

# Direct current driven by ac electric field in quantum wells

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It is shown that the excitation of charge carriers by ac electric field with zero average driving leads to a direct electric current in quantum well structures. The current emerges for both linear and circular polarization of the ac electric field and depends on the field polarization and frequency. We present a microscopic model and an analytical theory of such a nonlinear electron transport in quantum wells with structure inversion asymmetry. In such systems, the dc current is induced by ac electric field that has both the in-plane and out-of-plane components. The ac field polarized in the interface plane gives rise to a direct current if the quantum well is subjected to an in-plane static magnetic field.

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## I. INTRODUCTION

The excitation of charge carriers by ac electric field in noncentrosymmetric semiconductor structures may lead to a direct electric current even in the absence of dc driving. In the high-frequency spectral range, when mechanisms of the current formation involve quantum optical transitions, such effects are usually referred to as photogalvanic effects.<sup>1,2</sup> At present, they are intensively studied in various low-dimensional systems and provide the insight into the band-structure details as well as the kinetics of photoexcited carriers.<sup>3–8</sup> In the classical frequency range, i.e.,  $\omega \ll \bar{\epsilon}/\hbar$ , where  $\omega$  is the field frequency and  $\bar{\epsilon}$  is the mean kinetic energy of carriers, the photogalvanic effects can be treated fruitfully as nonlinear electron transport and included in the more general class of quantum ratchets (for a review, see Ref. 9). The ratchet transport of charge carriers induced by the ac electric field was studied both theoretically and experimentally for semiconductor structures with artificially fabricated asymmetric scatterers.<sup>10,11</sup> Recently, it has also been addressed theoretically for bulk wurtzite crystals and low-symmetry quantum wells (QWs) based on zinc-blende-lattice compounds.<sup>12,13</sup> However, the proposed mechanisms of the current formation require the multiband mixing of states and, therefore, vanish in the effective-mass approximation.

Here, we show that in conventional QWs, where the space inversion is lifted by structure asymmetry, the ratchet transport of free carriers emerges in the simple one-band model of size-quantized states. We develop a microscopic model and an analytical theory of the dc current generation by the ac electric field, which are valid for the classical range of the field frequency. It is shown that the electric current emerges for both linear and circular polarization of the ac field and its magnitude depends on the radiation frequency. We also study the effect of an in-plane static magnetic field on the electron transport and show that the magnetic field gives rise to additional contributions to the dc current, which have different polarization dependences. For simplicity, we neglect spin splitting of the conduction band and focus on the orbital mechanisms of the current formation.

Phenomenologically, the density of direct current  $\mathbf{j}$  induced by the ac electric field is described by

$$\mathbf{j}_\alpha = \sum_{\beta\gamma} \chi_{\alpha\beta\gamma} E_\beta E_\gamma^* + \sum_{\beta\gamma\delta} \phi_{\alpha\beta\gamma\delta} B_\beta E_\gamma E_\delta^*, \quad (1)$$

where  $\mathbf{E}$  is the complex amplitude of the electric field ,

$$\mathbf{E}(t) = \mathbf{E} \exp(-i\omega t) + \mathbf{E}^* \exp(+i\omega t), \quad (2)$$

assumed to be homogeneous, the indices  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  enumerate the Cartesian coordinates, and components of the tensors  $\chi$  and  $\phi$  satisfy the relations  $\chi_{\alpha\beta\gamma} = \chi_{\alpha\gamma\beta}^*$  and  $\phi_{\alpha\beta\gamma\delta} = \phi_{\alpha\delta\beta\gamma}^*$ , respectively, which follow from the reality of the current density  $\mathbf{j}$ . The third-rank tensor  $\chi$  describes the photogalvanic effect (or high-frequency nonlinear conductivity). The model and microscopic theory of this effect are presented in Sec. II. The fourth-rank tensor  $\phi$  is responsible for additional current contributions emerging in the presence of an external static magnetic field  $\mathbf{B}$ ; they are addressed in Sec. III.

## II. HIGH-FREQUENCY NONLINEAR CONDUCTIVITY

We consider a semiconductor QW with structure inversion asymmetry and assume that the well is isotropic in the interface plane ( $C_{\infty v}$  point-group symmetry). A straightforward symmetry analysis yields that the in-plane dc current in such structures can be induced only by the ac field, which has both the in-plane  $\mathbf{E}_\parallel = (E_x, E_y)$  and out-of-plane  $E_z$  components. The tensor  $\chi$  has nonzero components  $\chi_{xxz} = \chi_{yyz} = \chi_{xzx}^* = \chi_{yzy}^*$ , and the current density given by the first term in the right-hand side of Eq. (1) can be rewritten in the form

$$\mathbf{j} = L (\mathbf{E}_\parallel E_z^* + E_z \mathbf{E}_\parallel^*) + C i (\mathbf{E}_\parallel E_z^* - E_z \mathbf{E}_\parallel^*). \quad (3)$$

