendix A we present a detailed model of a SES. In Appendix B we calculate the current correlation function for a periodically driven mesoscopic scatterer connected to reservoirs biased with periodic voltages. In Appendix C the zero-frequency cross-correlation function for a circuit with two SESs is expressed in terms of single- and two-particle probabilities.

II. SINGLE-ELECTRON EMITTER AND BIASED METALLIC CONTACT AS AN ELECTRON SOURCE

We consider an electron collider with a SES in one branch and a MC with a potential $V_2^{(\sim)}(t) = V_2^{(\sim)}(t+\mathcal{T})$ periodic in time in another branch (see Fig. 1). The potential $U(t)$ driving the SES and $V_2^{(\sim)}(t)$ have the same period, $\mathcal{T} = 2\pi/\Omega$, which is assumed to be large enough to consider adiabatic transport and neglect relaxation and decoherence processes[21,22] relevant for high-energy excitations. In addition, the MC is biased with a constant potential V_2 with respect to the other contacts that all have the same chemical potential μ. The temperature is taken to be zero.

We utilize the scattering matrix approach[23] to transport in mesoscopic systems, and describe this circuit with the help of the frozen scattering matrix,

$$\hat{S}(t) = \begin{pmatrix} e^{ikL_{1S}} S^{\mathrm{SES}}(t) r_C & e^{ikL_{1V}} t_C \\ e^{ikL_{2S}} S^{\mathrm{SES}}(t) t_C & e^{ikL_{2V}} r_C \end{pmatrix}, \tag{1}$$

where $S^{\mathrm{SES}}(t)$ is the scattering amplitude of the SES [see Appendix A, Eq. (A5)], r_C (t_C) is the reflection (transmission) amplitude for the central quantum point contact C, and L_{jX} is the length from the SES ($X=S$) or the MC ($X=V$) to the contact $j = 1,2$, where the corresponding current $I_j(t)$ is measured. At zero temperature we need all quantities at the Fermi energy μ only.

We are interested in the zero-frequency correlation function[12] $\mathcal{P}_{12}$ of the currents $I_1(t)$ and $I_2(t)$ flowing into contacts 1 and 2 (see Fig. 1). The corresponding calculations are presented in Appendix B. In the adiabatic regime and at zero temperature we have $\mathcal{P}_{12} \equiv \mathcal{P}_{12}^{(\mathrm{sh,ad})}$ [see Eq. (B26)]:

$$\mathcal{P}_{12} = -\mathcal{P}_0 \sum_{q=-\infty}^{\infty} |\{S^{\mathrm{SES}}\Upsilon_2^*\}_q|^2 \left|\frac{eV_2}{\hbar\Omega} - q\right|, \tag{2}$$

where $\mathcal{P}_0 = e^2 R_C T_C/\mathcal{T}$ is the shot noise[2] produced by one particle (either an electron or a hole) emitted by the SES during the period $\mathcal{T}$. The oscillating potential at contact 2 appears in the form of a phase factor,

$$\Upsilon_2(t) = \exp\left[-i\frac{e}{\hbar}\int_{-\infty}^{t} dt'\, V_2^{(\sim)}(t')\right], \tag{3}$$

which multiplies the scattering amplitude of the SES. The symbol $\{\cdots\}_q$ indicates the qth Fourier component (in time) of these two amplitudes. Equation (2) illustrates that the correlation tests the coherence properties of the two sources.

Note that the decoherence processes, which we neglect in the present work, can lead to suppression of coherence.

A. Quantized voltage pulse

With a Lorentzian-shaped voltage pulse and a time integral quantized to a single flux quantum,[18,19] one can excite an electron from a Fermi sea without any other disturbance to the Fermi sea. The state for an excited electron has a Lorentzian density profile (the time-dependent current is a Lorentzian pulse), which is similar to the one[2] emitted by the SES in the adiabatic regime. Thus we can expect a ShNS effect if an electron is excited out of a metallic contact with a quantized voltage pulse, and an electron that is emitted by the SES collide at the central QPC. Below we show that this is really the case.

Thus let us assume that a periodic pulsed potential is applied to the MC,

$$eV_2(t) = \frac{2\hbar\Gamma}{(t-t_0)^2+\Gamma^2}, \quad 0 < t \leqslant \mathcal{T}, \tag{4}$$

where $\Gamma \ll \mathcal{T}$ is the half-width of the pulse, and t_0 is the time when the electron is excited. Such a pulse excites one electron during the period $\mathcal{T}$.

The potential $V_2(t)$ [Eq. (4)] has a dc component, $eV_2 = h/\mathcal{T}$, and a component that is periodic in time,

$$eV_2^{(\sim)}(t) = \frac{2\hbar\Gamma}{(t-t_0)^2+\Gamma^2} - \frac{h}{\mathcal{T}}, \quad 0 < t \leqslant \mathcal{T}. \tag{5}$$

The corresponding phase factor $\Upsilon_2(t)$ [Eq. (3)] (for $0 < t \leqslant \mathcal{T}$) is

$$\Upsilon_2(t) = \exp\left\{i\left[\Omega t - 2\left(\arctan\frac{t-t_0}{\Gamma} + \arctan\frac{t_0}{\Gamma}\right)\right]\right\}. \tag{6}$$

The result of a numerical evaluation of the shot noise based on Eq. (2), as a function of the time t_0, is given in Fig. 2. For almost all times t_0 the noise is $-3\mathcal{P}_0$, and only at a very special coincidence time is there a sharp reduction of the noise. The noise, $-3\mathcal{P}_0$, is produced by three uncorrelated particles emitted during a period: One electron is emitted by the metallic contact and two particles, an electron and a hole, are emitted by the SES. However, if the MC and the SES emit electrons at the same time, $t_0 = t_0^{(-)}$, then after colliding at the central QPC these electrons become correlated and do not contribute

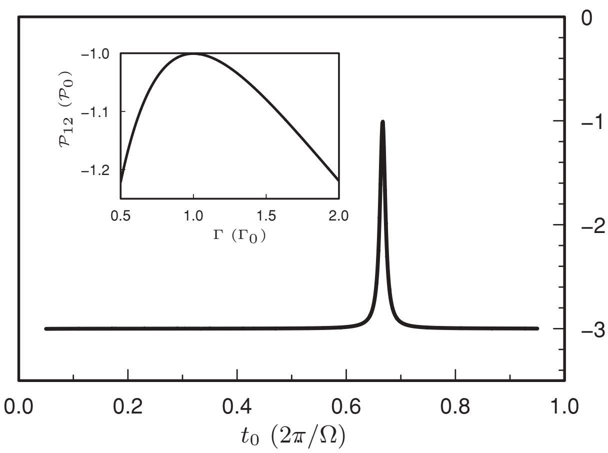

FIG. 2. The noise $\mathcal{P}_{12}$ [Eq. (2)] as a function of the time t_0 when an electron is excited out of the metallic contact by the voltage pulse $V_2(t)$. The plot assumes that the half-widths of the voltage pulse at the MC and the pulse of the SES are the same ($\Gamma = \Gamma_0$). The parameters of the SES [Eq. (A1)] are as follows: $T = 0.1$, $U_0 = 0.25$, $U_1 = 0.5$. Inset: The noise at the minimum as a function of Γ.

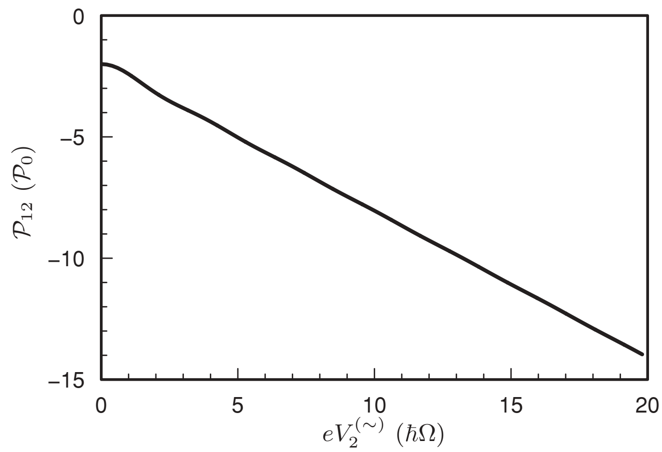

FIG. 3. The noise $\mathcal{P}_{12}$ [Eq. (2)] as a function of the amplitude $eV_2^{(\sim)}$ of the ac potential applied to the metallic contact. The parameters of the SES in Eq. (A1) are as follows: $T = 0.1$, $U_0 = 0.25$, $U_1 = 0.5$.

effectively to the shot noise. The remaining value, $-\mathcal{P}_0$, is owing to the hole emitted by the SES.