Here, the phenomenological parameter  $L = \text{Re } \chi_{xxz}$  describes the electric current that is induced by a linearly polarized ac field and is insensitive to the sign of radiation helicity for elliptical polarization. In contrast,  $C = \text{Im } \chi_{xxz}$  stands for the radiation-helicity-dependent electric current vanishing for the linearly polarized radiation. This is owing to the fact that  $i(E_\alpha E_\beta^* - E_\beta E_\alpha^*)$  ( $\alpha \neq \beta$ ) is nothing but a component of the pseudovector  $i[\mathbf{E} \times \mathbf{E}^*] = |\mathbf{E}|^2 (\mathbf{q}/q) P_{\text{circ}}$ , where  $\mathbf{q}$  and  $P_{\text{circ}}$  are the wave vector and the circular polarization degree of the electromagnetic wave.

The microscopic mechanisms of the current generation are illustrated in Figs. 1(a) and 1(b) for the linearly and circularly polarized ac field, respectively. We assume that the electric field  $\mathbf{E}(t)$  is polarized in the  $(x, z)$  plane and the electron mobility is limited by electron scattering from static impurities. The structure inversion asymmetry is modeled here by placing the  $\delta$  layer of impurities (dotted line) closer to the lower

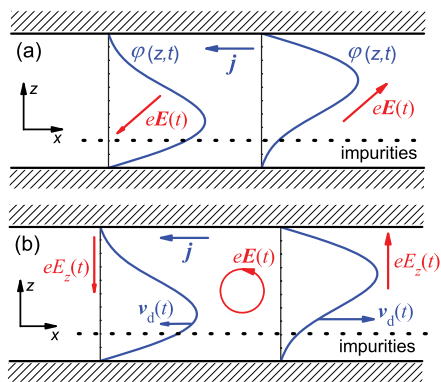


FIG. 1. (Color online) Microscopic model of dc current  $j$  generation by (a) linearly polarized and (b) circularly polarized ac electric field  $\mathbf{E}(t)$  in a QW.

interface rather than exactly in the QW center. However, the microscopic model presented below is also valid for QWs where nonequivalence of the  $z$  and  $-z$  directions is achieved by the asymmetry of confinement potential.

Figure 1(a) sketches the mechanism of dc current generation by the linearly polarized ac field, i.e., when the in-plane  $E_x(t)$  and out-of-plane  $E_z(t)$  components are cophased. The in-plane oscillating field  $E_x(t)$  causes an alternating current of electrons along the  $x$  axis. The time-average value of the force  $eE_x(t)$  acting upon carriers is zero, therefore, dc current driven solely by the in-plane electric field would vanish. However, the electric field  $\mathbf{E}(t)$  has also the out-of-plane component  $E_z(t)$ , which oscillates at the same frequency and acts upon the charge carriers as well. The force  $eE_z(t)$  pushes the carriers to the upper or lower interface depending on the force direction and thereby changes the electron wave function along the QW normal  $\varphi(z,t)$  [see Fig. 1(a)]. Such time dependence of the envelope function  $\varphi(z,t)$  results, in turn, in the modulation of the electron mobility  $\mu(t)$  at the frequency  $\omega$  because the impurities, which determine the mobility, are shifted off the QW center in our model. Therefore, when the carriers are driven in one direction in the QW plane their mobility and, hence, the drift velocity  $v_d$  are higher than those a half period later when the carriers flow in the opposite direction. Such an asymmetry in the drift of charge carriers along the  $x$  and  $-x$  directions implies a nonvanishing dc electric current.

For a circularly polarized ac electric field [Fig. 1(b)], the in-plane and out-of-plane components of  $\mathbf{E}(t)$  are phase shifted by  $\pm\pi/2$ : The component  $E_z(t)$  reaches maximum when  $E_x(t)$  is zero and vice versa. Now, the time-average product  $E_x(t)E_z(t)$  vanishes and no dc current emerges in the static limit  $\omega \rightarrow 0$ . The time-average flow of carriers along the  $x$  axis is obtained only at finite frequency if one takes into account the retardation of the drift velocity  $v_d(t)$  with respect to the in-plane field  $E_x(t)$ . Indeed, as is well known from the classical Drude theory of high-frequency conductivity,  $v_d(t)$  does not follow  $E_{\parallel}(t)$  exactly but is behind the field with the retardation phase shift of  $\arctan(\omega\tau_p)$ , where  $\tau_p$  is the momentum relaxation time. Owing to the retardation, the carriers keep moving in the QW plane even when  $E_x(t) = 0$  and the field component  $E_z(t)$  efficiently affects the mobility [see Fig. 1(b)]. Similarly to the case of linearly polarized field, such modulation of the mobility leads to a time-average drift

current along the  $x$  axis. An interesting feature of dc current induced by a circularly polarized ac field is that the current direction is opposite for right-handed ( $\sigma^+$ ) and left-handed ( $\sigma^-$ ) polarization. Indeed, for  $\sigma^+$  and  $\sigma^-$  polarizations the phase shift between the field components  $E_x(t)$  and  $E_z(t)$  has an opposite sign. Therefore, the inversion of radiation helicity changes the sign of the mobility oscillations and reverses the electric current.

The expression for dc electric current can be readily derived in the framework of the classical Drude theory. In this approach, the time evolution of the in-plane drift velocity  $v_d(t)$  is found from the Newton equation

$$\frac{d\mathbf{v}_d(t)}{dt} = \frac{e\mathbf{E}_{\parallel}(t)}{m^*} - \mathbf{v}_d(t)\gamma(t), \quad (4)$$

where  $m^*$  is the effective mass and  $\gamma(t)$  is the rate of velocity relaxation that depends on time owing to the effect of the out-of-plane field component  $E_z(t)$  on the function of size quantization (see Fig. 1). In the linear in  $E_z(t)$  regime, the time dependence of  $\gamma(t)$  follows  $E_z(t)$  and can be presented in the form

$$\gamma(t) = 1/\tau_p + \zeta eE_z(t), \quad (5)$$

where  $\tau_p$  is the velocity (momentum) relaxation time at  $E_z = 0$  and  $\zeta$  is a constant to be calculated below. Note that both  $E_{\parallel}(t)$  and  $E_z(t)$  oscillate at the same frequency [see Eq. (2)], therefore the drift velocity  $v_d(t)$  contains harmonics at zero as well as the double frequencies. To solve Eq. (4) we decompose the drift velocity into harmonics,

$$\mathbf{v}_d(t) = \sum_{n=0,\pm 1,\dots} \mathbf{v}_d^{(n)} \exp(-in\omega t), \quad (6)$$

and finally obtain within linear in  $E_z$  and  $E_{\parallel}$  approximation

$$\mathbf{v}_d^{(0)} = -\zeta e\tau_p (\mathbf{v}_d^{(1)} E_z^* + \mathbf{v}_d^{(-1)} E_z), \quad (7)$$

$$\mathbf{v}_d^{(1)} = \frac{e\mathbf{E}_{\parallel}/m^*}{1/\tau_p - i\omega}, \quad \mathbf{v}_d^{(-1)} = \frac{e\mathbf{E}_{\parallel}/m^*}{1/\tau_p + i\omega}. \quad (8)$$

The direct electric current is then found by multiplying  $\mathbf{v}_d^{(0)}$  by the electron charge  $e$  and the carrier density  $N_e$ , which yields

$$\mathbf{j} = -N_e \frac{\zeta e^3 \tau_p^2}{m^*} \left( \frac{\mathbf{E}_{\parallel} E_z^*}{1 - i\omega\tau_p} + \frac{\mathbf{E}_{\parallel}^* E_z}{1 + i\omega\tau_p} \right). \quad (9)$$

Equation (9) contains both linear and circular currents and can be rewritten in the form of Eq. (3) with the phenomenological parameters

$$L = -N_e \frac{\zeta e^3 \tau_p^2 / m^*}{1 + (\omega\tau_p)^2}, \quad C = -N_e \frac{\zeta e^3 \tau_p^3 \omega / m^*}{1 + (\omega\tau_p)^2}. \quad (10)$$

Shown in Fig. 2 are the frequency dependences of the electric currents driven by linearly polarized and circularly polarized radiation,  $\mathbf{j}_{\text{lin}}$  and  $\mathbf{j}_{\text{circ}}$ , respectively. The spectral behavior of  $\mathbf{j}_{\text{lin}}$  repeats that of the Drude absorption: It is maximal at zero frequency and decays as  $1/\omega^2$  at  $\omega\tau_p \gg 1$ . In contrast, the frequency dependence of the circular current is nonmonotonic. The current  $\mathbf{j}_{\text{circ}}$  is proportional to  $\omega$  at small frequencies, reaches maximum at  $\omega\tau_p = 1$ , and decays as  $1/\omega$  at higher frequencies.

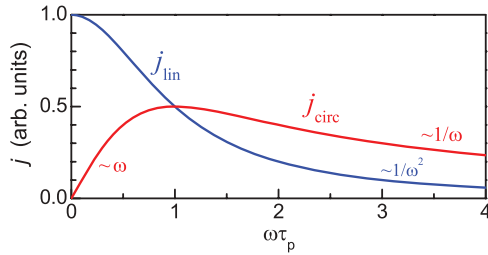


FIG. 2. (Color online) Frequency dependences of electric currents induced by a linearly polarized and circularly polarized ac electric field.