The ShNS effect depends on the overlap of wave packets in time (hence the times of emission should be the same) and in space (hence the width of wave packets should be the same). If the wave packets have a different width (see the inset of Fig. 2), there is some extra noise. Therefore, the ShNS effect provides a direct tool to compare the states of particles emitted from the sources of different types, not only from the similar sources. The ShNS of two SESs was already discussed in Ref. 2.

The most used source of electrons in mesoscopics is a biased (with dc or ac voltage) metallic contact. Now we show that the electron collider circuit with SES and a biased MC as an electron source does not show what we call the ShNS effect. Therefore, the state of electrons emanating from a biased MC is different from the state of an electron emitted by the SES.

B. dc bias

If no ac bias is applied, $V_2^{(\sim)}(t) = 0$, the phase factor is $|\Upsilon_2(t)|^2 = 1$ and only the Fourier coefficients, $|S_q^{\rm SES}|^2 = 4\Omega^2\Gamma_0^2 \exp(-2\Omega\Gamma_0|q|)$, enter Eq. (2). We recall that we assume an adiabatic limit of $\Omega\Gamma_0 \ll 1$, where $2\Gamma_0$ is the time during which an electron (a hole) is emitted by the SES. Because $|S_q|^2 = |S_{-q}|^2$, we conclude from Eq. (2) that in this case the shot noise is independent of the sign of the voltage. Therefore, the result will be the same regardless of whether the MC emits electrons, $eV > 0$, or holes, $eV < 0$. For definiteness we will use $eV_2 > 0$.

Evaluation of the cross correlator gives

$$\mathcal{P}_{12} = -\mathcal{P}_0\left\{\frac{eV_2}{\hbar\Omega} + 2e^{-2\Omega\Gamma_0\left(\frac{eV_2}{\hbar\Omega}\right)}\left[1 + 2\Omega\Gamma_0\left(\frac{eV_2}{\hbar\Omega}\right)\right]\right\}, \qquad (7)$$

where we have introduced the integer part $[eV_2/(\hbar\Omega)]$. This correlator has the following asymptotics:

$$\mathcal{P}_{12} = \begin{cases} -2\mathcal{P}_0, & eV_2 \ll \hbar\Omega, \\ -(e^3 V_2/h) R_C T_C, & eV_2 \gg \hbar\Gamma_0^{-1}. \end{cases} \qquad (8)$$

Here the first line is the shot noise owing to the SES emitting one electron and one hole during the period. The second line is the shot noise owing to a dc-biased metallic contact alone.[12] The latter noise is due to scattering at the quantum point contact C of extra electrons flowing out of a biased contact above the Fermi sea with a chemical potential μ. These electrons are emitted with a rate eV_2/h. Therefore, one could naively expect that, if the rate of emission of electrons from the SES and from the MC is the same, $\hbar\Omega = eV_2$, then each emitted electron will collide at the central QPC with an electron propagating within another edge state and the shot noise gets suppressed.

This is not the case. As follows from Eq. (7), the shot noise has no strong feature at $eV_2 \sim \hbar\Omega$. The shot noise is a monotonous function of the dc bias V_2. A possible reason for this is that the states of electrons emitted from the SES and from the dc-biased MC are quite different: The electrons emitted from the SES can be thought of as wave packets with a spatial extent proportional to the duration of emission Γ_0. In contrast, the electrons emitted by the metallic contact are rather plane-wave-like, extended along the whole edge state. Thus their overlap at the central QPC is minute, hence they do not acquire any significant correlations. The shot noise remains roughly the sum of the noises produced independently by the SES and by the dc-biased MC.

Next we show that the noise suppression effect is also absent if the metallic contact is driven by an ac bias.

C. ac bias

Next consider the case of a metallic contact with an ac bias, $V_2^{(\sim)}(t) = V_2^{(\sim)}\cos(\Omega t)$, $V_2 = 0$. In this case the metallic contact emits both electrons and holes. The cross correlator [Eq. (2)] as a function of amplitude $V_2^{(\sim)}$ is given in Fig. 3. There is a small feature at $eV_2^{(\sim)} = \hbar\Omega$ that is visible in Fig. 3, but this feature is minute compared to the huge dip of interest in this work. Therefore, there is no indication of a ShNS at $eV_2^{(\sim)} \sim \hbar\Omega$ when the rate of emission of particles from the SES and from the MC is the same.

This is in contrast to the case when sinusoidal voltages are applied to both inputs.[24] Then the discussion can best be cast into excitations of electron-hole pairs,[25] which create a shot noise that has been measured.[26] For two oscillating voltages, theory predicts significant two-particle correlations owing to the Hanbury-Brown–Twiss effect, which depend on the phase delay of the two oscillating voltages.[24]

One can wonder whether the absence of the ShNS effect is possibly due to fluctuations in emission of electrons from the biased metallic contact. Our expectation is that neither fluctuations nor a possible presence of multielectron (multihole) states play a crucial role. To show this, next we consider two circuits.

III. ShNS EFFECT WITH STOCHASTIC SESs

In Fig. 4 we show a circuit with two SESs, S_L and S_R, each emitting one electron and one hole per period $\mathcal{T}$. Initially the particle stream is regular. However, say, for particles emitted by the source S_L, at the quantum point contact L an electron (a hole) can be either reflected to the metallic collector 3 or be transmitted to the central part of a circuit. Thus, the SES, S_j, together with a corresponding quantum point contact $j = L,R$, comprise a stochastic SES that can either inject into the central part of a collider one electron (hole) during a given period or not.

We assume that all the metallic contacts are grounded and calculate the zero-temperature cross correlator $\mathcal{P}_{12} \equiv \mathcal{P}_{12}^{(\mathrm{sh,ad})}$ [Eq. (B26)]:

$$\mathcal{P}_{12} = \frac{e^2\Omega}{4\pi} \sum_{q=-\infty}^{\infty} |q| \sum_{\gamma,\delta=1}^{4} \{S_{1\gamma} S_{1\delta}^*\}_q \{S_{2\gamma} S_{2\delta}^*\}_q^*. \tag{9}$$

Because there is no bias, all the phase factors are $\Upsilon_\delta = 1$ in Eq. (B25) and $V_{\gamma\delta} = 0$ in Eq. (B26). The elements of the frozen scattering matrix are expressed in terms of the transmission (reflection) amplitude(s) t_i (r_i) for the quantum point contacts

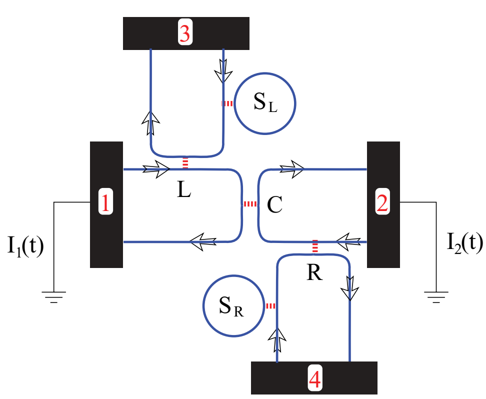

FIG. 4. (Color online) A mesoscopic electron collider circuit with two stochastic single-particle streams originating from the quantum point contacts L and R. In this case, two particles enter the central part of a circuit and they can collide at the quantum point contact C if the times of emission by the sources S_L and S_R are adjusted properly. Solid blue lines are edge states with direction of movement indicated by arrows. Short dashed red lines are quantum point contacts connecting different parts of the circuit. Black rectangles numbered by 1–4 are metallic contacts.