Equations (9) and (10) accurately describe the photocurrent provided  $\tau_p$  is independent of the electron energy, as it happens in the case of short-range scatterers, or the electron gas is degenerate. In the latter case,  $\tau_p$  should be taken at the Fermi energy. The more general expressions for the current can be derived by solving the Boltzmann kinetic equation

$$\frac{\partial f(\mathbf{p}, t)}{\partial t} + e\mathbf{E}_{\parallel}(t) \cdot \frac{\partial f(\mathbf{p}, t)}{\partial \mathbf{p}} = \text{St} f(\mathbf{p}, t) \quad (11)$$

for the electron distribution function  $f(\mathbf{p}, t)$ . Here,  $\mathbf{p} = (p_x, p_y)$  is the electron momentum and  $\text{St} f(\mathbf{p}, t)$  is the collision integral. In the case of elastic scattering, the integral has the form

$$\text{St} f(\mathbf{p}, t) = \sum_{\mathbf{p}'} [W_{\mathbf{p}'\mathbf{p}} f(\mathbf{p}', t) - W_{\mathbf{p}\mathbf{p}'} f(\mathbf{p}, t)], \quad (12)$$

where  $W_{\mathbf{p}'\mathbf{p}} = (2\pi/\hbar) \langle |V_{\mathbf{p}'\mathbf{p}}|^2 \rangle \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'})$  is the rate of scattering between the states  $\mathbf{p}$  and  $\mathbf{p}'$ ,  $V_{\mathbf{p}'\mathbf{p}}$  is the scattering matrix element, and the angle brackets denote averaging over the positions of impurities.

The effect of the out-of-plane field component  $E_z(t)$  on the function of size quantization can be treated as the field-induced admixture of excited electron states to the ground-subband wave function. To first order in the perturbation theory, the wave function of the ground subband  $e1$  has the form

$$\varphi(z, t) = \varphi_1(z) + eE_z(t) \sum_{\nu \neq 1} \frac{z_{\nu 1}}{\varepsilon_{\nu 1}} \varphi_{\nu}(z), \quad (13)$$

where  $\varphi_{\nu}(z)$  are the wave functions along  $z$  at zero electric field,  $z_{\nu 1} = \int \varphi_{\nu}(z) z \varphi_1(z) dz$  are the coordinate matrix elements,  $\varepsilon_{\nu 1}$  are energy separations between the subbands, and  $\nu$  is the subband index [the dominant contribution  $\propto E_z(t)$  comes usually from  $\nu = 2$ ]. Then, the scattering rate can be presented in the form

$$W_{\mathbf{p}'\mathbf{p}} = W_{\mathbf{p}'\mathbf{p}}^{(0)} + \delta W, \quad (14)$$

where  $W_{\mathbf{p}'\mathbf{p}}^{(0)} = (2\pi/\hbar) \langle |V_{11}(\mathbf{p}', \mathbf{p})|^2 \rangle \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'})$  is the scattering rate at zero electric field and  $\delta W$  is the linear in  $E_z(t)$  term given by

$$\delta W = \frac{8\pi e}{\hbar} E_z(t) \sum_{\nu \neq 1} \frac{z_{\nu 1}}{\varepsilon_{\nu 1}} \langle \text{Re} V_{11}^* V_{1\nu} \rangle \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'}), \quad (15)$$

with  $V_{11}$  and  $V_{1\nu}$  being the ‘‘intrasubband’’ and ‘‘intersubband’’ matrix elements of scattering. The rate  $W_{\mathbf{p}'\mathbf{p}}^{(0)}$  is determined by intrasubband scattering and may depend on the initial  $\mathbf{p}$  and final  $\mathbf{p}'$  electron momenta. In contrast, the scattering

processes between states described by different functions  $\varphi_{\nu}(z)$  require the transfer of momentum comparable to  $\pi\hbar/a$  ( $a$  is the QW width), which is much larger than the in-plane electron momentum. Such processes can be caused by short-range scatterers only, therefore, we assume that  $\delta W$  is independent of the directions of  $\mathbf{p}$  and  $\mathbf{p}'$ . In this case, the contribution to the collision integral (12) proportional to  $E_z(t)$  has the form

$$\delta \text{St} f(\mathbf{p}, t) = -\zeta e E_z(t) [f(\mathbf{p}, t) - \bar{f}(\mathbf{p}, t)], \quad (16)$$

where  $\bar{f}(\mathbf{p}, t)$  is the distribution function averaged over the directions of  $\mathbf{p}$  and

$$\zeta = \frac{4m^*}{\hbar^3} \sum_{\nu \neq 1} \frac{z_{\nu 1}}{\varepsilon_{\nu 1}} \langle \text{Re} V_{11}^* V_{1\nu} \rangle. \quad (17)$$

We note that the products  $z_{\nu 1} \langle \text{Re} V_{11}^* V_{1\nu} \rangle$  and, hence, the parameter  $\zeta$  are equal to zero in absolutely symmetric structures where the impurity profile is an even function with respect to the QW center and the functions  $\varphi_{\nu}$  are either odd or even.