$i = L,R,C$, time-dependent amplitudes $S_j^{\mathrm{SES}}(t)$ for sources S_j, $j = L,R$, and corresponding phase factors $e^{ikL_{\alpha\beta}}$, where $L_{\alpha\beta}$ is a length between metallic contacts α and β. For instance, $S_{13}(t) = e^{ikL_{13}} S_L^{\mathrm{SES}}(t) t_L r_C$. For $S_j^{\mathrm{SES}}(t)$ we use Eq. (A5) with emission times $t_0^{(\pm)}$ and a pulse half-width Γ_0 replaced by $t_j^{(\pm)}$ and Γ_j, respectively. Then we find

$$\mathcal{P}_{12} = -2\mathcal{P}_0\{(T_L - T_R)^2 + T_L T_R[\gamma(\Delta t^{(-)}) + \gamma(\Delta t^{(+)})]\}, \tag{10}$$

where $\Delta t^{(\pm)} = t_L^{(\pm)} - t_R^{(\pm)}$ with $t_j^{(\pm)}$ ($j = L,R$) the time of an electron-hole (e-h) emission by the SES j, and the suppression function

$$\gamma\,(\Delta t) = \frac{(\Delta t)^2 + (\Gamma_L - \Gamma_R)^2}{(\Delta t)^2 + (\Gamma_L + \Gamma_R)^2}. \tag{11}$$

If the SESs emit particles at different times, $\Delta t^{(\pm)} \gg \Gamma_L,\Gamma_R$, then the correlation is

$$\mathcal{P}_{12} = -2\mathcal{P}_0\left\{T_L^2 + T_R^2\right\}. \tag{12}$$

This expression results from the shot noise produced by the four uncorrelated particles (two electrons and two holes) emitted by the two sources during the period $\mathcal{T}$. Apparently the single-particle contribution (to the cross correlator) is negative. We call this regime *classical*, because the shot noise can be explained in terms of single-particle probabilities only (see Appendix C).

On the other hand, if pulses of the same width, $\Gamma_L = \Gamma_R$, are emitted at the same time, $t_L^{(-)} = t_R^{(-)}$ (for electrons) and $t_L^{(+)} = t_R^{(+)}$ (for holes), then the cross correlator is suppressed:

$$\mathcal{P}_{12} = -2\mathcal{P}_0\,(T_L - T_R)^2. \tag{13}$$

In addition, if the circuit is symmetric, $T_L = T_R$, then the cross correlator is suppressed down to zero.

This suppression is due to a positive two-particle contribution arising (in addition to negative single-particle contributions that are also present) when particles (either two electrons or two holes) collide at the quantum point contact C. Owing to such collisions, each of the particles loses information about its origin (i.e., about the source that emitted it), and the pair of particles propagating to contacts 1 and 2 in Fig. 4 becomes orbitally entangled.[3] We call this regime *a quantum regime*, because to describe a shot noise we additionally need to take into account the existence of both direct and exchange two-particle quantum-mechanical amplitudes for colliding particles [see Appendix C, Eq. (C6)].

Thus with this circuit we showed that the ShNS effect is sensitive to space-time confinement of electron states rather than to a regularity in appearance of electrons at the place (the QPC C) where they can overlap.

IV. ShNS EFFECT WITH SINGLE- AND TWO-PARTICLE SOURCES

Now we consider a circuit (see Fig. 5) that contains both a single-particle emitter S and a two-particle emitter S_2. As a two-particle source we use two SESs placed close to each other and emitting in synchronism.[27] In the adiabatic case of interest here, the scattering amplitude $S^{\mathrm{TES}}(t)$ is the product

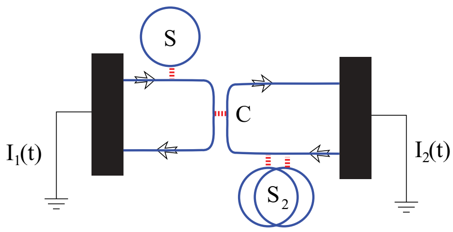

FIG. 5. (Color online) A mesoscopic electron collider circuit with a SES, S, and a two-particle source, S_2. At the quantum point contact C the particles emitted by different sources can collide if the times of emission were adjusted properly. Solid blue lines are edge states with direction of movement indicated by arrows. Short dashed red lines are quantum point contacts connecting different parts of a circuit. Black rectangles are metallic contacts.

of scattering amplitudes of SESs comprising a two-particle source. For simplicity, we assume both sources to be identical. Then S^{TES} is the square of the amplitude given by Eq. (A5) with $t_0^{(\pm)}$ and Γ_0 replaced with $t_2^{(\pm)}$ and Γ_2, respectively. At the time $t_2^{(-)}$ ($t_2^{(+)}$) the pair of electrons (holes) is emitted by the source S_2. The cross correlator $\mathcal{P}_{12}$ [Eq. (B26)] reads

$$\mathcal{P}_{12} = -\mathcal{P}_0 \sum_{q=-\infty}^{\infty} |q| |\{S^{\mathrm{SES}}(S^{\mathrm{TES}})^*\}_q|^2, \tag{14}$$

where $S^{\mathrm{SES}}(t)$ is the scattering amplitude [Eq. (A5)] for a SES, S, and $S^{\mathrm{TES}}(t)$ is the scattering amplitude for the two-electron (two-particle) source S_2 shown in Fig. 5.

Simple calculations yield

$$\begin{aligned}\mathcal{P}_{12} = -\mathcal{P}_0\{&\gamma^2(\Delta t^{(-)}) + 2\gamma(\Delta t^{(-)}) + \chi(\Delta t^{(-)})\\ &+ \gamma^2(\Delta t^{(+)}) + 2\gamma(\Delta t^{(+)}) + \chi(\Delta t^{(+)})\},\end{aligned} \tag{15}$$

where $\Delta t^{(\pm)} = t_0^{(\pm)} - t_2^{(\pm)}$. The function $\chi(\Delta t)$ is

$$\chi(\Delta t) = \frac{16\Gamma_2^2\Gamma_0^2}{[(\Delta t)^2 + (\Gamma_2 + \Gamma_0)^2]^2}, \tag{16}$$

and the suppression function $\gamma(\Delta t)$ is given in Eq. (11) with Γ_L and Γ_R replaced by Γ_0 (for a SES) and Γ_2 (for a two-particle source), respectively.

If all the particles are emitted at different times, $\Delta t^{(\pm)} \gg \Gamma_0, \Gamma_2$, the cross correlator, $\mathcal{P}_{12} = -6\mathcal{P}_0$, is determined by contributions of six uncorrelated particles (three electrons and three holes) emitted during the period $\mathcal{T}$, while for simultaneous emission, $\Delta t^{(\mp)} = 0$, the cross correlator is partially suppressed. If $\Gamma_2 = \Gamma_0$, the cross correlator is suppressed down to the level generated by two particles, $\mathcal{P}_{12} = -2\mathcal{P}_0$. So when the two-electron wave packet collides with a single-electron wave packet, two colliding electrons, one from each side, produce no noise while the remaining electron produces noise as if it had been propagated alone through the QPC. The same holds for hole wave packets.

V. CONCLUSION

A method to compare quantum states of initially uncorrelated electrons in mesoscopic circuits was proposed. The electron streams should be directed onto a quantum point contact from different sides and the cross correlator of currents flowing out of the QPC should be measured. In general, two uncorrelated streams produce additive noises. However, if the particles overlap at the QPC, they become correlated and the noise gets suppressed. The closer the quantum states of particles resemble each other, the better the overlap that can be achieved, and hence the noise is suppressed more strongly.

We considered several sources of electrons, in particular, (i) a metallic contact, emitting a rather continuous stream of electrons with a rate proportional to the bias, and (ii) a periodically driven quantum capacitor, a SES, emitting traveling wave packets of electrons that are rather localized in space and alternate with the wave packet of holes. We found that the streams produced by the MC biased with a dc (ac) voltage and by the SES remain almost uncorrelated after passing the QPC, even if the electrons are emitted with the same rate. Therefore, we conclude that the electrons of these streams are in quite different quantum states. On the other hand, if the periodic sequence of quantized voltage pulses is applied to a MC, then the resulting electron stream can be easily correlated with a stream emitted by the SES, resulting in a complete suppression of the shot noise. From this we can conclude that the electrons of these streams are in the same quantum states.

If the streams are fluctuating, then the shot noise can be suppressed by the amount proportional to the average number of particles overlapping at the QPC. We also found a partial suppression of the shot noise in the case of pulses carrying different numbers of particles. Basically the remaining noise results from the difference of the numbers of particles carried by the colliding pulses.

Note added in proof. Recently, a related discussion by Grenier *et al.*[38] appeared. That work is aimed at electron state tomography and it deals with a SES working in a non-adiabatic regime. In contrast, we assume an adiabatic regime for the SES and we are interested in a comparison of quantum states for electrons emitted from different sources.

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APPENDIX A: SCATTERING AMPLITUDE AND CURRENT OF A SES

As a SES we use a quantum capacitor[1,28–30] described by a model in which a single circular edge state of circumference L in a cavity is coupled via a QPC with transmission probability T to a linear edge state (see the upper left-hand corner of Fig. 1). A potential, $U(t) = U_0 + U_1\cos(\Omega t + \varphi)$, periodic in time is induced uniformly over the cavity with the help of a top gate. In the case of a slow potential, $\Omega\tau \ll T$, where τ is the time of one turn around the cavity, the (frozen[31]) scattering amplitude of a capacitor for an electron with incident energy E and propagating in the linear edge state at time t is

$$S^{\mathrm{SES}}(t,E) = e^{i\theta_r}\frac{\sqrt{1-T} - e^{i\phi(t,E)}}{1-\sqrt{1-T}e^{i\phi(t,E)}}. \tag{A1}$$

Here θ_r is the phase of the reflection amplitude $r = \sqrt{1-T}e^{i\theta_r}$ of the QPC connecting the circular edge state in the cavity to the linear edge state. $\phi(t,E) = \theta_r + \phi(E) - 2\pi eU(t)/\Delta$ is the phase accumulated by an electron with energy E during one trip along the cavity, and Δ is the level spacing in the cavity. The phase $\phi(E) = k_F L + (E-\mu)L/(\hbar v_D)$ with k_F as a constant and v_D as a drift velocity can be taken to depend linearly on the energy. In the following, we consider the scattering amplitude for electrons with Fermi energy, $S^{\mathrm{SES}}(t) \equiv S^{\mathrm{SES}}(t,\mu)$. We are interested in the limit of a small transparency, $T \to 0$, when the width of the levels in the cavity is much smaller than the level spacing Δ. The amplitude U_1 of the oscillating potential is chosen in such a way that during a period only one level of the cavity crosses the Fermi level μ in the linear edge state. The time of crossing t_0 is defined by the condition $\phi(t_0,\mu) = 0 \, mod \, 2\pi$. By introducing the deviation of the phase from its resonance value, $\delta\phi(t) = \phi(t,\mu) - \phi(t_0,\mu)$, we obtain the scattering amplitude

$$S^{\mathrm{SES}}(t) = -e^{i\theta_r}\frac{T + 2i\delta\phi(t)}{T - 2i\delta\phi(t)} + O(T^2). \tag{A2}$$

We keep only terms to leading order in $T \ll 1$.

There are two time moments when resonance conditions occur (two times of crossing). The first crossing time is the instant when the level rises above the Fermi level, and the second crossing time is when the level sinks below the Fermi level. We denote these as times $t_0^{(-)}$ and $t_0^{(+)}$, respectively. At the time $t_0^{(-)}$, one electron is emitted by the cavity into the linear edge state, while at the time $t_0^{(+)}$, one electron enters the cavity, and a hole is emitted.

We suppose that the constant part of the potential U_0 accounts for a detuning of the nearest electron level E_n in the SES from the Fermi level. Then the resonance times can be found from the following equation:

$$E_n + eU\big(t_0^{(\mp)}\big) = \mu_0 \Rightarrow U_0 + U_1\cos\big(\Omega t_0^{(\mp)} + \varphi\big) = 0. \tag{A3}$$

For $|eU_0| < \Delta/2$ and $|eU_0| < |eU_1| < \Delta - |eU_0|$ we find

$$t_0^{(\mp)} = \mp t_0^{(0)} - \frac{\varphi}{\Omega}, \quad t_0^{(0)} = \frac{1}{\Omega}\arccos\left(-\frac{U_0}{U_1}\right). \tag{A4}$$

The deviation from the resonance time, $\delta t^{(\mp)} = t - t_0^{(\mp)}$, can be related to the deviation from the resonance phase, $\delta\phi^{(\mp)} = \mp M\Omega\delta t^{(\mp)}$, where $\mp M = d\phi/dt|_{t=t_0^{(\mp)}}/\Omega = \mp 2\pi|e|\Delta^{-1}\sqrt{U_1^2 - U_0^2}$. With these definitions we can rewrite Eq. (A2) as follows:

$$S^{\mathrm{SES}}(t) = e^{i\theta_r}\begin{cases} \frac{t-t_0^{(+)}-i\Gamma_0}{t-t_0^{(+)}+i\Gamma_0}, & |t-t_0^{(+)}| \lesssim \Gamma_0, \\ \frac{t-t_0^{(-)}+i\Gamma_0}{t-t_0^{(-)}-i\Gamma_0}, & |t-t_0^{(-)}| \lesssim \Gamma_0, \\ 1, & |t-t_0^{(\mp)}| \gg \Gamma_0. \end{cases} \tag{A5}$$

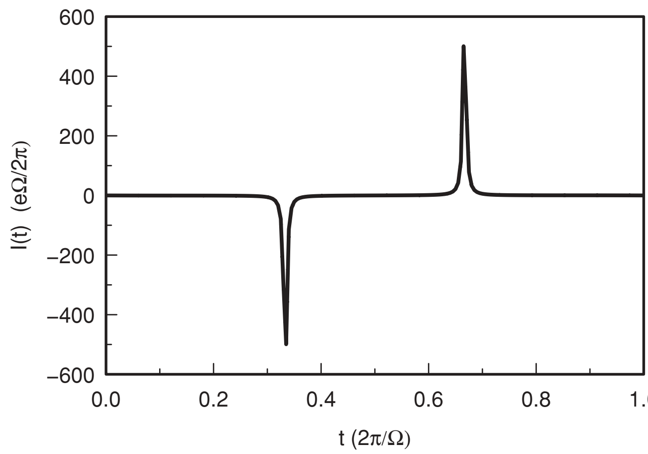

FIG. 6. The time-dependent current [Eq. (A8)] generated by the SES at zero temperature. The positive (negative) peak corresponds to emission of an electron (a hole). The parameters of the SES described by Eq. (A1) are as follows: $T = 0.1$, $U_0 = 0.25\Delta$, $U_1 = 0.5\Delta$, $\varphi = 0$.

Here Γ_0 is (half of) the time during which the level rises above or sinks below the Fermi level:

$$\Omega\Gamma_0 = \frac{T\Delta}{4\pi|e|\sqrt{U_1^2 - U_0^2}}. \tag{A6}$$

Equation (A5) assumes that the overlap between the resonances is small:

$$\big|t_0^{(+)} - t_0^{(-)}\big| \gg \Gamma_0. \tag{A7}$$

The basic equation for the time-dependent current is (see, e.g., Ref. 32)

$$I(t) = -\frac{e}{2\pi}\int dE\left(-\frac{\partial f_0}{\partial E}\right) S^{\mathrm{SES}}\frac{\partial (S^{\mathrm{SES}})^*}{\partial t}. \tag{A8}$$

By using Eq. (A5), we find the adiabatic current at zero temperature (for $0 < t < \mathcal{T}$):

$$I(t) = \frac{e}{\pi}\left[\frac{\Gamma_0}{\big(t-t_0^{(-)}\big)^2 + \Gamma_0^2} - \frac{\Gamma_0}{\big(t-t_0^{(+)}\big)^2 + \Gamma_0^2}\right]. \tag{A9}$$

In each time interval $2\pi/\Omega$, the current (shown in Fig. 6) consists of two pulses of Lorentzian shape with a half-width, Γ_0. The pulses correspond to the emission of an electron and a hole. Integrating over time, it is easy to check that the first pulse carries a charge e while the second pulse carries a charge $-e$.

APPENDIX B: CURRENT CORRELATION FUNCTION

A. General formalism

Let the scatterer be connected via one channel lead to reservoirs having different potentials,

$$V_\alpha(t) = V_\alpha + V_\alpha^{(\sim)}(t). \tag{B1}$$

Following the approach developed in Refs. 33 and 34, we include the potential $V_\alpha^{(\sim)}(t) = V_\alpha^{(\sim)}(t+\mathcal{T})$, $\mathcal{T} = 2\pi/\Omega$, oscillating with frequency Ω into the phase of the wave function for electrons injected into the circuit from reservoir α. The constant part of the potential changes the Fermi distribution function in contact α,

$$f_\alpha(E) = \frac{1}{1+\exp\frac{E-\mu_\alpha}{k_B T_\alpha}}, \quad \mu_\alpha = \mu_0 + eV_\alpha. \tag{B2}$$

We introduce the second quantization operator $\hat{a}'_\alpha(E)$ annihilating an electron in the state with energy E carrying a unit flux[35] in reservoir α. Then the corresponding distribution function is

$$\langle a'^{\dagger}_\alpha(E) a'_\alpha(E')\rangle = f_\alpha(E)\delta(E-E'). \tag{B3}$$