To solve the kinetic equation (11) with the scattering rate (14) we decompose the distribution function  $f(\mathbf{p}, t)$  into frequency  $n$  and angular  $m$  harmonics as follows:

$$f(\mathbf{p}, t) = \sum_{n, m} f_m^{(n)} \exp(im\theta_{\mathbf{p}} - in\omega t), \quad (18)$$

where  $\theta_{\mathbf{p}}$  is the polar angle of the vector  $\mathbf{p}$ . Then, for the time-independent asymmetric part of the distribution function  $\delta f(\mathbf{p}) = f_1^{(0)} \exp(i\theta_{\mathbf{p}}) + f_{-1}^{(0)} \exp(-i\theta_{\mathbf{p}})$ , which determines the dc current, we derive

$$\delta f(\mathbf{p}) = \zeta e^2 \tau_p^2 \left[ \frac{(\mathbf{E}_{\parallel} \cdot \mathbf{v}) E_z^*}{1 - i\omega\tau_p} + \frac{(\mathbf{E}_{\parallel}^* \cdot \mathbf{v}) E_z}{1 + i\omega\tau_p} \right] \frac{df_{\varepsilon}}{d\varepsilon}, \quad (19)$$

where  $f_{\varepsilon}$  is the equilibrium distribution function of carriers,  $\varepsilon = \mathbf{p}^2/(2m^*)$  and  $\mathbf{v} = \mathbf{p}/m^*$  are the electron kinetic energy and velocity, respectively, the momentum relaxation time is given by  $\tau_p^{-1} = \sum_{\mathbf{p}'} W_{\mathbf{p}'\mathbf{p}}^{(0)} (1 - \cos\theta_{\mathbf{p}'\mathbf{p}})$ , and  $\theta_{\mathbf{p}'\mathbf{p}} = \theta_{\mathbf{p}'} - \theta_{\mathbf{p}}$  is the angle between  $\mathbf{p}'$  and  $\mathbf{p}$ .

The electric current is obtained from Eq. (19) by multiplying  $\delta f(\mathbf{p})$  by the electron charge  $e$  and velocity  $\mathbf{v}$ , and summing up the result over the momenta  $\mathbf{p}$ . It gives

$$\mathbf{j} = \frac{2\zeta e^3}{m^*} \sum_{\mathbf{p}} \tau_p^2 \left( \frac{\mathbf{E}_{\parallel} E_z^*}{1 - i\omega\tau_p} + \frac{\mathbf{E}_{\parallel}^* E_z}{1 + i\omega\tau_p} \right) \frac{\varepsilon df_{\varepsilon}}{d\varepsilon}, \quad (20)$$

where the factor of 2 accounts for the spin degeneracy. Equation (20) accurately takes into account the possible dependence of the momentum relaxation time on energy and is more general than Eq. (9). In the case of short-range scattering, where the energy dependence of  $\tau_p$  vanishes, Eqs. (20) and (9) are equivalent to each other and can also be rewritten in the form of Eq. (3) with the parameters

$$L = -\frac{4N_e e^3 \tau_p / m^*}{1 + (\omega\tau_p)^2} \sum_{\nu \neq 1} \frac{z_{\nu 1}}{\varepsilon_{\nu 1}} \xi_{\nu}, \quad (21)$$

$$C = -\frac{4N_e e^3 \tau_p^2 \omega / m^*}{1 + (\omega\tau_p)^2} \sum_{\nu \neq 1} \frac{z_{\nu 1}}{\varepsilon_{\nu 1}} \xi_{\nu}, \quad (22)$$

where

$$\xi_v = \frac{\langle \text{Re } V_{11}^* V_{1v} \rangle}{\langle |V_{11}|^2 \rangle} = \frac{\int_{-\infty}^{\infty} \varphi_1^3(z) \varphi_v(z) u(z) dz}{\int_{-\infty}^{\infty} \varphi_1^4(z) u(z) dz}, \quad (23)$$

with  $u(z)$  being the profile of impurity distribution along the QW growth direction. Following Eqs. (3), (21), and (22), one can estimate the electric current magnitude. The estimate gives  $j \sim 10^{-3}$  A/cm for a GaAs-based QW with the electron density  $N_e = 10^{11}$  cm $^{-2}$ , the well width 20 nm, the structure asymmetry degree  $\xi_2 = 0.1$ , the electric-field amplitude  $E_z = E_{\parallel} = 1$  kV/cm, and the frequency  $\omega = 1/\tau_p$ .