If the reservoir α is subject to a periodic in time potential $V^{(\sim)}_\alpha(t)$, then the wave function for particles described by the operators $\hat{a}'_\alpha$ is a Floquet-type function having sidebands with energies $E_n = E + n\hbar\Omega$, $n = 0,\pm1,\pm2,\ldots$. The amplitudes of the sidebands are

$$\Upsilon_{\alpha,n} = \int_0^{\mathcal{T}} \frac{dt}{\mathcal{T}} e^{in\Omega t}\Upsilon_\alpha(t),$$
$$\Upsilon_\alpha(t) = \exp\left[-i\frac{e}{\hbar}\int_{-\infty}^{t} dt'\, V^{(\sim)}_\alpha(t')\right]. \tag{B4}$$

We suppose that there is no oscillating potential in the leads connecting the reservoirs to the scatterer. Then the operator for particles in lead α is[34]

$$\hat{a}_\alpha(E) = \sum_{n=-\infty}^{\infty} \Upsilon_{\alpha,n}\hat{a}'_\alpha(E_{-n}). \tag{B5}$$

If the scatterer is driven periodically, then it is characterized by the Floquet scattering matrix $\hat{S}_F$.[36] We assume that the scatterer is driven with the same period $\mathcal{T}$ as the reservoirs. The element $S_{F,\alpha\beta}(E_n,E)$ is a current scattering amplitude[35] for an electron incoming from the lead β with energy E to be scattered with energy $E_n = E + n\hbar\Omega$ into the lead α. With these amplitudes we find the operators for scattered particles,[31]

$$\hat{b}_\alpha(E) = \sum_\beta \sum_{m=-\infty}^{\infty} S_{F,\alpha\beta}(E,E_m)\hat{a}_\beta(E_m)$$
$$= \sum_\beta \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} S_{F,\alpha\beta}(E,E_m)\Upsilon_{\beta,n}\hat{a}'_\beta(E_{m-n}). \tag{B6}$$

Now we calculate the symmetrized current correlation function in a frequency representation,

$$P_{\alpha\beta}(\omega_1,\omega_2) = \tfrac{1}{2}\langle \Delta\hat{I}_\alpha(\omega_1)\Delta\hat{I}_\beta(\omega_2) + \Delta\hat{I}_\beta(\omega_2)\Delta\hat{I}_\alpha(\omega_1)\rangle, \tag{B7}$$

where $\langle\cdots\rangle$ stands for quantum-statistical averaging over the (equilibrium) state of reservoirs, $\Delta\hat{I}_\alpha(\omega) = \hat{I}_\alpha(\omega) - \langle\hat{I}_\alpha(\omega)\rangle$, and $\hat{I}_\alpha(\omega)$ is the operator for the current in lead α,

$$\hat{I}_\alpha(\omega) = e\int_0^\infty dE\{\hat{b}^\dagger_\alpha(E)\hat{b}_\alpha(E+\hbar\omega) - \hat{a}^\dagger_\alpha(E)a_\alpha(E+\hbar\omega)\}. \tag{B8}$$

By using Eqs. (B3) and (B5)–(B8) we find

$$P_{\alpha\beta}(\omega_1,\omega_2) = \sum_{l=-\infty}^{\infty} 2\pi\delta(\omega_1+\omega_2-l\Omega)\mathcal{P}_{\alpha\beta,l}(\omega_1),$$

$$\begin{aligned}\mathcal{P}_{\alpha\beta,l}(\omega_1) = \frac{e^2}{h}\int dE\Bigg[&\delta_{\alpha\beta} f_{\alpha\alpha}(E,E+\hbar\omega_1) - f_{\alpha\alpha}(E,E+\hbar\omega_1)\sum_n\sum_{p,q} S_{F,\beta\alpha}(E_{l+n},E_p)\Upsilon_{\alpha,p}S^*_{F,\beta\alpha}(E_n+\hbar\omega_1,E_q+\hbar\omega_1)\Upsilon^*_{\alpha,q}\\ &- f_{\beta\beta}(E,E+\hbar\omega_2)\sum_n\sum_{p,q} S_{F,\alpha\beta}(E_{l+n},E_p)\Upsilon_{\beta,p}S^*_{F,\alpha\beta}(E_n+\hbar\omega_2,E_q+\hbar\omega_2)\Upsilon^*_{\alpha,q}\\ &+ \sum_{\gamma,\delta}\sum_{n.m.s}\sum_{p,q,p_1,q_1} f_{\gamma\delta}(E_n,E_m+\hbar\omega_1)S_{F,\beta\gamma}(E_{l+s},E_{n+q})\Upsilon_{\gamma,q}S^*_{F,\alpha\gamma}(E,E_{n+p})\Upsilon^*_{\gamma,p}\\ &\times S_{F,\alpha\delta}(E+\hbar\omega_1,E_{m+q_1}+\hbar\omega_1)\Upsilon_{\delta,q_1}S^*_{F,\beta\delta}(E_s+\hbar\omega_1,E_{m+p_1}+\hbar\omega_1)\Upsilon^*_{\delta,p_1}\Bigg].\end{aligned} \tag{B9}$$

Here

$$f_{\alpha\beta}(E_1,E_2) = \tfrac{1}{2}\{f_\alpha(E_1)[1-f_\beta(E_2)] + f_\beta(E_2)[1-f_\alpha(E_1)]\}. \tag{B10}$$

We are interested in the zero-frequency limit of the equation given above, when the noise can be conveniently represented as the sum of the thermal noise $\mathcal{P}^{(\mathrm{th})}_{\alpha\beta}$ (vanishing at $k_B T_\alpha = 0$, $\forall\alpha$) and the shot noise $\mathcal{P}^{(\mathrm{sh})}_{\alpha\beta}$ (vanishing at $\Omega = 0$ and $eV_\alpha = eV_0$, $\forall\alpha$).

B. Zero-frequency noise power

At $l = 0$ and $\omega_1 = \omega_2 = 0$, Eq. (B9) can be represented as follows:

$$\mathcal{P}_{\alpha\beta} = \frac{e^2}{h}\int dE\left\{\mathcal{P}^{(\mathrm{th})}_{\alpha\beta}(E) + \mathcal{P}^{(\mathrm{sh})}_{\alpha\beta}(E)\right\}, \tag{B11a}$$

$$\mathcal{P}^{(\mathrm{th})}_{\alpha\beta}(E) = \delta_{\alpha\beta}\left\{f_{\alpha\alpha}(E,E) + \sum_\gamma F_{\alpha\gamma}(E)\right\} - F_{\alpha\beta}(E) - F_{\beta\alpha}(E),$$

$$\text{with} \quad F_{\alpha\gamma}(E) = f_{\gamma\gamma}(E,E)\sum_{n,p,q} S_{F,\alpha\gamma}(E_n,E_q)\Upsilon_{\gamma,q} \times S^*_{F,\alpha\gamma}(E_n,E_p)\Upsilon^*_{\gamma,p}, \tag{B11b}$$

$$\mathcal{P}^{(\mathrm{sh})}_{\alpha\beta}(E) = \frac{1}{2}\sum_{\gamma,\delta}\sum_{n.m.s}\sum_{p,q,p_1,q_1}\{f_\gamma(E_n) - f_\delta(E_m)\}^2 \times S_{F,\beta\gamma}(E_s,E_{n+q})\Upsilon_{\gamma,q}\,S^*_{F,\alpha\gamma}(E,E_{n+p})\Upsilon^*_{\gamma,p} \times S_{F,\alpha\delta}(E,E_{m+q_1})\Upsilon_{\delta,q_1}\,S^*_{F,\beta\delta}(E_s,E_{m+p_1})\Upsilon^*_{\delta,p_1}. \tag{B11c}$$

Now we show how Eq. (B9) was obtained.

C. Derivation of the current correlation function

To make the calculations more transparent it is convenient to represent the current as a sum, $\hat{I}_\alpha(\omega) = \hat{I}^{(\mathrm{out})}_\alpha(\omega) + \hat{I}^{(\mathrm{in})}_\alpha(\omega)$, of a current $I^{(\mathrm{out})}_\alpha$ carried by the scattered particles and a current $I^{(\mathrm{in})}_\alpha$ carried by the incident particles:

$$\hat{I}^{(\mathrm{out})}_\alpha(\omega) = e\int_0^\infty dE\,\hat{b}^\dagger_\alpha(E)\hat{b}_\alpha(E+\hbar\omega), \qquad \hat{I}^{(\mathrm{in})}_\alpha(\omega) = -\,e\int_0^\infty dE\,\hat{a}^\dagger_\alpha(E)\hat{a}_\alpha(E+\hbar\omega). \tag{B12}$$

Then $P_{\alpha\beta}(\omega_1,\omega_2)$ [Eq. (B7)] can be represented as the sum of four terms,

$$P_{\alpha\beta}(\omega_1,\omega_2) = \sum_{i,j=\mathrm{in,out}} P^{(i,j)}_{\alpha\beta}(\omega_1,\omega_2), \qquad P^{(i,j)}_{\alpha\beta} = \tfrac{1}{2}\big\langle \Delta\hat{I}^{(i)}_\alpha(\omega_1)\Delta\hat{I}^{(j)}_\beta(\omega_2) + \Delta\hat{I}^{(j)}_\beta(\omega_2)\Delta\hat{I}^{(i)}_\alpha(\omega_1)\big\rangle. \tag{B13}$$

We evaluate each of these four contributions separately.

1. Correlator for incoming currents

The first term in Eq. (B13) reads

$$P^{(\mathrm{in,in})}_{\alpha\beta}(\omega_1,\omega_2) = e^2\iint_0^\infty dE_1\,dE_2\,\frac{J^{(\mathrm{in,in})}_{\alpha\beta} + J^{(\mathrm{in,in})}_{\beta\alpha}}{2}, \tag{B14}$$

where

$$J^{(\mathrm{in,in})}_{\alpha\beta} = \langle\{\hat{a}^\dagger_\alpha(E_1)\hat{a}_\alpha(E_1+\hbar\omega_1) - \langle\hat{a}^\dagger_\alpha(E_1)\hat{a}_\alpha(E_1+\hbar\omega_1)\rangle\} \times\{\hat{a}^\dagger_\beta(E_2)\hat{a}_\beta(E_2+\hbar\omega_2) - \langle\hat{a}^\dagger_\beta(E_2)\hat{a}_\beta(E_2+\hbar\omega_2)\rangle\}\rangle.$$

In the correlation $J^{(\mathrm{in,in})}_{\beta\alpha}$ with the indices interchanged, the order of operators in each of the products contributing to $J^{(\mathrm{in,in})}_{\beta\alpha}$ is interchanged. By using Wick's theorem, we represent the average of the product of four operators via the average of pair products and find

$$J^{(\mathrm{in,in})}_{\alpha\beta} = \Pi^{(\mathrm{in,in})}_{\alpha\beta}\,\Xi^{(\mathrm{in,in})}_{\alpha\beta}, \quad \Pi^{(\mathrm{in,in})}_{\alpha\beta} = \langle\hat{a}^\dagger_\alpha(E_1)\,\hat{a}_\beta(E_2+\hbar\omega_2)\rangle, \quad \Xi^{(\mathrm{in,in})}_{\alpha\beta} = \langle\hat{a}_\alpha(E_1+\hbar\omega_1)\,\hat{a}^\dagger_\beta(E_2)\rangle. \tag{B15}$$

Then by using Eq. (B5) we obtain, after straightforward but slightly lengthy calculations,

$$P^{(\mathrm{in,in})}_{\alpha\beta}(\omega_1,\omega_2) = 2\pi\delta(\omega_1+\omega_2)\,\mathcal{P}^{(\mathrm{in,in})}_{\alpha\beta}(\omega_1), \qquad \mathcal{P}^{(\mathrm{in,in})}_{\alpha\beta}(\omega_1) = \delta_{\alpha\beta}\frac{e^2}{h}\int dE_1\,f_{\alpha\alpha}(E_1,E_1+\hbar\omega_1). \tag{B16}$$

This is exactly what could be expected for equilibrium electrons. Therefore, uniform oscillating potentials at the reservoirs in themselves do not produce additional noise.

2. Correlator between incoming and outgoing currents

The next term in Eq. (B13) is

$$P^{(\mathrm{in,out})}_{\alpha\beta} = -e^2\iint_0^\infty dE_1\,dE_2\,\frac{J^{(\mathrm{in,out})}_{\alpha\beta} + J^{(\mathrm{out,in})}_{\beta\alpha}}{2}, \tag{B17}$$

where

$$J^{(\mathrm{in,out})}_{\alpha\beta} = \Pi^{(\mathrm{in,out})}_{\alpha\beta}\,\Xi^{(\mathrm{in,out})}_{\alpha\beta}, \quad \Pi^{(\mathrm{in,out})}_{\alpha\beta} = \langle\hat{a}^\dagger_\alpha(E_1)\hat{b}_\beta(E_2+\hbar\omega_2)\rangle, \quad \Xi^{(\mathrm{in,out})}_{\alpha\beta} = \langle\hat{a}_\alpha(E_1+\hbar\omega_1)\hat{b}^\dagger_\beta(E_2)\rangle. \tag{B18}$$

In the correlation $J^{(\mathrm{out,in})}_{\beta\alpha}$ the order of operators in the averages of pairs is interchanged. By using Eqs. (B5) and (B6) we find

$$\Pi^{(\mathrm{in,out})}_{\alpha\beta} = \sum_{n,m,p}\Upsilon^*_{\alpha,n}S_{F,\beta\alpha}(E_2+\hbar\omega_2,E_{2,m}+\hbar\omega_2) \times\Upsilon_{\alpha,p}f_\alpha(E_{1,-n})\delta(E_{1,-n}-E_{2,m-p}-\hbar\omega_2),$$

$$\Xi^{(\mathrm{in,out})}_{\alpha\beta} = \sum_{n_1,m_1,p_1}\Upsilon_{\alpha,n_1}\Upsilon^*_{\alpha,p_1}[1-f_\alpha(E_{1,-n_1}+\hbar\omega_1)] \times S^*_{F,\beta\alpha}(E_2,E_{2,m_1})\delta(E_{1,-n_1}-E_{2,m_1-p_1}+\hbar\omega_1).$$

Next we integrate over energy E_2 by using the Dirac delta function in $\Pi^{(\mathrm{in,out})}_{\alpha\beta}$. In the reminder, we use $E_{2,m-p} = E_{1,-n} - \hbar\omega_2$ and find

$$P^{(\mathrm{in,out})}_{\alpha\beta} = -\frac{e^2}{\hbar}\int dE_1\sum_{n,m,p}\sum_{n_1,m_1,p_1}f_{\alpha\alpha}(E_{1,-n},E_{1,-n_1}+\hbar\omega_1) \times\Upsilon^*_{\alpha,n}\Upsilon_{\alpha,p}\Upsilon_{\alpha,n_1}\Upsilon^*_{\alpha,p_1}\delta(\omega_1+\omega_2-\Omega[p-n -m-p_1+n_1+m_1])S_{F,\beta\alpha}(E_{1,p-n-m},E_{1,p-n}) \times S^*_{F,\beta\alpha}(E_{1,p_1-n_1-m_1}+\hbar\omega_1,E_{1,p_1-n_1}+\hbar\omega_1).$$

We shift (under the integral over E_1) $E_1 \to E_1 + n\hbar\Omega$. Then we introduce $w = n - n_1$ instead of n_1. The sum over w gives us δ_{w0}. Then we introduce $l = p - m - p_1 + m_1$ instead of m and $r = p_1 - m_1$ instead of m_1. Finally we get

$$P^{(\mathrm{in,out})}_{\alpha\beta}(\omega_1,\omega_2) = \sum_{l=-\infty}^{\infty}2\pi\delta(\omega_1+\omega_2-l\Omega)\,\mathcal{P}^{(\mathrm{in,out})}_{\alpha\beta,l}, \tag{B19}$$

$$\mathcal{P}^{(\mathrm{in,out})}_{\alpha\beta,l}(\omega_1) = -\frac{e^2}{h}\int dE_1\,f_{\alpha\alpha}(E_1,E_1+\hbar\omega_1) \times\sum_{r,p,p_1}S_{F,\beta\alpha}(E_{1,l+r},E_{1,p})\Upsilon_{\alpha,p} \times S^*_{F,\beta\alpha}(E_{1,r}+\hbar\omega_1,E_{1,p_1}+\hbar\omega_1)\Upsilon^*_{\alpha,p_1}.$$

With similar steps we find that $P_{\alpha\beta}^{(\text{out,in})}$ can be obtained from $P_{\alpha\beta}^{(\text{in,out})}$ if one replaces $\alpha \leftrightarrow \beta$, $E_1 \leftrightarrow E_2$, and $\omega_1 \leftrightarrow \omega_2$. Therefore, from Eq. (B19), we immediately obtain

$$P_{\alpha\beta}^{(\text{out,in})}(\omega_1,\omega_2) = \sum_{l=-\infty}^{\infty} 2\pi\,\delta(\omega_1+\omega_2-l\Omega)\,\mathcal{P}_{\alpha\beta,l}^{(\text{out,in})}, \tag{B20}$$

$$\mathcal{P}_{\alpha\beta,l}^{(\text{out,in})}(\omega_2) = -\frac{e^2}{h}\int dE_2 f_{\beta\beta}(E_2,E_2+\hbar\omega_2) \times \sum_{r,p,p_1} S_{F,\alpha\beta}(E_{2,l+r},E_{2,p})\Upsilon_{\beta,p} \times S_{F,\alpha\beta}^{*}(E_{2,r}+\hbar\omega_2,E_{2,p_1}+\hbar\omega_2)\Upsilon_{\beta,p_1}^{*}.$$

Subsequently, to compare Eqs. (B19) and (B21) with Eq. (B9) we need to additionally redefine $r \to n$ and $p_1 \to q$.