The above developed microscopic theory describes the formation of dc current by ac electric field in the whole range of classical frequencies, no matter how large is  $\omega\tau_p$ . At even higher frequencies, when  $\hbar\omega$  becomes comparable to the mean electron energy  $\bar{\epsilon}$ , another approach involved quantum optical transitions is required. However, both classical and quantum approaches unite and should give the same results in the intermediate frequency range  $1/\tau_p \ll \omega \ll \bar{\epsilon}/\hbar$ . This is indeed the case: Our results for the circular current Eq. (22) in the limit  $\omega\tau_p \gg 1$  coincide with those obtained in Ref. 7, where virtual indirect optical transitions were considered. The linear current Eq. (21) decreases as  $1/(\omega^2\tau_p)$  at  $\omega\tau_p \gg 1$  and becomes much smaller than the circular current at  $\hbar\omega \approx \bar{\epsilon}$ . The microscopic theory of the linear photocurrent in this spectral range is a task for the future.

It is also worth mentioning that, in QWs grown from zincblende-type semiconductors, there are additional contributions to linear and circular currents caused by bulk inversion asymmetry. Those currents flow in different directions with respect to the currents caused by structure inversion asymmetry and can be easily discriminated in experiment.<sup>3</sup> The presented model of high-frequency nonlinear conductivity can be applied to evaluate those currents as well. In this case, one should consider the modulation of electron mobility caused by the electric-field-induced admixture of the valence-band states to the electron wave function.<sup>7</sup> Finally, we note that the effect of electric field on the mobility can originate not only from the change of scattering rate but also from the variation of the effective electron mass, as was proposed in Ref. 8.

### III. MAGNETIC-FIELD-INDUCED CURRENTS

The application of a static magnetic field  $\mathbf{B}$  in the QW plane enables the generation of a direct current even in the geometry where the ac electric field oscillates in the interface plane. Within a linear in  $\mathbf{B}$  approximation, such currents are described by the second term of the right-hand side of phenomenological Eq. (1). A symmetry analysis shows<sup>2,14</sup> that, for this particular geometry, the polarization dependence of the electric current caused by QW structure inversion asymmetry is given by

$$\begin{aligned} j_x &= M_1 [B_y (|E_x|^2 - |E_y|^2) - B_x (E_x E_y^* + E_y E_x^*)] \\ &\quad + M_2 B_y |E|^2 + M_3 B_x i (E_x E_y^* - E_y E_x^*), \\ j_y &= M_1 [B_x (|E_x|^2 - |E_y|^2) + B_y (E_x E_y^* + E_y E_x^*)] \\ &\quad - M_2 B_x |E|^2 + M_3 B_y i (E_x E_y^* - E_y E_x^*). \end{aligned} \quad (24)$$

Here, the parameter  $M_1$  describes the electric current whose magnitude and direction depend on the linear polarization of

the field,  $M_2$  describes the polarization-independent current, and  $M_3$  is responsible for the circular current sensitive to the radiation helicity.

Microscopically, magnetic-field-induced currents emerge owing to the modification of electron states in the in-plane magnetic field, which can be of both diamagnetic (pure orbital) and paramagnetic (spin-dependent) origins, see, e.g., Refs. 14–19 and references therein. To first order in the magnetic field, the diamagnetic and spin-related mechanisms additively contribute to the current generation and, therefore, can be considered independently. Spin-related mechanisms of the current formation are based on the Zeeman splitting of electron states together with spin-dependent electron scattering; they are studied for  $\omega\tau_p \gg 1$  in Refs. 20 and 21. Below we focus on diamagnetic mechanisms, which are less investigated and do not require spin-orbit coupling.

The diamagnetic mechanisms of the current generation originate from the Lorentz force acting upon charge carriers confined in QWs and modifying their energies and wave functions. To first order in  $\mathbf{B}$ , the diamagnetic corrections to the electron energy can be considered as displacements of electron subbands in the momentum space.<sup>15</sup> The shift of the ground subband alone does not disturb the symmetric distribution of carriers within the subband and, moreover, can be excluded by a proper choice of the coordinate origin. Besides the energy shift, the in-plane magnetic field changes the wave functions of free carriers, which leads to the asymmetry in electron scattering by impurities or phonons.<sup>22</sup> Such an asymmetry in electron scattering by impurities is illustrated in Fig. 3(a). Here, we assume that the  $\delta$  layer of impurities (dotted line) is placed closer to the QW lower interface, and the magnetic field  $\mathbf{B} \parallel y$ . Electrons with different velocities move in the QW plane and are pushed by the Lorentz force  $\mathbf{F}_L = (e/c)[\mathbf{v} \times \mathbf{B}]$  to the lower or upper interface depending on the sign of  $v_x$ . This leads to a variation of the electron function of size quantization that results, in turn, in the asymmetry of electron scattering: Carriers with  $v_x > 0$  are scattered by impurities at a higher rate than those with  $v_x < 0$ . The Lorentz force is proportional to both the magnetic field and the electron velocity, therefore, the small correction to the scattering rate is linear in  $\mathbf{p}$  and  $\mathbf{B}$ . In QWs with a structure inversion asymmetry, the