3. Correlator between outgoing currents

The last term in Eq. (B13) reads

$$P_{\alpha\beta}^{(\text{out,out})} = -e^2\iint_0^{\infty} dE_1\,dE_2\,\frac{J_{\alpha\beta}^{(\text{out,out})}+J_{\beta\alpha}^{(\text{out,out})}}{2}, \tag{B21}$$

where

$$J_{\alpha\beta}^{(\text{out,out})} = \Pi_{\alpha\beta}^{(\text{out,out})}\Xi_{\alpha\beta}^{(\text{out,out})},$$
$$\Pi_{\alpha\beta}^{(\text{out,out})} = \langle \hat{b}_\alpha^\dagger(E_1)\hat{b}_\beta(E_2+\hbar\omega_2)\rangle,$$
$$\Xi_{\alpha\beta}^{(\text{out,out})} = \langle \hat{b}_\alpha(E_1+\hbar\omega_1)\hat{b}_\beta^\dagger(E_2)\rangle.$$

In the correlation $J_{\beta\alpha}^{(\text{out,out})}$ the order of operators in the pair averages is interchanged.

By using Eqs. (B5) and (B6) we calculate

$$\Pi_{\alpha\beta}^{(\text{out,out})} = \sum_\gamma \sum_{n,m,p,q} \delta(E_{1,n-p}-E_{2,m-q}-\hbar\omega_2) \times f_\gamma(E_{1,n-p})S_{F,\alpha\gamma}^{*}(E_1,E_{1,n}) \times \Upsilon_{\gamma,p}^{*}S_{F,\beta\gamma}(E_2+\hbar\omega_2,E_{2,m}+\hbar\omega_2)\Upsilon_{\gamma,q},$$

$$\Xi_{\alpha\beta}^{(\text{out,out})} = \sum_{\gamma_1}\sum_{n_1,m_1,p_1,q_1} \delta(E_{1,n_1-p_1}+\hbar\omega_1-E_{2,m_1-q_1}) \times[1-f_{\gamma_1}(E_{1,n_1-p_1}+\hbar\omega_1)]S_{F,\beta\gamma_1}^{*}(E_2,E_{2,m_1}) \times\Upsilon_{\gamma_1,q_1}^{*}S_{F,\alpha\gamma_1}(E_1+\hbar\omega_1,E_{1,n_1}+\hbar\omega_1)\Upsilon_{\gamma_1,p_1}.$$

Then we integrate over energy E_2 by using the Dirac delta function in $\Pi_{\alpha\beta}^{(\text{out,out})}$. In the rest, we use $E_2 = E_{1,n-p-m+q} - \hbar\omega_2 = E_{1,n_1+q_1-p_1-m_1}+\hbar\omega_1$ and find

$$P_{\alpha\beta}^{(\text{out,out})} = \frac{e^2}{\hbar}\int dE_1 \sum_{\gamma,\gamma_1}\sum_{n,m,p,q}\sum_{n_1,m_1,p_1,q_1} \times f_{\gamma\gamma_1}(E_{1,n-p},E_{1,n_1-p_1}+\hbar\omega_1)\delta(\omega_1+\omega_2-\Omega \times[n+q-p-m-n_1-q_1+p_1+m_1]) \times S_{F,\alpha\gamma}^{*}(E_1,E_{1,n})\Upsilon_{\gamma,p}^{*}S_{F,\beta\gamma}(E_{1,n-p-m+q},E_{1,n-p+q}) \times S_{F,\beta\gamma_1}^{*}(E_{1,n_1+q_1-p_1-m_1}+\hbar\omega_1,E_{1,n_1+q_1-p_1}+\hbar\omega_1) \times \Upsilon_{\gamma,q}\Upsilon_{\gamma_1,q_1}^{*}S_{F,\alpha\gamma_1}(E_1+\hbar\omega_1,E_{1,n_1}+\hbar\omega_1)\Upsilon_{\gamma_1,p_1}.$$

To simplify, we introduce $t = n-p$ instead of n, $w = n_1 - p_1$ instead of n_1, $l = n+q-p-m-n_1-q_1+p_1+m_1$ instead of m, and $s = n_1+q_1-p_1-m_1$ instead of m_1. Then we get

$$P_{\alpha\beta}^{(\text{out,out})}(\omega_1,\omega_2) = \sum_{l=-\infty}^{\infty} 2\pi\,\delta(\omega_1+\omega_2-l\Omega)\,\mathcal{P}_{\alpha\beta,l}^{(\text{out,out})},$$

$$\mathcal{P}_{\alpha\beta,l}^{(\text{out,out})}(\omega_1) = \frac{e^2}{h}\int dE_1 \sum_{\gamma,\gamma_1}\sum_{s,t,w}\sum_{p,q,p_1,q_1} f_{\gamma\gamma_1}(E_{1,t},E_{1,w}+\hbar\omega_1) \times S_{F,\alpha\gamma}^{*}(E_1,E_{1,t+p})\Upsilon_{\gamma,p}^{*}S_{F,\beta\gamma}(E_{1,l+s},E_{1,t+q}) \times \Upsilon_{\gamma,q}S_{F,\beta\gamma_1}^{*}(E_{1,s}+\hbar\omega_1,E_{1,w+q_1}+\hbar\omega_1)\Upsilon_{\gamma_1,q_1}^{*} \times S_{F,\alpha\gamma_1}(E_1+\hbar\omega_1,E_{1,w+p_1}+\hbar\omega_1)\Upsilon_{\gamma_1,p_1}. \tag{B22}$$

Subsequently, to compare with Eq. (B9), we need to additionally redefine $t \to n$, $w \to m$, $p_1 \leftrightarrow q_1$, and $\gamma_1 \to \delta$.

Collecting together Eqs. (B16), (B19), (B21), and (B22), we arrive at Eq. (B9).

D. Adiabatic regime

In the adiabatic regime the Floquet scattering matrix elements to leading order in $\Omega \to 0$ are the Fourier coefficients for the frozen scattering matrix $\hat{S}(t,E)$,[36]

$$S_{F,\alpha\beta}(E_n,E_m) = S_{\alpha\beta,n-m}(E). \tag{B23}$$

Within this approximation we find, from Eqs. (B11b) and (B11c),

$$\mathcal{P}_{\alpha\beta}^{(\text{th,ad})}(E) = -f_{\alpha\alpha}(E,E)\overline{|S_{\beta\alpha}(E)|^2} - f_{\beta\beta}(E,E)\overline{|S_{\alpha\beta}(E)|^2} + \delta_{\alpha\beta}\left\{f_{\alpha\alpha}(E,E)+\sum_\gamma f_{\gamma\gamma}(E,E)\overline{|S_{\alpha\gamma}(E)|^2}\right\}, \tag{B24a}$$

$$\mathcal{P}_{\alpha\beta}^{(\text{sh,ad})}(E) = \frac{1}{2}\sum_{\gamma,\delta}\sum_{q=-\infty}^{\infty}\{f_\gamma(E_q)-f_\delta(E)\}^2\Phi_{\alpha,q}^{(\gamma\delta)}\Phi_{\beta,q}^{(\gamma\delta)*}, \tag{B24b}$$

where $\Phi_{\alpha,q}$ is a Fourier transform of

$$\Phi_\alpha^{(\gamma\delta)}(t) = S_{\alpha\gamma}^{*}(t,E)\Upsilon_\gamma^{*}(t)S_{\alpha\delta}(t,E)\Upsilon_\delta(t). \tag{B25}$$

Here the overbar stands for a time average, $\overline{X} = \int_0^{\mathcal{T}} dt X(t)/\mathcal{T}$. Calculating the shot noise, we made a shift of $E \to E - m\hbar\Omega$ and introduced $q = n-m$ instead of m.

One can see that the potentials oscillating at reservoirs have no effect on the thermal noise. Their effect on the shot noise in the adiabatic regime can be taken into account formally by changing the phase of the scattering elements $S_{\varphi\rho}(t,E)$ by the factor $\Upsilon_\rho(t)$ [Eq. (B4)].

E. Zero-temperature adiabatic regime

At zero temperatures there is no thermal noise. By calculating the shot noise, we take into account that, in the adiabatic regime, the frequency Ω is so small that we can neglect the energy dependence of the scattering matrix elements over the interval of order several $\hbar\Omega$.[31,36] In addition, we assume also that all the potential differences $V_{\alpha\beta} = V_\alpha - V_\beta$ are small as compared to the significant energy scales of the scattering matrix. Then, with Eq. (B24b), the integral over energy in Eq. (B11a) becomes trivial and we find

$$\mathcal{P}_{\alpha\beta}^{(\mathrm{sh,ad})} = \frac{e^2\Omega}{4\pi}\sum_{\gamma,\delta}\sum_{q=-\infty}^{\infty}\left|\frac{eV_{\gamma\delta}}{\hbar\Omega} - q\right|\Phi_{\alpha,q}^{(\gamma\delta)}\Phi_{\beta,q}^{(\gamma\delta)*}. \tag{B26}$$

Note the dc bias and ac bias enter this equation in a strongly nonequivalent way.

APPENDIX C: PROBABILITY DESCRIPTION OF THE CURRENT CROSS CORRELATOR FOR A CIRCUIT WITH SESs

The SES emits electrons and holes that are uncorrelated. Hence electrons (e) and holes (h) contribute to noise independently, $\mathcal{P}_{12} = \mathcal{P}_{12}^{(e)} + \mathcal{P}_{12}^{(h)}$. In the adiabatic regime we can neglect the energy dependence of the scattering matrix. Therefore, electrons and holes contribute to the noise equally, $\mathcal{P}_{12}^{(e)} = \mathcal{P}_{12}^{(h)} = 0.5\mathcal{P}_{12}$. Below we restrict ourselves to the electron contribution. We assume that the circuit has two inputs and two outputs, 1 and 2. In each input there is a SES emitting one electron per period.

A. Classical versus quantum regimes

It was noticed in Ref. 3 that the cross correlator $\mathcal{P}_{12}^{(e)}$ is related to the electron number correlator $\delta\mathcal{N}_{12}$ as follows:

$$\mathcal{P}_{12}^{(e)} = \frac{e^2\Omega}{2\pi}\delta\mathcal{N}_{12}, \tag{C1}$$

where

$$\delta\mathcal{N}_{12} = \mathcal{N}_{12} - \mathcal{N}_1\mathcal{N}_2. \tag{C2}$$

Here $\mathcal{N}_{12}$ is the probability to find one electron in output 1 and one electron in output 2 *during* the period $\mathcal{T}$, whereas $\mathcal{N}_j$ is the probability to find an electron in output $j = 1,2$ during the same period.

To determine the probabilities entering Eq. (C2) we need to consider a specific circuit. We consider the one given in Fig. 4. For this circuit, the quantum-mechanical amplitudes $\mathcal{A}_{ij}$ for an electron emitted by the source $j = L,R$ to arrive at the output $i = 1,2$ are the following:

$$\begin{aligned} \mathcal{A}_{1L} &= e^{ik_F L_{1L}} t_L r_C, \quad \mathcal{A}_{1R} = e^{ik_F L_{1R}} t_R t_C, \\ \mathcal{A}_{2L} &= e^{ik_F L_{2L}} t_L t_C, \quad \mathcal{A}_{2R} = e^{ik_F L_{2R}} t_R r_C. \end{aligned} \tag{C3}$$

With these amplitudes we find single-particle probabilities,

$$\begin{aligned} \mathcal{N}_1 &= |\mathcal{A}_{1L}|^2 + |\mathcal{A}_{1R}|^2 = T_L + T_C\,(T_R - T_L), \\ \mathcal{N}_2 &= |\mathcal{A}_{2L}|^2 + |\mathcal{A}_{2R}|^2 = T_L - T_C\,(T_R - T_L). \end{aligned} \tag{C4}$$

The calculation of the two-particle probability $\mathcal{N}_{12}$ depends crucially on whether electrons collide at the central QPC or not.

If electrons pass the QPC C at different times, $\Delta t^{(-)} \gg \Gamma_L,\Gamma_R$, then there are two independent processes contributing to $\mathcal{N}_{12}$ with amplitudes $\mathcal{A}_{\mathrm{I}}^{(2)} = \mathcal{A}_{1L}\mathcal{A}_{2R}$ and $\mathcal{A}_{\mathrm{II}}^{(2)} = \mathcal{A}_{1R}\mathcal{A}_{2L}$. Because the two-particle amplitudes factorize into the product of single-particle amplitudes, we term this the *classical* regime. With these amplitudes we find

$$\mathcal{N}_{12} = \left|\mathcal{A}_{\mathrm{I}}^{(2)}\right|^2 + \left|\mathcal{A}_{\mathrm{II}}^{(2)}\right|^2 = T_L T_R\left(R_C^2 + T_C^2\right). \tag{C5}$$

By using Eqs. (C4) and (C5) in Eq. (C2) we find the cross correlator $\mathcal{P}_{12}^{(e)}$ [Eq. (C1)] to be the same as the one given in Eq. (12) (times 0.5 to account for the electron contribution).

In contrast, if electrons can collide at the QPC C, $\Delta t^{(-)} = 0$, then the two-particle amplitude is given by the Slater determinant,

$$\mathcal{A}^{(2)} = \det\begin{vmatrix}\mathcal{A}_{1L} & \mathcal{A}_{1R}\\ \mathcal{A}_{2L} & \mathcal{A}_{2R}\end{vmatrix}. \tag{C6}$$

This is why we call this regime *quantum*. Then the two-particle probability reads

$$\mathcal{N}_{12} = |\mathcal{A}^{(2)}|^2 = T_L T_R. \tag{C7}$$

Note that this equation is independent of the parameters of the central QPC, which can be used as an indication of a quantum regime. We emphasize that in the quantum regime the two-particle probability becomes the Glauber joint detection probability,[37] because electrons after collision of the QPC C arrive at the outputs 1 and 2 simultaneously (disregarding a possible difference in arrival times owing to the different distances). With Eqs. (C7), (C4), (C2), and (C1), we recover the result given in Eq. (13).

B. Positive two-particle correlations in quantum regime

Let us show that, in the quantum regime, colliding electrons are positively correlated. To this end, we represent the single-particle probabilities as the sum of contributions owing to each of sources, $\mathcal{N}_i = \mathcal{N}_i^{(L)} + \mathcal{N}_i^{(R)}$ with $\mathcal{N}_i^{(j)} = |\mathcal{A}_{ij}|^2$, $i = 1,2$, $j = L,R$. Then we split the particle number correlator $\delta\mathcal{N}_{12}$ [Eq. (C2)] into the sum of three contributions,

$$\delta\mathcal{N}_{12} = \delta\mathcal{N}_{12}^{(LL)} + \delta\mathcal{N}_{12}^{(RR)} + \delta\mathcal{N}_{12}^{(\widehat{LR})}. \tag{C8}$$

Here the first two terms are contributions generated by either source alone, $\delta\mathcal{N}_{12}^{(jj)} = -\mathcal{N}_1^{(j)}\mathcal{N}_2^{(j)}$, $j = L,R$. Because the source emits single particles, this contribution to the cross correlator $\delta\mathcal{N}_{12}$ is definitely negative. The third contribution results from a joint action of both sources,

$$\delta\mathcal{N}_{12}^{(\widehat{LR})} = \mathcal{N}_{12} - \mathcal{N}_1^{(L)}\mathcal{N}_2^{(R)} - \mathcal{N}_1^{(R)}\mathcal{N}_2^{(L)}. \tag{C9}$$

In the classical regime we use Eq. (C5) and find $\delta\mathcal{N}_{12}^{(\widehat{LR})} = 0$, i.e., the particles emitted by different sources remain uncorrelated. In contrast, in the quantum regime, by using Eq. (C7), we get

$$\delta\mathcal{N}_{12}^{(\widehat{LR})} = 2T_L T_R R_C T_C. \tag{C10}$$

Therefore, in this regime the particles emitted by two sources and colliding at the central QPC C (see Fig. 4) become positively correlated. We stress that the total overall correlation $\mathcal{N}_{12}$ remains negative.

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