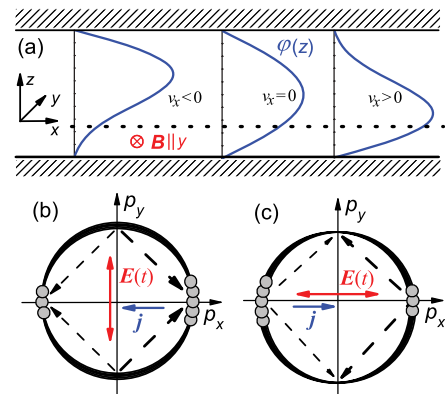


FIG. 3. (Color online) (a) Asymmetry in electron scattering by impurities caused by in-plane magnetic field. (b) and (c) Microscopic model of dc current generation owing to electric-field-induced alignment of electron momenta followed by asymmetric scattering.

rate of elastic electron scattering can be presented in the form<sup>22</sup>

$$W_{p'p} = W_{p'p}^{(0)} + w'[B_x(p_y + p'_y) - B_y(p_x + p'_x)], \quad (25)$$

where

$$w' = -\frac{4\pi e}{\hbar m^* c} \sum_{v \neq 1} \frac{z_{v1}}{\varepsilon_{v1}} \langle \text{Re } V_{11}^* V_{1v} \rangle \delta(\varepsilon_p - \varepsilon_{p'}). \quad (26)$$

The parameter  $w'$  is determined by the same matrix elements of scattering as  $\zeta$  [see Eq. (17)]. Below it is assumed that  $w'$  is independent of the directions of  $\mathbf{p}$  and  $\mathbf{p}'$ .

The asymmetry in electron scattering gives rise to a dc if the carriers are excited by an ac electric field. Figures 3(b) and 3(c) illustrate a mechanism of current formation for a linearly polarized electric field oscillating along the  $y$  and  $x$  axes, respectively. The ac electric field leads, in the second order in the field amplitude  $\mathbf{E}$ , to the alignment of electron momenta along the axis of field oscillations. In the case of  $\mathbf{E} \parallel y$ , the carriers populate predominantly the states with large  $|p_y|$ , which is indicated in Fig. 3(b) by the Fermi circle of variable thickness. The processes of electron scattering shown by dashed lines suppress the alignment tending to restore the isotropic distribution of carriers in  $\mathbf{p}$  space. However, in the presence of magnetic field  $B_y$ , the rates of electron scattering to the states with positive and negative  $p_x$  are different [see Eq. (25)]. Such a difference in scattering rates is illustrated in Fig. 3(b) by lines of different thicknesses. Therefore, the scattering events result in an imbalance of carrier population between positive and negative  $p_x$  (shown by full circles), giving rise to an electric current  $j_x$ .

Figure 3(c) sketches the same mechanism of the current formation for  $\mathbf{E} \parallel x$ . In this case, the ac electric field leads to the alignment of the electron momenta along the  $x$  axis. The subsequent asymmetric scattering of electrons in the magnetic field  $B_y$  also causes imbalance in the carrier population between positive and negative  $p_x$ , giving rise to a dc current  $j_x$ . However, as it follows from Figs. 3(b) and 3(c), the current directions are opposite for  $\mathbf{E} \parallel y$  and  $\mathbf{E} \parallel x$ . Thus, the illustrated mechanism of the current formation describes the polarization-dependent contribution to the current, which is given by the term  $\propto M_1$  in Eq. (24).

The analytical expression for magnetic-field-induced currents can be derived in the framework of kinetic theory by solving Eq. (11) with the scattering rate (25). To solve the equation, we decompose the distribution function  $f(\mathbf{p}, t)$  into frequency  $n$  and angular  $m$  harmonics according to Eq. (18) and obtain the system of coupled equations. Its solution for the time-independent asymmetric part of the distribution function  $\delta f(\mathbf{p})$ , which determines dc current, in the geometry  $\mathbf{B} \parallel y$  assumes the form

$$\begin{aligned} \delta f(\mathbf{p}) = & -[ (|E_x|^2 + |E_y|^2) v_x - (E_x E_y^* - E_y E_x^*) v_y ] \\ & \times \frac{\zeta \tau_p e^3 B_y}{4m^* c} \left( 2 + \varepsilon \frac{d}{d\varepsilon} \right) \frac{\tau_p \tau_2 df_\varepsilon / d\varepsilon}{(1 - i\omega\tau_p)(1 - i\omega\tau_2)} \\ & - [ (|E_x|^2 - |E_y|^2) v_x + (E_x E_y^* + E_y E_x^*) v_y ] \\ & \times \frac{\zeta \tau_p \tau_2 e^3 B_y}{4m^* c} \left( \varepsilon \frac{d}{d\varepsilon} \right) \frac{\tau_p df_\varepsilon / d\varepsilon}{1 - i\omega\tau_p} + \text{c.c.}, \quad (27) \end{aligned}$$

where  $\tau_2$  is the relaxation time of the second angular harmonic of the electron distribution function,  $\tau_2^{-1} = \sum_{p'} W_{p'p}^{(0)} (1 - \cos 2\theta_{p'p})$ .

The current density  $\mathbf{j}$  is obtained by multiplying  $\delta f(\mathbf{p})$  by the electron charge and velocity, and summing up the result over the momentum. This procedure yields Eq. (24) with the following parameters:

$$M_1 = \frac{\zeta e^4}{c m^{*2}} \sum_p \frac{\tau_p d(\tau_p \tau_2 \varepsilon^2) / d\varepsilon}{1 + (\omega\tau_p)^2} \frac{df_\varepsilon}{d\varepsilon}, \quad (28)$$

$$M_2 = \frac{\zeta e^4}{c m^{*2}} \sum_p \frac{(1 - \omega^2 \tau_p \tau_2) \tau_p \tau_2 \varepsilon^2 \tau'_p}{[1 + (\omega\tau_p)^2][1 + (\omega\tau_2)^2]} \frac{df_\varepsilon}{d\varepsilon}, \quad (29)$$

$$M_3 = -\frac{\zeta e^4}{c m^{*2}} \sum_p \frac{\omega \tau_p \tau_2 (\tau_p + \tau_2) \varepsilon^2 \tau'_p}{[1 + (\omega\tau_p)^2][1 + (\omega\tau_2)^2]} \frac{df_\varepsilon}{d\varepsilon}, \quad (30)$$

where  $\tau'_p = d\tau_p / d\varepsilon$ , and the factor 2 of spin degeneracy is already taken into account. It follows from Eqs. (24) and (28)–(30) that the polarization-independent and circular currents given by  $M_2$  and  $M_3$ , respectively, emerge owing to the energy dependence of the momentum relaxation time. If the energy dependence of  $\tau_p$  and  $\tau_2$  can be neglected and  $\tau_p = \tau_2$ , the parameter  $M_1$  assumes the form

$$M_1 = -4 \frac{N_e e^4 \tau_p^2 / (c m^{*2})}{1 + (\omega\tau_p)^2} \sum_{v \neq 1} \frac{z_{v1}}{\varepsilon_{v1}} \xi_v, \quad (31)$$

while  $M_2$  and  $M_3$  vanish. It is also worth mentioning that there are additional contributions to the polarization-independent current that originate from the energy relaxation of hot carriers in magnetic field.<sup>18,22</sup> These contributions depend on the energy relaxation mechanisms and are out of the scope of the present paper.

The magnetic-field-induced current (24) can be considerably larger than the current (3) excited at an oblique polarization of the ac electric field at  $B = 0$ . For linearly polarized radiation, the ratio between the current magnitudes is estimated as  $M_1 B / L = e B \tau_p / (m^* c)$ , which can exceed unity already at moderate magnetic fields.

Finally, we note that the presented theory describes orbital currents to first order in the in-plane magnetic field. Such an approximation is valid provided  $|\mathbf{B}| \ll B_0 = \varepsilon_{21} m^* c / (e \hbar)$ , where  $\varepsilon_{21}$  is the energy distance between the subbands  $e2$  and  $e1$ . The estimation shows that  $B_0 \sim 10^2$  T for a rectangular QW of 100 Å width with infinitely high barriers.

#### IV. SUMMARY

We have developed the microscopic theory of high-frequency nonlinear conductivity in QWs with a structure inversion asymmetry. It is shown that the excitation of free carriers in QWs by an ac electric field, which has both the in-plane and out-of-plane components, leads to a dc in-plane current, its direction and magnitude being determined by the field polarization and frequency. The frequency dependence of the current induced by the linearly polarized field repeats that of the free-carrier absorption: It is maximal at zero frequency and decays as  $1/\omega^2$  at frequencies  $\omega$  higher than

the momentum relaxation rate  $1/\tau_p$ . In contrast, the current induced by circularly polarized field increases linearly with the field frequency at small  $\omega$ , reaches maximum at  $\omega = 1/\tau_p$ , and then decays as  $1/\omega$ . The application of a static magnetic field  $\mathbf{B}$  in the QW plane enables the generation of a dc current even if the ac electric field oscillates in the interface plane. In such a geometry, the current depends linearly on  $\mathbf{B}$  at small magnetic fields and can be induced by a linearly or circularly polarized electric field.

The developed theory of high-frequency nonlinear transport can also be applied to study the generation of combination frequencies (frequency mixing).

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